# CSE 410 Section 2 Solutions – Integers and Bitwise Operators

Hi there! Welcome back to section, we're happy that you're here ☺

## IEC Prefixing System

We often need to express large numbers and the preferred tool for doing so is the IEC Prefixing System!

- Kibi- (Ki) $= 2^{10} \approx 10^3$
- Mebi- (Mi) $= 2^{20} \approx 10^6$
- Gibi- (Gi) $= 2^{30} \approx 10^9$
- Tebi- (Ti) $= 2^{40} \approx 10^{12}$
- Pebi- (Pi) $= 2^{50} \approx 10^{15}$
- Exbi- (Ei) $= 2^{60} \approx 10^{18}$
- Zebi- (Zi) $= 2^{70} \approx 10^{21}$
- Yobi- (Yi) $= 2^{80} \approx 10^{24}$

## Practice With Prefixes

Write the following numbers using IEC Prefixes:

$2^{16} =$ 64 Ki

$2^{34} =$ 16 Gi

$2^{27} =$ 128 Mi

$2^{61} =$ 2 Ei

$2^{43} =$ 8 Ti

$2^{47} =$ 128 Ti

$2^{36} =$ 64 Gi

$2^{58} =$ 256 Pi

Write the following numbers as powers of 2:

2 Ki $= 2^{11}$

256 Pi $= 2^{58}$

512 Ki $= 2^{19}$

64 Gi $= 2^{36}$

16 Mi $= 2^{24}$

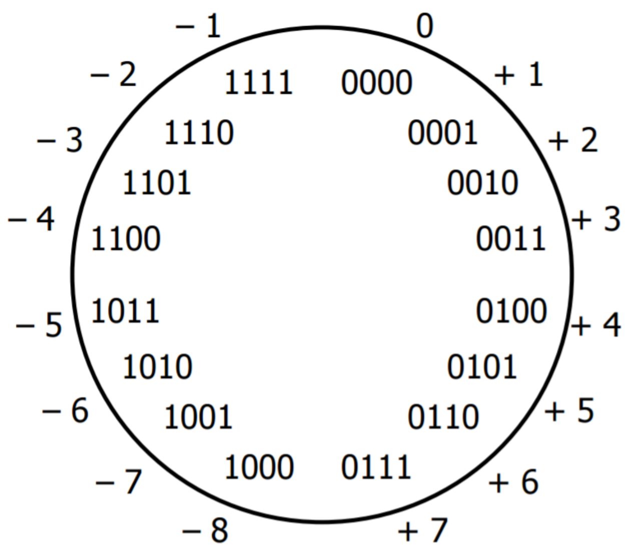
128 Ei $= 2^{67}$

## Signed Integers With Two's Complement

Two's complement is the standard solution for representing signed integers.
Remember that:

- The most significant bit has a negative value, all others have a positive value
- Otherwise exactly the same as unsigned integers

A neat trick for flipping the sign of a two's complement number: flip all the bits and add 1.
Two's Complement using 4 bit binary strings:



Notice how the largest number is 7 whereas the smallest number is -8. Also notice that there is a nice symmetry between numbers and their negative counterparts except -8, because its counterpart is taken to represent 0 in the circle.

- Addition is exactly the same as with an unsigned number.
- Arithmetic operations on binary numbers can lead to overflow. The easiest way to detect overflow is to keep in mind of the representational range of binary numbers.

**Two's Complement Practice**
For the following questions assume an 8 bit integer. Answer each question for the case of a two's complement number and unsigned number.

1. What is the largest integer? The largest integer + 1?

   (a) [Unsigned:] 255, 0
   (b) [Two's Complement:] 127, -128

2. How do you represent the numbers 0, 1, and -1?

   (a) [Unsigned:] 0b0000 0000, 0b0000 0001, N/A
   (b) [Two's Complement:] 0b0000 0000, 0b0000 0001, 0b1111 1111

3. How do you represent 17, -17?

   (a) [Unsigned:] 0b0001 0001, N/A
   (b) [Two's Complement:] 0b0001 0001, 0b1110 1111

4. Prove that the two's complement inversion trick is valid (i.e. that x and x ∓ 1 sum to 0).

   Note that for any x we have x + x = 0b1: : :1. A straightforward hand calculation shows that 0b1 : : : 1 + 0b1 = 0.

5. Explain why binary, decimal, or hexadecimal might be the preferred base of choice over the other bases. Give an example/context for your explanation!

   Decimal is the preferred radix for human hand calculations, likely related to the fact that humans have 10 fingers.

   Binary numerals are particularly useful for computers. Binary signals are less likely to be garbled than higher radix signals, as there is more \distance" (voltage or current) between valid signals. Additionally, binary signals are quite convenient to design circuits with, as we'll see later in the course.

   Hexadecimal numbers are a convenient shorthand for displaying binary numbers, owing to the fact that one hex digit corresponds exactly to four binary digits.

6. What is the largest positive number we can represent with a 12-bit signed two's complement integer? Write out the bit pattern and the value in decimal!

   Bit pattern: 0b 0111 1111 1111, Decimal : 2047

**Another Problem**
Let **x=0xE** and **y=0x7** be integers stored on a machine with a word size of **4 bits**.

1. What is the hex value of the result of adding these two numbers?

0x15, but since we only have 4 bits of presentation the 1 is truncated leaving us with 0x5

2. Interpreting these numbers as unsigned integers, what is the decimal result of adding x + y?

   0x5 in hex is 5 in decimal

3. Interpreting x and y as two's complement integers, what is the decimal result of computing x−y?

   0x7 in hex which is 7 in decimal

4. In one word, what is the phenomenon happening in question 2?

   Overflow

**Bitwise Operators and Masking**
C provides bitwise commands for AND(&), OR(|), XOR(^), and NOT(~). Ignoring NOT for now, let's see what happens when we reduce the 2-input gates to 1-input gates by fixing the second input.

Let x be the input. Fill in the following blanks with either 0, 1, x, or x(N̄OT x):

| | | | |
|---|---|---|---|
| x & 0 | = __0__ | x & 1 | = __x__ |
| x \| 0 | = __x__ | x \| 1 | = __1__ |
| x ^ 0 | = __x__ | x ^ 1 | = __~x__ |

**FACT:** Masking is very commonly used in bitwise operations. Masking helps to either clear or to extract bits from a bit string. AND a mask of all ones with the bit string to extract the bits. XOR a mask of all ones with the bit string give the complement of the bit string.

Consider the following mask, and two bit strings from which we want to extract the final bit:

mask = 00000001
value1 = 10011011
value2 = 10011100

Fill in the result:
mask & value1 = ____00000001________________

mask & value2 = ____00000000________________

The zeros in the mask *mask off* the first seven bits and only let the last bit show through. Masks can also be built up by operating on several flags, usually with inclusive OR:

flag1 = 00000001
flag2 = 00000010
flag3 = 00000100

mask = flag1 | flag2 | flag3 = ______00000111_________________

**Mask Practice**

Fill in the blank below to complete the function mod32 such that the function will return the remainder of x when divided by 32. The first blank is a bitwise operator. The second blank should be a decimal number.

```
int mod32(int x) {
        return x & 31;
}
```

Finish the function check nth bit. The function checks and returns the nth bit of the given argument x.

```
int check_nth_bit (int x, int n) {
        temp = x >> (n-1);
        return temp & 1;
}
```