Section 30: Vector Spaces

Def: Let F be a field. A *vector space* over F consists of an abelian group V under addition together with an operation of scalar multiplication of each element of V by each element of F on the left, such that for all $a, b \in F$ and $\alpha, \beta \in V$, the following conditions are satisfied:

1. $a\alpha \in V$
2. $a(b\alpha) = (ab)\alpha$
3. $(a + b)\alpha = (a\alpha) + (b\alpha)$
4. $a(\alpha + \beta) = (a\alpha) + (a\beta)$
5. $1\alpha = \alpha$