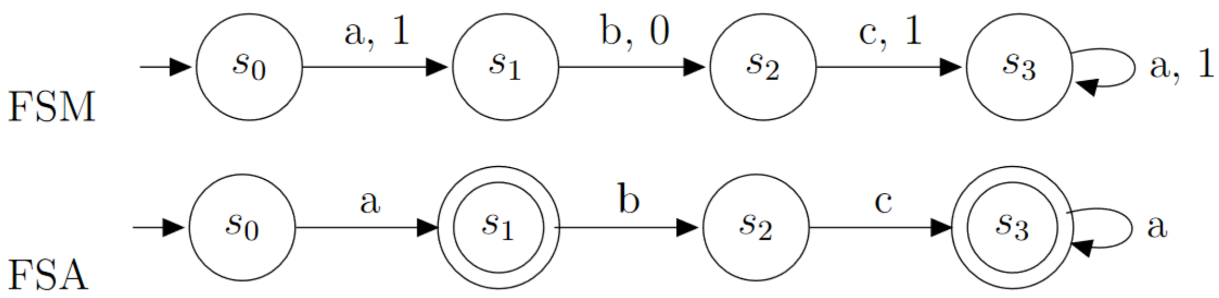
Finite-State Automata - 9/8/2021

Definition

A finite-state automaton (automata for plural form) is a kind of finite-state machine that does not include outputs for each transition, but rather has a list of final states. I.e. a *finite-state automaton* $M = (S, I, f, s_0, F)$ consists of the following:

- a finite set S of **states**
- a finite **input alphabet** I
- a **transition function** f that assigns each state and input pair a new state
- an **initial state** s_0
- (the difference between FSM and FSA) a set of **final states** $F \subseteq S$

Think of F as the set of all transitions whose output to get there is a 1 in a **Mealy machine**. Instead of using output, we just say that the machine recognizes a string that ends on that state. In a diagram, we'll represent a final state $s_f \in F$ with two circles around the state instead of one. E.g. see the following diagram:



To translate our FSM to an FSA, $S = \{s_0, s_1, s_2, s_3\}, I, f$, and s_0 all remain the same, but we now have $F = \{s_1, s_3\}$.

Language Recognition

Definition

To formalize our definition:

A string x is said to be *recognized* or *accepted* by the FSA $M = (S, I, f, s_0, F)$, if it takes the initial state s_0 to a final state. That is, $f(s_0, x)$ is a state in F. The **language recognized** by the machine M, denoted $L(M)$ is the set of all strings that are recognized by M.

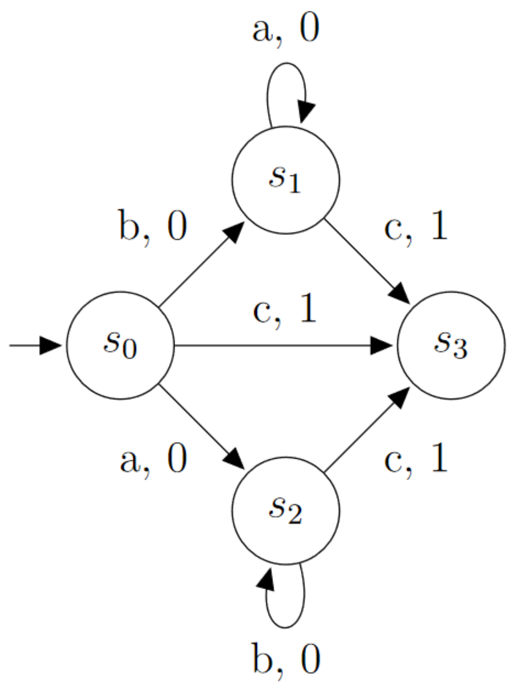
Finite-State Automata Equivalence

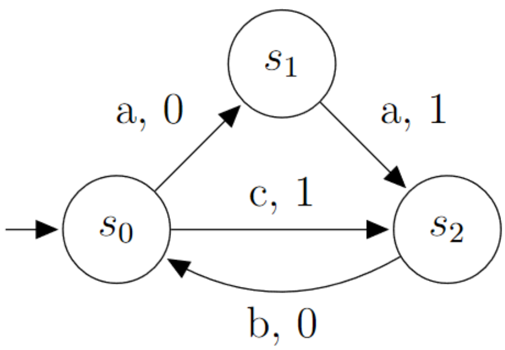
Definition

Same as for FSMs; two FSAs are **equivalent** if they recognize the same language.

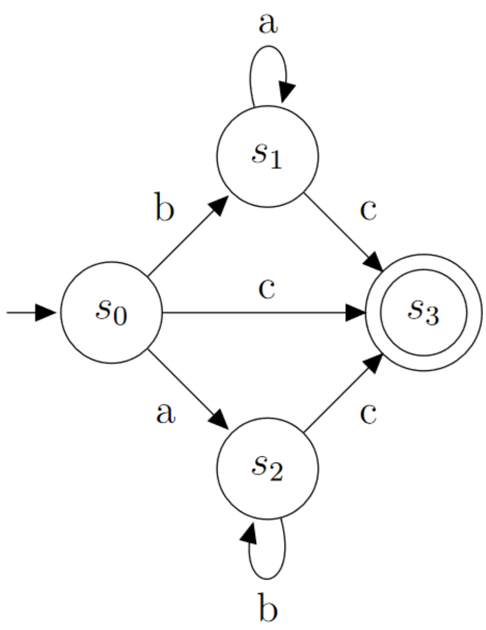
More examples:

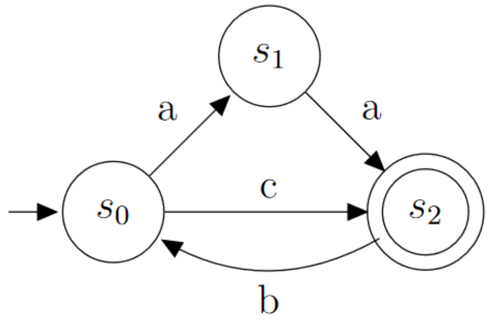
FSM:





FSA:


a
s_1
b
c
c
s_0
s_3
a
c
s_2
b


s_1
a
a
c
s_0
s_2
b