# Chapter 6 Trajectory Following Homework

## 1、MPC Controller Design

### 1.1 System Model

Due to low vehicle speed, we ultilze the kinematic model as the system model:

$$\begin{cases} \dot{p}_x = v\cos(\phi) \\ \dot{p}_y = v\sin(\phi) \\ \dot{\phi} = \frac{v}{L}\tan(\delta) \\ \dot{v} = a \end{cases}$$

State: $X = [x\ y\ \phi\ v]^T$, Input: $U = [a\ \delta]$

- $Linearization$

  For improving the solving efficiency, we linearize the nonlinear system model. There are two different method.

  - Linearization along the state trajectory:

    We can linearize the system model based on the result of last run.

  - **Linearization along the reference trajectory**

    we ca linearize the system mode based on the reference state.

Define the linearization point as follows:

$\bar{X} = [\bar{p_x}\ \bar{p_y}\ p\bar{h}i\ \bar{v}]^T$ , $\bar{U} = [\bar{a}\ \bar{\delta}]^T$

The linear model:

$$\begin{bmatrix} \dot{p_x} \\ \dot{p_y} \\ \dot{\phi} \\ \dot{v} \end{bmatrix} = \begin{pmatrix} 0 & 0 & -\bar{v}\sin\bar{\phi} & \cos\bar{\phi} \\ 0 & 0 & \bar{v}\cos\bar{\phi} & \sin\bar{\phi} \\ 0 & 0 & 0 & \frac{\tan\bar{\delta}}{L} \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} p_x \\ p_y \\ v \\ \phi \end{bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{\bar{v}}{L}\frac{1}{\cos^2\bar{\delta}} \\ 1 & 0 \end{pmatrix} \begin{bmatrix} a \\ \delta \end{bmatrix} + \begin{pmatrix} \bar{v}\sin\bar{\phi} \\ -\bar{v}\bar{\phi}\cos\bar{\phi} \\ -\bar{v}\frac{\bar{\delta}}{L\cos^2\bar{\delta}} \\ 0 \end{pmatrix}$$

- Discretizaton

  For computer calculation, we discretize the continuous system with forward Euler difference:

$$\begin{aligned} A_{\text{discrete}} &= I + \Delta T \times A_{\text{linearized}} \\ B_{\text{discrete}} &= \Delta T \times B_{\text{linearized}} \\ g_{\text{discrete}} &= \Delta T \times g_{\text{linearized}} \end{aligned}$$

### 1.2 Define Cost Function

The objective is to follow the trajectory accuracy, the cost function can define as follow:

$$
\begin{aligned}
& \min \sum_{k=1}^{N} (x_k - \bar{x}_k)^T Q (x_k - \bar{x}_k) + u_{k-1}^T R u_{k-1} \\
& \text{s.t. } x_0 = x, \\
& x_{k+1} = f(x_k, u_k), \quad k = 0, \ldots, N-1 \\
& \underline{x} \le x_k \le \bar{x}, \quad k = 1, \ldots, N \\
& \underline{u} \le u_k \le \bar{u}, \quad k = 0, \ldots, N-1
\end{aligned}
$$

## 1.3 Prediction

$$\eta = AAx_0 + BBU + G$$

$\eta = [x_1 \; x_2 \ldots x_N]^T, U = [u_0 \; u_1 \; u_2 \ldots u_{N-1}]^T$

$$AA = \begin{bmatrix} A_0 \\ A_1 A_0 \\ \vdots \\ \prod_{k=0}^{N-1} A_k \end{bmatrix}$$

$$BB = \begin{bmatrix} B_0 & 0 & \ldots & 0 & 0 \\ A_1 B_0 & B_1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \prod_{k=1}^{N-1} A_k B_0 & \prod_{k=2}^{N-1} A_k B_1 & \ldots & A_{N-1} B_{N-2} & B_{N-1} \end{bmatrix}$$

$$G = \begin{bmatrix} g_0 \\ A_1 g_0 + g_1 \\ \vdots \\ \sum_{n=0}^{N-2} \left( \prod_{k=n+1}^{N-1} A_k \right) g_n + g_{N-1} \end{bmatrix}$$

Thus we can reomove the equility constrain of the optimize problem.

## 1.4 OSQP Solver

we adopt the QP solver OSQP to solve the QP problem.

OSQP solves convex quadratic programs (QPs) of the form

$$
\begin{array}{ll}
\text{minimize} & \frac{1}{2} x^T P x + q^T x \\
\text{subject to} & l \le Ax \le u
\end{array}
$$

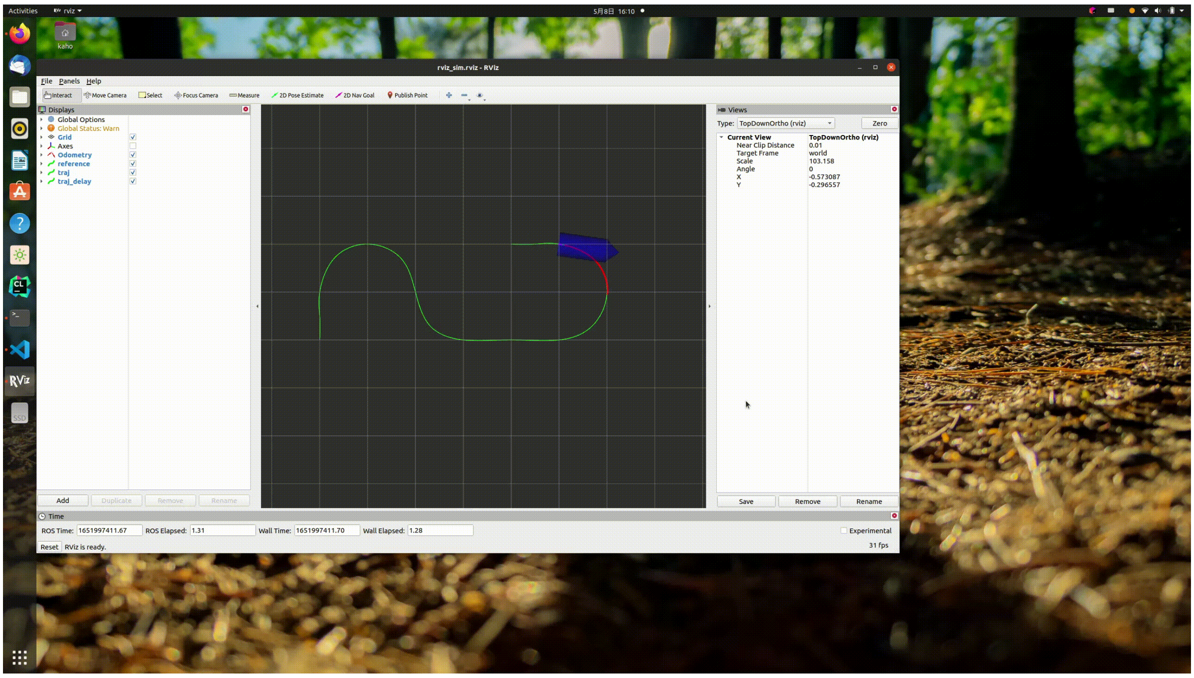
where $x \in \mathbf{R}^n$ is the optimization variable. The objective function is defined by a positive semidefinite matrix $P \in \mathbf{S}_+^n$ and vector $q \in \mathbf{R}^n$. The linear constraints are defined by matrix $A \in \mathbf{R}^{m \times n}$ and vectors $l$ and $u$ so that $l_i \in \mathbf{R} \cup \{-\infty\}$ and $u_i \in \mathbf{R} \cup \{+\infty\}$ for all $i \in \{1, \ldots, m\}$.

Subtitutiing the predciton model into the cost function, we have

$$U^T (BB^T QBB) U + 2(x_0 AA^T QBB + GG^T QBB - q_x QBB) U$$

$q_x = [\bar{x}_1, \; \ldots, \bar{x}_N]$

# 2、Result



Activities
rviz
rviz_sim.rviz - RViz
File Panels Help
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Displays
Global Options
Global Status: Warn
Grid
Axes
Odometry
reference
traj
traj_delay
Add
Duplicate
Remove
Rename
Views
Type: TopDownOrtho (rviz)
Zero
Current View
TopDownOrtho (rviz)
Near Clip Distance
0.01
Target Frame
world
Scale
103.158
Angle
0
X
-0.573087
Y
-0.296557
Save
Remove
Rename
Time
ROS Time: 1651997411.67
ROS Elapsed: 1.31
Wall Time: 1651997411.70
Wall Elapsed: 1.28
Experimental
Reset
RViz is ready.
31 fps