for the DFT, all signals and spectra are length *N* measured in samples per second (our sample rate.) a length *N* sequence *x* can be denoted by *x(n), n = 1, 2, … N – 1*, where *x(n)* may be real (*x* ∈ ℝ*N*) or complex (*x* ∈ ℂ*N*). wishing to regard *x* as a vector *x* in an *N* dimensional vector space. that is each sample *x(n)* is regarded as a coordinate in that space. a vector *x* is mathematically a single point in *N*-space represented by a list of coordinates (*x0, x1, x2, …, xn-1*) called an *N*-tuple. (the notation *xn* here being the same as *x(n)*.) it can be interpreted as an arrow in *N*-space from the origin *0* ≜ (*0, 0, …, 0*) to the point *x* ≜ (*x0, x1, x2, …, xn-1*)*.* the following is defined as equivalent:

*x* ≜ *x* ≜ *x* (⋅)≜(*x0, x1, …, xn-1*)≜[*x0, x1, … xn-1*] ≜ [*x0, x1, ⋯, xn-1*]

where *x(n) ≜ xn* is the *n*th sample of the signal (vector) *x*. i.e. unless stated otherwise, all signals are length *N* samples.

abiding by standard vector notation, using scalars denoted as:

*αx* ≜ (*αx1*, *αx2*, …, *αxn*)

where for example, altering each sample by a constant number, for example a 6dB boost being a gain factor resulting in α = 2, and likewise an attenuation being a scalar less than one.

given that a linear combination of vectors is a sum of scalar multiples of those vectors

*y = α1x1 + α2x2 ⋯, αMxM = αixi*

this would be the equivalent of the mix output. therefore we can create a linear vector space (α ∈ ℝ *N*) and (α ∈ ℂ*N*) although we can’t construct a vector space combining real vectors with complex scalars.

the average power of a signal *x* is the energy per sample, given by:

also described as, when *x* is real the mean square, i.e.

the root mean square (RMS) level of a signal *x* is

the L2 norm of a signal x is defined as the square root of its total energy:

we think of ∥*x*∥ as the length of the vector *x* in *N*-space, and ∥*x-y*∥ as the distance between x and y.the norm can also be thought of as the “absolute value” or “radius” of a vector.

combining Pythagorean theorem with normed vector space math gives:

so giving the projection of y ∈ ℂ*N* onto *x* ∈ ℂ*N*

denoting the set of all linear combinations of vectors from ℂ*N* as a vector space *M,* if the set of vectors is *s0, …, sM-1* then , also, given any basis set in ℂ*N* a new basis set can be formed by rotating all vectors in ℂ*N* by the same angle.

the DFT can be viewed as a change in coordinates from coordinates relative to the natural basis in ℂ*N* ,

, to coordinates relative to the sinusoidal basis for ℂ*N*, ,

where sk(n)= ej𝜔ktn

the sinusoidal basis set for ℂ*N* consists of length *N* sampled complex sinusoids at frequencies *𝜔k = 2πkfs/N, K = 0, 1, 2, …, N-1*

any scaling of these vectors in ℂ*N* by complex scale factors could also be chosen as the sinusoidal basis (i.e. any nonzero amplitude and any phase will do.)

the time-domain samples of a signal are its coordinates relative to the natural basis for ℂ*N*.

the spectral coefficients of a signal are its coordinates relative to the sinusoidal basis for ℂ*N*.

And the reconstruction:

(sinusoids being any function of the form *x(t) = Asin*(*𝜔t + 𝜙*) where

*A* = peak amplitude (nonnegative)

*𝜔* = *2πf* = radian frequency (radians/sec)

*t* = time (sec)

*f* = frequency (Hz)

*𝜙 =* initial phase (radians). (*𝜔t + 𝜙*) = instantaneous phase (radians). )

Fundamental theorem of algebra:

Every *n*-th order polynomial possesses exactly *n* complex roots

where the points *zi* are the polynomial roots.

also,

and,

and,

Euler’s Identity:

De Moivre’s theorem:

Fourier Transform:

|  |
| --- |
|  |

DFT:

*K = 0, ±1, ±2, ...* where *𝜔k* ≜ *2πk/P* is the *k*th harmonic frequency (radians/sec)

The normalisation *1/P* is included to make the Fourier series coefficients, *X*(*𝜔k*), independent of the fundamental frequency 1/P thereby depending on only one period of the time waveform.