



FACULTY OF COMPUTER  
SCIENCE AND ENGINEERING



# RELATIONAL ALGEBRA AND RELATIONAL CALCULUS

DATABASES - lectures



# Outline

## ➤ Relational Algebra

### ➤ Unary Relational Operations

- SELECT (symbol:  $\sigma$  (sigma))
- PROJECT (symbol:  $\pi$  (pi))
- RENAME (symbol:  $\rho$  (rho))

### ➤ Relational Algebra Operations From Set Theory

- UNION ( $\cup$ ), INTERSECTION ( $\cap$ ), DIFFERENCE (or MINUS,  $-$ )
- CARTESIAN PRODUCT ( $\times$ )

### ➤ Binary Relational Operations

- JOIN (several variations of JOIN exist)
- DIVISION

### ➤ Additional Relational Operations

- AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)
- GROUPING FUNCTIONS
- OUTER JOINS

## ➤ Relational Calculus

### ➤ Tuple Relational Calculus

### ➤ Domain Relational Calculus

# Relational Algebra Overview

- Relational algebra is the basic set of operations for the relational model
- These operations enable a user to specify **basic retrieval requests** (or **queries**)
- The result of an operation is a *new relation*, which may have been formed from one or more *input* relations
  - This property makes the algebra “closed” (all objects in relational algebra are relations)

# Relational Algebra Overview (2)

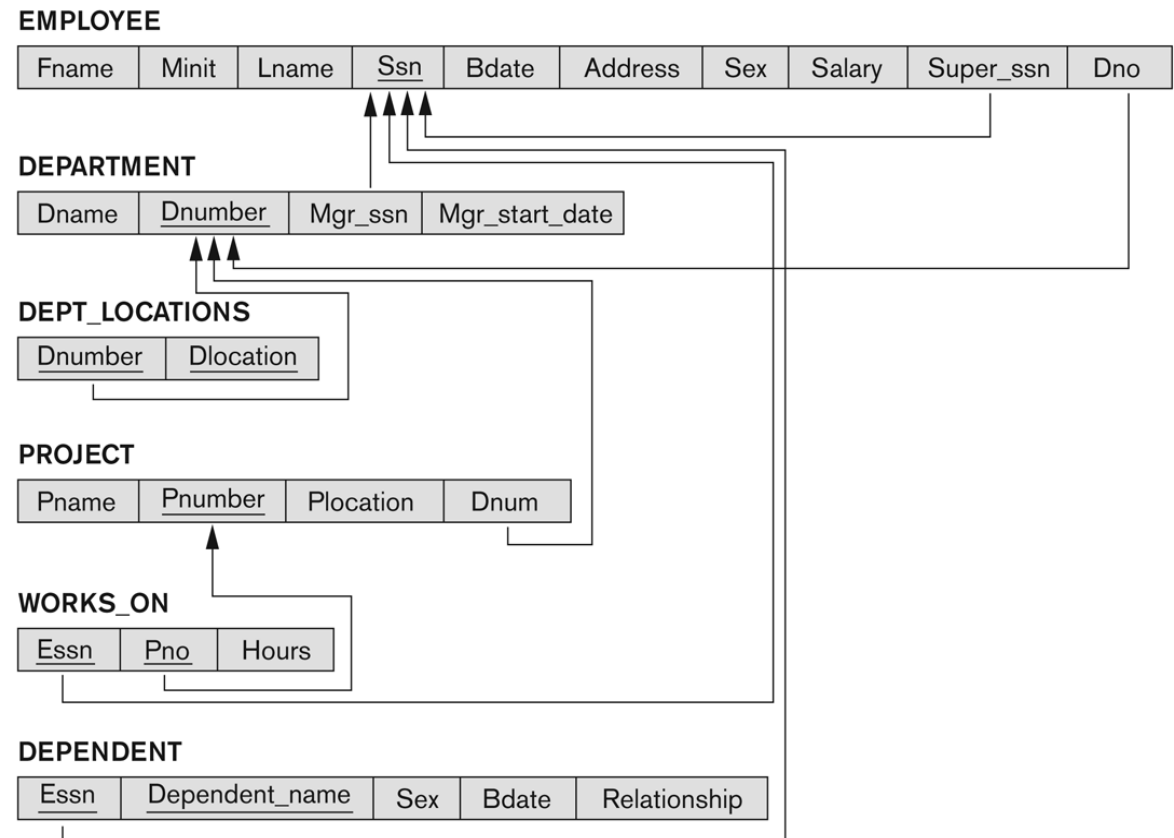
- The **algebra operations** thus produce new relations
  - These can be further manipulated using operations of the same algebra
- A sequence of relational algebra operations forms a **relational algebra expression**
  - The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

# Database State for COMPANY (1)

➤ All examples discussed below refer to the COMPANY database shown here

**Figure 5.7**

Referential integrity constraints displayed on the COMPANY relational database schema.



# Database State for COMPANY (2)

➤ All examples discussed below refer to the COMPANY database shown here

Figure 5.6

One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT\_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS\_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse



# Unary Relational Operations: SELECT

$$\sigma_{\langle \text{selection condition} \rangle}(R)$$

- The SELECT operation (denoted by  $\sigma$  (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.
  - The selection condition acts as a **filter**
  - Keeps only those tuples that satisfy the qualifying condition
  - Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)
  - The result is a new relation

# Unary Relational Operations: SELECT

- Select the EMPLOYEE tuples whose department number is 4 :

$$\sigma_{DNO = 4} (EMPLOYEE)$$

- Select the employee tuples whose salary is greater than \$30,000 :

$$\sigma_{SALARY > 30,000} (EMPLOYEE)$$

EMPLOYEE

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1





# Unary Relational Operations: SELECT

## ➤ SELECT Operation Properties

- The SELECT operation  $\sigma_{\langle \text{selection condition} \rangle}(R)$  produces a relation S that has the same schema (same attributes) as R
- SELECT  $\sigma$  is commutative:
  - $\sigma_{\langle \text{condition1} \rangle}(\sigma_{\langle \text{condition2} \rangle}(R)) = \sigma_{\langle \text{condition2} \rangle}(\sigma_{\langle \text{condition1} \rangle}(R))$
- Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
  - $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(\sigma_{\langle \text{cond1} \rangle}(R)))$
- A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
  - $\sigma_{\langle \text{cond1} \rangle}(\sigma_{\langle \text{cond2} \rangle}(\sigma_{\langle \text{cond3} \rangle}(R))) = \sigma_{\langle \text{cond1} \rangle \text{ AND } \langle \text{cond2} \rangle \text{ AND } \langle \text{cond3} \rangle}(R))$
- The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

# Unary Relational Operations: PROJECT

$$\pi_{\langle \text{attribute list} \rangle}(R)$$

- PROJECT Operation is denoted by  $\pi$  (pi). This operation keeps certain *columns* (attributes) from a relation and discards the other columns. The remaining columns form the resulting relation
- PROJECT creates a vertical partitioning
  - The list of specified columns (attributes) is kept in each tuple
  - The other attributes in each tuple are discarded
  - If the list of attributes **does not include a key of R**, then the project operation *removes any duplicate tuples*
    - Mathematical sets *do not allow* duplicate elements

# Unary Relational Operations: PROJECT

- To list each employee's first and last name and salary, the following is used:

$\pi_{\text{LNAME, FNAME, SALARY}}(\text{EMPLOYEE})$

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1



# Unary Relational Operations: PROJECT

## ➤ PROJECT Operation Properties

- The number of tuples in the result of projection  $\pi_{\langle \text{list} \rangle}(R)$  is always less or equal to the number of tuples in  $R$ 
  - If the list of attributes includes a *key* of  $R$ , then the number of tuples in the result of PROJECT is *equal* to the number of tuples in  $R$
- PROJECT is *not* commutative
- Moreover,
  - $\pi_{\langle \text{list1} \rangle}(\pi_{\langle \text{list2} \rangle}(R)) = \pi_{\langle \text{list1} \rangle}(R)$ 
    - as long as  $\langle \text{list2} \rangle$  contains the attributes in  $\langle \text{list1} \rangle$

# Examples of applying SELECT and PROJECT operations

➤ Answer the following questions:

- Find the employees who either work in department 4 and make over \$25,000 per year, or work in department 5 and make over \$30,000?
- Generate the payroll for all employees (name, surname and salary).
- Return the sex and salary for all employees.



# Examples of applying SELECT and PROJECT operations

**Figure 6.1**

Results of SELECT and PROJECT operations. (a)  $\sigma_{(Dno=4 \text{ AND } Salary > 25000) \text{ OR } (Dno=5 \text{ AND } Salary > 30000)}(EMPLOYEE)$ . (b)  $\pi_{Lname, Fname, Salary}(EMPLOYEE)$ . (c)  $\pi_{Sex, Salary}(EMPLOYEE)$ .

(a)

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Example



# Relational Algebra Expressions

- We may want to apply several relational algebra operations one after the other
  - Either we can write the operations as a single **relational algebra expression** by nesting the operations, or
  - We can apply one operation at a time and create **intermediate result relations**.
    - In this case, we **must give names to the relations** that hold the intermediate results.

# Relational Algebra Expressions

- To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation.

Example

- We can write a *single relational algebra expression* as follows :

$$\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO}=5}(\text{EMPLOYEE}))$$

- OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation :

$$\begin{aligned} \text{DEP5\_EMPS} &\leftarrow \sigma_{\text{DNO}=5}(\text{EMPLOYEE}) \\ \text{RESULT} &\leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS}) \end{aligned}$$



# Unary Relational Operations: RENAME

$$\rho_S (B_1, B_2, \dots, B_n)(R)$$

- The RENAME operator is denoted by  $\rho$  (rho)
- In some cases, we may want to *rename* the attributes of a relation or the relation name or both
  - Useful when a query requires multiple operations
  - Necessary in some cases (see JOIN operation later)

# Unary Relational Operations: RENAME

- The general RENAME operation  $\rho$  can be expressed by any of the following forms:
  - $\rho_S(B_1, B_2, \dots, B_n)(R)$  changes both:
    - the relation name to  $S$ , *and*
    - the column (attribute) names to  $B_1, B_1, \dots, B_n$
  - $\rho_S(R)$  changes:
    - the *relation name* only to  $S$
  - $\rho_{(B_1, B_2, \dots, B_n)}(R)$  changes:
    - the *column (attribute) names* only to  $B_1, B_1, \dots, B_n$

# Unary Relational Operations: RENAME

- For convenience, we also use a *shorthand* for renaming attributes in an intermediate relation:
  - If we write:
    - $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}} (\text{DEP5\_EMPS})$
    - RESULT will have the *same attribute names* as DEP5\_EMPS (same attributes as EMPLOYEE)
  - If we write:
    - $\text{RESULT (F,M,L,S,B,A,SX,SAL,SU,DNO)} \leftarrow \rho_{\text{RESULT (F,M,L,S,B,A,SX,SAL,SU,DNO)}} (\text{DEP5\_EMPS})$
    - The 10 attributes of DEP5\_EMPS are *renamed* to F, M, L, S, B, A, SX, SAL, SU, DNO, respectively

# Unary Relational Operations: RENAME

(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Write down the renaming operations for b)

(b)

TEMP

Fname	Minit	Lname	<u>Ssn</u>	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

**Figure 6.2**

Results of a sequence of operations.

(a)  $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno}=5}(\text{EMPLOYEE}))$ .

(b) Using intermediate relations and renaming of attributes.

Example



# Relational Algebra Operations from Set Theory: UNION

$$R \cup S$$

- Binary operation, denoted by  $\cup$
- The result of  $R \cup S$ , is a relation that includes all tuples that are either in R or in S or in both R and S
- Duplicate tuples are eliminated
- The two operand relations R and S must be “type compatible” (or UNION compatible)

# Relational Algebra Operations from Set Theory: UNION

- Type Compatibility of operands is required for the binary set operations (UNION  $\cup$ , INTERSECTION  $\cap$ , and SET DIFFERENCE  $-$ )
- $R_1(A_1, A_2, \dots, A_n)$  and  $R_2(B_1, B_2, \dots, B_n)$  are type compatible if:
  - they have the same number of attributes, and
  - the domains of corresponding attributes are type compatible
$$\text{dom}(A_i) = \text{dom}(B_i) \text{ for } i=1, 2, \dots, n$$
- The resulting relation for  $R_1 \cup R_2$  (also for  $R_1 \cap R_2$ , or  $R_1 - R_2$ , see next slides) has the same attribute names as the *first* operand relation  $R_1$  (by convention)

# Relational Algebra Operations from Set Theory: UNION

- Retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below).

## Example

- We can use the UNION operation as follows :

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO}=5} (\text{EMPLOYEE})$$
$$\text{RESULT1} \leftarrow \pi_{\text{SSN}} (\text{DEP5\_EMPS})$$
$$\text{RESULT2} \leftarrow \pi_{\text{SUPERSSN}} (\text{DEP5\_EMPS})$$
$$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$$

# Relational Algebra Operations from Set Theory: UNION

## Example

**Figure 6.3**

Result of the UNION operation  
 $RESULT \leftarrow RESULT1 \cup RESULT2$ .

**RESULT1**

Ssn
123456789
333445555
666884444
453453453

**RESULT2**

Ssn
333445555
888665555

**RESULT**

Ssn
123456789
333445555
666884444
453453453
888665555



# Relational Algebra Operations from Set Theory: INTERSECTION

$$R \cap S$$

- INTERSECTION is denoted by  $\cap$
- The result of the operation  $R \cap S$ , is a relation that includes all tuples that are in both R and S
  - The attribute names in the result will be the same as the attribute names in R
- The two operand relations R and S must be “type compatible”

# Relational Algebra Operations from Set Theory: SET DIFFERENCE

$$R - S$$

- SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –
- The result of  $R - S$ , is a relation that includes all tuples that are in  $R$  but not in  $S$ 
  - The attribute names in the result will be the same as the attribute names in  $R$
- The two operand relations  $R$  and  $S$  must be “type compatible”

# Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE

(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

**Figure 6.4**

The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b)  $\text{STUDENT} \cup \text{INSTRUCTOR}$ . (c)  $\text{STUDENT} \cap \text{INSTRUCTOR}$ . (d)  $\text{STUDENT} - \text{INSTRUCTOR}$ . (e)  $\text{INSTRUCTOR} - \text{STUDENT}$ .



# Some properties of UNION, INTERSECT, and DIFFERENCE

- Notice that both union and intersection are *commutative* operations; that is
  - $R \cup S = S \cup R$ , and  $R \cap S = S \cap R$
- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is
  - $R \cup (S \cup T) = (R \cup S) \cup T$
  - $(R \cap S) \cap T = R \cap (S \cap T)$
- The minus operation is not commutative; that is, in general
  - $R - S \neq S - R$

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

$$R \times S$$

## ➤ CARTESIAN (or CROSS) PRODUCT Operation

- This operation is used to combine tuples from two relations in a combinatorial fashion.
- Denoted by  $R(A_1, A_2, \dots, A_n) \times S(B_1, B_2, \dots, B_m)$
- Result is a relation  $Q$  with degree  $n + m$  attributes:
  - $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order.
- The resulting relation state has one tuple for each combination of tuples—one from  $R$  and one from  $S$ .
- Hence, if  $R$  has  $n_R$  tuples (denoted as  $|R| = n_R$ ), and  $S$  has  $n_S$  tuples, then  $R \times S$  will have  $n_R * n_S$  tuples.
- The two operands do NOT have to be "type compatible"

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

R	
A	B
1	2
1	4
2	5

X

S	
B	C
2	6
2	4
3	5
4	8

=

Q			
A	B	B	C
1	2	2	6
1	2	2	4
1	2	3	5
1	2	4	8
1	4	2	6
1	4	2	4
1	4	3	5
1	4	4	8
2	5	2	6
2	5	2	4
2	5	3	5
2	5	4	8



# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

- Generally, CROSS PRODUCT is not a meaningful operation
  - Can become meaningful when followed by other operations

## Example

- Example (not meaningful):
  - $\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$
  - $\text{EMP\_NAMES} \leftarrow \pi_{\text{FNAME, LNAME, SSN}}(\text{FEMALE\_EMPS})$
  - $\text{EMP\_DEPENDENTS} \leftarrow \text{EMP\_NAMES} \times \text{DEPENDENT}$
- EMP\_DEPENDENTS will contain every combination of EMP\_NAMES and DEPENDENT
  - Regardless of whether or not they are actually related

# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

➤ To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows

➤ Example (meaningful):

➤  $\text{FEMALE\_EMPS} \leftarrow \sigma_{\text{SEX}='F'}(\text{EMPLOYEE})$

➤  $\text{EMP\_NAMES} \leftarrow \pi_{\text{FNAME, LNAME, SSN}}(\text{FEMALE\_EMPS})$

➤  $\text{EMP\_DEPENDENTS} \leftarrow \text{EMP\_NAMES} \times \text{DEPENDENT}$

➤  $\text{ACTUAL\_DEPS} \leftarrow \sigma_{\text{SSN}=\text{ESSN}}(\text{EMP\_DEPENDENTS})$

➤  $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT\_NAME}}(\text{ACTUAL\_DEPS})$

➤ RESULT will now contain the name of female employees and their dependents





# Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

**Figure 6.5**  
The CARTESIAN PRODUCT (CROSS PRODUCT) operation.

## FEMALE\_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

## EMPNames

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

## EMP\_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	...
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	...
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	...
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	...
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	...
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	...
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	...
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	...
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	...
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	...
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	...
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	...
Joyce	English	453453453	333445555	Alice	F	1986-04-05	...
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	...
Joyce	English	453453453	333445555	Joy	F	1958-05-03	...
Joyce	English	453453453	987654321	Abner	M	1942-02-28	...
Joyce	English	453453453	123456789	Michael	M	1988-01-04	...
Joyce	English	453453453	123456789	Alice	F	1988-12-30	...
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	...

## ACTUAL\_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

## RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

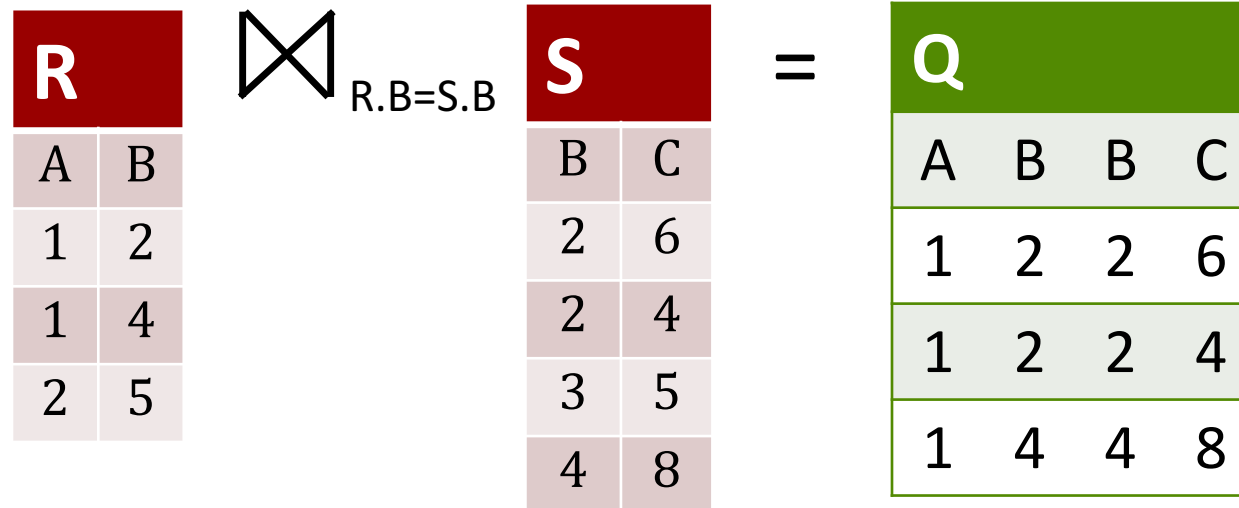
Example

# Binary Relational Operations: JOIN

$$R \bowtie_{\langle \text{join condition} \rangle} S$$

- JOIN Operation (denoted by  $\bowtie$ )
  - The sequence of CARTESIAN PRODUCT followed by SELECT is used quite commonly to identify and select **related tuples** from two relations
  - A special operation, called JOIN combines this sequence into a single operation
  - This operation is very important for any relational database with more than a single relation, because it allows us to *combine related tuples* from various relations
  - R and S can be any relations that result from general *relational algebra expressions*.

# Binary Relational Operations: JOIN



# Binary Relational Operations: JOIN

➤ Suppose that we want to retrieve the name of the manager of each department.

➤ To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple.

DEPT\_MGR  $\leftarrow$  DEPARTMENT  $\bowtie$  <sub>MGRSSN=SSN</sub> EMPLOYEE

➤ MGRSSN=SSN is the join condition

➤ Combines each department record with the employee who manages the department

➤ The join condition can also be specified as  
DEPARTMENT.MGRSSN= EMPLOYEE.SSN



# Binary Relational Operations: JOIN

Example

**DEPT\_MGR**

Dname	Dnumber	Mgr_ssn	...	Fname	Minit	Lname	Ssn	...
Research	5	333445555	...	Franklin	T	Wong	333445555	...
Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
Headquarters	1	888665555	...	James	E	Borg	888665555	...

**Figure 6.6**

Result of the JOIN operation

DEPT\_MGR ← DEPARTMENT  MGRSSN=SSN EMPLOYEE

# Some properties of JOIN

➤ Consider the following JOIN operation:

$$\begin{array}{ccc} \text{➤ } R(A_1, A_2, \dots, A_n) & \bowtie & S(B_1, B_2, \dots, B_m) \\ & R.A_i = S.B_j & \end{array}$$

➤ Result is a relation Q with degree  $n + m$  attributes:

➤  $Q(A_1, A_2, \dots, A_n, B_1, B_2, \dots, B_m)$ , in that order.

➤ The resulting relation state has one tuple for each combination of tuples— $r$  from  $R$  and  $s$  from  $S$ , but *only if they satisfy the join condition*  $r[A_i] = s[B_j]$

➤ Hence, if  $R$  has  $n_R$  tuples, and  $S$  has  $n_S$  tuples, then the join result will generally have *less than*  $n_R * n_S$  tuples.

➤ Only related tuples (based on the join condition) will appear in the result

# Some properties of JOIN

- The general case of JOIN operation is called a Theta-join:

$$R \bowtie_{\theta} S$$

- The join condition is called *theta* or  $\Theta$
- *Theta* can be any general boolean expression on the attributes of R and S; for example:
  - $R.A_i < S.B_j \text{ AND } (R.A_k = S.B_l \text{ OR } R.A_p < S.B_q)$
- Most join conditions involve one or more equality conditions “AND”ed together; for example:
  - $R.A_i = S.B_j \text{ AND } R.A_k = S.B_l \text{ AND } R.A_p = S.B_q$

# Binary Relational Operations: EQUIJOIN

- The most common use of join involves join conditions with *equality comparisons* only (comparison of a primary and a foreign key)
- Such a join, where the only comparison operator used is =, is called an **EQUIJOIN**.
  - In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - The JOIN seen in the previous example was an EQUIJOIN.



# Binary Relational Operations: NATURAL JOIN

- Another variation of JOIN called **NATURAL JOIN** — denoted by  $*$  — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
  - because one of each pair of attributes with identical values is superfluous
- The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, *have the same name* in both relations
  - If this is not the case, a renaming operation is applied first.

# Binary Relational Operations: NATURAL JOIN

➤ Retrieve the departments and their locations?

➤ To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT\_LOCATIONS, it is sufficient to write:

$$\text{DEPT\_LOCS} \leftarrow \text{DEPARTMENT} * \text{DEPT\_LOCATIONS}$$

➤ An implicit join condition is created based on this attribute:

$$\text{DEPARTMENT.DNUMBER} = \text{DEPT\_LOCATIONS.DNUMBER}$$

➤  $Q \leftarrow R(A,B,C,D) * S(C,D,E)$

➤ The implicit join condition includes *each pair* of attributes with the same name, “AND”ed together:

➤  $R.C = S.C \text{ AND } R.D = S.D$

➤ Result keeps only one attribute of each such pair :

➤  $Q(A,B,C,D,E)$

Example

# Binary Relational Operations: NATURAL JOIN

(a)

PROJ\_DEPT

Pname	<u>Pnumber</u>	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT\_LOCS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

**Figure 6.7**

Results of two NATURAL JOIN operations.

(a) PROJ\_DEPT  $\leftarrow$  PROJECT \* DEPT.

(b) DEPT\_LOCS  $\leftarrow$  DEPARTMENT \* DEPT\_LOCATIONS.



# Binary Relational Operations: DIVISION

$$R(Z) \div S(X)$$

- $R(Z) \div S(X)$ , where  $X \subseteq Z$ . Let  $Y = Z - X$  (and hence  $Z = X \cup Y$ ); that is, let  $Y$  be the set of attributes of  $R$  that are not attributes of  $S$ .
- The result of DIVISION is a relation  $T(Y)$  that includes a tuple  $t$  if tuples  $t_R$  appear in  $R$  with  $t_R[Y] = t$ , and with  $t_R[X] = t_s$  *for every tuple*  $t_s$  in  $S$ .
- For a tuple  $t$  to appear in the result  $T$  of the DIVISION, the values in  $t$  must appear in  $R$  in combination with *every* tuple in  $S$ .

# Binary Relational Operations: DIVISION

Example

(a)

**SSN\_PNOS**

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

**SMITH\_PNOS**

Pno
1
2

**SSNS**

Ssn
123456789
453453453

(b)

**R**

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

**S**

A
a1
a2
a3

**T**

B
b1
b4

**Figure 6.8**

The DIVISION operation. (a) Dividing SSN\_PNOS by SMITH\_PNOS. (b)  $T \leftarrow R \div S$ .

# Binary Relational Operations: DIVISION

- Retrieve the SSNs of the employees that work on the same projects as 'John Smith'

## Example

- $SMITH \leftarrow \sigma_{Fname='John' \text{ AND } Lname='Smith'}(EMPLOYEE)$
- $SMITH\_PNOS \leftarrow \pi_{Pno} (WORKS\_ON \bowtie_{Essn=ssn} SMITH)$
- $SSN\_PNOS \leftarrow \pi_{Essn, Pno} (WORKS\_ON)$
- $SSNS \leftarrow SSN\_PNOS \div SMITH\_PNOS$

# Complete Set of Relational Operations

➤ The set of operations including SELECT  $\sigma$ , PROJECT  $\pi$ , UNION  $\cup$ , DIFFERENCE  $-$ , RENAME  $\rho$ , and CARTESIAN PRODUCT  $\times$  is called a **complete set** because any other relational algebra expression can be expressed by a combination of these five operations.

➤ For example:

➤  $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$

➤  $R \bowtie_{\langle \text{join condition} \rangle} S = \sigma_{\langle \text{join condition} \rangle} (R \times S)$

# Recap of Relational Algebra Operations

**Table 6.1**

Operations of Relational Algebra

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation $R$ .	$\sigma_{\langle \text{selection condition} \rangle}(R)$
PROJECT	Produces a new relation with only some of the attributes of $R$ , and removes duplicate tuples.	$\pi_{\langle \text{attribute list} \rangle}(R)$
THETA JOIN	Produces all combinations of tuples from $R_1$ and $R_2$ that satisfy the join condition.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$
EQUIJOIN	Produces all the combinations of tuples from $R_1$ and $R_2$ that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\langle \text{join condition} \rangle} R_2$ , OR $R_1 \bowtie_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of $R_2$ are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\langle \text{join condition} \rangle} R_2$ , OR $R_1 *_{(\langle \text{join attributes 1} \rangle), (\langle \text{join attributes 2} \rangle)} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in $R_1$ or $R_2$ or both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both $R_1$ and $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in $R_1$ that are not in $R_2$ ; $R_1$ and $R_2$ must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of $R_1$ and $R_2$ and includes as tuples all possible combinations of tuples from $R_1$ and $R_2$ .	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in $R_1$ in combination with every tuple from $R_2(Y)$ , where $Z = X \cup Y$ .	$R_1(Z) \div R_2(Y)$





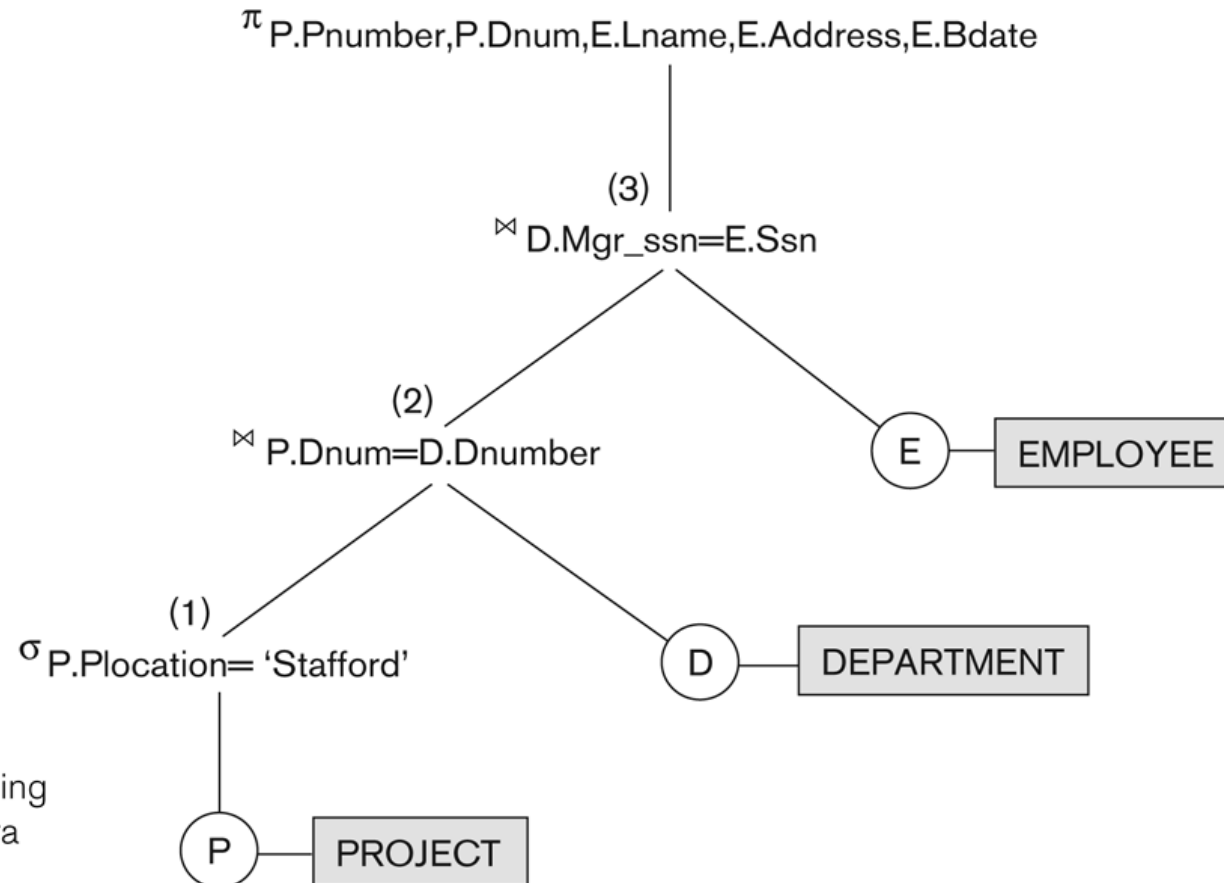
# Query Tree Notation

## ➤ Query Tree

- An internal data structure to represent a query
- Standard technique for estimating the work involved in executing the query, the generation of intermediate results, and the optimization of execution
- Nodes stand for operations like selection, projection, join, renaming, division, ....
- Leaf nodes represent base relations
- A tree gives a good visual feel of the complexity of the query and the operations involved

# Query Tree Notation

What is the relational algebra expression represented with the given query tree?



**Figure 6.9**

Query tree corresponding to the relational algebra expression for Q2.

# Additional Relational Operations

➤ The additional relational operations covered are:

- Aggregate functions
- Grouping functions
- OUTER JOIN operation

# Aggregate Function Operation

- A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
  - Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples.
    - These functions are used in simple statistical queries that summarize information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values.

# Aggregate Function Operation

$\mathcal{F}$  aggregate function (R)

- Use of the Aggregate Functional operation  $\mathcal{F}$  and a corresponding aggregate function:
  - $\mathcal{F}_{\text{MAX Salary}}$  (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{MIN Salary}}$  (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{SUM Salary}}$  (EMPLOYEE) retrieves the sum of the Salary from the EMPLOYEE relation
  - $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$  (EMPLOYEE) computes the count (number) of employees and their average salary
- Note: count just counts the number of rows, without removing duplicates

# Using Grouping with Aggregation

grouping attributes  $\mathcal{F}$  aggregate function (R)

- The previous examples all summarized one or more attributes for a set of tuples
  - Maximum Salary or Count (number of) Ssn
- Another common type of request involves grouping the tuples in a relation by the value of some of their attributes (called *grouping* attributes) and then applying an aggregate function *independently to each group*.
- A variation of aggregate operation  $\mathcal{F}$  allows this:
  - Grouping attributes placed to left of symbol
  - Aggregate functions to right of symbol
  - DNO  $\mathcal{F}$  COUNT SSN, AVERAGE Salary (EMPLOYEE)

# Examples of applying aggregate functions and grouping

**Figure 6.10**

The aggregate function operation.

(a)  $\rho_{R(Dno, No\_of\_employees, Average\_sal)} (Dno \bowtie \text{COUNT Ssn, AVERAGE Salary (EMPLOYEE)})$ .

(b)  $Dno \bowtie \text{COUNT Ssn, AVERAGE Salary (EMPLOYEE)}$ .

(c)  $\text{COUNT Ssn, AVERAGE Salary (EMPLOYEE)}$ .

**R**

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125



# Examples of applying aggregate functions and grouping

## Grouping by Dno

Example

(a)

Fname	Minit	Lname	<u>Ssn</u>	...	Salary	Super_ssn	Dno
John	B	Smith	123456789		30000	333445555	5
Franklin	T	Wong	333445555		40000	888665555	5
Ramesh	K	Narayan	666884444		38000	333445555	5
Joyce	A	English	453453453	...	25000	333445555	5
Alicia	J	Zelaya	999887777		25000	987654321	4
Jennifer	S	Wallace	987654321		43000	888665555	4
Ahmad	V	Jabbar	987987987		25000	987654321	4
James	E	Bong	888665555		55000	NULL	1

Dno	Count (*)	Avg (Salary)
5	4	33250
4	3	31000
1	1	55000

Result of Q24

Grouping EMPLOYEE tuples by the value of Dno



# Additional Relational Operations: OUTER JOIN

## ➤ The OUTER JOIN Operation

- In NATURAL JOIN and EQUIJOIN, tuples without a *matching* (or *related*) tuple are eliminated from the join result
  - Tuples with **null** in the join attributes are also eliminated
  - This amounts to loss of information.
- A set of operations, called OUTER joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.

# Additional Relational Operations: OUTER JOIN

- The left outer join operation keeps every tuple in the first or left relation R in  $R \bowtie L S$ ; if no matching tuple is found in S, then the attributes of S in the join result are filled or “padded” with null values.
- A similar operation, right outer join, keeps every tuple in the second or right relation S in the result of  $R \bowtie R S$ .
- A third operation, full outer join, denoted by  $\bowtie$  keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

# Additional Relational Operations: OUTER JOIN

Example

## RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

**Figure 6.12**

The result of a  
LEFT OUTER JOIN  
operation.

Retrieve the names of the  
employees with the  
department name they  
manage

# Examples of Queries in Relational Algebra

- **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

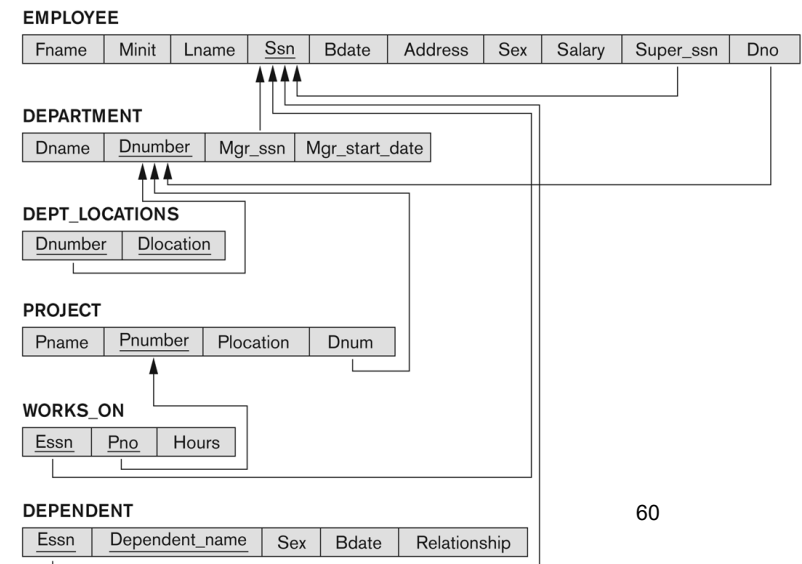
$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{DNAME}='Research'}(\text{DEPARTMENT})$

$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{DNUMBER}=\text{DNOEMPLOYEE}} \text{EMPLOYEE})$

$\text{RESULT} \leftarrow \pi_{\text{FNAME}, \text{LNAME}, \text{ADDRESS}}(\text{RESEARCH\_EMPS})$

**Figure 5.7**

Referential integrity constraints displayed on the COMPANY relational database schema.



# Examples of Queries in Relational Algebra

➤ **Q6: Retrieve the names of employees who have no dependents.**

$$\text{ALL\_EMPS} \leftarrow \pi_{\text{SSN}}(\text{EMPLOYEE})$$

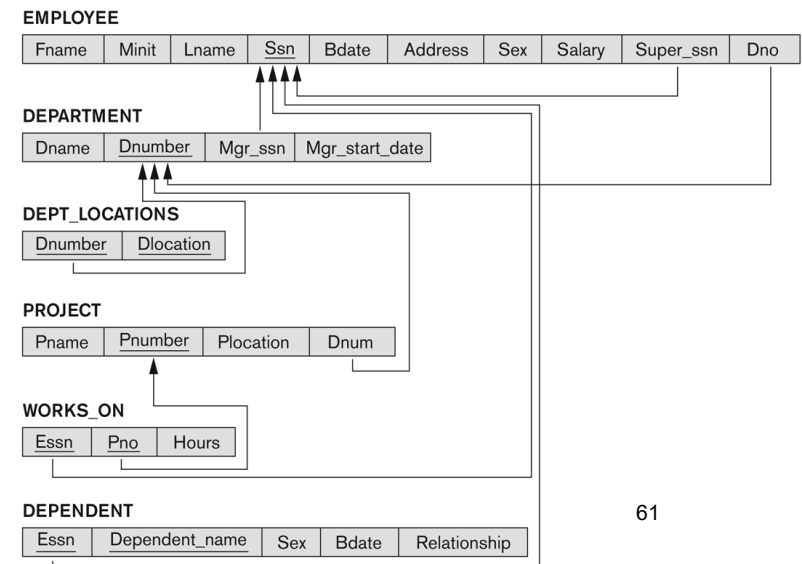
$$\text{EMPS\_WITH\_DEPS}(\text{SSN}) \leftarrow \pi_{\text{ESSN}}(\text{DEPENDENT})$$

$$\text{EMPS\_WITHOUT\_DEPS} \leftarrow (\text{ALL\_EMPS} - \text{EMPS\_WITH\_DEPS})$$

$$\text{RESULT} \leftarrow \pi_{\text{LNAME, FNAME}}(\text{EMPS\_WITHOUT\_DEPS} * \text{EMPLOYEE})$$

**Figure 5.7**

Referential integrity constraints displayed on the COMPANY relational database schema.



# Examples of Queries in Relational Algebra

➤ As a single expression, previous queries become:

➤ **Q1: Retrieve the name and address of all employees who work for the 'Research' department.**

$\pi_{\text{Fname, Lname, Address}} (\sigma_{\text{Dname} = \text{'Research'}} (\text{DEPARTMENT} \bowtie_{\text{Dnumber} = \text{Dno}} (\text{EMPLOYEE})))$

➤ **Q6: Retrieve the names of employees who have no dependents.**

$\pi_{\text{Lname, Fname}} ((\pi_{\text{Ssn}} (\text{EMPLOYEE}) - \rho_{\text{Ssn}} (\pi_{\text{Essn}} (\text{DEPENDENT}))) * \text{EMPLOYEE})$

The choice of the solving approach is up to you.

The most important thing is to know how you can transform one approach to the other!

# Relational Calculus

- A **relational calculus** expression creates a new relation, which is specified in terms of variables that range over rows of the stored database relations (in **tuple calculus**) or over columns of the stored relations (in **domain calculus**).
- In a calculus expression, there is *no order of operations* to specify how to retrieve the query result—a calculus expression specifies only what information the result should contain.
  - This is the main distinguishing feature between relational algebra and relational calculus.

# Relational Calculus

- Relational calculus is considered to be a **nonprocedural** or **declarative** language.
- This differs from relational algebra, where we must write a *sequence of operations* to specify a retrieval request; hence relational algebra can be considered as a **procedural** way of stating a query.



# Tuple Relational Calculus

- The tuple relational calculus is based on specifying a number of tuple variables.
- Each tuple variable usually ranges over a particular database relation, meaning that the variable may take as its value any individual tuple from that relation.
- A simple tuple relational calculus query is of the form

$$\{t \mid \text{COND}(t)\}$$

- where  $t$  is a tuple variable and  $\text{COND}(t)$  is a conditional expression involving  $t$ .
- The result of such a query is the set of all tuples  $t$  that satisfy  $\text{COND}(t)$ .

# Tuple Relational Calculus

- Example: To find the first and last names of all employees whose salary is above \$50,000, we can write the following tuple calculus expression:

**$\{t.FNAME, t.LNAME \mid EMPLOYEE(t) \text{ AND } t.SALARY > 50000\}$**

- The condition  $EMPLOYEE(t)$  specifies that the **range relation** of tuple variable  $t$  is  $EMPLOYEE$ .
- The first and last name (PROJECTION  $\pi_{FNAME, LNAME}$ ) of each  $EMPLOYEE$  tuple  $t$  that satisfies the condition  $t.SALARY > 50000$  (SELECTION  $\sigma_{SALARY > 50000}$ ) will be retrieved.

# The Existential and Universal Quantifiers

- Two special symbols called quantifiers can appear in formulas; these are the universal quantifier ( $\forall$ ) and the existential quantifier ( $\exists$ ).
- Informally, a tuple variable  $t$  is bound if it is quantified, meaning that it appears in an  $(\forall t)$  or  $(\exists t)$  clause; otherwise, it is free.
- If  $F$  is a formula, then so are  $(\exists t)(F)$  and  $(\forall t)(F)$ , where  $t$  is a tuple variable.
  - The formula  $(\exists t)(F)$  is true if the formula  $F$  evaluates to true for some (at least one) tuple assigned to free occurrences of  $t$  in  $F$ ; otherwise  $(\exists t)(F)$  is false.
  - The formula  $(\forall t)(F)$  is true if the formula  $F$  evaluates to true for every tuple (in the universe) assigned to free occurrences of  $t$  in  $F$ ; otherwise  $(\forall t)(F)$  is false.

# The Existential and Universal Quantifiers

- $\forall$  is called the universal or “for all” quantifier because every tuple in “the universe of” tuples must make  $F$  true to make the quantified formula true.
- $\exists$  is called the existential or “there exists” quantifier because any tuple that exists in “the universe of” tuples may make  $F$  true to make the quantified formula true.

# The Existential and Universal Quantifiers

- Retrieve the name and address of all employees who work for the 'Research' department. The query can be expressed as :

$\{t.FNAME, t.LNAME, t.ADDRESS \mid EMPLOYEE(t) \text{ and } (\exists d) (DEPARTMENT(d) \text{ and } d.DNAME='Research' \text{ and } d.DNUMBER=t.DNO) \}$

- The only *free tuple variables* in a relational calculus expression should be those that appear to the left of the bar (  $\mid$  ).
  - In the above query,  $t$  is the only free variable; it is then *bound successively* to each tuple.
- If a tuple *satisfies the conditions* specified in the query, the attributes FNAME, LNAME, and ADDRESS are retrieved for each such tuple.
  - The conditions  $EMPLOYEE(t)$  and  $DEPARTMENT(d)$  specify the range relations for  $t$  and  $d$ .
  - The condition  $d.DNAME = 'Research'$  is a selection condition and corresponds to a SELECT operation in the relational algebra, whereas the condition  $d.DNUMBER = t.DNO$  is a JOIN condition.

# The Existential and Universal Quantifiers

- Find the names of employees who work on *all* the projects controlled by department number 5. The query can be:

**{e.LNAME, e.FNAME | EMPLOYEE(e) and ( (  $\forall x$  )(not(PROJECT(x)) or not(x.DNUM=5)  
OR (  $\exists w$  )(WORKS\_ON(w) and w.ESSN=e.SSN and x.PNUMBER=w.PNO))))}**

- Exclude from the universal quantification all tuples that we are not interested in by making the condition true *for all such tuples*.
  - The first tuples to exclude (by making them evaluate automatically to true) are those that are not in the relation R of interest.
- In the query above, using the expression **not(PROJECT(x))** inside the universally quantified formula evaluates to true all tuples x that are not in the PROJECT relation.
  - Then we exclude the tuples we are not interested in from R itself. The expression not(x.DNUM=5) evaluates to true all tuples x that are in the project relation but are not controlled by department 5.
- Finally, we specify a condition that must hold on all the remaining tuples in R.

**(  $\exists w$  )(WORKS\_ON(w) and w.ESSN=e.SSN and x.PNUMBER=w.PNO)**

# Domain Relational Calculus

- Another variation of relational calculus called the domain relational calculus, or simply, domain calculus is equivalent to tuple calculus and to relational algebra.
- Domain calculus differs from tuple calculus in the type of variables used in formulas:
  - Rather than having variables range over tuples, the variables range over single values from domains of attributes.
- To form a relation of degree  $n$  for a query result, we must have  $n$  of these domain variables— one for each attribute.

# Domain Relational Calculus

➤ An expression of the domain calculus is of the form

$$\{ x_1, x_2, \dots, x_n \mid \text{COND}(x_1, x_2, \dots, x_n, x_{n+1}, x_{n+2}, \dots, x_{n+m}) \}$$

- where  $x_1, x_2, \dots, x_n, x_{n+1}, x_{n+2}, \dots, x_{n+m}$  are domain variables that range over domains (of attributes)
- and COND is a condition or formula of the domain relational calculus



# Domain Relational Calculus

Retrieve the birthdate and address of the employee whose name is 'John B. Smith'.

$$\{uv \mid (\exists q) (\exists r) (\exists s) (\exists t) (\exists w) (\exists x) (\exists y) (\exists z) (\text{EMPLOYEE}(qrstuvwxyz) \text{ and } q='John' \text{ and } r='B' \text{ and } s='Smith')\}$$

- **Abbreviated notation**  $\text{EMPLOYEE}(qrstuvwxyz)$  uses the variables without the separating commas:  $\text{EMPLOYEE}(q,r,s,t,u,v,w,x,y,z)$
- Ten variables for the employee relation are needed, one to range over the domain of each attribute in order.
  - Of the ten variables  $q, r, s, \dots, z$ , only  $u$  and  $v$  are free.
- Specify the *requested attributes*, BDATE and ADDRESS, by the free domain variables  $u$  for BDATE and  $v$  for ADDRESS.
- Specify the condition for selecting a tuple following the bar ( | )
  - namely, that the sequence of values assigned to the variables  $qrstuvwxyz$  be a tuple of the employee relation and that the values for  $q$  (FNAME),  $r$  (MINIT), and  $s$  (LNAME) be 'John', 'B', and 'Smith', respectively.

# Summary

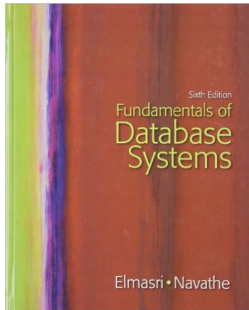
## ➤ Relational Algebra

- Unary Relational Operations
- Relational Algebra Operations From Set Theory
- Binary Relational Operations
- Additional Relational Operations

## ➤ Relational Calculus

- Tuple Relational Calculus
- Domain Relational Calculus

# Bibliography



➤ **Chapter 6**



➤ **Chapter 2**

➤ **Chapter 5**