DATA130062: Homework 1

Due via eLearning at 23:59 on September 24, 2024

1. Rizzo book (2nd edition) Exercises 3.5, 3.6, 3.9, 3.10, 3.12, 3.13, and 3.14.

2. Prove the Box-Muller transformation: if U, V are independent Unif$(0, 1)$, then

$$Z_1 = \sqrt{-2 \log U} \cos(2\pi V), \qquad Z_2 = \sqrt{-2 \log U} \sin(2\pi V)$$

   are independent standard normal variables.

3. Given a partition of a d-dimensional Gaussian distribution $X = (X_1, X_2)^\top$. Correspondingly, the mean and covariance of X are specified as

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

   Prove that the conditional distribution of X_1 given $X_2 = x_2$ is still Gaussian. Derive its conditional mean and covariance.