a function $f(x)$ is said to be periodic with period p $f(x) = f(x + np)$ for $1,\ 2,\ \ldots$ for example, the sine function $\sin(x)$ illustrated above, is periodic with period 2π (as well as with period, -2π, 4π, 6π, etc.)

in music, if a note has frequency f , integer multiples of that frequency, $2f$, $3f$, $4f$ and so on, are known as *harmonics*. as a result, the mathematical study of overlapping waves is called harmonic analysis.

Harmonic analysis is a diverse field, with branches such as Fourier analysis, Fourier series and signal processing using harmonic analysis extensively.

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the Fourier transform (FT) is a transform that converts a function into a form that describes the frequencies present in the original function.

the output of the transform is a complex-valued function of frequency.

the term Fourier transform refers to both this complex-valued function and the mathematical operation.

when a distinction needs to be made the Fourier transform is sometimes called the frequency domain representation of the original function.

the Fourier transform is analogous to decomposing the sound of a musical chord into terms of the intensity of its constituent pitches.

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a Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of sines and cosines.

the computation and study of Fourier series is known as harmonic analysis.

it is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in and solved individually.

these simplified terms can then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical.

examples of successive approximations to common functions using Fourier series are illustrated below.

Fourier series make use of the orthogonality relationships of the sine and cosine functions.

Two functions f(x) and g(x) are orthogonal over the interval $a \le x \le b$ with weighting function $w(x)$ if

$< f(x)|g(x) >\equiv| \int_a^b f(x)g(x)w(x)dx = 0$

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an ordinary differential equation, ODE , is an equality involving a function and its derivatives. An ODE of order n is an equation of the form $F\left(x,\ y,\ y',\ \ldots\ ,\ y^{(n)}\right) = 0$.

seriously, thats all you need or want to know...

the 'superposition principle' states that for a linear homogeneous ODE , if $y_1(x)$ and $y_2(x)$ are solutions, then so is $y_1(x) + y_2(x)$.

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in particular, since the superposition principle holds for solutions of a linear homogeneous ODE , if such an equation can be solved in the case of a single sinusoid, the solution for an arbitrary function is immediately available by expressing the original function as a Fourier series and then plugging in the solution for each sinusoidal component.

in some special cases where the Fourier series can be summed in closed form, this technique can even yield analytic solutions.

any set of functions that form a complete orthogonal system have a corresponding generalized Fourier series analogous to the Fourier series.

for example, using orthogonality of the roots of a Bessel function of the first kind gives a so-called Fourier-Bessel series.

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using the method for a generalized Fourier series, the usual Fourier series involving sines and cosines is obtained by taking $f_1(x) = \cos x$ and $f_2(x) = \sin x$.

since these functions form a complete orthogonal system over [-π,π], the Fourier series of a function $f(x)$ is given by

$f(x)) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \quad a_n \cos(nx) + \sum_{n=1}^{\infty} \quad b_n \sin(ny).$

where

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) \, dx$$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) \, dx$

and n=1, 2, 3, ... Note that the coefficient of the constant term a_0 has been written in a special form compared to the general form for a generalized Fourier series in order to preserve symmetry with the definitions of a_n and b_n.
