Trends in Extreme Value Index

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Background

- Classic extreme value analysis assumes that the observations are i.i.d.
- Recent studies aim at dealing with the case when observations are drawn from different distributions.
- This paper considers a continuously changing extreme value index and try to estimate the functional extreme value index accurately.

Model Setting

- Consider a set of distributions $F_s(x)$ for $s \in [0, 1]$ and independent random variables $X_i \sim F_{\frac{i}{n}}(x)$ for $i = 1, \ldots, n$.
- Here $F_s(x)$ is assumed to be continuous with respect to s and x. And assume that $F_s \in D_{\gamma(s)}$.
- This article considers the case that the function γ is positive and continuous on $[0, 1]$.
- The goal is to estimate the function γ and test the hypothesis that $\gamma = \gamma_0$ for some given function γ_0.

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Methodology

- The idea for estimating $\gamma(s)$ locally is similar to the kernel density estimation.
- More specifically, use only observations X_i in the h-neighborhood of s, that is
$$i \in I_n(s) = \left\{ \left| \frac{i}{n} - s \right| \le h \right\},$$
where $h := h(n)$ is the bandwidth such that as $n \to \infty$, $h \to \infty$ and $nh \to \infty$.
- Correspondingly, there will be $[2nh]$ observations for $s \in [h, 1-h]$.

Methodology

- To apply any known estimators for extreme value index, such as Hill estimator, choose am intermediate sequence $k = k(n)$ such that $k \to \infty, k/n \to 0$ as $n \to \infty$.
- Then one can use the top $[2kh]$ order statistics among the $[2nh]$ local observations in the h-neighborhood of s to estimate $\gamma(s)$.
- The local Hill estimator for $\gamma(s)$ is defined as follows. Rank the $[2nh]$ observations X_i with $i \in I_n(s)$ as $X^{(s)}_{1,[2nh]} \leq \cdots \leq X^{(s)}_{[2nh],[2nh]}$. Then

$$\hat{\gamma}_H(s) := \frac{1}{[2kh]} \sum_{i \in I_n(s)} \left(\log X_i - \log X_{[2nh]-[2kh],[2nh]} \right)^+ .$$

Second order condition

To obtain the asymptotic theory, the following conditions are required.

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