

Cumulative Spectral Analysis for Transient Decaying Signals in a Transmission System Including a Feedback Loop*

YOSHINORI TAKAHASHI, AES Associate Member, MIKIO TOHYAMA, AES Member, AND**
 (takahashi@acoust.rise.waseda.ac.jp) (m_tohyama@waseda.jp)

YOSHIO YAMASAKI
 (yamasaki@acoust.rise.waseda.ac.jp)

Waseda University, Tokyo, Japan

Cumulative spectral analysis (CSA) of transient decaying signal portions is an effective approach to detecting spectral distortion and to determine quickly the principal resonant frequency of a public-address system before it starts howling. Spectral distortion, so-called coloration, due to periodic delays in a feedback loop, which might cause howling of the loop, could be detected by observing a spectral-accumulation process of the signals. CSA was originally proposed by Berman and Fincham for transient analysis of loudspeakers. The cumulative spectral process is investigated by introducing a spectral accumulation function into CSA, called cumulative harmonic analysis (CHA), so that the spectral accumulation process might be visualized effectively. The spectral accumulation effect of signals or impulse responses revealed by CSA is a little less than that found when using CHA. Consequently while a spectral-frequency distribution of the dominant frequency components picked up by CHA for decaying speech-signal portions clearly displays the coloration due to feedback speech signals, it can nevertheless be only slightly perceived by listening. Thus frequency distribution analysis by CHA or by conventional CSA for short decaying segments of signal samples can be useful in the blind prediction of the howling frequency without detailed specifications of the transfer functions and the original input signals under in situ conditions. As future work is concerned, it is necessary to investigate how long an observation interval would be required, and what kind of accumulation function is effective to predict howling frequencies. In particular, simulation experiments for multiple input and output systems, including time-variant closed loops under reverberation conditions, would be necessary for evaluating the proposed method from a practical point of view.

0 INTRODUCTION

Time-frequency analysis plays a fundamental role in signal representation and the detection of spectral distortions. This study investigates the time-dependent frequency distortion of speech signals through a feedback loop, such as an audio public-address system, which is

commonly called spectral coloration because of the system resonance. Even if the spectral coloration can be only slightly perceived by listening, the free-oscillation components of the resonance could be a main factor in howling. Therefore spectral-coloration analysis is significant for predicting the howling frequency of a closed loop and realizing a stable acoustic system. It is desirable that we represent the frequency characteristics, including the coloration, for a sound systems under in situ conditions. We will see that cumulative spectral analysis (CSA) [1] is a way to determine quickly what the principal resonant frequency of a public-address system is before it starts howling.

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**Now with Kogakuin University, Tokyo, Japan.

CSA was originally proposed by Berman and Fincham [1] and formulated by means of their Fourier transform of an impulse response record using a unit-step time-window function. It performs a time-frequency analysis of the transient response of a linear system such as a loudspeaker corresponding to a tone-burst input signal. Bunton and Small [2] extended the original CSA so that the transient responses to filtered tone-burst signals without abrupt changes might be analyzed. Although various approaches to time-frequency representations of signals have been proposed [3] since CSA, the authors evaluate the advantage of the spectral accumulation by CSA for predicting howling frequencies as opposed to a conventional viewpoint of CSA.

The spectral-accumulation effect can be emphasized by substituting a spectral-accumulation function for the unit-step time-window function of CSA. The authors call this substituted formula cumulative harmonic analysis (CHA). The spectral-accumulation process, such as a growing spectral peak inherent in signals, might be effectively displayed by CSA or CHA. CHA, however, visualizes the spectral-accumulation process by increasing the order of the resonance poles of the transfer functions.

A feedback loop, in principle, yields periodic signals. The harmonic spectral structures of periodic signals are typical examples of spectral-accumulation processes. A periodic signal can be expressed by the convolution of a cycle of a waveform and a periodic pulse sequence. The periodic pulse sequence yields the harmonic structure as the length of the periodic sequence increases. Thus the harmonic structure of a periodic signal is the result of the superposition of repeated spectral records with a fixed phase lag. That is, as the periodic-signal length becomes longer, spectral peaks grow due to the in-phase accumulation of the harmonic sinusoidal components corresponding to the period.

However, if the superposition makes the resultant signal unstable beyond the steep but stable spectral peaks, then the system that produces the superposition starts howling. Howling can be interpreted as a change from the spectral periodicity of signals to the spectral selectivity dominated by only a few components. Estimating the spectral-accumulation process that reaches howling is quite important for making a public-address system stable. Samples of simulation results for speech signals, as typical examples of wide-band signals, through a feedback loop contained in a public-address system, are presented. We will see that an accumulated spectral distribution for short samples, including decaying signal portions, reveals the spectral coloration due to feedback signals, which indicates the howling frequency.

A decaying signal portion might be useful to characterize the free oscillations of an acoustic system [4], [5]. The free-oscillation component that causes howling could be displayed by visualizing the spectral accumulation process according to CSA or CHA, both of which visualize how fast and which frequency component is growing. Thus CSA or CHA is a possible effective way of time-frequency analysis for estimating the howling frequency by means of the spectral-accumulation process. However,

from a theoretical point of view CHA emphasizes the effect of resonance poles better than CSA.

This engineering report is organized as follows. In Section 1 CSA is reviewed and CHA is formulated using the Fourier transform from the point of view of the spectral accumulation function. We confirm that CHA increases the order of the resonance poles by using an impulse-response model for a single-degree-of-freedom system. In Section 2 speech samples in the feedback loop are analyzed by CSA, CHA, and conventional DFT analysis. Both CSA and CHA work well in estimating the howling frequency, even for speech materials. CHA visualizes the harmonic structure a little more clearly than CSA. In Section 3 we discuss a possibility of predicting howling at an early stage under in situ conditions. We introduce the spectral-frequency distribution of decaying speech portions picked up by CHA. The spectral coloration due to feedback speech, which indicates the howling frequency, is displayed using spectral-frequency distributions for short segments of a speech material that includes decaying portions. In Section 4 we summarize that the frequency-distribution analysis based on CHA has great potential in the prediction of howling under in situ conditions. In future work simulation experiments under time-variant and reverberation conditions, including multiple inputs and outputs, are necessary to evaluate the proposed method from a practical point of view.

1 FORMULATION OF CUMULATIVE SPECTRAL ANALYSIS

Berman and Fincham [1] defined the cumulative spectral analysis (CSA) of an impulse response record for characterizing the transient response of a loudspeaker system. Suppose that a sinusoidal signal in a complex function form such as

$$s(n) \equiv u(n)e^{j\Omega n} \quad (1)$$

is an input signal for a system with the impulse response $h(n)$. CSA is formulated as

$$\begin{aligned} \text{CSA}(n, e^{-j\Omega}) &= \sum_{m=0}^n u(n-m)e^{j\Omega(n-m)}h(m) \\ &\equiv e^{j\Omega n} \sum_{m=0}^n H(m, e^{-j\Omega}). \end{aligned} \quad (2)$$

Here

$$H(m, e^{-j\Omega}) \equiv u(n-m)h(m)e^{-j\Omega m}, \quad 0 \leq m \leq n \quad (3)$$

where $u(n)$ is a unit-step function corresponding to input signal conditions such as

$$u(n) = \begin{cases} 1, & 0 \leq n \\ 0, & n < 0. \end{cases} \quad (4)$$

Bunton and Small [2] investigated the effect of input signal conditions on CSA by filtering the input tone-burst

signals. We look at the spectral-accumulation property of CSA rather than the input signal conditions. The spectral-accumulation effect is emphasized by substituting a spectral-accumulation function for the unit-step window function defined by Eq. (4). Suppose that we have a signal sequence $x(n)$ and define a spectral-accumulation function $w(n)$. We define the cumulative harmonic analysis (CHA) of $x(n)$ as

$$\text{CHA}(n, e^{-j\Omega}) \equiv \sum_{m=0}^n w(m)x(m)e^{-j\Omega m}. \quad (5)$$

Introducing an example of a simple spectral-accumulation function such as

$$w(m) \equiv m + 1, \quad 0 \leq m \leq n \quad (6)$$

into $w(n)$, we have

$$\begin{aligned} \text{CHA}(n, e^{-j\Omega}) &\equiv \text{CHA}(n, z^{-1})|_{z=e^{j\Omega}} \\ &= \sum_{m=0}^n (m+1)x(m)z^{-m} \\ &= 1x(0)z^{-0} + 2x(1)z^{-1} + 3x(2)z^{-2} \\ &\quad + \dots + nx(n)z^{-n} \end{aligned} \quad (7)$$

using the z transform, where we set $z = e^{j\Omega}$.

The effect of the transfer function pole on the frequency characteristics can be emphasized by CHA. Suppose that we have the sequence

$$h(n) \equiv a^n, \quad n = 0, 1, 2, \dots; \quad 0 < |a| < 1. \quad (8)$$

If we take a limit such as

$$H(z^{-1}) \equiv \sum_{n=0}^{\infty} (n+1)a^n z^{-n} \quad (9)$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \text{CHA}_H(n, z^{-1}) &\equiv \lim_{n \rightarrow \infty} \sum_{m=0}^n (m+1)a^m z^{-m} \\ &= \frac{1}{(1 - az^{-1})^2} \end{aligned} \quad (10)$$

then we can see that CHA increases the order of the poles.

Fig. 1 is a schematic of CHA with the accumulation function $w(n)$ given by Eq. (6). Using a discrete Fourier transform (DFT) for discrete signal analysis, CHA can be written as

$$\text{CHA}(n, k) \equiv \sum_{m=0}^n w(m)x(m)e^{-j\frac{2\pi k}{N}m} \quad (11)$$

where we set N to include zero padding after the signal record of interest so that frequency interpolation might be

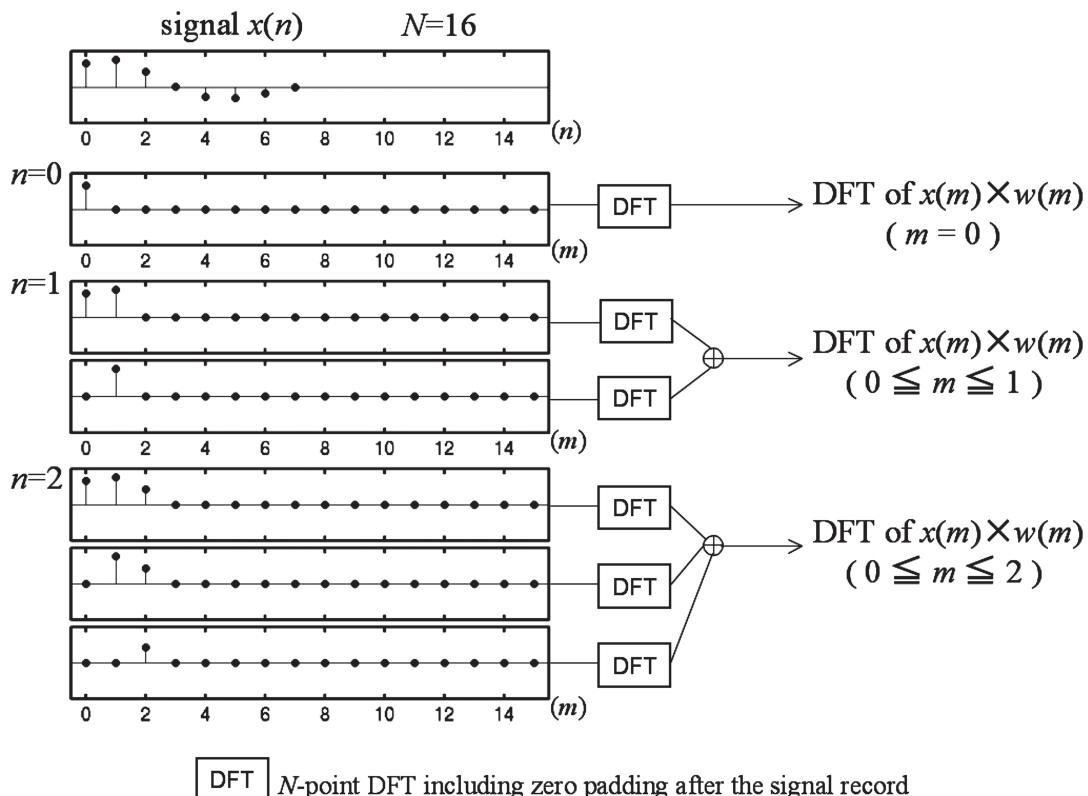


Fig. 1. Schematic of cumulative harmonic analysis (CHA) using a numerical example. $w(m) = m + 1$.

adequately performed. We can see that the time-window function $w(n)$ can be interpreted as a spectral-accumulation function.

Fig. 2 is a sample of the CSA of an impulse response for a single-degree-of-freedom system. The locations of the complex-conjugate poles are shown in Fig. 2(a). The impulse response is illustrated in Fig. 2(b), and magnitude records for CSA and CHA are displayed in Fig. 2(c) and (d). The maximum magnitude is normalized to unity at every instant. We can confirm that the spectral-

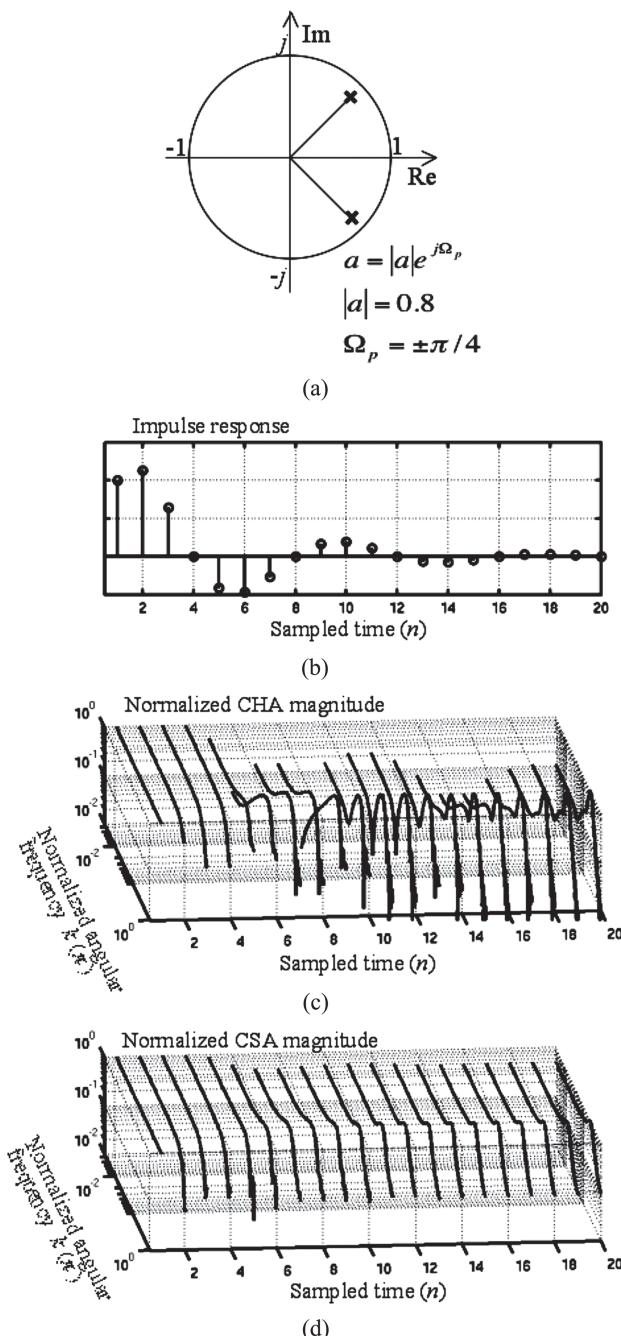


Fig. 2. Cumulative spectral analysis of impulse response for a single-degree-of-freedom resonance system. (a) Locations of complex-conjugate poles of transfer function located at $\pi/4$ in z plane. (b) Sequence of impulse responses. (c), (d) Maximum CHA and CSA magnitudes, normalized to unity at every instant. $N = 800$.

accumulation process where the resonant peak is growing is emphasized by CHA.

2 CUMULATIVE ANALYSIS OF SPEECH SIGNALS IN A FEEDBACK LOOP

Howling control is critical to a stable public-address system. To estimate the howling frequency and avoid unstable amplification, it is useful to visualize under stable conditions, prior to howling, how the frequency components of the input signals, including feedback, can be narrowed down to a single element. If it is possible to test a system including a feedback loop, then observation of the reverberation sound might be a good way to diagnose the system.

Fig. 3 illustrates a system with a stable feedback loop. Fig. 3(a) shows an open-loop impulse response and its magnitude frequency response from the loudspeaker to the microphone. Similarly, Fig. 3(b) shows the responses including the stable feedback loop.

Let us take a speech signal as a typical example of wide-band signals fed to a sound system. Fig. 4(a) shows speech signals in the original (left), a stable feedback loop (center), and an unstable loop (right). The impulse response for the stable loop is the same as that shown in Fig. 3. Fig. 4(b) and (c) visualizes the accumulated spectral magnitude by CSA and CHA, and Fig. 4(d) shows the result from a conventional spectrogram using STFT. The maximum magnitude is normalized to unity in the figure at every instant.

CHA emphasizes the harmonic structure of the speech sample, as shown by the left column of Fig. 4(c). The harmonic structure of the speech signal builds up after the onset of the speech signal. We can clearly see the spectral-accumulation process making the harmonic structure as the time goes by. The impulse response for the feedback loop, however, breaks the harmonic structure inherent in the speech signal, as shown in the center column. In the unstable feedback loop the spectral property is destroyed to the extent that no periodic property is displayed in the right column of the figure. Instead, only a single major component is illustrated. This single component corresponds to the howling frequency, which is the highest spectral peak of the magnitude response in Fig. 2(d).

The CSA magnitude [left column in Fig. 4(b)] also reveals the harmonic structure of the original speech sample, but the spectral-accumulation effect by CSA in the process where the dominant frequencies are selected is a little less than that of CHA. We can see the harmonics more clearly by CHA rather than by CSA in a stable feedback loop. The howling frequency is predictable by both CSA and CHA for a speech sample in the feedback loop, as we see in the right column, while we can see no clear indication from the conventional spectrogram in Fig. 4(d).

It is desirable that we can predict the frequency under in situ conditions for a system that includes a feedback loop such as a public-address system. The center column, however, implies that estimating the howling frequency might be difficult under stable-loop conditions, even by CSA or CHA. But if we look closer at the displays for the signal-

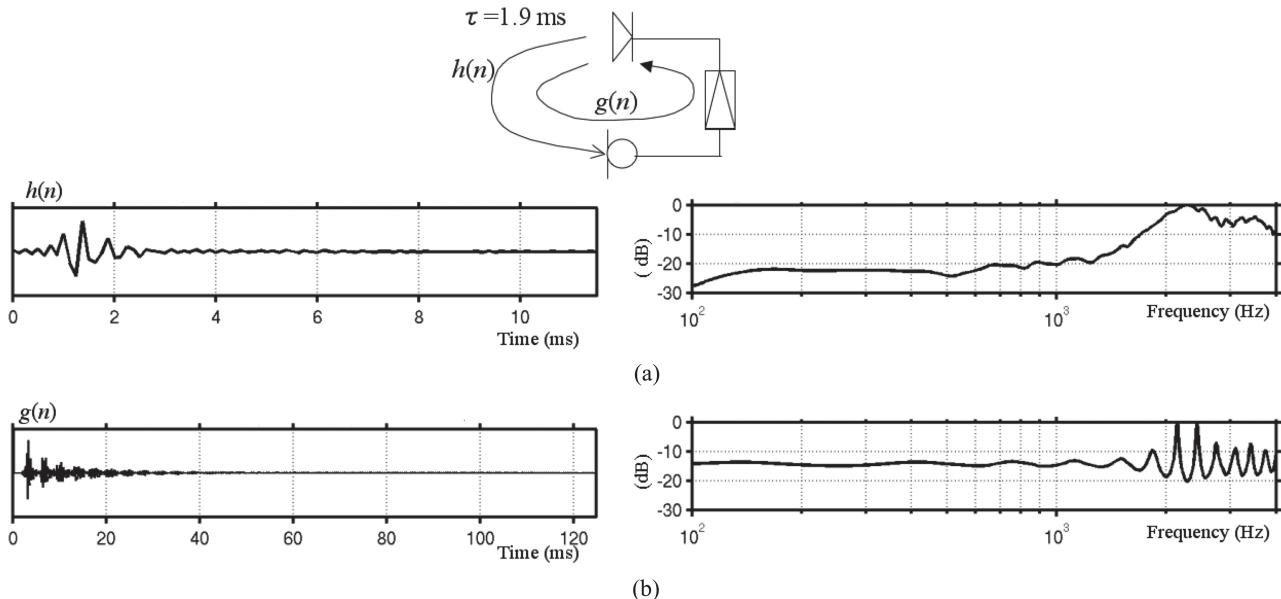


Fig. 3. Impulse responses and frequency characteristics. (a) Open-loop simulation. (b) Feedback-loop simulation.

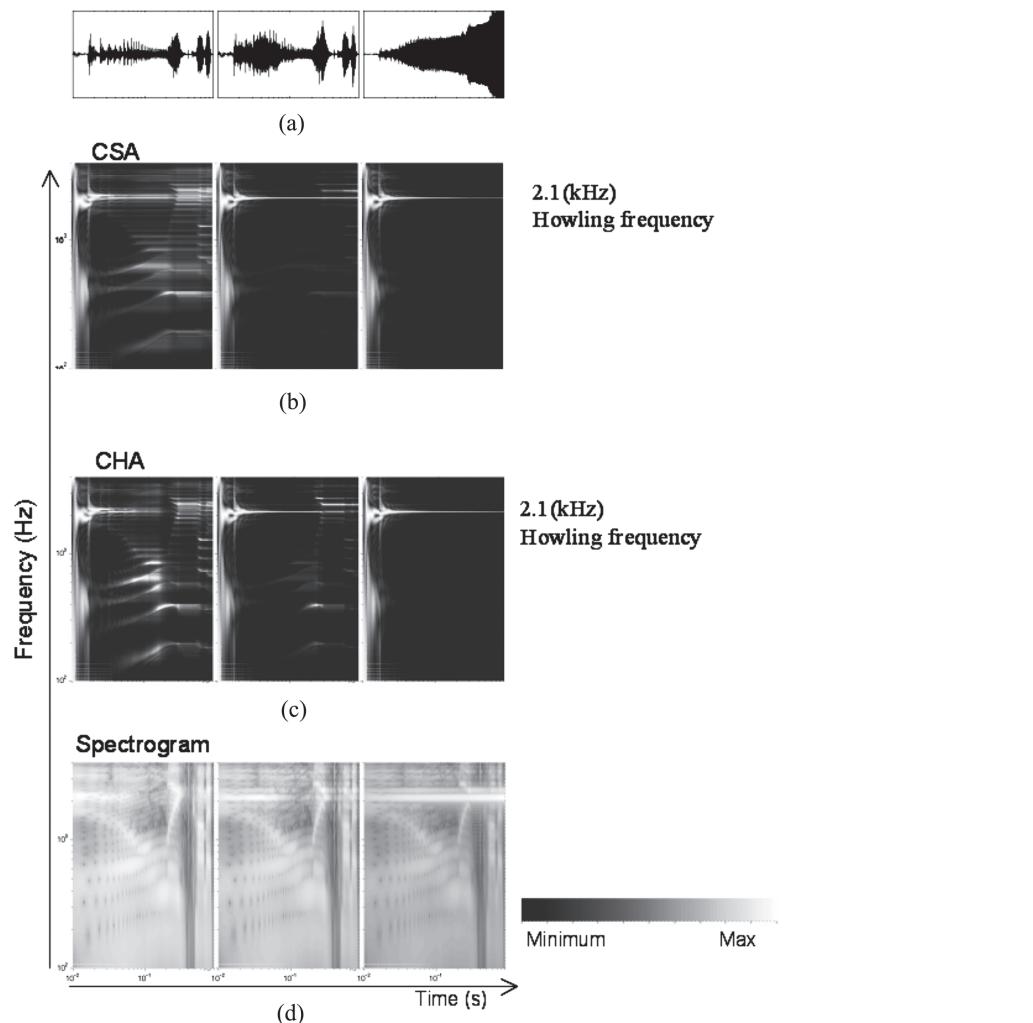


Fig. 4. Spectral analysis for speech samples. Left—original; center—stable feedback loop; right—unstable feedback loop. (a) Speech waveforms. (b) CSA magnitude. (c) CHA magnitude. (d) Spectrogram by STFT, where STFT is calculated for 12.5-ms frame length with frequency interpolation by zero-padding and hop size set to 1/10 frame length. Length of DFT, $N = 8192$ points, sampling frequency $f_s = 8$ kHz.

decaying portion, we can get a clear indication about the howling frequency. The decaying portions of the signal in principle are composed of free oscillations of a linear system of interest. Therefore there are clear differences between the left (original) and center (stable-loop) columns for the decaying portions of the signal. The decaying portions are also useful in estimating the reverberation time in reverberation space under in situ condition [4].

The spectral distortion of a signal due to a feedback loop is commonly called coloration [6]. If we can detect spectral coloration, then it might be possible to predict the howling frequency at an early stage prior to howling. For that purpose we will investigate accumulated spectral distributions picked up by CHA for the transient decaying signal portions under feedback-loop conditions. We expect that the frequency distribution will display a clear difference between the feedback loop and the original signal.

3 FREQUENCY DISTRIBUTIONS FOR DECAYING SPEECH-SIGNAL PORTIONS IN A STABLE LOOP

Frequency distributions of a signal observed through a linear system are useful for the blind characterization of the system transfer function [4], [5]. We assume that there is a difference between the frequency distribution of a signal with a transfer function and the original signal, when we observe the signal through a linear system. Here we regard the difference as the spectral coloration through the linear system from a perceptual point of view, although there are no nonlinear distortions in terms of linear system theory. This spectral coloration is due to the spectral selectivity in the spectral-accumulation process for the linear system.

Fig. 5 is an example of short frames of decaying or stationary portions of a speech sample which are observed

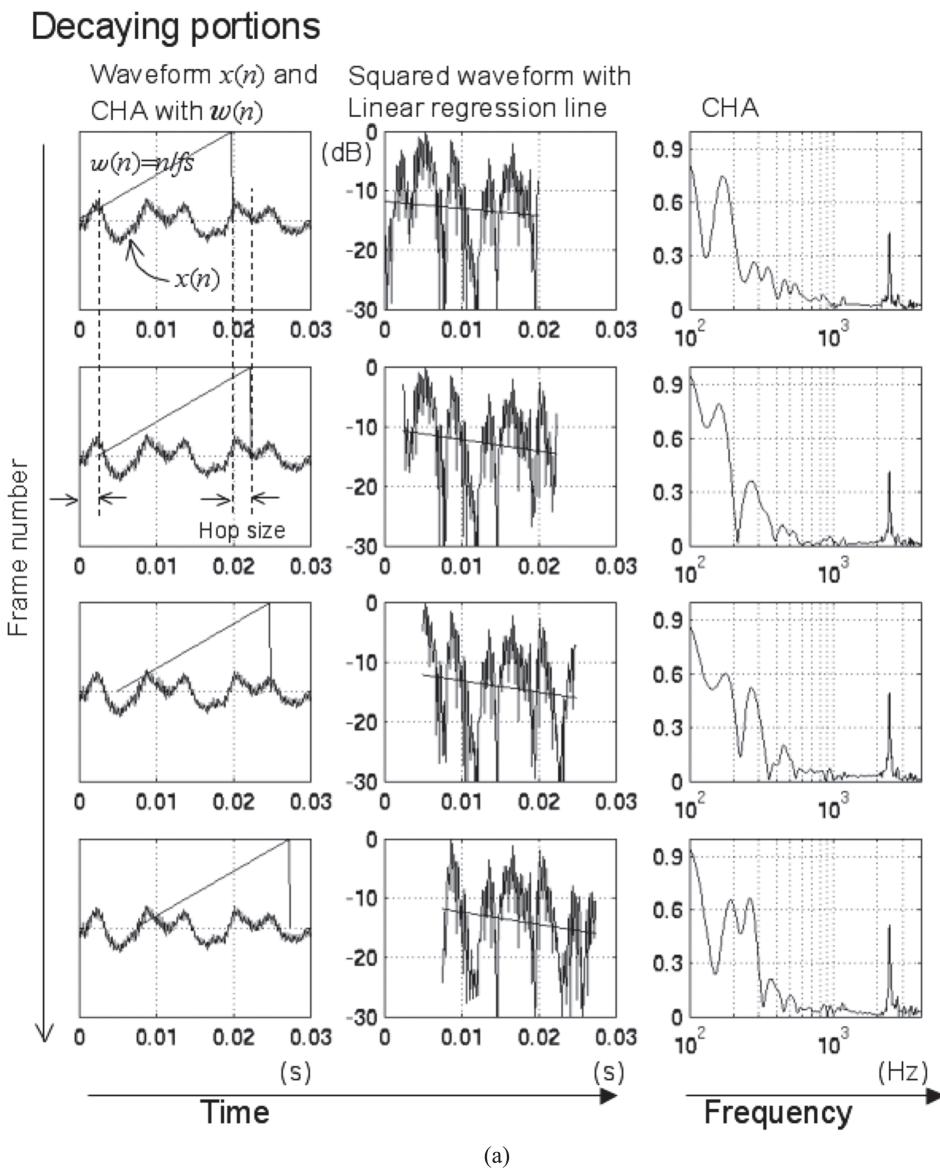


Fig. 5. Samples of CHA magnitude of four short observation frames. (a) Decaying portion. (b) Stationary portion. Left—waveforms per frame; center—squared waveforms and linear-regression lines; right—CHA magnitude plots obtained after spectral accumulation over a single frame (20 ms) using $w(n)$, where maximum magnitude is normalized to unity at every panel. Frame length = 20 ms; $w(m) \equiv m + 1$ ($0 \leq m \leq n$, $n \leq 0.02 f_s$); hop size = 1/8 frame length; length of DFT, $N = 4096$ points; sampling frequency $f_s = 8$ kHz.

through the stable feedback loop whose impulse response was shown in Fig. 3. The left column shows short segments of the speech sample in the feedback loop. The speech sample is observed on a frame-by-frame basis, where the frame length is 20 ms and frames are taken every 2.5 ms (with a hop size of 1/8 frame length). Every frame was classified into decaying, stationary, or other, according to the linear regression analysis for the squared sequence on a dB scale, as shown in the center column. The method of frame classification is summarized in Fig. 6. CHA is performed only for decaying or stationary portions, and thus spectral accumulation is carried out within a single frame at 20-ms intervals. CHA plots show only the results every 20 ms, that is, the right column illustrates only the CHA magnitude obtained after spectral accumulation at the last instant of a single frame of interest. The maximum magnitude is normalized to unity in every CHA panel.

Examples using four short frames are displayed in Fig. 5, with Fig. 5(a) showing the decaying and Fig. 5(b) the

stationary portions. We can see a remarkable difference between the two portions in the CHA magnitude records. CHA records for the decaying portions have an isolated spectral peak around 2 kHz, clearly different from those in the stationary portions. This difference indicates the spectral distortion due to the feedback loop, and the frequency of the spectral peak corresponds to the howling frequency shown in Fig. 4.

According to the results shown in Fig. 5 we obtain histograms for the dominant spectral components for the decaying and the stationary short signal frames. The method for counting the histograms is summarized in Fig. 6. We take the dominant components only in a frame where the number of spectral components with CHA magnitudes greater than 0.5 is smaller than 4. Here the CHA magnitude is observed after spectral accumulation at the last instant of a single frame of interest, similar to the right columns of Fig. 5(a) and (b). The spectral components located at the same frequency band (with ± 5 Hz) for four successive frames are picked up as the dominant frequencies.

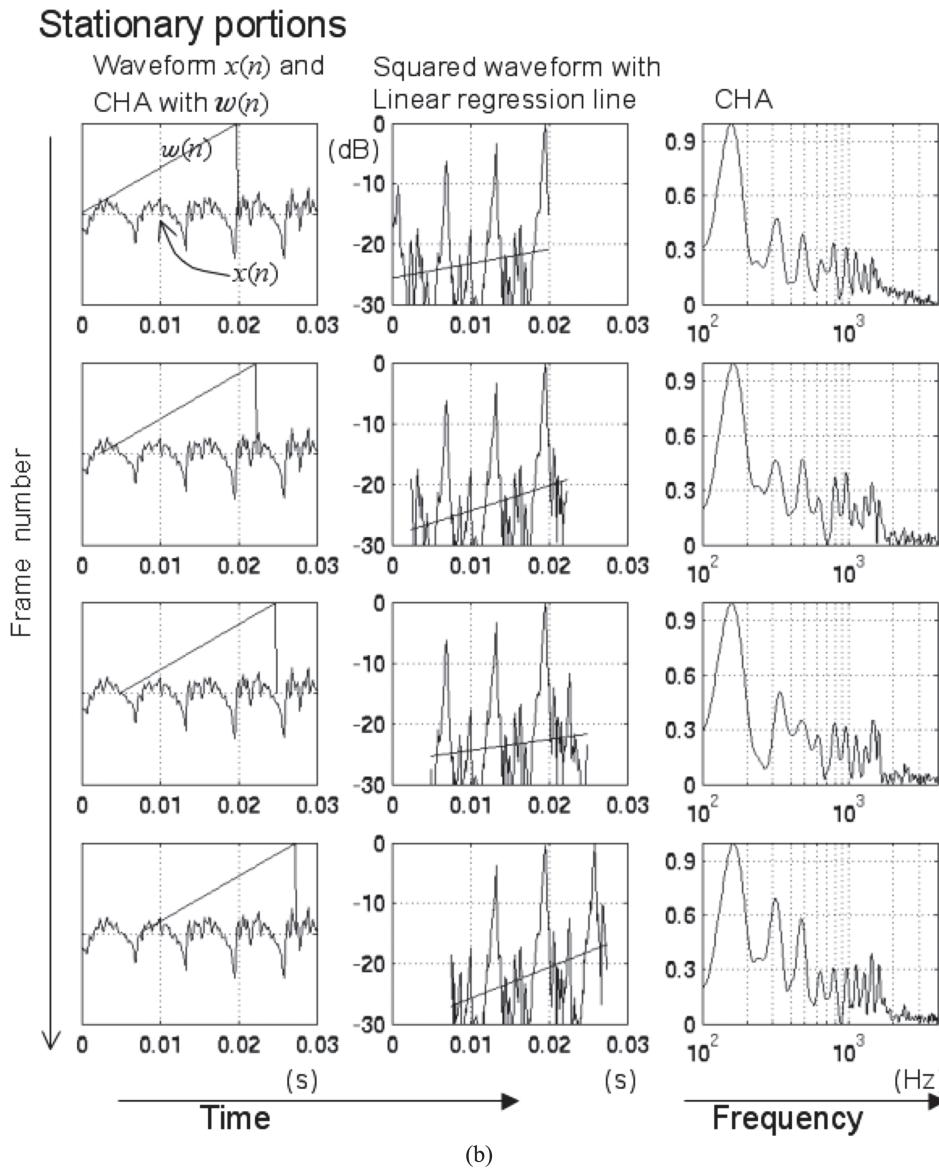


Fig. 5. *Continued*

Fig. 7 shows examples of histograms for the dominant components (decaying and stationary segments) picked up from the original speech samples (top) as well as the those in the stable closed loop (bottom). The total record length of the speech material used in this analysis is 90 s, including any pauses. Here the results for the original speech are illustrated only for reference. The original samples could not be observed for a public-address system under in situ conditions.

Fig. 7 confirms the differences in the histograms between the decaying and the stationary frames. The histograms for the decaying portions has an isolated spectral peak around 2 kHz, which indicates the spectral distortion due to the feedback loop, and the frequency of the spectral peak corresponds to the howling frequency shown in Fig. 4. If we observe the histogram for the original speech for the reference, then we see that for the stationary segments there are no significant differences between the original and the feedback-loop samples. On the contrary the differences around 2 kHz in the decaying portions could also be confirmed between original and feedback samples. Although making the histograms for the dominant components requires a time interval that is a little longer, the results displayed in Fig. 7 suggest that there is a possibility to predict the howling frequency in situ and under blind

conditions without a specified knowledge of the transfer function, including a feedback loop and the original signals. Fig. 8 shows the histograms for 90/4 s during 90 s including feedback speech. This figure suggests that the prediction algorithm for the howling frequency using CHA is able to work even in a period shorter than 90/4 s. We could predict the howling frequency by using the histogram of the cumulative analysis of decaying parts in our simulation experiments where the system was time invariant and included a single input and a single output with a closed loop of short reverberations. If we wish to prevent howling, it is desirable to predict the howling frequency under stable and time-invariant conditions prior to howling. Therefore our simulation results obtained for stable conditions suggest that the proposed method could be effective in the prediction of howling.

4 SUMMARY

We have developed a possibility to predict the howling frequency by investigating a way to quickly determine the principal resonant frequency of a public-address system before it starts howling. Resonance or howling can be interpreted as the stable or unstable spectral accumulation

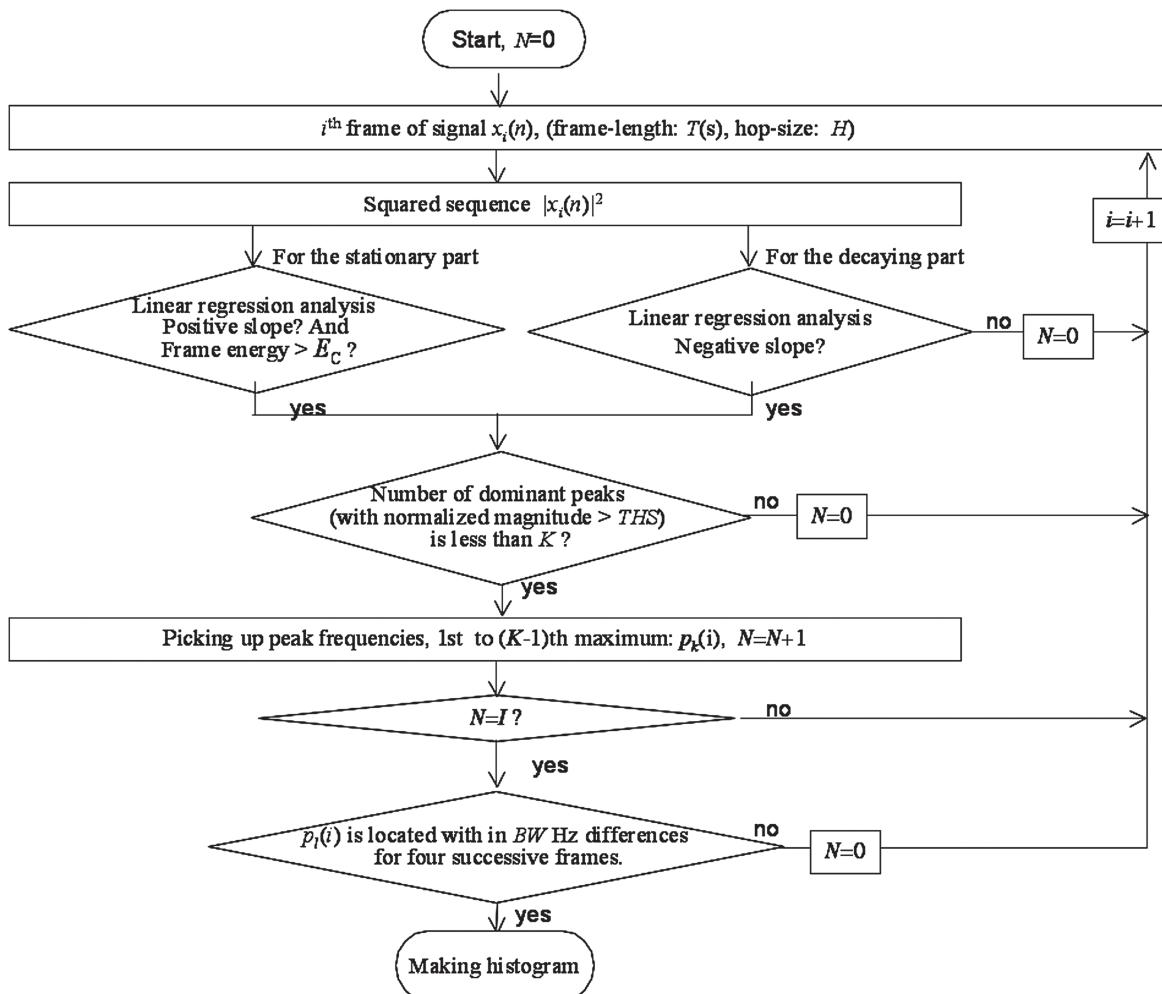


Fig. 6. Method for counting dominant CHA magnitude for histogram analysis where histogram counting is performed for decaying and stationary portions. In Figs. 5 and 7, $T = 0.02$ s, $H = 1/8$ frame length; $E_c = 1/5$ per frame squared sample average; THS = 0.5, where maximum CHA magnitude is normalized to unity; $K = 4$, $I = 4$, $BW = 5$ Hz.

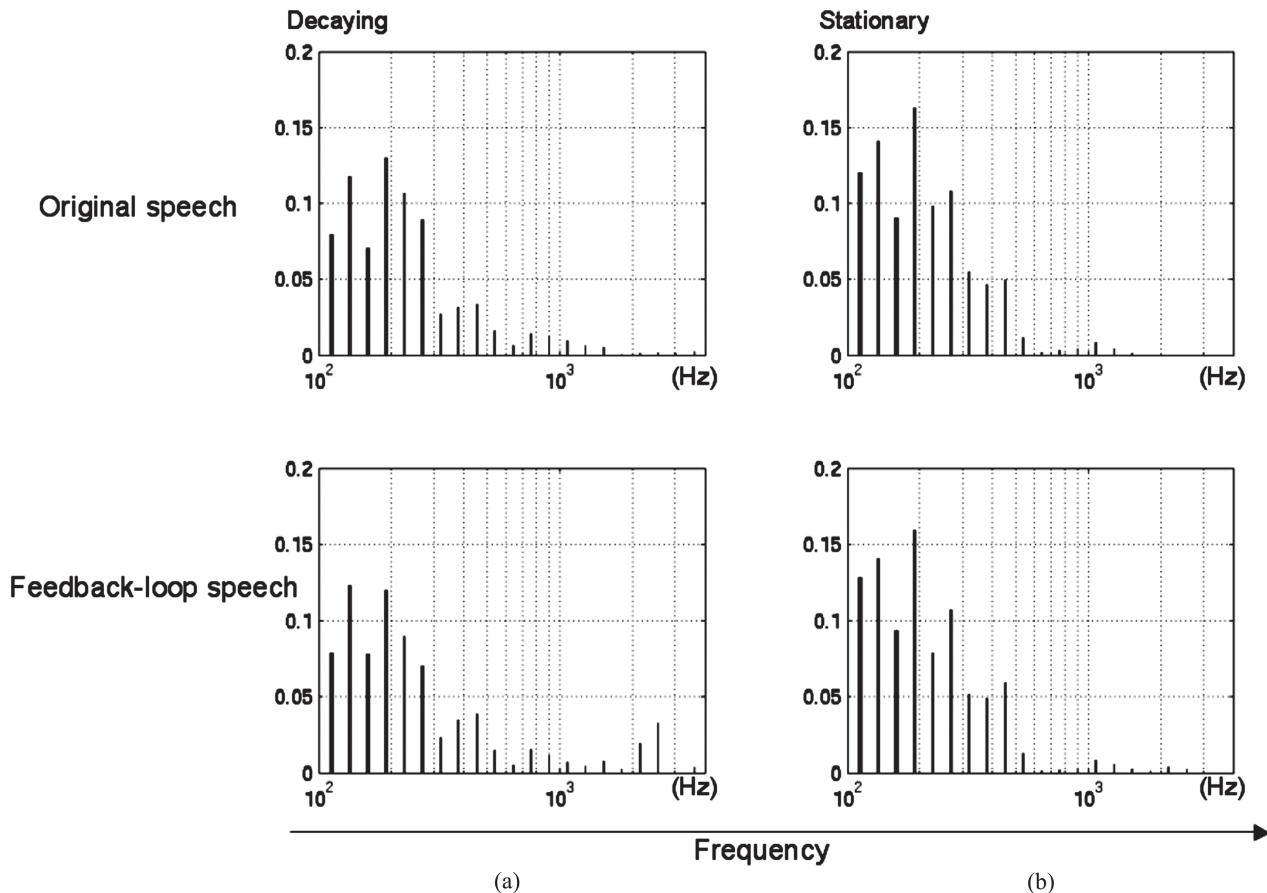


Fig. 7. Histograms of dominant spectral components picked up by CHA for short samples of speech signals analyzed following procedure shown in Fig. 6. (a) Decaying portions. (b) Stationary portions. Total record length of the speech 90 s. Results for original speech samples are illustrated only for reference.

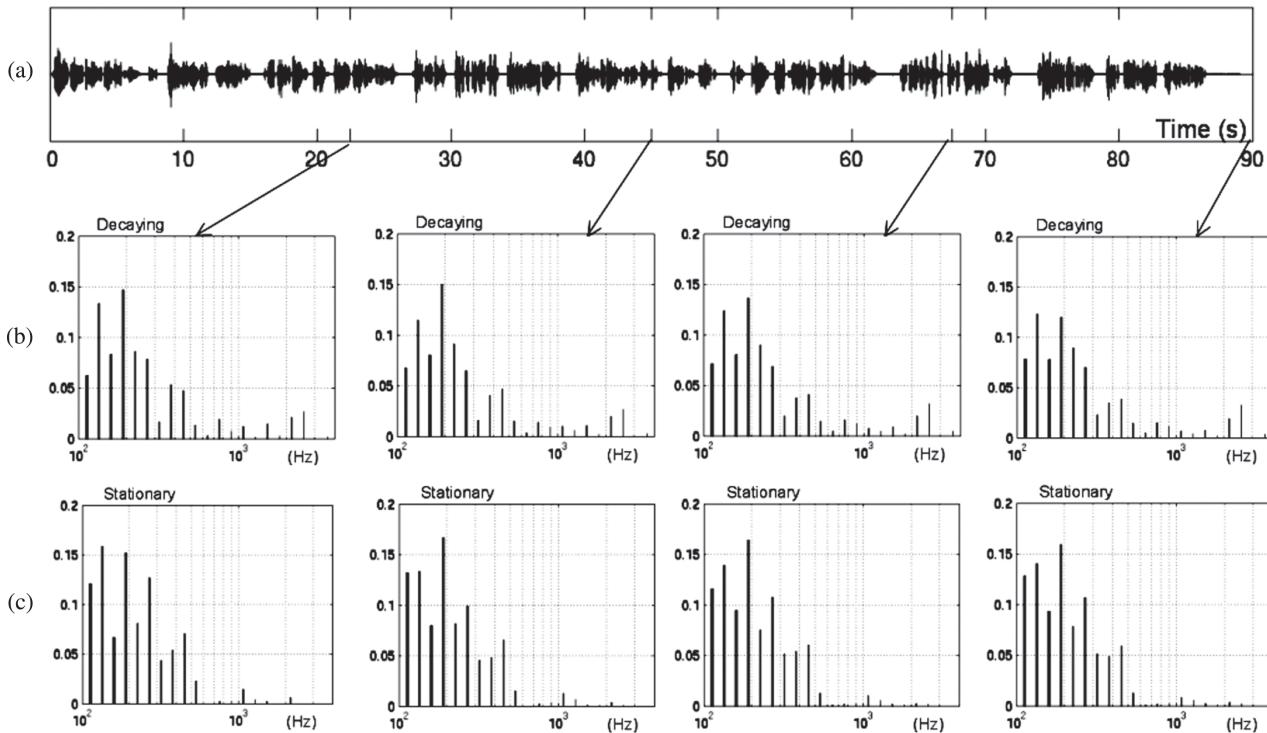


Fig. 8. Histograms of dominant spectral components for feedback-loop speech. (a) Waveform of feedback-loop speech used for Fig. 7. (b), (c) Histograms accumulated every 90/4 s for decaying and stationary parts.

process of signals through a linear system, where the dominant frequencies are selected and focused. The spectral accumulation process can be displayed by CHA (cumulative harmonic analysis), which is formulated by introducing a spectral accumulation function into the conventional cumulative spectral analysis (CSA). Both conventional CSA and CHA are effective in predicting the howling frequency. However, the spectral-accumulation effect revealed by CSA is a little less than that of CHA. CHA displays and emphasizes the accumulation process by increasing the order of resonance poles. Frequency distributions (histograms) constructed by the dominant frequency components picked up by CHA indicated a significant difference between decaying and stationary short segments of a speech material observed in a stable feedback loop. This result suggests that the proposed analysis method for the prediction of howling frequency might be able to work even in a period shorter than 90/4 s. It is necessary to investigate how long an observation time is effective to find the difference in the frequency distribution. An effective and smart method for the short-frame segmentation of signals into decaying or stationary frames under feedback-loop conditions is still under study, and the effect of external noise or reverberation on the coloration analysis for howling frequency prediction is a future problem. In particular, simulation experiments under time-variant and reverberation conditions, including multiple

inputs and outputs are necessary for evaluating the proposed method from a practical point of view. In addition, we used only a simple example of the spectral accumulation function, and therefore the optimization of the accumulation function is also left for future study.

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THE AUTHORS



Y. Takahashi



M. Tohyama



Y. Yamasaki

Yoshinori Takahashi received a Doc.Sc. degree from Waseda University, Tokyo, Japan, in 2006. His dissertation deals with acoustic transfer function and phase representation using poles and zeros.

He is a visiting researcher in the Department of Computer Science, Kogakuin University, Tokyo, Japan. His research interests are communication acoustics and acoustic signal processing.

Dr. Takahashi is an associate member of the Audio Engineering Society and a member of the Institute of Electrical and Electronics Engineers, the Acoustic Society of Japan, and the Institute of Electronics, Information, and Communication Engineers of Japan.

Mikio Tohyama received a Doc.Eng. degree from Waseda University, Tokyo, Japan, in 1975.

He was involved in the fundamental research of room acoustics and acoustic signal processing at Nippon Telecommunication and Telephone Corporation Laboratories

until 1993. He was a professor at the Kogakuin University, Tokyo, Japan, from 1993 to 2003, and he has been a visiting professor at the Waseda University and the University of York, UK, since 2003.

Dr. Tohyama is a member of the Audio Engineering Society, the Acoustical Society of America, the Institute of Electrical and Electronics Engineers, and the IOA in the UK.

Yoshio Yamasaki received a Doc.Eng. degree from Waseda University, Tokyo, Japan, in 1990.

He has been involved in acoustics, digital signal processing, and architectural acoustics in the Science and Engineering Laboratory, Waseda University; the Department of Information and Computer Science, Chiba Institute of Technology, Japan; and the Advanced Research Institute for Science and Engineering, Waseda University. He is a professor at the Graduate School of Global Information and Telecommunication Studies at the Waseda University.