Who cares about Inflation?

Endogenous Expectation Formation of Heterogeneous Households

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Abstract.

This paper studies the effect of wealth levels on households' inflation expectations. Using data from the DNB Household Survey, we show absolute forecast errors as well as the dispersion of expectations across households to be decreasing in assets and debt. These patterns can be rationalized in a consumption-savings model with endogenous expectation formation, where households can exert effort to reduce uncertainty about future price changes. The implied consumption response to news about inflation is hump shaped in wealth: Wealthier households pay closer attention and update their expectations more in response to a signal received, but change their consumption less after any given update in expectations due to the income effect of future inflation. In a quantitative exercise, we show this mechanism to reduce the on-impact aggregate consumption response to forward guidance policies up to 55% compared to an attentive counterfactual.

Keywords: Household Heterogeneity, Expectation Formation, Inflation Expectations.

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1 Introduction

What effect do wealth levels have on how households form their inflation expectations? Answering this question, the present paper studies heterogeneity in households' forecast errors. Surveys on household-level expectations exhibit substantial dispersion in inflation forecasts, questioning the assumption of uniform and fully rational expectations. US data exhibit an average interquartile range of 4 percentage points in one-year-ahead inflation expectations. The observed dispersion suggests that a large fraction of households makes significant errors when forecasting future prices. We report these errors as well as the dispersion in expectations to be decreasing in absolute wealth and argue that these patterns can be explained by the influence of wealth on the incentives to form precise expectations.

It is well understood that expected inflation is important for the consumption-savings choice of forward looking agents due to its impact on expected real interest rates. Less studied so far is the reverse effect of asset levels on households' expectation formation. As to other aggregate risks, heterogeneous households are not exposed to inflation uniformly. Their wealth is an important determinant of how much households are affected by inflation: Through its influence on real interest rates, inflation is only relevant to economic agents engaging in intertemporal optimization, deciding on how much to spend, save, or borrow between periods.[1] Agents who consume their entire income in every period (living hand-to-mouth) are not exposed to real interest rates and are therefore not exposed to the effect of inflation on these. If expectation formation is in any way costly, we can hence not assume heterogeneous households to be equally willing to cover these cost. To fully understand the formation of households' inflation expectations, it therefore has to be studied jointly with their wealth accumulation.

Furthermore, wealth also influences a household's response to changes in expectations. Any given change in (expected) inflation rates will have different consequences for

[1] In this paper, the focus lies on inflation as a risk only to the real interest rate that affects all saving and borrowing uniformly. Wages are real and all households face the same effective inflation rate. This assumption is discussed in later sections.

households, depending on their wealth level. While an increase in expected inflation is good news for debtors since it reduces the real value of their future repayments, it is bad news for savers who would be the recipients of those payments. This heterogeneous income effect impacts households' consumption response to changes in their inflation expectations.

Our paper formalizes the intuitive arguments above in a theoretical framework, disciplining it with empirical observations on expectation errors along the wealth distribution. While a developing literature studies the interaction of household heterogeneity and aggregate risk, it is still widely assumed that households' expectations about aggregate variables, such as inflation, are homogeneous.[2] Our modelling approach is novel in considering the joint formation of inflation expectations and savings decisions, incorporating interactions in both directions.

We begin by developing a model of households' inflation expectations in Section 2, allowing for endogenous expectation formation. Households are assumed to understand the underlying inflation process and to be uncertain only about future inflation rates. To reduce this uncertainty, they can exert effort. Analytical results show that this model of expectation formation implies the standard deviation of forecast errors as well as the mean absolute error across a group of otherwise identical households to be decreasing in the common effort they exert. The results allow to interpret differences in these cross-sectional statistics at different points of the wealth distribution as differences in exerted effort.

Section 3 provides empirical observations on the joint distribution of inflation expectations and wealth, making use of the Dutch National Bank's Household Survey. We find the standard deviation of forecast errors and the mean absolute error across households to be decreasing in absolute wealth. Both richer as well as indebted households have more precise and less disperse expectations compared to those around zero net wealth. The findings are robust to the inclusion of age, education, and fundamental

[2]See e.g. Krusell and Smith (1998), Krueger et al. (2016) and Ahn et al. (2018).

disagreement about the long-run mean of inflation as additional controls. They can be replicated using data from the Michigan Survey of Consumers.

Section 4 integrates our model of expectation formation into an infinite horizon consumption-savings problem. The setup allows us to study the interaction between expectation formation and wealth accumulation, generating endogenously a joint distribution of wealth and expectations. The calibrated model matches the empirically observed pattern of expectations along the wealth distribution. It allows us to back out the consumption response to a signal about future inflation rates – the marginal propensity to consume on signal (MPCS) – from households' optimal policy functions, which depends on two factors: How much a household updates his expectation in response to a signal received, and how he reacts to any given change in his expectations.

Households' consumption response to any given change in expectations is decreasing in wealth due to an expected income effect. In contrast, as richer households' are endogenously paying closer attention to inflation rates, they updated their expectations more in response to any signal received. Combined, these two forces yield a hump shaped pattern for MPCS along the wealth distribution. To highlight the importance of our findings at the aggregate level, we conduct a forward guidance exercise within our framework. We show that under endogenous expectation formation, forward guidance can miss out on up to 55% of the effect it could have if all households would chose to be as informed as the most attentive. This result is driven by low wealth households, who are potentially most responsive to a change in their inflation expectations, but fail to update their expectations in response to the policy as they do not pay close attention to news about future price changes.

1.1 Related Literature

An evolving literature discusses models for the formation of economic agents' inflation expectations. Important work including Coibion and Gorodnichenko (2012), Patton and Timmermann (2010), or Mankiw et al. (2003) has made progress along these lines but has studied expectation formation in isolation and abstracted from any influence of wealth on expectations. The same holds true for most empirical work on the drivers behind households' inflation expectations, which has focussed e.g. on personal experiences (Malmendier and Nagel, 2015), perceived past inflation (Axelrod et al., 2018), or cognitive ability (D'Acunto et al., 2019), but not considered wealth levels. As an exception, Ben-David et al. (2018) study the relation between uncertainty about macroeconomic variables such as inflation or house prices and socio-economic status of households. Their data does not include households' wealth but they find uncertainty about macroeconomic variables to decrease in income and employment.[3] Another strand of the empirical literature considers the impact of expectations on wealth levels. Among others, Crump et al. (2015), Dräger and Nghiem (2018), or Vellekoop and Wiederholt (2019) evaluate the consistency of households' choices with their expectations. We extend their work by including the reverse impact of households' choices on their expectation formation.

Recent theoretical work is beginning to study the implications of information frictions in economies with heterogeneous agents. Both Carroll et al. (2020) and Auclert et al. (2020) show that incorporating such frictions can reconcile high individual MPCs with smoothness of aggregate consumption dynamics. While in their economies expectation formation is once again exogenous, Broer et al. (2018) discuss the endogenous choices of heterogeneous households to use precise laws of motion for aggregate capital in a Krusell and Smith (1998) economy. They find substantial heterogeneity in the utility loss from not using full information. In their model, choices are based on simulated lifetime utilities and forecasting capital impacts forecasts about both returns and wages, accounting for the difference to our findings. Closely related to our approach, Yin (2018) discusses the savings problem of single agents in a rational inattention framework and finds the optimal amount of attention to be decreasing in the initial endowment of agents. In contrast to our model, here capital income is the only form of income. To the best of our knowledge, we are the first to discipline a heterogeneous agent model with data on households' inflation expectations.

[3]Unrelated to inflation but in line with our theory, Fuster et al. (2018) find more exposed participants to be willing to pay a higher cost for information about future house prices in an experiment.

2 Modelling Endogenous Inflation Expectations

Our model of households inflation expectations allows for endogenous expectation formation. To maintain computational tractability, households are assumed to understand the underlying process and perfectly observe current inflation but to be uncertain about the shock component to future inflation rates. This uncertainty can be reduced endogenously by households exerting costly effort. The setup yields heterogeneous effort choices if the gains of reducing uncertainty about future inflation rates are distributed unevenly across households. In this section, focus lies on how effort transmits into individual expectations and cross-sectional moments of expectation errors. Section 4 analyses households' effort choice.

Assume inflation follows a first-order autoregressive process

$$\pi_{t+1} = (1-\rho)\mu + \rho\pi_t + e_{t+1} \qquad e_{t+1} \sim \mathcal{N}(0, \sigma_e^2). \tag{1}$$

π_t is inflation in period t, μ is the long run mean of inflation and ρ its persistence across periods. e_t is a shock to inflation, which is i.i.d across time.

Households know that inflation follows (1) and agree about (the true) μ, ρ and σ_e^2. In contrast to most of the literature on expectation formation, [4] households perfectly observe current and all past inflation rates. In period t, π_τ is known for all $\tau \leq t$. This assumption keeps the state space of the household problem small. We believe this is justified, given that information about current and past inflation rates is easily accessible online.[5] Furthermore, when embedding the expectation formation process in a consumption-savings model, it will be important for households to know current prices in order to pin down their budget set in real terms. Therefore, households are assumed to be uncertain only about future inflation rates.

[4] See e.g. Vellekoop and Wiederholt (2019).

[5] One can reinterpret our assumption as the first marginal bit of effort providing full information about present and past inflation rates. With the assumptions imposed below on the cost of effort, the first marginal bit of information is costless and hence always obtained.

Households form expectations with respect to the shock to future inflation, e_{t+1}. In period t, household i can exert some effort n_t^i to influence the noise in a signal $\hat{e}_{t+1}^i$ he receives about next period's shock. The signal he receives follows

$$\hat{e}_{t+1}^i = e_{t+1} + s_{t+1}^i \qquad s_{t+1}^i \sim \mathcal{N}(0, \sigma_s^2(n_t^i)), \tag{2}$$

where the noise component s_{t+1}^i can be influenced by households' effort choice. We assume its standard deviation to be a decreasing but convex function of effort ($\sigma_s'(n) < 0,\ \sigma_s''(n) > 0$) and s_{t+1}^i to be pure noise, i.e.

$$e_t \perp\!\!\!\perp s_t^i \ \forall i, t \qquad s_t^i \perp\!\!\!\perp s_t^j \ \forall i, j, t \qquad s_t^i \perp\!\!\!\perp s_{t+s}^i \ \forall i, t, s. \tag{3}$$

Households have identical priors about the shock given by $e_{t+1} \sim \mathcal{N}(0, \sigma_e^2)$, i.e. corresponding to the true unconditional distribution.[6] Based on the signal received, the household updates his prior belief according to Bayes Rule. Let $\omega_{t+1}^i(n_t^i) = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_s^2(n_t^i)}$ be the weight he attaches to the signal, yielding his posterior belief about the shock as

$$e_{t+1}|_{\hat{e}_{t+1}^i, n_t^i} \sim \mathcal{N}\left(\omega_{t+1}^i(n_t^i)\hat{e}_{t+1}^i, \omega_{t+1}^i(n_t^i)\sigma_s^2(n_t^i)\right). \tag{4}$$

Household i's expected value for inflation is determined by his expectation about the future shock and is given as

$$\mathbb{E}_t[\pi_{t+1}|\hat{e}_{t+1}^i, n_t^i] = (1-\rho)\mu + \rho\pi_t + \omega_{t+1}^i(n_t^i)\hat{e}_{t+1}^i \tag{5}$$

implying an ex-post forecast error of

$$err_{t+1}^i = \mathbb{E}_t[\pi_{t+1}|\hat{e}_{t+1}^i, n_t^i] - \pi_{t+1} = \omega_{t+1}^i(n_t^i)s_{t+1}^i - (1 - \omega_{t+1}^i(n_t^i))e_{t+1}. \tag{6}$$

The first term in the error captures households' over-reaction to noise while the second term yields under-reaction to news contained in the signal, as is standard in models with Bayesian updating.

The subjective uncertainty a household attaches to his inflation forecast is driven solely by his uncertainty about the future shock. We define it as the standard deviation

[6] Heterogeneity in prior variance, especially if correlated with households wealth, would complicate the analysis. Assuming a common (unbiased) prior about the mean of the shock is without loss of generality. Relaxing this assumption would yield similar results as introducing heterogeneous beliefs about μ, see appendix A.1.

of the posterior distribution

$$SU^i_{t+1} = \sqrt{\omega^i_{t+1}(n^i_t)\sigma^2_s(n^i_t)} = \sqrt{\frac{\sigma^2_e\sigma^2_s(n^i_t)}{\sigma^2_e + \sigma^2_s(n^i_t)}}. \tag{7}$$

Under the assumptions that $\sigma'_s(n) < 0,\ \sigma''_s(n) > 0$, one can show that the subjective uncertainty a household faces is a decreasing and convex function of his effort.

We can also derive theoretical moments for a group g of households with equal choices for $n^i_t = \bar{n}^g_t$. An equal choice for n implies identical weights $\omega^i_{t+1}(n^i_t) = \omega^g_{t+1}(\bar{n}^g_t)$. The variance of their forecasts errors, across households and time, will be given as

$$\text{Var}^g(err^i_{t+1}) = (\omega^g_{t+1}(\bar{n}^g_t))^2\sigma^2_s(\bar{n}^g_t) + (1-\omega^g_{t+1}(\bar{n}^g_t))^2\sigma^2_e = \frac{\sigma^2_e\sigma^2_s(\bar{n}^g_t)}{\sigma^2_e + \sigma^2_s(\bar{n}^g_t)} = \overline{SU}^{g2}_{t+1}, \tag{8}$$

where we use the assumption that noise is uncorrelated across households. Hence the within group standard deviation of forecast errors across households exerting effort $\bar{n}^g_t$ can be interpreted as the subjective uncertainty about future inflation of households' in that group, $\overline{SU}^g_{t+1}$. With our model, disagreement among households becomes a measure of the subjective uncertainty faced by individual households.

As a measure of forecast precision, we can derive the mean absolute error of households by using the fact that s and e are normally distributed and uncorrelated with each other and over time. The theoretical distribution for the expectation error among a group of households is then normal with mean zero and variance given in (8). This means, by the properties of folded normal distributions, that the average absolute error is given as

$$\mathbb{E}^g[|err^i|] = \sqrt{\text{Var}^g(err^i_{t+1})\frac{2}{\pi}} = \overline{SU}^g_{t+1}\sqrt{\frac{2}{\pi}}. \tag{9}$$

Therefore, our model predicts a strong co-movement of the standard deviation and mean absolute error for a group of households exerting identical effort, driven by the uncertainty they face with respect to future inflation. This co-movement across separate groups of households can therefore be seen as a disciplining feature of our model when comparing it against the data.

To conclude, the proposed model of expectation formation has clear empirical predictions. (7) suggests that households who chose to exert less effort to form precise inflation expectations face higher subjective uncertainty. (8) predicts that higher uncertainty should translate into larger disagreement among households assumed to exert similar effort, as measure by the standard deviation of errors. (8) jointly with (9) implies that the mean absolute error and the standard deviation taken across such groups of households and time should co-move. If effort is costly, we should therefore expect the group of households with the largest potential gains from precise expectations to exhibit the lowest standard deviation of errors and mean absolute errors among them.

3 Expectations Along the Wealth Distribution

Having specified a model of expectation formation, we now turn to some empirical observations on households' inflation expectations along the wealth distribution. We begin by outlining the data used and methodology applied before presenting our baseline results and some additional robustness tests.

3.1 Data and Methodology

To gain insight into the joint distribution of inflation expectations and wealth we use data from the Dutch National Bank's Household Survey (DHS). This dataset is unique in providing comprehensive data on both households' wealth and their inflation expectations. We combine observations from the survey waves 2010-2018. The choice of period reflects changes made to the questionnaire on inflation expectations in the 2008 wave and excludes the financial crisis episode. We use data at the individual level, as presented in the DHS, but restrict our sample to household heads to avoid within household correlations and take heads' answers to be representative for their household.

In the survey, households are simultaneously asked about their current wealth and their expectations of next year's inflation. We interpret their answers as beginning of period t wealth and expectations about inflation between t and $t+1$. As will become clear from our model in later sections, period t wealth is the relevant measure influencing the formation of expectations between t and $t+1$.[7]

[7] Even if we could observe households' savings decisions between t and $t+1$, this would be endogenous to their expectation formation.

Households' net financial wealth is computed as the sum of all forms of wealth less liabilities reported in the DHS, except for houses and related mortgages, business equity and vehicles. Whenever referring to "wealth" in the remainder of this paper, we apply this definition. We focus on financial wealth as we believe it to capture best the resources out of which the household decides to consume or save in response to changes in inflation rates. Using the described wealth measure, we construct decile groups based on households' position in the wealth distribution in the year of observation. We pool observations across waves that are in the same wealth decile for their wave. Table A.1 in the appendix reports summary statistics for decile groups. For the interpretation of our empirical findings it is important to note that the cut-off between the first and second wealth decile group is around zero net financial wealth, i.e. net debtors are concentrated in the first decile group.

Participants in the DHS are asked to report a point forecast for the inflation rate over the following 12 months, choosing from the set of whole numbers between 1 and 10. Ex-post errors are computed by subtracting the realized inflation rate over the next 12 months from this forecast. As the exact month of the observation is unknown (the survey generally takes place between April and October each year), we subtract June-to-June inflation as an approximation to the forecasted rate. As an example, for an observation of the 2016 wave inflation is the change in the Dutch CPI between June 2016 and June 2017.[8]

3.2 Empirical Observations

Our focus is on observations at the wealth decile group level and we assume that households within the same decile group behave similarly in terms of their expectation formation, i.e. form together one group g as described in Section 2. For each wealth decile group, we report the within group standard deviation and the mean absolute error. Figure 1 presents our baseline empirical results. Both the mean absolute error and the within-group standard deviation increase between the first and second decile

[8]None of the results is altered qualitatively if June-to-June inflation is replaced by inflation in the current year (annual inflation in 2016 in our example) or the following year (annual inflation in 2017).

and decline as wealth increases further until reaching a stable level in the upper half of the wealth distribution. At its lowest level, both variables are about 0.6 pp. lower than at their peak in the second decile. Both the initial increase and the subsequent decline are statistically significant at the 95% level.

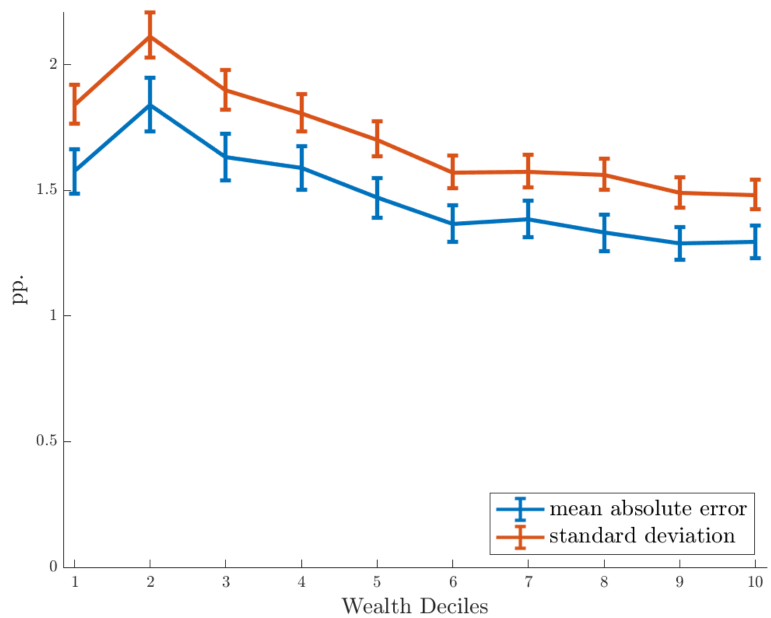


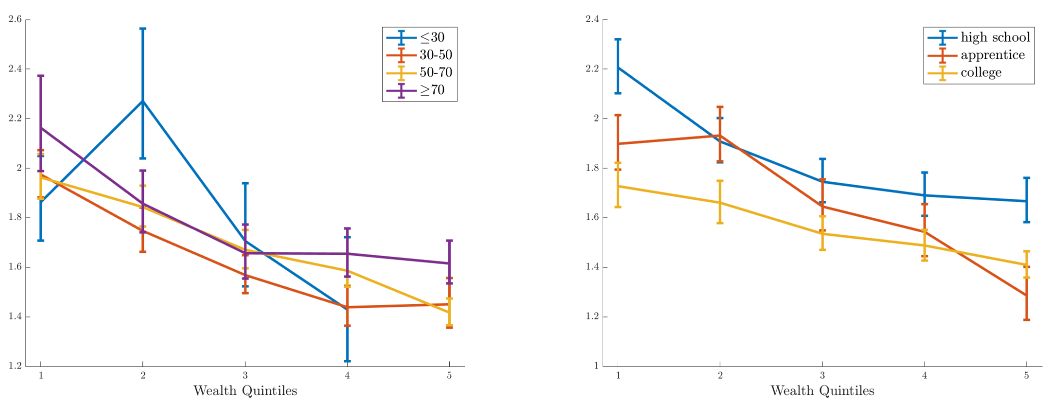
Figure 1: Expectation Errors by Wealth Decile Groups

The figure plots the within-decile group standard deviation of errors and the mean absolute forecast error. Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from DNB Household Survey waves 2010-2018.

The findings highlighted above suggest that wealthier, as well as indebted (first decile) households have more precise expectations compared to households around zero wealth (second decile), measured by their mean absolute error. Furthermore, wealthier and indebted households disagree less about future inflation, as measured by the standard deviation of their forecast errors. Interpreting these findings through our model of expectation formation suggests that richer and indebted households chose to exert more effort in order to form precise inflation expectations. This will yield lower subjective uncertainty about future inflation rates. The lower uncertainty, on average, translates into lower absolute errors and a lower standard deviation of errors. The close co-movement of error standard deviation and mean absolute error is consistent with the proposed model for expectation formation.

The role of age and education

It is well established that other demographic characteristics are highly correlated with positions in the wealth distribution.[9] The two most important for our analysis are age and education. An argument can be made that more experienced (as older) people could be better at forming expectations. Similar argument applies for more educated individuals. As education and age correlate positively with wealth this could be driving the finding in Figure 1. We therefore repeat our analysis controlling for age and education respectively.



(a) by age (b) by education

Figure 2: Standard Deviation of Expectation Errors by Wealth Quintiles – Controls

The figure plots the within-quintile group standard deviation of errors by age (a) and education groups (b). Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from DNB Household Survey waves 2010-2018. Combination of youngest age and highest wealth quintile omitted due to lack of observations.

Testing for robustness towards age and education, we look at the data on quintile group level to allow for a sufficient number of observations within each age/wealth and education/wealth cell. Figure 2 reports the within wealth quintile group standard deviation of errors by age groups and education groups. The general downward trend of disagreement in wealth persists after controlling for either age or education. Age appears

[9]See e.g. Cooper and Zhu (2016).

to have little explanatory power beyond the impact of wealth, providing an argument against experience as a driving force for expectation formation. College education, however, appears to decrease disagreement compared to less educated groups. Similar findings hold for the mean absolute expectation error.[10]

Allowing for fundamental disagreement

In our baseline model we abstract from fundamental disagreement about the underlying model of inflation and its parameters or any other exogenously imposed heterogeneity in beliefs to focus on endogenous expectation formation.[11] Nevertheless, additional sources of disagreement among households can be important to capture moments of the data our baseline model fails to explain, such as e.g. a positive mean error[12] or a positive error covariance. We therefore extend our empirical analysis in order to evaluate the potential impact of other sources of heterogeneity in expectations on our findings.

It can be shown that under fundamental disagreement about the long run mean μ subjective uncertainty is identified as

$$\overline{SU}^g_{t+1} = \sqrt{\mathrm{Var}^g(err^i_{t+1}) - \mathrm{Cov}^g(err^i_{t+1}, err^i_t)}. \tag{10}$$

Intuitively, the covariance of errors is a sufficient statistic to measure heterogeneity in beliefs about the long run mean as we assume noise to be uncorrelated over time. For a detailed derivation of this result refer to Appendix A.1. We apply equation (10) to our data and compute the implied subjective uncertainty of households by subtracting for each wealth decile group the error covariance over time from the within group variance. The result is presented in Figure 3. The implied subjective uncertainty exhibits a similar pattern as our benchmark results. It is slightly increasing between the first and second decile group and broadly decreasing for further increases in wealth. The covariance, which according to the extended model is driven by the dispersion of beliefs about the long-run mean, is equally higher among households with lower wealth and

[10]These results are presented in Figure A.5 in the appendix.

[11]The endogenous formation of beliefs about the underlying model and its parameter values lies beyond the scope of this paper but provides a promising avenue for future research.

[12]See Figure A.4 in the appendix.

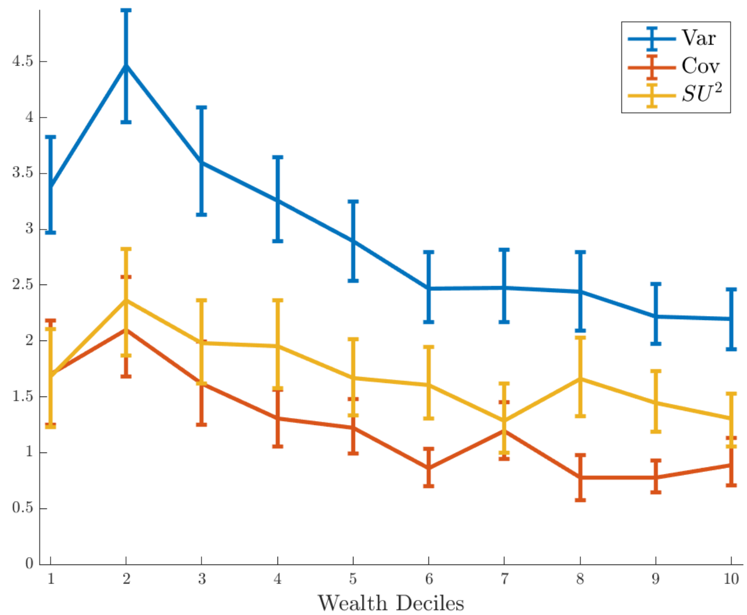


Figure 3: Expectation Error Variance – Decomposition

The figure decomposes the variance of expectation errors across households (Var) by wealth decile groups into error covariance (Cov) and the square of subjective uncertainty (SU^2) as in (20). Data from DNB Household Survey waves 2010-2018. Bootstrapped 95% confidence intervals.

decreasing alongside subjective uncertainty. Both the decreases in subjective uncertainty and covariance between their respective peaks and lowest points are significant at the 95% level . Of the overall drop in the variance of expectation errors across households, about half is attributable to the fall in exogenous disagreement about the long run mean (covariance) and half to a fall in endogenous subjective uncertainty. We take these findings as evidence that existence of the mechanism in our benchmark model is robust to incorporating fundamental disagreement.

Additional data

To test for robustness to data selection, we extend our analysis to include housing and mortgages into our wealth measure and repeat it for data from the Michigan Survey of Consumers (MSC). Including housing and related mortgages into our measure of wealth leaves the results broadly unchanged, as Figure 4 shows. The peak of both mean absolute error and standard deviation of errors remains in the second decile group (again around zero net wealth). Both decline to either side and flatten out over the upper part of the wealth distribution. The overall decline between peak and trough in both variables is of similar magnitude as before. Different from previous results are an additional small decline in the highest decile groups and the starting point of the flat part being moved to the 4th (compared to previously the 6th) decile group when including housing in wealth.

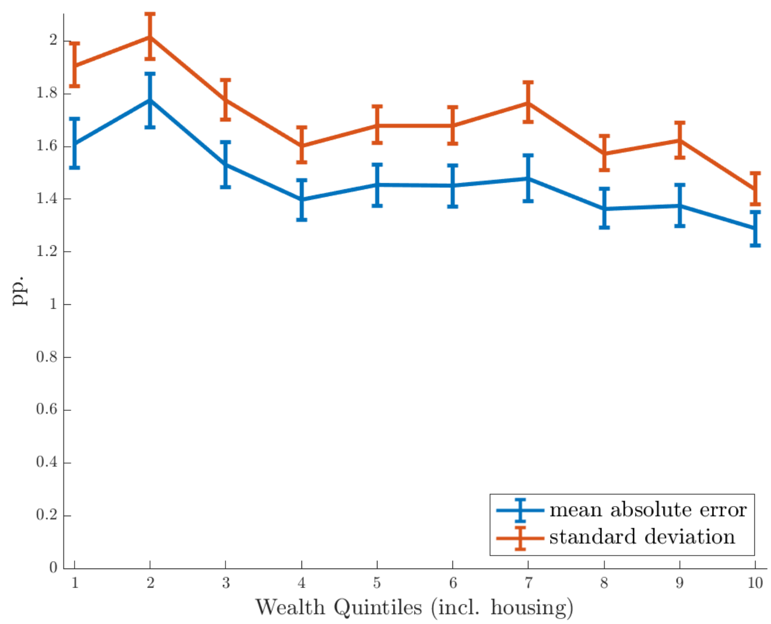


Figure 4: Expectation Errors by Wealth Decile Groups – Housing

The figure plots the within-group standard deviation of errors and mean absolute errors by net financial wealth decile groups, including housing and mortgages in the wealth measure. Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from DNB Household Survey waves 2010-2018.

Compared to the DHS data, the MSC contains substantially less information on household wealth. Therefore, reported stock market investment is used as an approximation of net financial wealth. Detailed findings are provided in Appendix A.2. The patterns reported for the DHS are strongly supported by findings from the Michigan Survey of Consumers.

Portfolio Composition

In line with our modelling choices, we abstract from a detailed empirical analysis of households' portfolio composition.[13] The share of wealth held through checking, savings or deposit accounts, savings certificates and deposit books in the DHS provides empirical justification for this choice. These assets arguably have predetermined interest rates and therefore carry a one-for-one exposure to inflation. In our sample the average share of these assets over total assets is close to 80% and drops to about 50% only for the richest 10% of households. As (adjustable rate) mortgages are excluded from our analysis, the liability side of households balance sheets equally carries a one-for-one exposure. We conclude that, except for the very top, portfolio composition is of negligible importance for changes in exposure to inflation rates along the wealth distribution. A more detailed consideration of households' balance sheets is unlikely to alter findings derived under the assumption that all assets and liabilities carry a one-for-one exposure to inflation rates.

Taking stock

Our empirical findings show that both the standard deviation as well as the mean absolute error across households co-move with households' wealth in a meaningful way, richer and indebted households disagreeing less and having more precise inflation expectations compared to zero net wealth households. This finding is robust to an array of potentially confounding factors, namely covariation with age or education, introducing fundamental disagreement and including housing wealth. It can be replicated with data from the Michigan Survey of Consumers. The empirical findings can be justified based on our model of expectation formation, if richer and more indebted households chose to exert larger effort in order to form more precise expectations. We move on to incorporating expectation formation into a standard consumption-savings problem to show that this result emerges endogenously.

[13] On the role of households balance sheets for their exposure to aggregate variables such as inflation see Doepke and Schneider (2006), Auclert (2019), and Tzamourani (2019).

4 Savings Choice and Endogenous Expectations

To conclude our analysis, the expectation formation presented in Section 2 is incorporated into an infinite-horizon consumption-savings model. The model explicitly considers the effect of wealth on households' expectation formation. The dynamic setting generates a joint distribution of expectations and wealth, allowing for a reciprocal relationship between the two. We proceed by outlining the model and calibrating it to our empirical findings before discussing potential aggregate implications of the influence of wealth on expectation formation.

4.1 Household Problem with Endogenous Expectations

At the beginning of each period, a household knows the assets carried over from the previous period a and learns about his real income y as well as the current inflation rate π, which are both stochastic over time. Together, these variables determine the available resources for consumption and saving. Based on this information, the household decides on his effort n. After deciding on n, he receives a signal about the shock to inflation between the current and the next period and updates his belief about future inflation. He will base his choice over consumption today and savings on the updated belief. Households' income is assumed to follow a Markov process with transition matrix Π_y. We assume income y to be real income.[14] Savings and borrowing are subject to a nominal interest rate. We abstract from interest rate risk and assume the nominal interest between any two periods to be constant at r^n. We do so to discuss the effect of inflation risk in isolation. The qualitative findings presented below rely on this assumption only to the extent that nominal interest rates do not move one-for-one with inflation.[15] A constant nominal interest rate together with real income define inflation effectively as a risk only to the real interest rate.

[14]This choice is motivated by the fact that labour income, the largest component of non-financial income, for the Netherlands over our sample period is to a large extend protected from inflation through collective bargaining agreements. According to OECD data, collective bargaining coverage in the Netherlands was well above 80% for the period under study.

[15]Our assumption is supported by the empirical fact that, as discussed above, most assets and liabilities in our sample have a one-for-one exposure to inflation as their interest rates are pre-determined.

With all other notation as introduced above, households' information choice problem is given as

$$\tilde{V}(a, y, \pi) = \max_{n \in [0, \bar{n}]} \mathop{\mathbb{E}}_{\hat{e}}[V(a, y, \pi, n, \hat{e})|n] \tag{11}$$

where we restrict the choice of effort to be positive and impose an upper limit $\bar{n}$ on how much the households can learn about future inflation to rule out perfect foresight.

The subsequent consumption-savings choice, conditional on chosen effort n and received signal $\hat{e}$, can be described as the solution to

$$\begin{aligned} V(a, y, \pi, n, \hat{\epsilon}) = \max_{c, a'} & \left(c^{1-\gamma} + \beta \left(\mathop{\mathbb{E}}_{\pi', y'} [\tilde{V}(a', y', \pi')^{1-\alpha} | \hat{e}, n, \pi, y] \right)^{\frac{1-\gamma}{1-\alpha}} \right)^{\frac{1}{1-\gamma}} \\ s.t. \quad c + a' = & \frac{1 + r^n}{1 + \pi} a + y - \mathcal{F}(n) \\ a' \geq & \bar{a}, \; c \geq 0 \end{aligned} \tag{12}$$

where the budget constraint is written in real terms, a is today's nominal asset level divided by yesterdays prices and $\bar{a}$ is the borrowing limit.[16] Expectations over π' are based on households' updated belief taking into consideration π, $\hat{e}$ and the previous choice for n. The law of motion of inflation and the expectation formation based on the signal are as presented in Section 2. Preferences of the household are recursive as in Epstein and Zin (1989), allowing for independence of risk aversion and intertemporal substitution.

We model the cost of effort as monetary cost, representing both the opportunity cost of spending time on forming expectations as well as the cost of acquiring information. For the cost of effort and the relationship between effort and noise in the signal, we assume functional forms

$$\sigma_s(n) = \frac{\chi}{1 + n} \qquad \mathcal{F}(n) = (\theta n)^{\phi}. \tag{13}$$

These choices yield convex cost of and convex gains from exerting effort.[17] Note that with these functional forms, χ is the variation in the noise if zero effort is exerted, i.e. the maximum variation possible, and that zero effort implies zero cost.

[16] For details see appendix B.1.

[17] $\sigma_s'(n) < 0, \; \sigma_s''(n) > 0$ and $\mathcal{F}'(n) > 0, \; \mathcal{F}''(n) \geq 0, \;$ iff $\phi \geq 1$.

4.2 Calibration

The calibration of the model aims to replicate the patterns presented in Figure 1. Our calibration strategy is twofold: First, a range of parameters is set exogenously. These include preference parameters $\gamma = 1.5$ and $\alpha = 8$ which we chose in line with previous work.[18] We furthermore assume the cost of information to be quadratic ($\phi = 2$). The inflation process is estimated from Dutch annual inflation rates for the period 1988-2018. This yields a long run mean of about 2% and an annual persistence of about 0.5, similar to the estimates of Vellekoop and Wiederholt (2019). The nominal interest rate r^n is set at 4% for a steady state real rate of 2%. Second, we calibrate β, $\bar{a}$, θ, χ and $\bar{n}$ as well as the process for y jointly for the model to fit the data on households' wealth and their expectation errors along the wealth distribution. Calibration targets include the position of the peak of households' errors in the second decile, the beginning of the flattened part of the standard deviation of errors in the sixth decile, the magnitude of the drop in error standard deviation of 0.57pp[19] as well as the share of wealth held by each quintile of the wealth distribution. All parameters (and their interaction) influence a wide range of model statistics. Nevertheless, β and $\bar{a}$ are particularly important to determine the lower end of the wealth distribution while θ, χ and $\bar{n}$ reproduce the slope and level of errors along the wealth distribution. Bounding n generates a flat standard deviation of errors across high wealth groups. Under our calibration, the maximum possible effort $\bar{n}$ reduces the standard deviation of noise to 0.5pp, half the standard deviation of shocks to the inflation rate. Exerting effort $\bar{n}$ comes at a cost of less than 0.1% of average income, speaking to the fact that little is necessary to deter households from acquiring information about future inflation. As in Castañeda et al. (2003), the process for y is calibrated to generate the distribution of wealth. Similar to their results,

[18]Papers applying Epstein-Zin preferences in a consumption-savings framework with idiosyncratic risk include Cooper and Zhu (2016), Ampudia et al. (2018), Campanale and Sartarelli (2018) and Kaplan and Violante (2014). They agree about the intertemporal elasticity of substitution. We chose the risk aversion from the lower end of the range of their estimates, a conservative choice closer to standard CRRA preferences.

[19]We target the difference between the standard deviation of errors in the second decile group versus the average over deciles 6-10.

one high earnings state with lower persistence is necessary to generate a long right tail of the wealth distribution. Table 1 summarizes our parameter choices.

Table 1: Dynamic Model – Calibration

	Parameter	Value	Target
intertemp. substitution	γ	1.5	Literature
risk aversion	α	8.0	Literature
time preference	β	0.9779	fraction of debtors
borrowing limit	$\bar{a}$	-7.5	total debt
income states	y	[0.45 1 8]	wealth distribution
income transition	Π_y	$\begin{bmatrix} 0.975 & 0.025 & 0 \\ 0.057 & 0.931 & 0.012 \\ 0 & 0.15 & 0.85 \end{bmatrix}$	wealth distribution
nominal interest rate	r^n	0.04	2% SS real rate
persistence inflation	ρ	0.5	Dutch data, 1988-2018
long-run mean inflation	μ	0.02	Dutch data, 1988-2018
std. inflation shocks	σ_e	0.01	Dutch data, 1988-2018
curvature cost of effort	ϕ	2	quadratic cost of effort
scale cost of effort	θ	0.0015	range flat std. errors
maximum std. of noise	χ	0.1	peak std. errors
upper bound on effort	$\bar{n}$	17.5	low std. errors

Table 2 presents the fit of our model with respect to the wealth distribution. The model performs reasonably well along this dimension. It struggles to match the strong concentration of wealth at the top as well as the total amount of debt. We argue, that the failure to match the concentration at the top has negligible relevance for our results regarding expectations, since the model performs much better in matching the total fraction of wealth held by the flat part of the expectation distribution (wealth quintiles 3-5 jointly). As expectation formation is similar within this range, not matching the correct distribution of wealth within the 3rd to 5th quintile will not have consequences for our results regarding expectation formation. The failure to match the total amount of debt stems from the difficulty to match jointly total debt as well as the fraction of debtors. The model does well with respect to the fraction of indebted households, a feature important to match the position of the peak in the expectations distribution. It falls short in fully matching the amount of net liabilities of indebted agents. This imprecision is biasing the standard deviation of expectation errors and mean absolute errors in the first wealth decile upwards, as we will see below.

Table 2: Wealth Distribution

Quintile	Data	Model
1	-6.15%	-3.52%
2	1.37%	2.90%
3	5.96%	10.15%
4	16.08%	22.64%
5	82.74%	67.84%

Data refers to net financial wealth in the DNB Household Survey (waves 2010-2018). Compared to simulated, model implied wealth distribution.

4.3 Endogenous Expectations along the Wealth Distribution

Figure 5 presents the model implied equivalent to our baseline empirical findings in Figure 1. The model matches well qualitatively and quantitatively the behaviour of both the standard deviation of errors across households and the mean absolute forecast error along the wealth distribution: A peak in the second wealth decile, a flattening over wealth deciles 6-10 and the quantitative magnitude of the decline between deciles 2 and 6. The model captures qualitatively the untargeted decline in both the standard deviation and mean absolute error for the first wealth decile vis-à-vis the second. As in the data, the first decile consists of households with negative net wealth. The shortfall in reproducing the quantitative magnitude of this decline is due to the left tail of the wealth distribution in the model being less spread out compared to the left tail of the net wealth distribution in the data. Where the model is off by the largest margin quantitatively is the level of both the standard deviation and the mean absolute error. In the data, both curves are about one percentage point higher than in the model. Note, however, that in order to isolate the effect of the proposed mechanism we abstract entirely from any exogenous dispersion in beliefs such as e.g. fundamental disagreement about the long run mean μ or heterogeneous biases in the signal. The level of error dispersion is in line with the fraction attributed to our mechanism in Figure 3 after controlling for disagreement in long-run means. Exogenously imposing additional sources of dispersion would likely shift the reported measures up and towards the data equivalent.

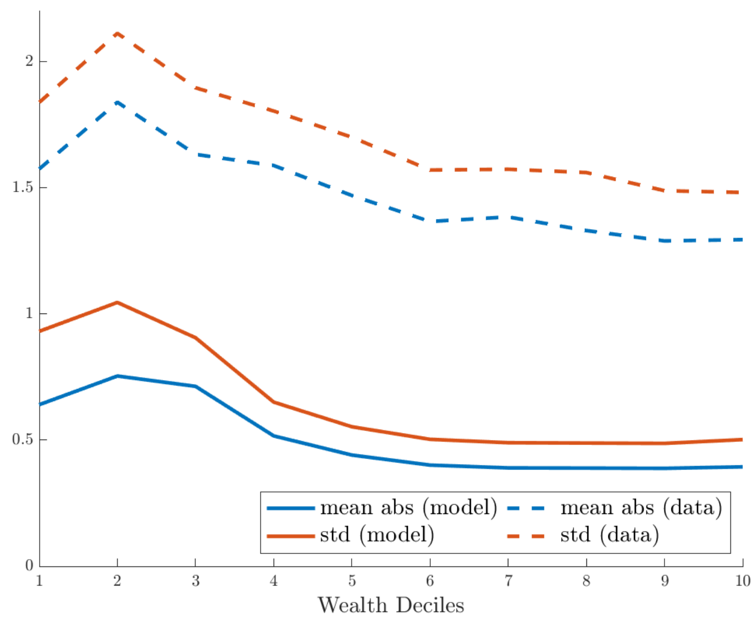


Figure 5: Expectation Errors by Wealth Decile Groups

Simulated, model implied statistics versus targeted data moments from Figure 1.

We have shown in Section 2 that, with our model of expectation formation, the driving force behind changes in mean absolute errors and standard deviation of errors is the subjective uncertainty that households face and hence the effort they choose. Our quantitative findings show that, indeed, wealthier and indebted households endogenously choose to exert more effort, enabling the model to replicate the empirical patterns. But why does the choice of effort vary with wealth? When inflation is a risk to the real interest rate, the more an agent wants to save or borrow between periods, the more he is exposed to fluctuations in the inflation rate. As future savings are positively correlated with current wealth, the richer (or the more indebted) a household is today, the more he will expose himself to inflation going forward. This exposure drives the incentives of households to exert effort and reduce the perceived uncertainty about future inflation. Importantly, it is the resolution of uncertainty rather than the change in optimizing consumption choices in response to changes in expected inflation rates that drives our results. Households value precise expectations since under risk aversion their lifetime utility decreases if future consumption is uncertain.[20] As we will see in the following section, the households who decide to exert the highest effort are not those who adjust their consumption choice the most in response to a change in the signal they receive.

Before we highlight potential consequences of the endogenous effects of wealth on expectation formation by studying households' consumption responses to signals about future inflation, we revisit four key assumptions underlying our results. First is our choice of preferences. The qualitative finding of effort choices increasing in absolute wealth levels does not rely on the assumption of recursive preferences, it persists also under more standard CRRA utility. A sufficient level of risk aversion is, however, important to quantitatively generate a steep decline in the standard deviation of errors as it leads to a stronger increase of the gains from effort with wealth. Epstein-Zin preferences allow for high risk aversion without marginalizing the intertemporal elasticity of substitution, which is important for our analysis of consumption responses to signals below.

Second is our empirical measure of wealth. What ultimately matters for the formation of households' inflation expectation in the model are beginning of period resources $\frac{1+r^n}{1+\pi}a + y$. These determine the *potential* exposure to inflation until the next period as they pin down the general range of future savings/borrowing. The *actual* exposure will then be given by the realized savings/borrowing choice within this range, but this happens only after the household has formed his expectations and is therefore endogenous to his effort choice. Motivated by the states relevant to households' expectation formation in the model, beginning of period wealth is the model-consistent empirical measure to consider.

Third, we have abstracted from modelling portfolio composition. Households' exposure to future inflation is independent of the composition of beginning of period

[20] We provide a more detailed discussion of the exposure effect in a two period framework in Appendix C.

wealth as long as this composition is adjustable going forward.[21] Households' balance sheets going forward are endogenous to their expectation formation. Including a portfolio choice into the model is a promising avenue for future research. It is likely to only strengthen results as the benefits from information in the presence of portfolio choice are increasing in wealth as shown e.g. in Peress (2003). His results suggest that when aggregate risk is distorting the relative returns of different assets, households with larger portfolios can gain more from acquiring information and rebalance their asset holdings optimally. Therefore, if inflation is distorting relative asset returns, again richer households would have higher incentives to form precise expectations.

Fourth, we have also abstracted from any exposure of non-asset (labour) income to inflation risk. Our results rely on this assumption to the extent that the exposure of labour income to inflation has to be sufficiently below the exposure of asset income. "Sufficiency" is determined by the levels of absolute risk aversion along the wealth distribution. What is important for our findings is that the residual absolute exposure to inflation, the exposure households face after controlling for all indexation of wages and asset returns to inflation, increases enough along the wealth distribution to outweigh the decrease in absolute risk aversion.[22] The high collective bargaining coverage along with the low portfolio share of potentially indexed assets or debt in our sample provide evidence for a sufficient difference in residual absolute exposure.

4.4 The Marginal Propensity to Consume upon Signal

To highlight some implications of endogenising expectations, this section focusses on households' marginal propensity to consume in response to a signal about future inflation rates. We will refer to this metric as MPCS and define it as the relative change in a household's consumption policy in response to a change in the signal he receives about tomorrows shock to inflation $\hat{\epsilon}$, when holding all other variables (a,y,π,n) constant. This

[21]We argue that this is the case for financial wealth at annual frequency, the time horizon at which we have data and to which we calibrate the model.

[22]For an extended discussion of the interplay between absolute risk aversion and exposure in a two period example see Appendix C.

measure has two components,

$$\text{MPCS} = \underbrace{\frac{1}{c}\frac{\partial c}{\partial\, \mathbb{E}[\epsilon]}}_{\text{MPCE}} \times \left.\frac{\partial\, \mathbb{E}[\epsilon]}{\partial \hat{\epsilon}}\right|_n . \tag{14}$$

The first term refers to the marginal propensity to consume in response to a change in expectations (MPCE). It is the percentage change in current consumption in response to a change in expectations about future inflation rates. The second term captures the change in expectations in response to a change in the signal, where $\mathbb{E}[\epsilon]$ stands for households' full subjective distribution over the future shock. Since we are concerned here only with a change in the value of a signal but not the fact that a household receives a signal and further under assumptions as above, the only moment of the subjective distribution affected is the posterior mean.[23] Applying Bayesian updating as before, in our framework the change in the subjective mean is given explicitly as

$$\left.\frac{\partial \bar{\epsilon}}{\partial \hat{\epsilon}}\right|_n = \omega(n) d\hat{\epsilon} = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_s^2(n)} d\hat{\epsilon}. \tag{15}$$

It becomes clear immediately how the response of expectations to a signal depends on effort n: The more effort is exerted, i.e. the less noisy a signal is perceived to be, the more a household will respond to this signal by updating his expected mean inflation - a standard result of Bayesian updating.

We analyse households' MPCS' quantitatively and compute the change in current consumption for each household if he receives a signal of $\hat{\epsilon} = 0.01$ instead of $\hat{\epsilon} = 0$. We distinguish four different scenarios, defined by how noisy households perceive the signal to be. An *endogenous* scenario follows our baseline model where noise is determined by the endogenous choice of effort and heterogeneous across households. We compare this benchmark to three scenarios in which noise is equalized across households: An *inattentive* scenario, setting the perceived noise of all households equal to that of the endogenously least informed. An *attentive* scenario, assigning to all agents the noise of the endogenously most informed households. A *flat* scenario, in which the noise of

[23]The standard deviation of the posterior distribution responds only to the fact that a signal is received and to the noise attached to such signal but is independent of the value the signal takes. Under our assumptions on how households' form their expectations, the change in mean and standard deviation are sufficient to characterize the response of the entire distribution.

all agents is chosen in order to match the unconditional standard deviation of errors in our baseline economy. These three cases have in common that the second term in (14) is constant across households and forces them to update their expectations in response to the signal in the same way, isolating differences in their MPCE. In the endogenous scenario, we also let $\left.\frac{\partial \mathbb{E}[\epsilon]}{\partial \hat{\epsilon}}\right|_n$ vary according to the endogenous effort choices of households.

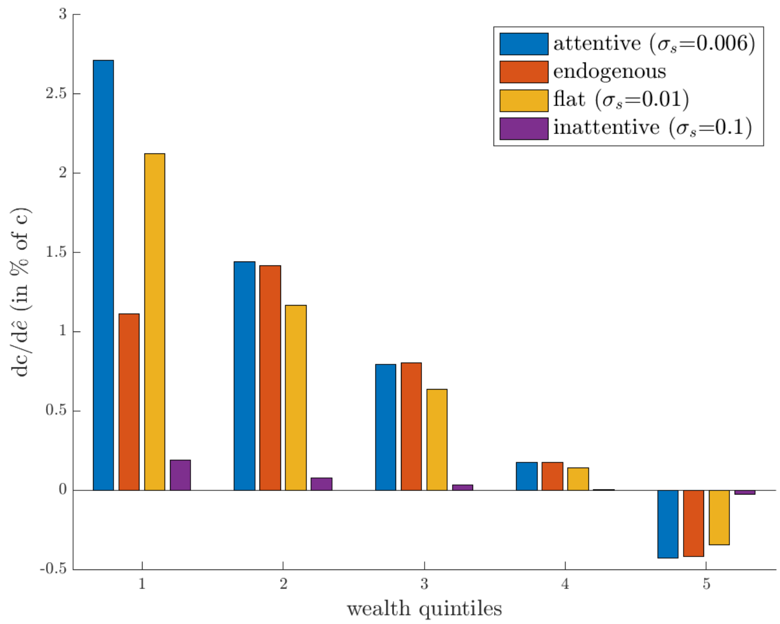


Figure 6: Marginal Propensity to Consume on Signal

Percentage change in consumption (aggregated by wealth quintile) on impact if $\pi = 2$ and $\hat{e}$ changes from 0 to 1pp. Endogenous: Noise as endogenously chosen. Attentive: All HHs σ_s=0.006. Inattentive: All HHs σ_s=0.1. Flat: All HHs σ_s=0.01.

Figure 6 plots the MPCS' aggregated by quintile of the wealth distribution.[24] We begin by looking at the three cases in which we keep $\left.\frac{\partial \mathbb{E}[\epsilon]}{\partial \hat{\epsilon}}\right|_n$ constant across households. The figure shows the MPCS for the inattentive, attentive and flat scenarios to be decreasing in wealth. Remember that from equation (14) MPCS = MPCE $\times \left.\frac{\partial \mathbb{E}[\epsilon]}{\partial \hat{\epsilon}}\right|_n$. Following this decomposition, it has to be the MPCE that is declining in wealth. This is due to the interaction of income and substitution effects in expectation of future

[24]To aggregate, we use the stationary wealth distribution of the converged economy if inflation is constant at two percent.

inflation rates. For a household who previously would not have held any savings or debt between periods, a change in expected inflation comes down to a change in the expected relative price of consumption today versus tomorrow, generating a substitution effect on current consumption. For a household who initially would have held either savings or debt, the substitution effect is accompanied by an income effect as a change in expected inflation implies a change in expected real financial income in the future. This income effect counteracts the substitution effect for saving households while it reinforces the substitution effect for borrowing households. For the case considered in Figure 6, a change in the signal from zero to one percentage point reveals to saving households that they will tomorrow be poorer than previously expected, hence diminishing their consumption response compared to households with little savings or debt. A good predictor for future savings in the model is the current asset level of a household, implying a MPCE declining in wealth.

The difference in the magnitude of MPCS' between the three cases with constant noise across households is driven by how much they update their expectations in response to the signal. The least informed ("inattentive") households choose a standard deviation of noise (σ_s) as high as 0.1 compared to a standard deviation of 0.01 of the actual shock (σ_e). Therefore, they attach little weight to any signal they receive ($\omega^{inatt} \approx 0.01$), do not update their beliefs in response and hence do not change their consumption behaviour. This is why the MPCS for inattentive households remains low. For the flat scenario, σ_s decreases to 0.01 and hence $\omega^{flat} \approx 0.5$. For the attentive scenario we assume the standard deviation of the noise to be 0.006. Therefore, they attach more weight to any signal they receive ($\omega^{att} \approx 0.74$) and respond stronger in terms of consumption. The increase in the MPCS is not linear in ω across scenarios since a change in effort also affects household's uncertainty, i.e. the standard deviation of their inflation expectations.

In the endogenous scenario, both terms in equation (14) interact. From our analysis so far we know the MPCE to be decreasing in wealth. From section 4.2 we now households effort choice and hence $\left.\frac{\partial \mathbb{E}[\epsilon]}{\partial \hat{\epsilon}}\right|_n$ to be increasing in wealth. The interaction between these two trends yields a hump shaped pattern of MPCS' along the wealth distribution. At the lowest wealth levels the increase in effort following an increase of resources outweighs the decline in MPCEs. From the second decile onwards the decline in MPCEs dominates as effort is almost constant over the upper half of the wealth distribution. The figure shows that at low levels of wealth endogenous effort leads to a substantially lower consumption response compared to the counterfactual attentive scenario. This gap is how the influence of wealth on expectation formation has an impact on macroeconomic aggregates.

4.5 A Forward Guidance Exercise

Campbell et al. (2012) famously coined the terms of *odyssean* and *delphic* forward guidance, the former referring to policy makers commitment to some future policy action and the latter standing in for an attempt to influence expectations about the future path of economic variables. Our model naturally lends itself to a discussion of the channel behind delphic forward guidance, as it provides an understanding into how heterogeneous households respond to signals about future inflation rates. More specifically, we can provide an approximation to how much endogenous expectation formation can decrease the effectiveness of such forward guidance policies. While a full general equilibrium analysis is beyond the scope of our setup, we will be able to capture the initial consumption response to a change in households' inflation expectations. Since any demand shock can be decomposed into such a partial equilibrium consumption response on impact and a general equilibrium multiplier,[25] our results should be interpreted as capturing the initial increase in aggregate demand which is then amplified through a general equilibrium multiplier.

To highlight households' response to delphic forward guidance we conduct a quantitative exercise: Assume the economy to be stationary at $\pi = 0.02$. In this economy we shift the signal of every household by 0.01, such that all signals are drawn from $\mathcal{N}(0.01, \sigma_s^2(n_t^i))$ instead of $\mathcal{N}(0, \sigma_s^2(n_t^i))$. For each household we compute the change

[25] See Rognlie and Auclert (2020).

in consumption compared to the original signal and obtain an aggregate response using the stationary distribution of households. We do so under two different assumptions about how noisy households perceive their signals to be: Our benchmark scenario, where households' choose their effort endogenously, as well as the counterfactual attentive scenario, where all households' are as informed as the most informed inside the model economy. The attentive scenario provides an upper bound on how effective forward guidance could be as it assumes all households to attach the highest possible weight to any signal received and hence a strong updating of expectations. The difference between the two scenarios provides us an estimate for the potential consumption response that forward guidance misses out on due to some households not paying attention to inflation.

Table 3: Forward Guidance Exercise

calibration	**attentive**	**endogenous**	**missing potential**
baseline	0.20	0.09	0.11 (55%)
adjusted	0.13	0.08	0.05 (42%)

The table reports aggregated MPCS' in pp as defined in (14) if signals are drawn from $\mathcal{N}(0.01, \sigma_s^2(n_t^i))$ instead of $\mathcal{N}(0, \sigma_s^2(n_t^i))$ when the economy is stationary at $\pi = 0.02$. The first row reports results for our baseline calibration, the second row for an alternative calibration with $\bar{n} = 10$.

The first row in table 3 presents results for our baseline calibration. It shows that due to endogenous expectation formation forward guidance loses approximately 55% of its consumption response on impact, a sizeable drop in the partial equilibrium response necessary to trigger any general equilibrium effects. As outlined in the previous section and especially Figure 6, the missing potential lies with households around zero net wealth who exert little effort in forming precise expectations, perceive any signal about future inflation as noisy, and hence do not update their expectations despite having the largest potential consumption response if they would do so. Any higher order (general equilibrium) effects that rely on this initial trigger will also be attenuated. Reaching those households' to whom higher future inflation does not imply a decrease in future income from asset holdings could therefore substantially increase the effectiveness of delphic forward guidance policies. Central banks should take this into account when designing the communication of their policies.

Our baseline calibration has attributed the entire decline in the standard deviation of errors along the wealth distribution to endogenous factors and therefore provides an upper bound on the effect of endogenous expectations on forward guidance. To test the robustness of our estimate to this assumption, we adjust the calibration to match the decline in subjective uncertainty after controlling for dispersion in beliefs about the long-run mean of inflation μ (see Figure 3). This provides some lower bound as it assumes any decline in dispersed beliefs about μ to be entirely exogenous, restricting the endogenous gap of attention between high and low wealth households. Instead of a decline of 0.57 between peak and low of the standard deviation of errors, we now target a drop of only 0.34. This target is met by adjusting $\bar{n}$ to 10 and keeping all other parameters as they were before. The second row of Table 3 presents results for this alternative calibration. While in general the response of consumption is weaker than under the baseline calibration due to the reduced attentiveness (and hence reduced updating of expectations upon a signal) of the most informed households, forward guidance still loses about 42% of its initial effect on consumption when moving from maximum attention of all households' to endogenous expectation formation.

5 Concluding Remarks

This paper provides a framework to discuss the joint formation of households' inflation expectations and savings choices. We argue that wealth levels are important for both the formation of expectations and households' response to expected inflation. Looking at empirical observations from the DHS dataset, the standard deviation of forecast errors and mean absolute errors are declining in absolute wealth. We exploit changes in these cross-sectional statistics along the wealth distribution to discipline a consumption-savings problem with endogenous expectation formation, where households can exert effort to reduce uncertainty about future inflation rates. The model matches the empirical observations. The mechanism behind this finding works through the heterogeneous exposure to inflation that households at different points in the wealth distribution face. The model allows us to back out marginal propensities to consume in response to signals about future inflation. These MPCS' are hump shaped in wealth, driven by a negative correlation between households' consumption response to expected inflation and the change in their expectations in response to signals. At the aggregate level, small MPCS' of low wealth households (due to a lack of attention to inflation) can substantially reduce the effectiveness of forward guidance policies.

While an empirical analysis of MPCS' lies beyond the scope of this paper, others have conducted related work in the DHS dataset: Lieb and Schuffels (2019) find the likelihood of durable consumption in response to higher inflation expectations to be decreasing in wealth. Similarly, Coibion et al. (2019) report a stronger decline in durable consumption in response to (exogenously) higher inflation expectations for households with higher wealth levels. This can be seen as support for MPCEs declining in wealth due to the interaction of income and substitution effects. More work along these lines is necessary for a full empirical evaluation of our theory, especially with regard to the effect wealth has on how expectations respond to signals.

Our paper also leaves room for further theoretical work on the topic. One possible addition to the analysis presented here can be to include a portfolio choice into our model. As mentioned before, such an extension is unlikely to alter the findings presented in this paper. It might nevertheless yield interesting additional results on the implications of costly inflation expectations for wealth inequality, as suggested by the findings of Peress (2003) and Lei (2019). While we focus on uncertainty and endogenous expectations about the shock to inflation rates, the model can be extended to other sources of heterogeneity in expectations such as learning about the underlying model. Our extension to include fundamental disagreement in Section 3.2 provides a starting point for work in this direction. More importantly, a computationally demanding but interesting application of the mechanism described in this paper would be to introduce our model of expectation formation into a general equilibrium environment as in Carroll et al. (2020) or Auclert et al. (2020). These papers rely so far on exogenous updating of expectations. It would be important to understand the impact of our findings on MPCS' in their general equilibrium setting. We leave these extensions for future research.

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Appendix

A Empirical Observations

A.1 Extension - Fundamental Diasgreement

To test for robustness towards including fundamental disagreement, we adjust our baseline model of expectation formation to incorporate heterogeneity in beliefs about the long run mean of inflation μ. Household i's belief about μ is denoted μ^i and assumed to be distributed normally among households. Furthermore, we assume $\mu^i \perp\!\!\!\perp s_t^i\ \forall i, t$. With all other notation as before, household i's inflation expectation and expectation error are now given as

$$\mathbb{E}_t^i[\pi_{t+1}|\hat{e}_{t+1}^i, n_t^i] = (1-\rho)\mu^i + \rho\pi_t + \omega_{t+1}^i(n_t^i)\hat{e}_{t+1}^i \tag{16}$$

$$\begin{aligned} err_{t+1}^i &= \mathbb{E}_t^i[\pi_{t+1}|\hat{e}_{t+1}^i, n_t^i] - \pi_{t+1} \\ &= (1-\rho)(\mu^i - \mu) + (\omega_{t+1}^i(n_t^i)s_{t+1}^i - (1-\omega_{t+1}^i(n_t^i))e_{t+1}). \end{aligned} \tag{17}$$

The error now includes an additional term accounting for households' misperception of the long run mean. Denote the average belief about the long term mean of a group g of households as $\bar{\mu}^g$ and its variance as σ_μ^{g2}. Assuming, as before, that households in group g exert the same effort $\bar{n}_t^g$, the variance of errors across households in group g and over time becomes

$$\begin{aligned} \mathrm{Var}^g(err_{t+1}^i) &= (1-\rho)^2\mathrm{Var}(\mu^i) + (\omega_{t+1}^g(\bar{n}_t^g))^2\sigma_s^2(\bar{n}_t^g) + (1-\omega_{t+1}^g(\bar{n}_t^g))^2\sigma_e^2 \\ &= (1-\rho)^2\sigma_\mu^{g2} + \frac{\sigma_e^2\sigma_s^2(\bar{n}_t^g)}{\sigma_e^2 + \sigma_s^2(\bar{n}_t^g)} = (1-\rho)^2\sigma_\mu^{g2} + \overline{SU}_{t+1}^{g2} \end{aligned} \tag{18}$$

where now the endogenous subjective uncertainty term $\overline{SU}_{t+1}^{g2}$ is adjusted by the within-group fundamental disagreement about μ. Disagreement among households can hence be decomposed into disagreement about the long run mean and households' subjective uncertainty. We can also compute the covariance of the ex-post errors across time. This is given as

$$\mathrm{Cov}^g(err_{t+1}^i, err_t^i) = (1-\rho)^2\,\mathbb{E}[(\mu^i-\mu)^2] - (1-\rho)^2(E[(\mu^i-\mu)])^2 = (1-\rho)^2\sigma_\mu^{g2}. \tag{19}$$

Together, (18) and (19) allow us to identify the endogenous component of error dispersion in the presence of fundamental disagreement from the difference between variance and covariance of forecast errors as

$$\overline{SU}^g_{t+1} = \sqrt{\mathrm{Var}^g(err^i_{t+1}) - \mathrm{Cov}^g(err^i_{t+1}, err^i_t)}. \tag{20}$$

With fundamental disagreement, the mean absolute error is given as

$$\mathbb{E}^g[|err^i|] = \sqrt{\mathrm{Var}^g(err^i_{t+1})\frac{2}{\pi}}e^{-\frac{\overline{err}^{g2}}{2\mathrm{Var}^g(err^i_{t+1})}} - \overline{err}^g\left(1 - 2\Phi\left(\frac{-\overline{err}^g}{\sqrt{\mathrm{Var}^g(err^i_{t+1})}}\right)\right). \tag{21}$$

A.2 Robustness - Michigan Survey of Consumers

The Michigan Survey of Consumers (MSC) is one of the most established sources for data on households' expectations. Compared to our main data source it has a disadvantage in that it does not provide comprehensive data on the wealth of participants. It only reports the current value of individuals' stock market portfolios. We use this value as a proxy for financial wealth and repeat part of the analysis on DHS data for the Michigan Survey.

An advantage of the MSC is the long time series for which consistent data are available. Data on inflation expectations and stock investment are continuously provided since September 1998. Furthermore, the data is available at monthly frequency. This does not only increase the number of observations along the time dimension, but also allows for a more precise computation of the forecast error as we can pin down the exact month of the observation. Applying the same approach as discussed above for the DHS data, we assign observations to investment quintile groups based on their position in the stock portfolio distribution in the month of their observation. Note that the first quintile now begins at zero investment as naturally there are no observations reporting a negative value of their portfolio. We compute the expectation error as the reported forecast minus the realized inflation rate in the 12 months following the month of observation.

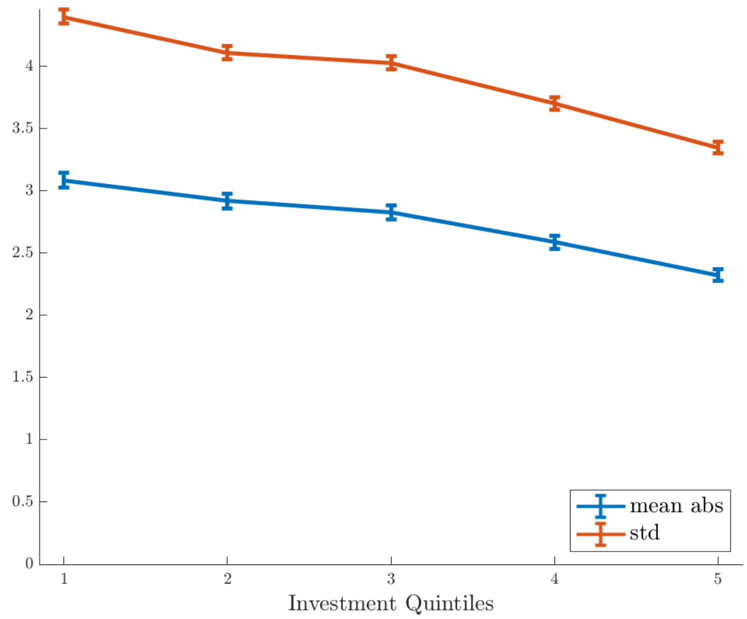


Figure A.1: Expectation Errors by Investment Quintiles (Michigan Data)

The figure plots the within quintile group standard deviation of errors and the mean absolute forecast error. Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from Michigan Survey of Consumers waves 09/1998-04/2018.

Figure A.1 reports the within quintile group standard deviation of expectation errors as well as the mean absolute forecast error by quintile group. Similar to the DHS data both are declining in investment value, a pattern that is statistically significant. We cannot observe any drop for negative wealth levels, as there are no such observations in our sample. Interestingly, we also cannot observe a flattening out of the decline for high levels of stock investment. Again, the pattern is robust to controlling for age or education.[26] Figure A.2 provides the decomposition of error variance across households by investment quintile group into covariance and subjective uncertainty, as of equation (20). In the MSC, even after allowing for fundamental disagreement almost all of the decline of within quintile group error variance is attributed to a decline in subjective uncertainty, while error covariance declines only modestly. Hence, the additional data source generally supports our initial findings.

[26]See Figure A.6 and Figure A.7 in Section A.3.

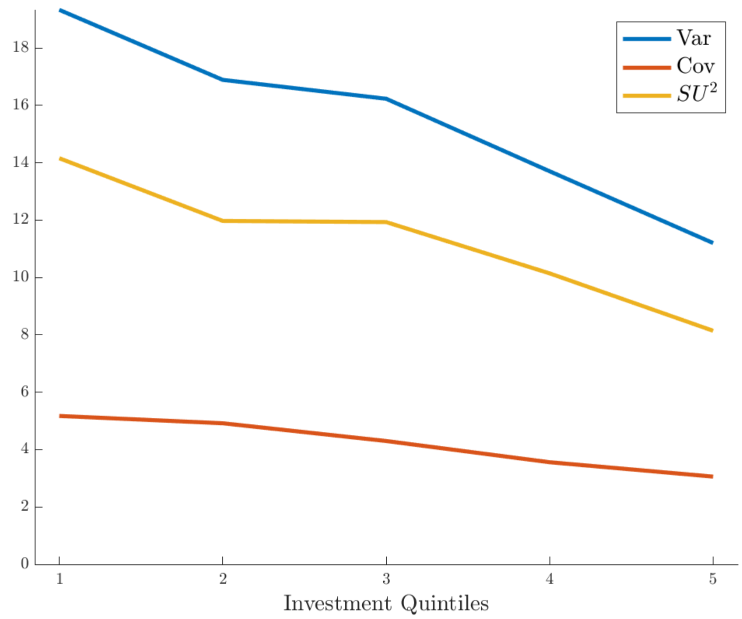


Figure A.2: Expectation Error Variance – Decomposition (Michigan Data)

The figure decomposes the cross-sectional variance of expectation errors (Var) by investment quintile groups into error covariance (Cov) and subjective uncertainty (SU) as in (20). Data from the Michigan Survey of Consumers waves 09/1998-04/2018.

A.3 Additional Empirical Results

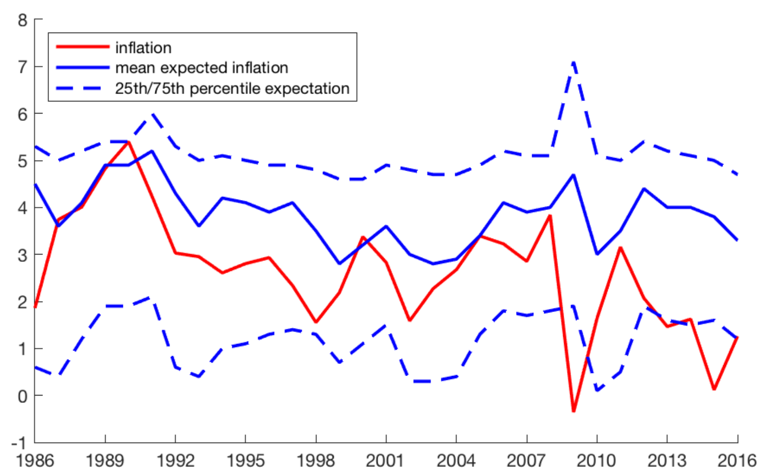


Figure A.3: Time Series of Inflation Expectations

Realized inflation rates and cross-sectional distribution of inflation expectations. Own computations based on data from Michigan Survey of Consumers and FRED.

Table A.1: Net Financial Wealth Decile Groups – Summary

Decile	**N**	**Min**	**Max**	**1pct**	**99pct**	**N err missing**
1	1,272	-1,791,382	-150	-245,100	-542	149
2	1,267	-3,109	1,235	-2,309	925	170
3	1,262	321	4,500	375	4,250	128
4	1,264	2,076	9,245	2,200	8,547	123
5	1,261	5,488	16,935	5,607	16,513	122
6	1,270	10,475	26,750	10,803	26,196	110
7	1,260	18,284	42,274	18,612	40,100	104
8	1,265	28,873	74,089	30,000	70,944	84
9	1,264	50,833	133,425	53,503	128,073	66
10	1,260	105,452	3,702,125	109,500	1,524,455	53
Total	12,645	-1,791,382	3,702,125	-49,545	511,557	1,109

Data from DNB Household Survey waves 2010-2018. Summary statistics for net financial wealth by wealth decile groups. Net financial wealth refers to net wealth ex housing, mortgages, businesses and vehicles. Decile groups overlap due to differences in cut-offs across waves.

Table A.2: Expectation Error by Decile Groups – Summary

Decile	**Mean**	**Std**	**Mean (abs)**	**N**
1	1.18	1.84	1.58	1,123
2	1.47	2.11	1.84	1,097
3	1.27	1.90	1.63	1,134
4	1.24	1.80	1.59	1,141
5	1.04	1.70	1.47	1,139
6	0.99	1.57	1.37	1,160
7	1.02	1.57	1.39	1,156
8	0.97	1.56	1.33	1,181
9	0.86	1.49	1.29	1,198
10	0.91	1.48	1.30	1,207
Total	1.09	1.72	1.47	11,536

Data from DNB Household Survey waves 2010-2018. Summary statistics for expectation error by net financial wealth decile groups, computed as ex-ante point forecast for inflation minus ex-post realized inflation rate. Mean (abs) refers to the mean of the absolute error.

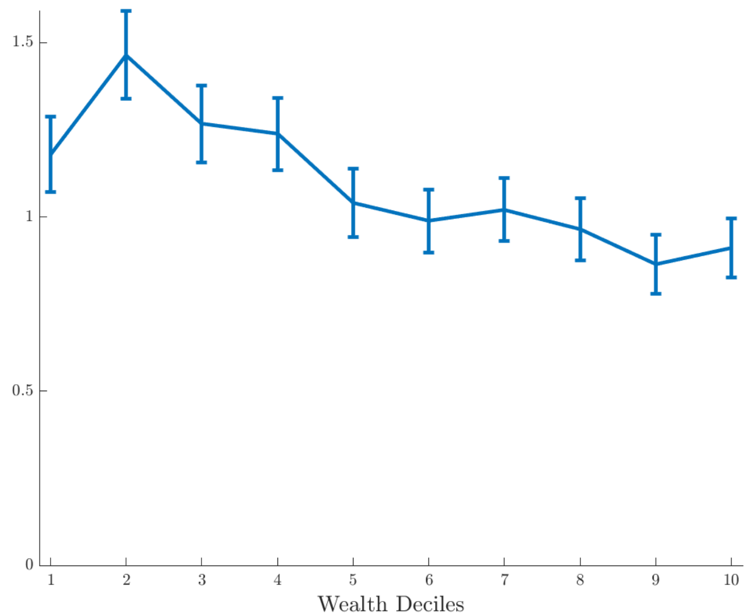
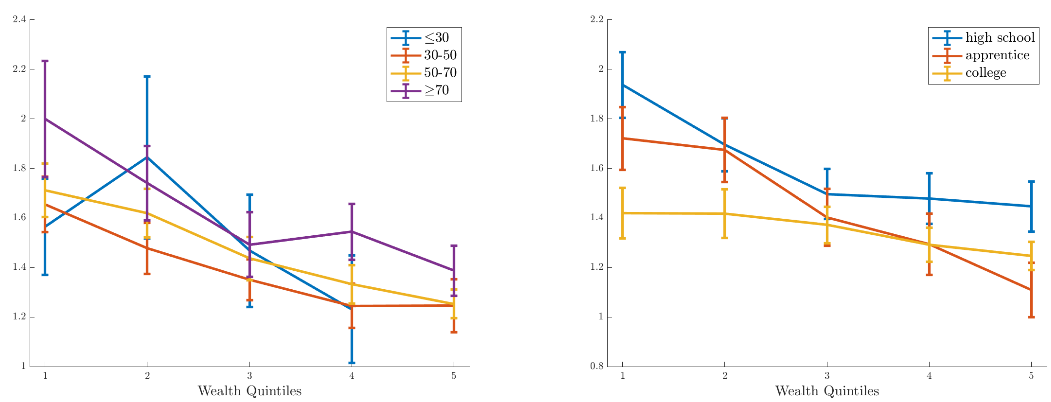


Figure A.4: Expectation Errors by Wealth Decile Groups – Mean

The figure plots the average expectation error by net financial wealth decile group. Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from DNB Household Survey waves 2010-2018.



(a) by age

(b) by education

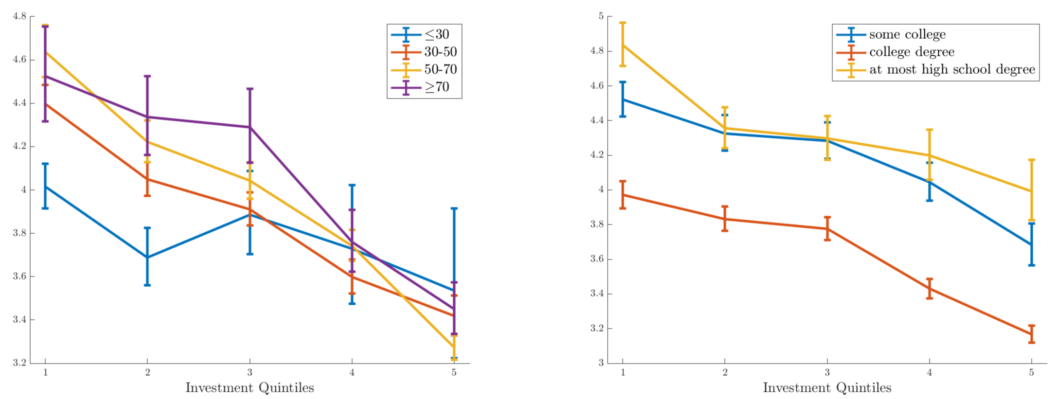
Figure A.5: Mean Absolute Expectation Error by Wealth Quintiles – Controls

The figure plots the mean absolute forecast error for each wealth quintile group by age (a) and education groups (b). Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from DNB Household Survey waves 2010-2018. Combination of youngest age and highest wealth quintile omitted due to lack of observations.

Table A.3: Expectation Error by Quintile Groups (Michigan Data) – Summary

Quintile	**Mean**	**Std**	**Mean (abs)**	**N**
1	1.41	4.48	3.15	11,027
2	1.29	4.17	2.98	10,886
3	1.05	4.08	2.88	10,912
4	0.87	3.77	2.65	10,438
5	0.58	3.42	2.38	9,917
Total	1.05	4.02	2.82	53,180

Data from Michigan Survey of Consumers waves 09/1998-04/2018. Summary statistics for expectation error by investment quintiles, computed as ex-ante point forecast for inflation minus ex-post realized inflation rate. Mean (abs) refers to the mean of the absolute error.



(a) by age

(b) by education

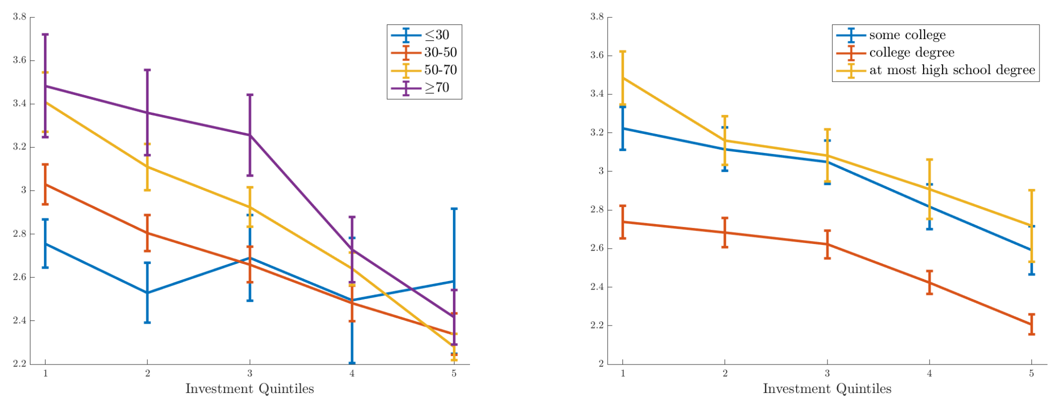
Figure A.6: Standard Deviation of Expectation Errors by Investment Quintiles (Michigan Data) – Controls

The figure plots the within quintile group standard deviation of errors by age (a) and education groups (b). Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from Michigan Survey of Consumers waves 09/1998-04/2018.

Table A.4: Derived Distributions – Consistency with Point Forecast

Distribution	Correlation with point forecast
discrete	0.866
conservative discrete	0.905
continuous	0.712

Data from DNB Household Survey waves 2010-2018. Correlation of means from derived distributions and reported point forecast.



(a) by age

(b) by education

Figure A.7: Mean Absolute Expectation Error by Investment Quintiles (Michigan Data) – Controls

The figure plots the mean absolute forecast error for each investment quintile group by age (a) and education groups (b). Bars provide confidence bands at the 95% level. Expectation errors are ex-ante point forecasts minus ex-post realizations. Data from Michigan Survey of Consumers waves 09/1998-04/2018.

B Dynamic Model

B.1 Dynamic Budget Constraint - From Nominal to Real

Starting with nominal assets $\hat{a}$

$$Pc + \hat{a}' = (1 + r^n)\hat{a} + y - P\mathcal{F}(n, i)$$
$$c + \frac{\hat{a}'}{P} = (1 + r^n)\frac{\hat{a}}{P} + y - \mathcal{F}(n, i)$$

Define $a' = \frac{\hat{a}'}{P}$, i.e. tomorrow's nominal assets in today's real consumption, and inflation rate $1 + \pi = \frac{P}{P_{-1}}$

$$c + a' = (1 + r^n)\frac{P_{-1}}{P}a + y - \mathcal{F}(n, i)$$
$$c + a' = \frac{1 + r^n}{1 + \pi}a + y - \mathcal{F}(n, i)$$

C Endogenous Expectations in a Two Period Model

To highlight the mechanism through which households' wealth levels impact their expectation formation, it is instructive to analyse the properties of a two period model. In the interest of a simpler exposition, we abstract from inflation entirely and focus directly on risk to the real interest rate. This is without loss of generality, since fluctuations in inflation translate into fluctuations in the real interest rate as long as nominal rates are not assumed to adjust one-for-one with inflation. Furthermore, their impact on real interest rates is the only channel through which fluctuations in inflation are relevant to the household's problem as long as additional (labour) income is assumed to be in real terms. These are the same assumptions we impose in the dynamic model where we consider inflation explicitly, making the two approaches comparable.

C.1 A Two Period Model

A household lives for two periods and maximizes utility by choosing consumption in both periods (c_1 and c_2) as well as savings a between periods. In both periods he receives a deterministic and constant income y. Additionally, at the beginning of the first period the household receives initial assets A. Preferences of the household are recursive, following Epstein and Zin (1989).

The real interest rate r between the two periods is stochastic. Before choosing savings in period 1, the household receives a noisy signal $\hat{r}$ about the interest rate. The distribution of the interest rate and the signal are given as

$$r \sim \mathcal{N}(\bar{r}, \sigma_r^2) \qquad \hat{r} = r + s \qquad s \sim \mathcal{N}(0, \sigma_s^2(n)), \tag{22}$$

where s is pure noise. Before receiving the signal, the household can influence the variance of the noise by exerting some effort n, for which he has to incur a monetary cost $\mathcal{F}(n)$. Based on the signal, the household forms a Bayesian posterior belief about the true interest rate r, attaching weight $\omega(n)$ to the signal received. Hence, conditional on n and $\hat{r}$, the posterior distribution is given as

$$\begin{aligned} r_{|n,\hat{r}} &\sim \mathcal{N}((1-\omega(n))\bar{r} + \omega(n)\hat{r}, \omega(n)\sigma_s^2(n)) \\ \omega(n) &= \frac{\sigma_r^2}{\sigma_r^2 + \sigma_s^2(n)}. \end{aligned} \tag{23}$$

We will refer to the standard deviation of a household's posterior belief about r (given by $\sqrt{\omega(n)\sigma_s^2(n)}$) as his subjective uncertainty about the future interest rate.

The household's effort choice problem is then given as

$$\tilde{V}(A) = \max_n \mathop{\mathbb{E}}_{\hat{r}}[V(A, n, \hat{r})|n]. \tag{24}$$

Conditional on having exerted effort n and receiving signal $\hat{r}$, the consumption-savings problem is given by

$$\begin{aligned} V(A, n, \hat{r}) &= \max_a \left(c_1^{1-\gamma} + \beta \left(\mathop{\mathbb{E}}_r[c_2^{1-\alpha}|\hat{r}, n] \right)^{\frac{1-\gamma}{1-\alpha}} \right)^{\frac{1}{1-\gamma}} \\ c_1 &= A + y - a - \mathcal{F}(n) \\ c_2 &= (1+r)a + y \quad \forall\ r \end{aligned} \tag{25}$$

For the cost of effort and the relationship between effort and noise in the signal we assume functional forms

$$\sigma_s(n) = \frac{\chi}{1+n} \qquad \mathcal{F}(n) = (\theta n)^{\phi}. \tag{26}$$

These choices yield convex cost of and convex gains from exerting effort.[27] Note that with these functional forms χ is the variation in the noise if zero effort is exerted, i.e. the maximum variation possible, and that zero effort implies zero cost.

To highlight some properties of the proposed mechanism, we calibrate the model outlined above. The calibration is ad-hoc and for instructive purposes only. It is provided in Table C.1.

Table C.1: Two Period Model – Calibration

Parameter	Value
γ	2
α	2
β	0.98
y	4
$\bar{r}$	0.02
σ_r	0.01
ϕ	2
θ	0.005
χ	0.03

Calibration for the two period model. Values are ad-hoc and only for instructive purpose.

C.2 Information Incentives

To study households' incentives to form precise expectations, we begin by taking the effort choice n as exogenously given. In order to do so, we drop the max-operator in (24) and set the cost in (25) to $\mathcal{F}(n) = 0 \; \forall \; n$. After solving the households' problem for given n we can compute a certainty equivalence consumption level cec_n, satisfying

$$\tilde{V}_n(A) = \left(cec_n^{1-\gamma} + \beta \left(cec_n^{1-\alpha} \right)^{\frac{1-\gamma}{1-\alpha}} \right)^{\frac{1}{1-\gamma}}, \tag{27}$$

where $\tilde{V}_n(A)$ is the value of (24) for exogenously given n and zero cost of effort. We use this certainty equivalent to construct a measure of the benefit of decreasing the noise in the signal as

$$\Delta cec_n = \frac{cec_n}{cec_0} - 1, \tag{28}$$

[27] $\sigma_s'(n) < 0, \; \sigma_s''(n) > 0$ and $\mathcal{F}'(n) > 0, \; \mathcal{F}''(n) \geq 0, \;$ iff $\phi \geq 1$.

which is the percentage change in the certainty equivalence consumption level if effort is increased from 0 to n, and hence the standard deviation of the noise is decreased from χ to $\sigma_s(n)$.

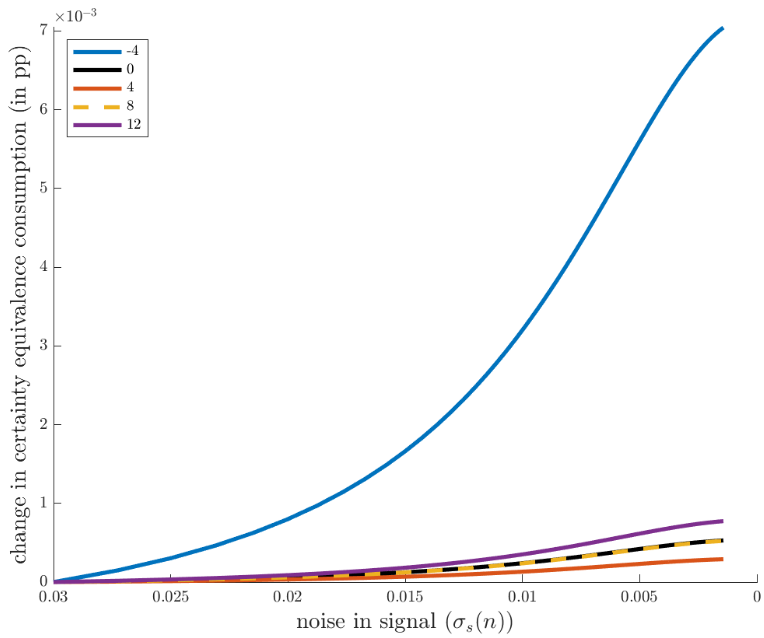


Figure C.1: Change in Certainty Equivalence Consumption

The figure plots the percentage gain in certainty equivalence consumption (cec_n, as defined in (27)) of decreasing the standard deviation of the noise in the signal from χ to $\sigma_s(n)$. Each line represents a different initial asset level A.

Figure C.1 plots results from the calibrated model for a range of initial asset values A. The gain from decreasing the variation in noise is highest for households starting with debt. It decreases as initial asset levels increase towards zero and modestly positive values for A and increases again once A becomes substantially positive. Note that the gains of decreasing the variation in the noise are small, for the given calibration below 0.01% of the certainty equivalent consumption level. This is evidence that already small cost of forming precise expectations might deter households from doing so.

The pattern of noise in wealth can be explained by two forces, governing households' incentives to form precise expectations: Exposure and absolute risk aversion. Exposure is given by the absolute value of a household's savings or borrowing between the two periods.

It determines the relevance of the risk for a household. The higher absolute savings, the larger are expected fluctuations in period 2 consumption due to fluctuations in the interest rate. In the presence of risk aversion, fluctuations in future consumption reduce expected utility. Hence households with larger fluctuations in their future consumption due to the risk have stronger incentives to reduce the perceived risk and form more precise expectations. The exposure effect is therefore higher for households with either higher initial debt or higher (positive) initial assets, who engage in borrowing/saving between periods, but low for households with A close to zero, as these households save/borrow little between $t = 1$ and $t = 2$. Absolute risk aversion, as usual, implies that any absolute fluctuation in consumption has higher cost in terms of expected utility to households with a lower average consumption level. This effect is hence highest for households with higher debt (A substantially negative), as these households have the lowest consumption levels, and decreases as A increases.

To highlight the two effects on the change in certainty equivalence consumption, we conduct two quantitative experiments. For the first, we eliminate differences in the absolute risk aversion of households with different A to focus solely on exposure. This is achieved by compensating each household to obtain the same average consumption level as a benchmark household, which we chose to be a household with initial assets $A = -4$. More specifically, we fix the savings choice of a household at the optimal choice without any compensation. Conditional on the exogenously set effort n and the signal received $\hat{r}$, each household receives a deterministic transfer for both periods, satisfying

$$\begin{aligned} \Delta c_1(A, n, \hat{r}) &= c_1(-4, n, \hat{r}) - c_1(A, n, \hat{r}) \\ \Delta c_2(A, n, \hat{r}) &= \mathop{\mathbb{E}}_r[c_2(-4, n, \hat{r})|n, \hat{r}] - \mathop{\mathbb{E}}_r[c_2(A, n, \hat{r})|n, \hat{r}]. \end{aligned} \tag{29}$$

As this equalizes consumption levels across households, any difference in the remaining effect on the certainty equivalence consumption should be due to different exposure.

Figure C.2 plots the quantitative results. As expected, the change in the certainty equivalence consumption level is monotonically increasing in the absolute value of A, which is directly related to the absolute value of households' savings between periods.

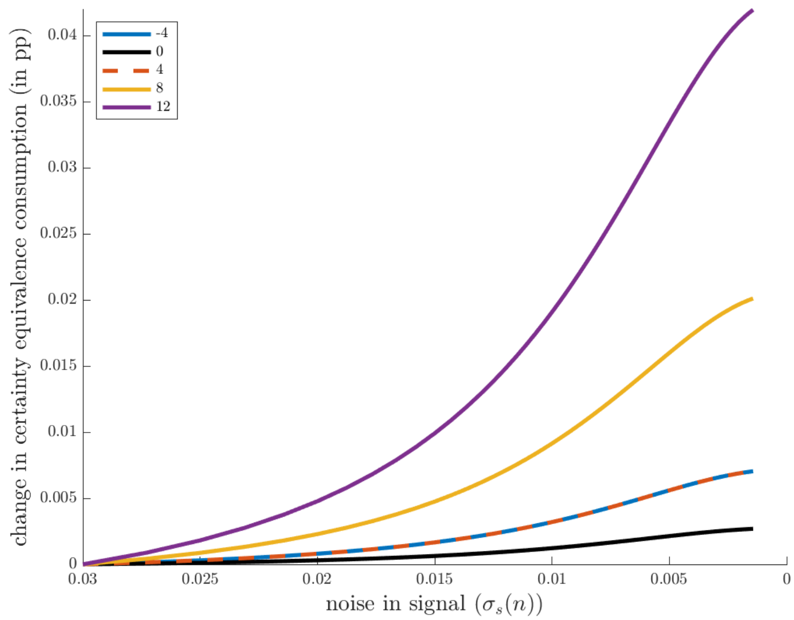


Figure C.2: Change in Certainty Equivalence Consumption – Exposure

The figure plots the adjusted percentage gain in certainty equivalence consumption (cec_n, as defined in (27)) of decreasing the standard deviation of the noise in the signal from χ to $\sigma_s(n)$. Adjustment equalizes average consumption levels across households to the level of a households with $A = -4$, as given in (29), while leaving the savings choice unchanged. Each line represents a different initial asset level A.

Note that the effect of decreasing the variation in noise is almost identical for households with $A = 4$ and $A = -4$. This reflects their, in absolute values and on average across signals, almost identical savings choices, implying a similar exposure to interest rate risk.

To control for the exposure effect and highlight the influence of absolute risk aversion, we can conduct a similar experiment by normalizing households savings choice. We assign every households the savings choice of a household with $A = 10$ (i.e. $s(10, n, \hat{r})$), controlling for n and $\hat{r}$. We additionally assign transfers, such that the household has the same average consumption level as before. These are given as

$$\begin{aligned} \tilde{\Delta} c_1(A, n, \hat{r}) &= s(10, n, \hat{r}) - s(A, n, \hat{r}) \\ \tilde{\Delta} c_2(A, n, \hat{r}) &= \mathop{\mathbb{E}}_r[c_2(A, n, \hat{r})|n, \hat{r}] - \mathop{\mathbb{E}}_r[c_2(10, n, \hat{r})|n, \hat{r}]. \end{aligned} \tag{30}$$

The results can be interpreted as the gain from decreasing the variation in noise for households with identical savings choice but varying consumption levels.

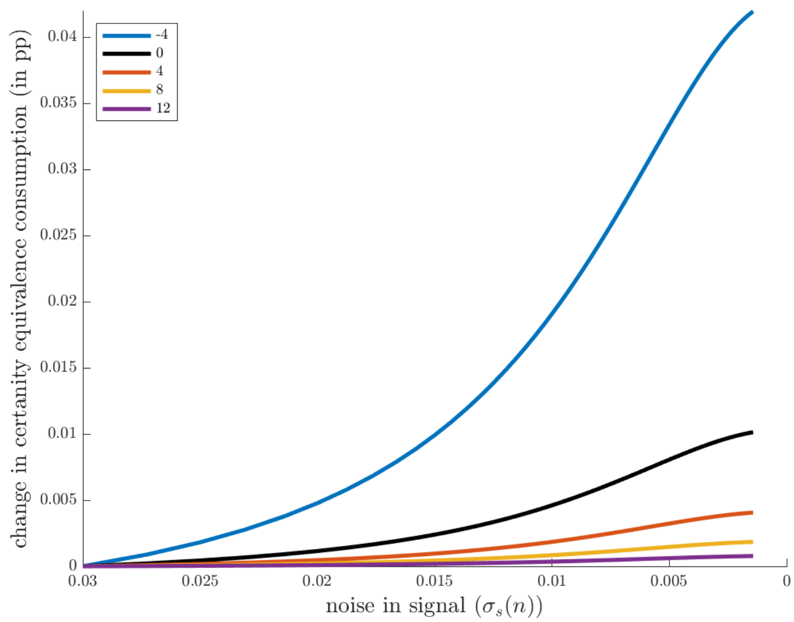


Figure C.3: Change in Certainty Equivalence Consumption – Absolute Risk Aversion

The figure plots the adjusted percentage change in certainty equivalence consumption (cec_n, as defined in (27)) of decreasing the standard deviation of the noise in the signal from χ to $\sigma_s(n)$. Adjustment equalizes savings across households to the level of a households with $A = 10$ as given in (30) while leaving the average consumption level of the household unchanged. Each line represents a different initial asset level A.

Figure C.3 plots the quantitative results. Unsurprisingly, when controlling for the savings choice, households with lower consumption level (and hence higher absolute risk aversion) profit more from a reduction of uncertainty. The gain from increasing n / reducing $\sigma_s(n)$ is decreasing in A.

C.3 Information Choice

We can summarize the findings above to make predictions about how households decide on effort n when the choice is endogenous. The exposure effect is increasing in households absolute initial wealth, as their future absolute savings will be equally increasing. This implies, that starting at a wealth level of zero, the further away we move in any direction along the wealth distribution the more effort households should want to exert due to the exposure effect. This effect is almost symmetric for positive and negative values of initial assets A. Absolute risk aversion is, however, monotonically decreasing in wealth. It reinforces the exposure effect, but more so for negative asset levels. The effect of absolute risk aversion is hence asymmetric in positive/negative wealth. We should hence expect the chosen noise in the signal to peak around zero wealth, decline as we move away from zero wealth in any direction, but decline steeper for negative wealth than for positive wealth. All discussion above assumes that effort is equally costly for all households. With the specification for effort to have monetary cost, this is not true in utility terms, as the same monetary costs transmit into higher utility cost for households with lower consumption levels. This adds an additional dimension of heterogeneity in incentives.

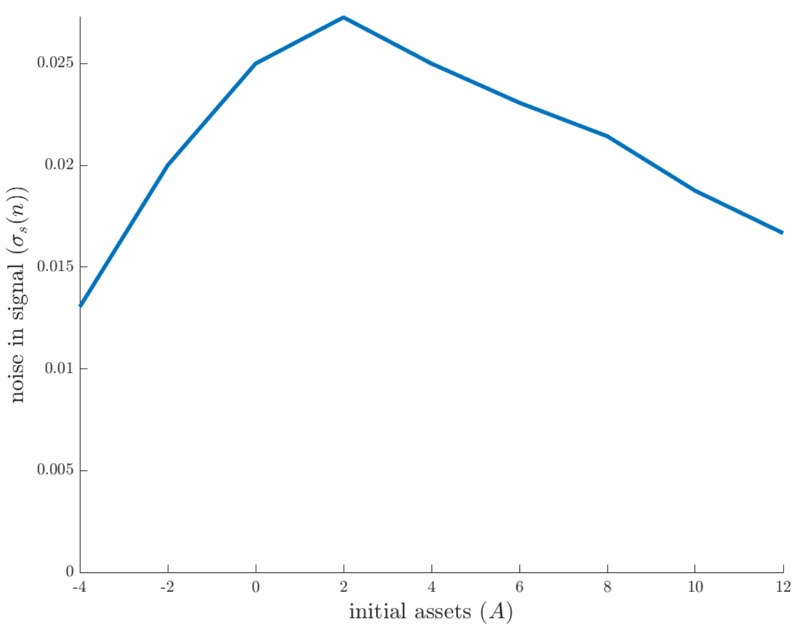


Figure C.4: Endogenous Effort – Chosen Standard Deviation of Noise

The figure plots the standard deviation of the noise implied by the endogenous choice for n, solving (24) for given initial asset level A.

We confirm the predictions of our exercise by moving on to an endogenous choice of effort according to (24) and (25), subject to the cost function and return to effort as outlined in (26). The calibration remains the same as before. Figure C.4 plots the standard deviation of the noise implied by effort choice $n(A)$ across a range of initial asset level A. The findings confirm our conjecture from Section C.2. With increasing absolute wealth level (positive or negative), households decide to exert more effort to reduce the noise in the signal, driven by the exposure effect. Additionally, households with negative wealth choose to exert more effort (reduce the noise further) than households with similar positive wealth. This is due to the asymmetric impact of absolute risk aversion which has equally been discussed above.