Section 51: Separable Extensions

Def: Let $f(x) \in F[x]$. An element $\alpha \in \bar{F}$ such that $f(\alpha) = 0$ is a *zero of multiplicity* v, if v is the greatest integer such that $(x - \alpha)^v$ is a factor of $f(x)$ in $\bar{F}[x]$.

Thm. Let $f(x)$ be irreducible in $F[x]$. Then all the zeros of $f(x)$ in $\bar{F}$ have the same multiplicity.

Corollary: If $f(x)$ is irreducible in $F[x]$, then $f(x)$ has a factorization in $\bar{F}[x]$ of the form
$$a \prod_i (x - \alpha_i)^v$$
where the α_i are the distinct zeros of $f(x)$ in $\bar{F}$ and $a \in F$.