Real Analysis

Summary: The fundamental concepts in calculus are limits, continuity, differentiation, and integration. In turn, all these ideas are built off ϵ and δ based reasoning. A good foundation in real analysis will cover up to the uniform convergence of functions.

- **Basic Object:** The Real numbers.
- **Basic Maps:** Continuous and Differentiable functions.
- **Basic Goal:** The Fundamental Theorem of Calculus.

Def: A function $f : \mathbb{R} \to \mathbb{R}$ has a *limit* L at the point a if given $\epsilon > 0$ there is a $\delta > 0$ such that $\forall x$ with $0 < |x - a| < \delta$ we have $|f(x) - L| < \epsilon$. This is denoted by

$$\lim_{x \to a} f(x) = L$$

Note: The definition of the limit says that in order for $f(x)$ to be close to L, x must be close to a. We can take the required restriction for ϵ, $|f(x) - L| < \epsilon$, and find what our δ must be starting from $0 < |x - a| < \delta$.

Def: A function $f : \mathbb{R} \to \mathbb{R}$ is *continuous* at a if

$$\lim_{x \to a} f(x) = f(a)$$

Note: In order for the limit to exist, the right and left limits must be the same value.