Grammars: Table-Driven Parsing

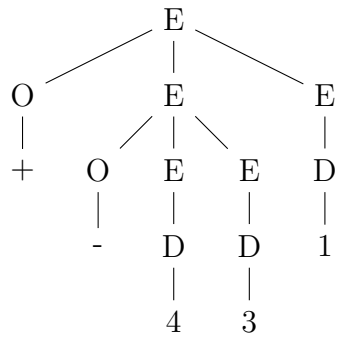
CS236 - Discrete Structures
Instructor: Brett Decker
Fall 2020

Systematic Parsing

We have not yet discussed a systematic way to create parse-trees. Having an algorithm that defines a systematic approach is vital in order for us to design code to create parse-trees. A naive approach would be to use a full search algorithm: create all possible parse-trees and see if any matches the terminal string we're trying to parse. This would be computationally expensive and inefficient. Let's develop an algorithm to automate parsing. To do this, let's consider the following grammar (it represents arithmetic expressions in prefix, or Polish, notation, see Section 11.3.4*):

$G = (V, T, S, P)$ where
$N = \{E, O, D\}$
$T = \{0, 1, 2, \ldots, 9, +, -, *, /\}$
$S = E$
$P = \{$
$E \rightarrow D \mid OEE$
$O \rightarrow + \mid - \mid * \mid /$
$D \rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9$
$\}$

Let's create the parse tree for the terminal string '+ - 4 3 1':



We evaluate from bottom to top, so we evaluate '4 - 3' first, and then the result, 1 added with '1', thus '1 + 1 = 2.' Therefore, our prefix notation terminal string '+ - 4 3 1' evaluates to 2.

Is there a systematic way to create this parse-tree? How did we know to choose the production rule $E \rightarrow OEE$ instead of $E \rightarrow D$? We looked at the first terminal in our terminal

string, which is '+.' Then we look to see which production rule would lead to the same terminal. That's the rule we choose. Thus we see the start of our algorithm. We check if we can match the current symbol of our terminal string with a producible terminal. If so, we choose that production. If the current nonterminal cannot produce the current symbol, then we must choose a production with a starting nonterminal. In order to choose the correct production, we have to look ahead at what each possible nonterminal produces. When we find a match with our current symbol, we've found the production. Let's make this concrete with our example above.

Our first symbol in our terminal string is '+.' Starting with our start symbol, E, we check if E has a production that starts with '+.' It does not. Now we must look at the productions from E that start with a nonterminal. Our choices are O and D. Now we check to see if either O or D has a production that starts with '+.' O does; D does not. Thus we choose the production OEE. We continue this process until we have parsed the entire terminal string.

LL Grammars

What happens if there are more than one possible production rule that produces the same terminal? We would have to guess at which production to take and backtrack if we later find out the production was the wrong one. Backtracking is expensive, so we'd like to avoid this. There is a subset of grammars, called *LL grammars* that do not require backtracking. An LL grammar is a special context-free grammar that can be parsed with an *LL parser*. An LL parser parses the input from left to right and produces a leftmost derivation (thus LL) without backtracking (note: a leftmost derivation is a strategy where the leftmost nonterminal is chosen as the next nonterminal to rewrite). An LL(k) parser is able to parse an LL grammar with just k look-ahead characters. That means it only needs to know the next k terminals, starting from the left in order to pick the correct next production rule. What is k for the prefix notation grammar, G?

$G = (V, T, S, P)$ where
$N = \{E, O, D\}$
$T = \{0, 1, 2, \ldots, 9, +, -, *, /\}$
$S = E$
$P = \{$
$E \to D \mid OEE$
$O \to + \mid - \mid * \mid /$
$D \to 0 \mid 1 \mid 2 \mid \ldots \mid 9$
$\}$

The prefix grammar above is an LL(1) grammar, thus $k = 1$, because we only have to look at the first character of the terminal string to choose the correct production rule. Let's look at another example. Starting at the start symbol, E, what production should be chosen if the terminal string is '6'?

E does not produce '6' directly, so we must look at the first nonterminal of each production.

D produces '6' (and thankfully O does not – if it did, we wouldn't have an LL(1) grammar). So we must choose $E \rightarrow D$. Note that for all digits, we choose this production, and that for all operators we choose $E \rightarrow OEE$.

FIRST Sets

By definition, with LL(1) grammars we only have to look at the first character of the terminal string to choose the correct production. Let us formalize this. We say that the $FIRST$ set of a nonterminal is the set of all first terminals it can *eventually* produce (since we are dealing with leftmost derivations, we mean the first, leftmost terminal produced by any current or future production). Consider again the LL(1) grammar we saw above:

$G = (V, T, S, P)$ where
$N = \{E, O, D\}$
$T = \{0, 1, 2, \ldots, 9, +, -, *, /\}$
$S = E$
$P = \{$
$E \rightarrow D \mid OEE$
$O \rightarrow + \mid - \mid * \mid /$
$D \rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9$
$\}$

The $FIRST$ set of the nonterminal D, denoted as $FIRST(D)$ is $\{0, 1, 2, \ldots, 9\}$, since those are the first, leftmost terminals that can be produced from the nonterminal D. $FIRST(O) = \{+, -, *, / \}$. What is the $FIRST(E)$? There are no terminals in any production rule when E is replaced. But starting from E the first nonterminals produced are either D or O. To find the $FIRST(E)$ we simply take the union of the $FIRST(O)$ and the $FIRST(D)$. Thus $FIRST(E) = FIRST(O) \cup FIRST(D) = \{+, -, *, /, 0, 1, 2, \ldots, 9\}$.

Parse Tables

Because LL(k) grammars can be systematically parsed by an algorithm, they can make use of *parse tables* for efficient parsing. A parse table contains the label for which production to use for every possible terminal character at every point in the parsing. It contains both the new nonterminals and terminals to produce, and the label for the production in each cell. Recall our prefix notation grammar production rules shown below (we'll reduce the operators and digits for clarity):

$P = \{$
$E \rightarrow D_{(1)} \mid OEE_{(2)}$
$O \rightarrow +_{(3)} \mid *_{(4)}$
$D \rightarrow 0_{(5)} \mid 1_{(6)} \mid 2_{(7)} \mid 3_{(8)}$
$\}$

Notice that we have added a numeric label to each production. We use these labels, and the $FIRST$ sets to build our parse table:

	+	*	0	1	2	3	#
E	(OEE, 2)	(OEE, 2)	(D, 1)	(D, 1)	(D, 1)	(D, 1)	
O	(+, 3)	(*, 4)					
D			(0, 5)	(1, 6)	(2, 7)	(3, 8)	
+	AdPop						
*****		AdPop					
0			AdPop				
1				AdPop			
2					AdPop		
3						AdPop	
#							Accept

To build the parse table we create a column for every terminal and the extra symbol # (we'll explain the pound – no, it's not a hashtag – shortly). We create a row for each symbol in the vocabulary (all nonterminals and terminals) and the extra symbol #. Keep the order of the terminal consistent for both rows and columns (just as done above).

How do we use the table? Note the 'AdPop' and 'Accept' entries in the table. These are present because we use the table with a pushdown automaton. Recall that a pushdown automaton (PDA) makes use of a stack. Before we start parsing a terminal string, we will push a pound symbol ('') to the stack. Each cell in the parse table tells the machine what to do next. The empty cells tell the PDA to reject the terminal string. The cells with 'AdPop' tell the PDA to pop the top element off the stack and advance the input. The PDA only accepts the terminal string if the cell with 'Accept' is reached. There is one more type of cell command in our table. Let's give a concrete example to explain how it works. Consider the cell for row E, column +, which is $(OEE, 2)$. This cell tells the PDA that if we are parsing the nonterminal E and see a '+' as the current character (in the terminal string we are parsing), then we replace the E with OEE and output '2.' Notice that this corresponds to the production rule labeled 2 in our grammar: $E \to OEE_{(2)}$.

Parse Table Example:

Let's do an example. We will use the parse table above to parse the terminal string '*+123' (the input to our PDA). We call this a *trace* (the up arrow, ↑, will keep track of where we are in the input string). The output column is the production label used at each step – the pop action does not have a label, thus it does not affect the output.

Action	**Stack**	**Input**	**Output**
Initialize, push '#' and start symbol	$E\#$	$\uparrow * + 123\#$	
Action$(E, *)$=Replace$[E, OEE]$	$OEE\#$	$\uparrow * + 123\#$	2
Action$(O, *)$=Replace$[O, *]$	$*EE\#$	$\uparrow * + 123\#$	24
Action$(*, *)$=AdPop	$EE\#$	$*\uparrow + 123\#$	24
Action$(E, +)$=Replace$[E, OEE]$	$OEEE\#$	$*\uparrow + 123\#$	242
Action$(O, +)$=Replace$[O, +]$	$+EEE\#$	$*\uparrow + 123\#$	2423
Action$(+, +)$=AdPop	$EEE\#$	$*+\uparrow 123\#$	2423
Action$(E, 1)$=Replace$[E, D]$	$DEE\#$	$*+\uparrow 123\#$	24231
Action$(D, 1)$=Replace$[D, 1]$	$1EE\#$	$*+\uparrow 123\#$	242316
Action$(1, 1)$=AdPop	$EE\#$	$*+1\uparrow 23\#$	242316
Action$(E, 2)$=Replace$[E, D]$	$DE\#$	$*+1\uparrow 23\#$	2423161
Action$(D, 2)$=Replace$[D, 2]$	$2E\#$	$*+1\uparrow 23\#$	24231617
Action$(2, 2)$=AdPop	$E\#$	$*+12\uparrow 3\#$	24231617
Action$(E, 3)$=Replace$[E, D]$	$D\#$	$*+12\uparrow 3\#$	242316171
Action$(D, 3)$=Replace$[D, 3]$	$3\#$	$*+12\uparrow 3\#$	2423161718
Action$(3, 3)$=AdPop	$\#$	$*+123\uparrow \#$	2423161718
Action$(\#, \#)$=Accept		$*+123\#\uparrow$	Done!

Let's run another trace for an invalid terminal string '1+2':

Action	**Stack**	**Input**	**Output**
Initialize, push '#' and start symbol	$E\#$	$\uparrow 1 + 2\#$	
Action$(E, 1)$=Replace$[E, D]$	$D\#$	$\uparrow 1 + 2\#$	1
Action$(D, 1)$=Replace$[D, 1]$	$1\#$	$\uparrow 1 + 2\#$	16
Action$(1, 1)$=AdPop	$\#$	$1\uparrow +2\#$	16
Action$(\#, +)$=Reject	$\#$	$1\uparrow +2\#$	Reject!

This terminal string will be rejected because the row pound, column '+' is empty – there is no possible production to use for this input.

Conclusion

Using a table for parsing is inefficient in code. There is a better algorithm for parsing LL(k) grammars: recursive-descent parsing. We will study this next. Parsing grammars is important in the fields of programming languages, program analysis, and formal verification. Thanks to Dr. Michael Goodrich for the prefix notation grammar and examples.

**Discrete Mathematics and Its Applications*, by Kenneth H. Rosen.