Preparation Midterm Exam

CS236 - Discrete Structures
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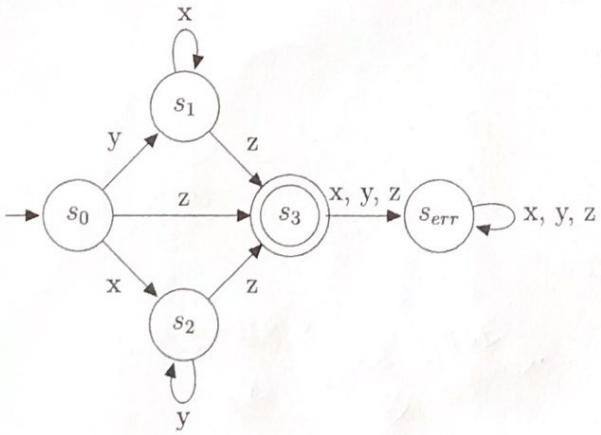
Fall 2021, All Sections

Instructions

Complete every problem using whatever resources will best help you learn. See the course announcement on LS for instructions on how the *preparation exam* questions relate to the *actual exam* questions.

Question 1

Consider the following Finite State Automaton (FSA):



What is the regular expression that defines all valid strings in the accepted language for the above FSA?

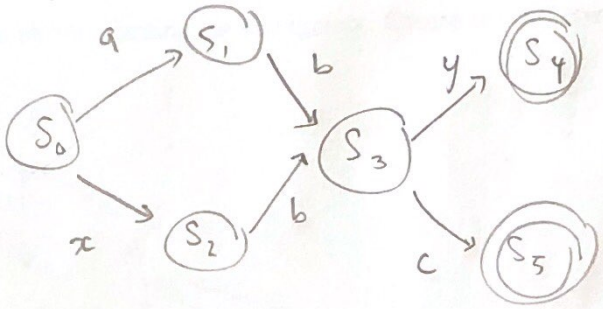
$(yx^* \cup xy^*)z$

$((y \wedge x^*) \cup (x \wedge y^*)) \wedge z$

Question 2

Consider the following Regular Expression: $(a \cup x)b(y \cup c)$

Draw the corresponding FSA for the above regular expression. Show all necessary states and transitions. The FSA must accept all strings in the language generated by the above regular expression and most not accept any string not generated by the above regular expression.

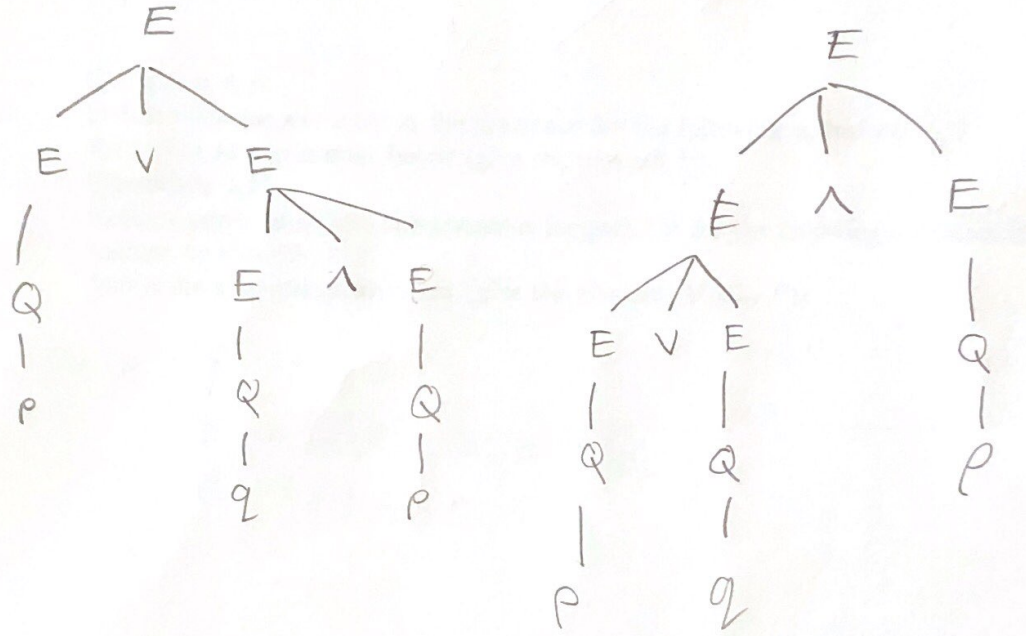


Question 3 (25 pts)

Consider the following grammar (production labels follow each production):

$V = \{E, Q, p, q, r, \wedge, \vee\}$
$T = \{p, q, r, \wedge, \vee\}$
$S = E$
$P = \{ \quad E \to Q \mid E \wedge E \mid E \vee E$
$\quad\quad Q \to p \mid q \mid r \}$

The above grammar is ambiguous. Create two different parse-trees for the string $p \vee q \wedge p$.



Question 4

Perform the following two steps to modify the grammar to remove ambiguity and enforce precedence.

$V = \{E, Q, p, q, r, \wedge, \vee\}$
$T = \{p, q, r, \wedge, \vee\}$
$S = E$
$P = \{$
$\quad E \to Q \mid E \wedge E \mid E \vee E$
$\quad Q \to p \mid q \mid r$
$\}$

Question 4.A
Enforce left-associativity in the grammar for the following operators: $\wedge, \vee$
Write the new grammar below (give the new set P):
Question 4.B
Enforce precedence into the grammar for part 4.A for the following operators (in order from highest to lowest): $\wedge, \vee$
Write the new grammar below (give the new sets V and P):

(A) $P = \{$
$\quad E \to Q \mid E \wedge Q \mid E \vee Q$
$\quad Q \to p \mid q \mid r$
$\}$

(B) $P = \{$ $\qquad V = \{E, A, Q, p, q, r, \wedge, \vee\}$
$\quad E \to A \mid E \wedge A$
$\quad A \to Q \mid E \vee Q$
$\quad Q \to p \mid q \mid r$
$\}$

Question 5

Use the grammar below for this question:

$V = \{A, B, !, a, b, c\}$
$T = \{!, a, b, c\}$
$S = A$
$P = \{$
$A \rightarrow aBB$
$B \rightarrow b \mid !c \mid a!a$
$\}$

FIRST(A) = {a}
FIRST(B) = {b, !, a}

Question 5.A

For the parse table below, fill in the first row with the symbols that can appear in the parser input and the first column with the symbols that can appear in the parser stack.

Question 5.B

Fill in the cells that need values to complete the parse table below (leave blank if there is no possible production; ignore cells with dotted lines):

-------	!	a	b	c	#
A		aBB			------
B	!c	a!a	b		------
!	AdPop				------
a		AdPop			------
b			AdPop		------
c				AdPop	------
#	------	------	------	------	accept

c)

Stack	Input	Action
A#	↑a!cb#	B → aBB
aBB#	↑a!cb#	AdPop
BB#	a↑!cb#	B → !c
!cB#	a↑!cb#	AdPop
cB#	a!↑cb#	AdPop
B#	a!c↑b#	B → b
b#	a!c↑b#	AdPop
#	a!cb↑#	Accept.

Question 5.C

Using the parse table created in 5.B, write the trace for the following word: $a!cb$

Stack	Input
$A\#$	$\uparrow a!cb\#$

Answered on previous page.

Question 6

Consider the following grammar: non-terminals $N = \{S, A, D, T\}$, terminals $T = \{a, b, 0, 1\}$, the start non-terminal S, and the following productions:

$$\begin{aligned} &S \rightarrow aAb \mid bAa \\ &A \rightarrow DT \\ &T \rightarrow A \mid \lambda \\ &D \rightarrow 0 \mid 1 \end{aligned}$$

Question 6.A
Give the FIRST set for $A \rightarrow DT$.

FIRST $(A \rightarrow DT) = \{0, 1\}$

Question 6.B
Give the FOLLOW set for T.

FOLLOW $(T) = \{a, b\}$

Question 7

Create the truth table to show Conditional-Disjunction Equivalence is a tautology: $(p \to q) \leftrightarrow (\neg p \vee q)$

p	q	$p \to q$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Question 8

Consider the following set of premises:

1. e
2. $\ell \vee \neg s$
3. $s \vee \neg r$
4. $r \vee \neg h$
5. $h \vee \neg m$
6. $m \vee \neg g$
7. $g \vee \neg e$

Use Proof by Contradiction with Resolution to prove ℓ.

8. $\neg \ell$ negate the conclusion
9. $\neg s$ resolution on 8 & 2
10. $\neg r$ resolution on 9 & 3

...

13. $\neg g$ resolution on 12 & 6
14. $\neg e$ resolution on 13 & 7
15. F resolution on 14 & 1

This is a contradiction
$\therefore \ell$

Question 9

Use the following predicates, $P(x)$, $Q(x)$, and $R(x)$ and their truth tables to answer the questions below (the domain is $\{1, 2, 3, 4, 5\}$):

$P(x)$	Value
$P(1)$	T
$P(2)$	F
$P(3)$	T
$P(4)$	F
$P(5)$	F

$Q(x)$	Value
$Q(1)$	T
$Q(2)$	T
$Q(3)$	T
$Q(4)$	T
$Q(5)$	T

$R(x)$	Value
$R(1)$	T
$R(2)$	T
$R(3)$	F
$R(4)$	T
$R(5)$	T

Question 9.A

Given the tables above, what is the following: $Q(4) \wedge R(4)$ - True or False (circle one)

Question 9.B

Given the tables above, what is the following: $P(1) \vee P(5)$ - True or False (circle one)

Question 9.C

Given the tables above, what is the following: $\exists x P(x)$ - True or False (circle one)

Question 9.D

Given the tables above, what is the following: $\forall y Q(y)$ - True or False (circle one)

Question 9.E

Identify a value of x that shows the following is true:
$\exists x[Q(x) \wedge R(x) \rightarrow P(x)]$

$a \rightarrow b \quad (!a) \vee (b)$

when $x = 1$

$Q(1) \wedge R(1) \rightarrow P(1)$

$T \wedge T \rightarrow T$

T

QED.

Question 10

Question 10.A

Consider the following sets A, B, C, and D:

$A = \{a, b, c, d, f, g, h\}$
$B = \{b, c, d, e, f, g, h, i\}$
$C = \{a, b, c, d, e, f, g, h\}$
$D = \{a, b, c, d, e, g, h\}$

Which of the following is true?

$A \subseteq B$ False
$A \subseteq C$ True
$B \subseteq C$ False
$B \subseteq A$ False

Question 10.B

What is the power set of A, $\mathcal{P}(A)$, given that $A = \{a, b, c\}$?

$\mathcal{P}(A) = \{\emptyset, \{a, b, c\}, \{a, b\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \{a, c\}\}$

Question 10.C (10 pts)

Given A and B what is the result of the operation $A - B$?

$A = \{a, b, c, e, f, g\}$
$B = \{b, e, g, i, k\}$

$A - B = \{a, c, f\}$

Question 10.D

Given A and B what is the result of the operation $A \cap B$?

$A = \{a, b, c, e, f, g\}$
$B = \{b, e, g, i, k\}$

$A \cap B = \{b, e, g\}$

Question 11

Consider the following relations:

Q:

A	B
5	1
6	1
4	2
3	4

R:

B	C
1	4
2	4
2	5
3	6
3	9

S:

C	D	E
4	1	1
4	2	1
3	3	2
2	4	2

T:

C	D
1	2
2	4

Question 11.A Evaluate $\sigma_{A<4}\ Q \times R$
Question 11.B Evaluate $\pi_A\ \sigma_{B<A}\ Q$
Question 11.C Evaluate $\pi_{BE}\ (\rho_{C \leftarrow A}\ R \times \pi_{CE}\ S)$
Question 11.D Evaluate $T \cup \pi_{CD}\ S$
Question 11.E Evaluate $\pi_{CDE}\ (\sigma_{B<2}\ R \times \rho_{C \leftarrow E}\ T) - S$
Question 11.F Evaluate $R|\times|S|\times|T$

(A)

$(\sigma_{A<4}\ Q) \times R$

A	B
3	4

× R =

A	B	C
3	1	4
3	2	4
3	2	5
3	3	6
3	3	9

UNDEFINED BC BOTH HAVE B columns.

(B)

$\pi_A(\sigma_{B<A}\ Q)$

$\sigma_{B<A}\ Q =$

A	B
5	1
6	1
4	2

$\pi_A(\sigma_{B<A}\ Q) =$

A
5
6
4

(C) $\pi_{BE}\ (\rho_{C \leftarrow A} R \times \pi_{CE} S)$

$\rho_{C \leftarrow A} R =$

B	A
1	4
2	4
2	5
3	6
3	9

$\pi_{CE} S =$

C	E
4	1
4	1
3	2
2	2

$\pi_{BE}(\leftarrow) =$

B	E
1	1
1	2
2	1
2	2
3	1
3	2

(D) $T \cup \pi_{CD} S$

T =

C	D
4	1
4	2
3	3
2	4
1	2

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Q11)

(E) $\pi_{CDE}(\sigma_{B<2}R \times \rho_{C\leftarrow E T}) - S$

$\sigma_{B<2}R =$

B	C
1	4

$\rho_{C\leftarrow E T} =$

E	D
1	2
2	4

$X =$

B	C	E	D
1	4	1	2
1	4	2	4

$\pi_{CDE}(\leftarrow) =$

C	D	E
4	2	1
4	4	2

$- S =$

C	D	E
4	4	2

Q12) $R \bowtie S \bowtie T = (R \bowtie S) \bowtie T$

$R \bowtie S =$

B	C	D	E
1	4	1	1
2	4	2	1

$\bowtie T =$

B	C	D	E