

## Lebesgue Integration

**Def:** For the interval  $E = [a, b]$ , the *length* of  $E$  is

$$\ell(E) = b - a$$

**Note:** Finding the length of intervals is easy, but we can also find the length of sets that are not intervals based on what we know about the easy lengths.

**Def:** Let  $E$  be any subset of  $\mathbb{R}$ . A countable collection of intervals  $\{I_n\}$ , with each  $I_n = [a_n, b_n]$ , *covers* the set  $E$  if

$$E \subset \bigcup I_n$$

**Def:** For any set  $E$  in  $\mathbb{R}$ , the *outer measure* of  $E$  is

$$m^*(E) = \inf \left\{ \sum (b_n - a_n) \right\}$$

such that the collection of intervals  $\{[a_n, b_n]\}$  covers  $E$ .