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Wavelet Analysis: Mother Wavelet Selection Methods

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Abstract. Wavelet analysis, being a popular time-frequency analysis method has been applied in various fields to analyze a wide range of signals covering biological signals, vibration signals, acoustic and ultrasonic signals, to name a few. With the capability to provide both time and frequency domains information, wavelet analysis is mainly for time-frequency analysis of signals, signal compression, signal denoising, singularity analysis and features extraction. The main challenge in using wavelet transform is to select the most optimum mother wavelet for the given tasks, as different mother wavelet applied on to the same signal may produces different results. This paper reviews on the mother wavelet selection methods with particular emphasis on the quantitative approaches. A brief description of the proposed new technique to determine the optimum mother wavelet specifically for machinery faults diagnosis is also presented in this paper.

Introduction

Wavelet analysis is increasingly used for vibration analysis that includes faults detection and diagnosis. Peng et al. [1] compared the performance of three methods (frequency spectrum, scalogram, reassigned scalogram) in the detection of rotor to stator rub impact, oil whirl, coupling misalignment with actual measurement data from a Pumped Storage Power Generator Unit. The reassigned scalogram proved to be more suitable for fault features extraction when the fault is at an early stage. Lim and M. Salman [2] applied wavelet analysis to detect blade faults in gas turbines. They found that rotor dynamic wavelet map was more sensitive and effective to detect blade faults in gas turbines. This paper presents a review of mother wavelet selection to offer guidance on the wavelets available and to select the most appropriate mother wavelets.

Wavelets analysis is a mathematical procedure that cut up data into different frequency components, and then study each component with a resolution matched to its scale. In wavelet analysis, the signal is decomposed into its “wavelet”, scaled and shifted version of the original (mother) wavelet. Generally, wavelet transform are divided into, discrete wavelet transform (DWT), wavelet packet transform (WPT) and continuous wavelet transform (CWT). The discrete wavelet transform (DWT) can be efficiently realized by decomposing the signal into approximation (low frequency) and detail (high frequency) coefficients. After the first level, only the approximation is decompose into a higher level. For wavelet packet transform (WPT), after the first level, both the detail and approximation are decomposed into further level. Unlike the discrete wavelet transform, the CWT can operate at every scale. During computation, the wavelet is scaled and shifted over the full domain of the analyzed signal.

In wavelet analysis, there are many types of mother wavelet which can be used for wavelet analysis. Different mother wavelet used to analyze the same signal will produced different results. Generally, mother wavelets are characterized by properties such as orthogonality, compact support, symmetry and vanishing moment. Based on previous study, properties of mother wavelet are considered in selecting a mother wavelet. However, more than one mother wavelet with the same properties often exists. To overcome this, the similarity between signal and mother wavelet are considered in selecting a mother wavelet.

Recently, researchers have developed various methods to determine the similarity between signal and mother wavelet based on quantitative approaches. Till now, there is no standard or general method to select mother wavelet.

Selection of Mother Wavelet Based on Qualitative Approaches

The selection of the mother wavelet for analysis is dependent on the properties of mother wavelet or the similarity between signal and mother wavelet. Based on symmetry properties of mother wavelet, it was concluded that biorthogonal 6.8 was the best mother wavelet to separate the surface profiles into its multi-scale representation [3]. By taking into account the regularity, vanishing moment and degree of shift variance, the biorthogonal wavelets were selected for texture characterization [4]. In the study of analyzing power system transients, the properties of compact support and vanishing moment were used to select to most optimal mother wavelet for this analysis. Safavian et al. [5] concluded that the db4, coiflet and b-spline were equally suitable in detecting power system transients. In the area of biomedical engineering, the properties of compact support, vanishing moment and orthogonality were considered by Wang et al. [6] to separate burst and tonic components in the compound surface electromyogram (EMG) signals recorded from patients with dystonia. In the area of image processing, Ahuja et al. [7] concluded that the b-spline wavelets are the most suitable mother wavelet for image sequence superresolution after taking into account the properties of orthogonality, symmetry, regularity, explicit expression and compact support.

Selection of mother wavelet based on similarity between signal and mother wavelet was another selection criterion based on qualitative approaches. Normally the shape matching by visual inspection is applied to pick up the most proper mother wavelet. Martha Flanders [8] investigated the efficiency of different mother wavelet shapes in measuring the timing of multiunit bursts in surface electromyograms (EMG) from single trials. The db2 wavelet was chosen as the most similar mother wavelet to the EMG signal. In detection of actual three phase voltage sags, Mohamed Fuad and Azah Mohamed [9] commented that the selection of a mother wavelet that closely matches the signal is important in the detection of voltage sags. They also concluded that Gauss wavelet as the most suitable mother wavelet in the detection of voltage sag events in three phase networks. By visual inspection, Majid Ahadi and Mehrdad Sharif [10] found that the gauss mother wavelets are potentially most similar to the measured acoustic emission leakage signal-signatures. They also commented that the spectrograms produced by properly chosen mother wavelets can be effectively used for practical leak detection systems. Tang et al. [11] applied morlet wavelet to denoise vibration signals for wind turbine fault diagnosis as morlet wavelet is similar to mechanical impulse signal. They also commented that to increase the amplitude of the generated wavelet coefficients related to the fault impulses and enhance the fault detection process, the selected mother wavelet should be similar to the mechanical impulse response in characteristics. It was however difficult to visually match the shape of the signal to the shape of mother wavelet.

Selection of Mother Wavelet Based on Quantitative Approaches

In order to justify the similarity between signal and mother wavelet with a more accurate method, quantitative approaches have been proposed in recent years. The measure of minimum description length (MDL) was proposed by N. Saito [12] as a criterion to select the most optimal mother wavelet for noise suppression and signal compression. The MDL principle suggested that the ‘best’ model among the given collection of models is the one giving the shortest description of the data and the model itself. Effrina et al. [13] also applied MDL as a guideline to select the most optimal mother wavelet for compression of power disturbance data. Using the MDL criterion, the symlet 7 appeared superior than other mother wavelets for most power disturbance signals.

In the study of three phase interior permanent motor protection, Khan et al. [14] also applied MDL criteria to select the most optimal mother wavelet for this analysis. As the result, the db3 was selected as the mother wavelet used for wavelet packet transforms. Using the same mother wavelet selection criterion, they developed a self-tuning multiresolution proportional integral derivative controller for the interior permanent motor synchronous motor drive system [15].

In a study of partial discharge (PD) detection, the db4 was selected as the mother wavelet for the UHF signal denoising because it had the maximum cross correlation coefficient between the UHF signal and the selected mother wavelet [16]. Wenjie Li [17] also applied the maximum cross correlation coefficient criterion as a guideline to select the most suitable mother wavelet for PD signal extraction. Besides that, maximum cross correlation coefficient criterion also applied for denoising of an ECG signal. Brij N. Singh and Arvind K. Tiwari [18] studied the denoising of ECG signal using db8 which is selected based on the maximum cross correlation coefficient criterion.

Lei Zhang et al. [19] proposed two criteria to select an optimal mother wavelet for image denoising. The first criterion was the information extraction criterion and the second criterion was the distribution error criterion. The smallest the ratio of information extraction criterion to distribution error criterion means that the more similar the selected mother wavelet to the shape of the signal being analyzed. Using the above mentioned ratio as a criterion, it was found that the bior1.3 wavelet was capable to give the best performance for image denoising. Patrick P.C. Tsui and Otman A. Basir [20] proposed relative entropy as the measure of wavelet coefficients similarity to select the most suitable mother wavelet for automatic ultrasound non-destructive foreign body (FB) detection and classification. They found that the best mother wavelet for FB shape classification is biorthogonal 3.1, while haar (or symlet 1 or reverse biorthogonal 1.1) and reverse biorthogonal 3.9 are the best for spherical and rectangular FB material classifications respectively.

R. Yan [21] proposed the energy to Shannon entropy ratio criterion and MinMax information criterion to select the most optimal mother wavelet for bearing faults detection. The energy to Shannon entropy ratio criterion means maximum amount of energy while minimizing the Shannon entropy of the corresponding wavelet coefficients. While the MinMax information criterion was a criterion that considered the minimum joint entropy criterion, the minimum condition entropy criterion, the minimum elative entropy criterion, the maximum mutual information criterion and the maximum correlation coefficient criterion. Using the energy to Shannon entropy ratio criterion and MinMax information criterion, the reverse biorthogonal 5.5 was selected as the mother wavelet for analysis using discrete wavelet transform. On the other hand, the complex morlet wavelet was selected for analysis using continuous wavelet transform. The energy to Shannon entropy ratio criterion was also applied by Kankar et al. [22] for bearing fault detections. Besides that, they also proposed another mother wavelet selection criterion based on the information about relative energy with associated frequency bands. Using the bearing signals, continuous wavelet coefficients were calculated using three real mother wavelets and three complex mother wavelets. The meyer wavelet was selected based on the energy to Shannon entropy ratio criterion while the complex morlet wavelet was selected based on the maximum relative wavelet energy. Results showed that the mother wavelet selected using energy to Shannon entropy ratio criterion can provide better classification effectively.

J. Rafiee et al. [23] proposed the selection of the most similar mother wavelet for gear signals by calculating the variances of the continuous wavelet coefficients. The more variance that are available meant a greater the ability to properly classify failures. The summation of five element with highest value from different faults were calculated and called as ‘‘SUMVAR’’ for simplicity. The mother wavelet that had the highest value of SUMMAR was selected as the most similar mother wavelet to gear signals. Among the 324 mother wavelets, the db44 was selected for analysis because db44 was the most similar mother wavelet to gear signals. The same mother wavelet selection criterion was applied for bearing and gear faults diagnosis. It was found that although db44 had the most similar mother wavelet function across the bearing and gear signals, it was not the proper function for all wavelet-based processing [24]. Selection of mother wavelet based on genetic algorithm also proposed by J. Rafiee et al. [25] for gear faults diagnosis. Genetic algorithm approach was applied to optimally search daubechies order, decomposition level of the signals, and the number of neurons in hidden layer. Results showed that db11, level 4 and 14 neurons that were selected had the best values for daubechies order, decomposition level, and the number of nodes in hidden layer respectively.

In the study of harmonic distortion in the power system, Walid and El-Hawary [26] proposed a novel mother wavelet selection criterion based on the energy of the wavelet coefficients at each level in order to choose the most suitable wavelet family and the most suitable mother wavelet for analyzing power system steady-state harmonic distorted waveforms with minimum spectral leakage. It was found for low distorted level, the most suitable mother wavelet family was the db and the accuracy increases with increasing the wavelet order. For high distortion level the most suitable wavelet family was the coiflet and the mother wavelet with low order give more accurate results. In the study of contrast enhancement, Cheng et al. [27] proposed a novel approach to automatic selecting mother wavelet and parameters which is an important issue for implementing wavelet algorithms. Using the proposed method, total of 66 mother wavelets were pre-selected based on the properties of vanishing moment, shift variance, and regularity. By calculating the measurement of horizontal and vertical edges, the most optimal mother wavelet was selected based on the smallest value.

In the area of biomedical, Angkoon Phinyomark et al. [28] presented a novel mother wavelet selection algorithm that is robust to noises for EMG feature extraction. The mean squared error (MSE) of wavelet coefficients and reconstructed signals were calculated to determine the most suitable mother wavelet. J. Rafiee et al. [29] proposed evaluation criterion (EC) as the mother wavelet selection criterion for biomedical signals. The absolute value of continuous wavelet coefficient was calculated in each scale. The summation of this value in all scales was then calculated for different biomedical signals. The average was calculated for each type of signal, and referred to as evaluation criterion (EC) for simplicity. Using the evaluation criterion, the db44 was selected to analyze EMG, EEG and VPA signals using continuous wavelet transform. It was suggested that the similarity between signal and mother wavelet function was not always proper for signal processing based on wavelet transform and wavelet-based processing methods based on the resemblance between signals and mother functions was more appropriate.

Conclusion and Future Study

It was shown that the main challenge in wavelet analysis is the selection of the most optimum mother wavelet. Selection of mother wavelet based on qualitative approaches and quantitative approaches have been proposed and applied in various areas. Almost all of the selection methods proposed was based on the similarity between signal and mother wavelet. Previous studies also showed that the most similar mother wavelet across the signals is not the proper function for all wavelet-based processing. Further work is recommended for selection of mother wavelet based on the accuracy of wavelet results instead of current practices based solely on similarity between signal and mother wavelet.

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