Normalization requirements

For a given relation:
R(A, B, C, D, E, F)

there are functional dependencies
E → B, C
D → E, F

1. Find the closures {E}+, {D,E}+.
2. Identify the super keys.
3. Decompose the relation in multiple relations if needed to achieve BCNF.

Solution

Step 1: Compute Closures

Functional Dependencies:

1. E→B,C
2. D→E,F

Compute {E}+

Start with E+={E}.

1. From E→B,C add B,C to E+
   E+={E,B,C}

No further functional dependencies can be applied.

Result: {E}+={E,B,C}

Compute {D,E}+
Start with {D,E}+={D,E}

1. From D→E,F add D and E

{D,E}+={D,E,F}

2. From E→B,C add B,C (since E∈{D,E}+).

{D,E}+={D,E,F,B,C}

No further functional dependencies can be applied.

Result: {D,E}+={D,E,F,B,C}

Step 2: Identify Superkeys

A **superkey** is any set of attributes whose closure contains **all attributes** of the relation R(A,B,C,D,E,F).

Check Possible Superkeys

1. **{D,E}:**

{D,E}+={A,B,C,D,E,F} (contains all attributes)
{D,E} is a superkey.

2. **{D}:**

D+={D,E,F,B,C}. (missing A)

{D} is not a superkey.

3. **{E}:**

E+={E,B,C} (missing A,D,F)

{E} is not a superkey.

4. **{D,E,A}:** Since {D,E}+ already contains all attributes, adding A is redundant.
   {D,E,A} is a superkey.

5. **If we start from {D} and add A to it (look at case 2 above):**

{D,A}+={A,B,C,D,E,F} (contains all attributes)
Hence, {D,A} is a superkey.

Step 3: Decompose to Achieve BCNF

BCNF Condition

A relation is in **BCNF** if, for every functional dependency X→Y:

1. X is a superkey, or
2. Y⊆X.

Check Given Functional Dependencies

1. **E→B,C:**

- E is **not a superkey** (as $E^+ \neq R$).
- Decompose the relation based on this FD.

2. **D→E,F:**
   - D is **not a superkey** (as $D^+ \neq R$).
   - Decompose the relation based on this FD.

Decomposition Steps

1. Decompose R(A,B,C,D,E,F) using E→B,C:
   - R_1(E,B,C) (contains E and its attributes).
   - R_2(A,D,E,F) (remaining attributes).
2. Check R_2(A,D,E,F) using D→E,F:
   - Decompose R_2(A,D,E,F) into:
     - R_3(D,E,F) (contains D and its attributes).
     - R_4(A,D) (remaining attributes).

Final Decomposed Relations

1. R_1(E,B,C)
2. R_3(D,E,F)
3. R_4(A,D)

Each of these relations satisfies BCNF because:

- All functional dependencies within each relation have superkeys as determinants.