FEM21011 Assignment 1: Arbitrage Trading Strategies

Luca Leimbeck Del Duca (591608), Antoni Guàrdia Sanz (779906),
Melvin Bazeille (779913), Panagiotis Makris (696297)

Part A: Dual Listing

Question 1

(i) In this exercise, we are asked to check whether an arbitrage opportunity exists between the stock and its dual listing. In principle, their prices should be identical since both represent the same ownership rights. If prices diverge, a trader can profit by buying the cheaper instrument (long at best ask) and selling the more expensive one (short at best bid). Over time, the expectation is that prices will converge.

Using the provided book snapshots, the stock book quotes 19.40 (bid) @ 19.60 (ask), while the dual listing shows 19.00 @ 19.30. Since the stock bid exceeds the dual ask, an arbitrage opportunity arises. The maximum tradable size is limited by depth on the dual ask, which shows 50 units available. One can therefore buy 50 units of the dual at 19.30 and simultaneously sell 50 units of the stock at 19.40. The profit is:

$$\pi = (19.40 - 19.30) \cdot 50 = 0.10 \cdot 50 = 5.0\,.$$

In theory, this is risk-free because both orders are executed simultaneously, removing exposure to price movements. In practice, however, orders must be placed sequentially. To minimize execution risk, the less liquid dual listing should be traded first, as fills are less certain there. The more liquid stock market is more likely to provide a fill even with a slight delay. Using an immediate-or-cancel (IOC) order ensures only the available 50 units are executed. After this trade, the next dual ask moves to 19.40, equal to the stock bid, eliminating further arbitrage opportunities.

(ii) We now extend the analysis by deriving a trading rule based on the mid-points of the two order books. Let b_S, a_S denote the best bid and ask in the stock book, and b_D, a_D those in the dual listing. For exchange $i \in S, D$, define the mid-point and the spread as:

$$m_i = \frac{a_i + b_i}{2}, \qquad s_i = a_i - b_i\,.$$

A natural starting point is the mid-point rule: buy on the exchange with the lower mid-point and sell on the one with the higher mid-point. However, since mid-points are not directly tradable, execution requires crossing the spread. A buy executes at the ask (half a spread above the mid-point), and a sell at the bid (half a spread below the mid-point). Hence, arbitrage only arises if the mid-point difference is large enough to overcome half-spreads and transaction costs. Formally:

$$m_S - m_D > \frac{s_S + s_D}{2} + (c_S + c_D) \quad \Leftrightarrow \quad b_S - a_D > c_S + c_D,$$

$$m_D - m_S > \frac{s_S + s_D}{2} + (c_S + c_D) \quad \Leftrightarrow \quad b_D - a_S > c_S + c_D,$$

where c_S and c_D denote transaction costs in the stock and dual exchange, respectively (here assumed to be zero). If the first condition holds, one sells the stock at b_S and buys the dual at a_D. If the second holds, one sells the dual at b_D and buys the stock at a_S. Applying this to the current books, we obtain:

$$b_S - a_D = 19.40 - 19.30 = 0.10 > 0.$$

Therefore one can buy the dual at 19.30 and sell the stock at 19.40, consistent with (i). Note that, as shown, using mid-points and spreads would yield the same end result.

(iii) Suppose transaction fees are $c_S = c_D = 0.05$ per share. Then, for the above strategies, where we buy on one exchange and sell on the other, the total per-share transaction cost becomes $c_S + c_D = 0.10$. This exactly offsets the gross arbitrage profit of $19.40 - 19.30 = 0.10$. Similarly, the mid-point condition becomes:

$$b_S - a_D = 19.40 - 19.30 = 0.10 \not> 0.10,$$

so the trading signal vanishes. Thus, once costs are included, the strategy ceases to be profitable.

(iv) Now assume trading is possible only in the dual listing D. In this case, strict arbitrage is no longer possible. Instead, other market participants who can access both exchanges will exploit arbitrage opportunities, pushing D upward relative to S and closing the gap within transaction costs. Therefore, we can take a long position in D when it appears undervalued, and exit once prices converge.

For example, suppose $c_D = 0.05$. At time t_0 the books are as in part (i). Since $a_D^{(t_0)} < b_S^{(t_0)}$, D appears undervalued. Therefore, we buy 50 shares in D at 19.30. By time t_1, prices converge towards the fair value and the dual book improves to 19.40 @ 19.50 with sufficient depth. Selling 50 shares at 19.40 yields a gross profit per share of:

$$b_D^{(t_1)} - a_D^{(t_0)} = 19.40 - 19.30 = 0.10.$$

Accounting for costs on both entry and exit, net profit is zero. In other words, the cost-aware condition derived in subquestion (ii) is not met.

Unlike the earlier case, this strategy carries risk. While waiting for convergence, an adverse move in the underlying stock or in market sentiment could lower the fair value, leaving us with a loss. This is called positional risk.

(v) Such positional risk can lead to significant losses. To mitigate it, several approaches are available:

- **Direct hedge:** If the stock is tradable, short (long) the stock against the long (short) dual position. Under equal economic exposure (same currency and 1:1 equivalence), this corresponds to a 1:1 hedge, leading to a delta-neutral position.

- **Derivative hedge:** If the stock is inaccessible, open offsetting positions in liquid derivatives (e.g., single-stock futures or options), targeting delta neutrality. This reduces positional risk but introduces basis risk (futures) or option sensitivities (vega, gamma, theta).

- **Stop-losses:** If neither stocks nor derivatives are tradable, risk must be managed via stop-loss orders (e.g., limit orders that trigger a sale at a given stop price below entry). While not a true hedge, this caps downside in line with risk tolerance.

Question 2

Before turning to the algorithmic implementation, we specify our testing framework. Each trade is executed in a fixed lot size of one unit per instrument, and the algorithm iterates once every 0.2 seconds. Performance is assessed over runs of 30 minutes. At every iteration we enforce exchange constraints: tick size of 0.10, maximum position of ± 100 lots per instrument, ≤ 200 outstanding orders, ≤ 25 order updates per second, and we start each sub-question from a flat book.

To evaluate results, we employ: total traded volume, total PnL, PnL per trade, mean and standard deviation of 1-minute PnL increments, and the Sharpe ratio. The Sharpe ratio is constructed as follows. Let $\Delta\pi_t$ denote the sequence of 1-minute PnL increments, with sample mean μ and standard deviation σ. Assuming 252 trading days per year and 390 minutes per trading day, the annualized Sharpe ratio is:

$$\text{Sharpe}_{\text{annual}} = \text{Sharpe}_{1min} \cdot \sqrt{252 \times 390} = \frac{\mu}{\sigma} \cdot \sqrt{252 \times 390}.$$

Annualization aligns with industry practice, ensuring direct comparability with benchmarks in both literature and practice.

(i) We now implement the arbitrage rule derived in A1(ii). In the absence of transaction costs, the entry signals simplify to:

$$b_S > a_D \Rightarrow \text{Buy dual (undervalued)}, \qquad a_S < b_D \Rightarrow \text{Short dual (overvalued)}.$$

In addition to entry signals, it is important to specify how positions are closed. Rather than holding until the opposite signal arises, it is more prudent to exit once convergence has taken place. At that point, the price discrepancy between the two order books has disappeared, and any remaining exposure simply carries positional risk without expected reward. Denoting by p_D the outstanding dual position, the exit conditions are:

$$p_D < 0 \wedge a_S \geq a_D \Rightarrow \text{Buy dual to cover short}, \qquad p_D > 0 \wedge b_S \leq b_D \Rightarrow \text{Sell dual to close long}.$$

This completes the definition of the strategy. As mentioned above, all trades are submitted as IOC orders. In this question, only two instruments are tradable, namely `ASML_DUAL` and `SAP_DUAL`. Table 1 summarizes the performance of this strategy.

Table 1: Performance of baseline strategy (Stock listing not tradable).

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
542.4	18.1	45.2	0.40	12.40

(ii) The baseline implementation from part (i) trades only the dual listing and is therefore exposed to delta risk. As discussed in A1(v), this is dangerous because although we profit from short-term mispricings, an uncovered dual position is still sensitive to large moves in the underlying stock. If both the stock and the dual drift strongly upward or downward, our PnL can deteriorate sharply.

To mitigate delta risk, we extend the baseline algorithm by directly hedging dual trades with offsetting positions in the stock. Concretely, at each iteration, we first check for arbitrage opportunities in the dual as in part (i). If an order is executed in the dual, we immediately attempt to offset the resulting exposure by sending an order in the stock at the best available price. Formally, if the combined exposure across dual and stock is $p = p_D + p_S$, then $p > 0$ triggers a sell of one unit in the stock, and $p < 0$ triggers a buy of one unit in the stock. If $p \neq 0$ in the next iteration, we retry the hedge. This ensures positions remain delta-neutral.

Because the dual is less liquid, we always trade it first and only attempt the hedge if execution is confirmed. This strategy is expected to lower the average PnL, since hedge trades may execute at less favorable prices. At the same time, it should reduce return volatility by neutralizing directional exposure. As can be seen in Table 2, these expectations are correct. However, the Sharpe ratio has been reduced, indicating that the reduction in volatility is not substantial enough to offset the loss in PnL.

Table 2: Performance with delta-hedging.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
411.7	13.7	36.1	0.38	119.13

(iii) As mentioned above, execution risk must be considered. Even if an arbitrage signal is present, IOC orders only trade against currently available liquidity. If another participant consumes the liquidity first (for example due to better latency), our order expires. This is particularly problematic for the less liquid dual listing, though even in the stock market trades can occasionally fail. Therefore we:

- Verify dual fill before hedging, to avoid opening stock positions unnecessarily.
- Since hedge orders can fail, after each iteration, check if $p_D + p_S = 0$. If not, hedge did not execute. Hence, the hedge order is resubmitted in the next iteration.

This sequencing slightly slows down the algorithm but ensures that the hedge truly neutralizes the exposure. Therefore, the main consequence is missed profitable opportunities rather than incorrect hedging.

(iv) So far, arbitrage orders were placed exactly at the dual's best bid or ask. While optimal in price, these are less likely to execute, particularly under thin liquidity. To increase fill probability, we use less aggressive quotes (i.e., prices that are more likely to execute immediately). Formally, we adjust our submission prices as follows:

$$\text{Price if dual undervalued} = \max\big(a_D,\ b_S - n * \delta\big), \qquad \text{Price if dual overvalued} = \min\big(b_D,\ a_S + n * \delta\big),$$

where $\delta = 0.01$ (one tick) and n is an integer. The idea is to sacrifice a small part of the spread to (substantially) increase execution rates, thereby leading to more frequent, albeit individually less profitable, trades. Multiple values of n were tested to locate the best performing. The lower the value of n, the higher the percentage of trades that are priced at the potentially suboptimal prices $b_S - n * \delta$ or $a_S + n * \delta$. This percentage is noted as A. Table 3 summarizes the performance of this strategy.

Table 3: Performance metrics for different values of $n * \delta$

$n * \delta$	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)	A
0.1	17.2	27.6	0.62	194.37	100%
0.2	17.1	30.8	0.56	175.56	66%
0.3	19.7	37.3	0.53	166.15	47%
0.4	16.4	39.2	0.42	131.67	18%

As the percentage A approaches 0%, this algorithm converges to the algorithm used in part iii, with all trades occurring at optimal prices. However, it can be seen that lowering $n * \delta$ and therefore increasing A is strictly an improvement. As such, we can conclude that it is always best to insert orders with the lowest possible profit margin. Trading successfully as frequently as possible is more important than extracting as much value from each trade, due to the high speed with which bids and asks are fulfilled.

Part B : Stock vs Futures

Question 1

(i) In this exercise, we examine whether an arbitrage opportunity exists between the stock and its futures contract. The stock book quotes 19.20 @ 19.40, while the futures book quotes 19.70 @ 20.00. To compare these directly, we apply the no-arbitrage relationship to convert futures prices into synthetic stock equivalents (one could equivalently convert stock prices into synthetic futures). Using the cost-of-carry relation $F = Se^{r\tau}$ with $r = 0.03$ and $\tau = 0.5$, , the best synthetic stock bid $\tilde{b}_S$ and ask $\tilde{a}_S$ become:

$$\tilde{b}_S = 19.70 \cdot e^{-0.03 \cdot 0.5} \approx 19.4067, \qquad \tilde{a}_S = 20.00 \cdot e^{-0.03 \cdot 0.5} \approx 19.7022,$$

respectively. Since $\tilde{b}_S > a_S = 19.40$, the future is slightly overpriced relative to spot. Therefore, one can arbitrage by shorting the future at 19.70 and buying the stock at 19.40. Given the exchange position limit of 100 contracts per instrument, the profit is:

$$\pi = (19.4067 - 19.40) \cdot 100 \approx 0.67.$$

Although positive, this margin is negligible. Once fees, slippage, and latency are considered, the opportunity is likely n exploitable in practice.

(ii) We generalize by constructing a combined stock–futures order book. The idea is to apply the no-arbitrage pricing relationship not just at the top of book, but to the entire futures order book, producing a synthetic stock book for direct comparison. Let $\tilde{\mathbf{b}}_\mathbf{S}$ and $\tilde{\mathbf{a}}_\mathbf{S}$ denote synthetic futures bid/ask vectors, respectively. The, these are computed as:

$$\tilde{\mathbf{b}}_\mathbf{S} = \mathbf{b}_\mathbf{F} \cdot e^{-0.03 \cdot 0.5}, \qquad \tilde{\mathbf{a}}_\mathbf{S} = \mathbf{a}_\mathbf{F} \cdot e^{-0.03 \cdot 0.5},$$

where $\mathbf{b}_\mathbf{F}$ and $\mathbf{a}_\mathbf{F}$ represent futures bid/ask vectors. Table 4 reports the actual stock book alongside the synthetic stock (futures-based) book.

Table 4: Combined stock and synthetic stock (futures based) order book.

Stock Order Book		
# Bid	Price	# Ask
	19.90	7000
	19.80	6000
	19.70	5000
	19.60	4000
	19.50	3000
	19.40	1500
	19.30	
2000	19.20	
3000	19.10	
2000	19.00	
3000	18.90	
4000	18.80	

Synthetic Stock (Futures) Order Book (T = 0)			
# Bid	Synthetic Price	Futures Price	# Ask
	19.8993	20.20	600
	19.8007	20.10	500
	19.7022	20.00	400
	19.6037	19.90	
	19.5052	19.80	
150	19.4067	19.70	
200	19.3082	19.60	
	19.2097	19.50	
300	19.1112	19.40	
400	19.0127	19.30	
500	18.9141	19.20	
600	18.8156	19.10	

Let c_F denote the per-unit transaction cost in the futures exchange. Comparing the stock order book with the synthetic stock (futures-based) order book, arbitrage occurs if the following conditions hold:

$$\tilde{b}_S - a_S > c_F + c_S \quad \Leftrightarrow \quad b_F \cdot e^{-r\tau} - a_S > c_F + c_S\,,$$

$$b_S - \tilde{a}_S > c_F + c_S \quad \Leftrightarrow \quad b_S - a_F \cdot e^{-r\tau} > c_F + c_S\,.$$

If the first condition holds, one shorts the future at b_F and buys the stock at a_S, leading to a per-unit profit of $\tilde{b}_S - a_S - (c_F + c_S)$. On the other hand, if the second condition holds, one buys the future at a_F and shorts the stock at b_S, leading to a per-unit profit of $b_S - \tilde{a}_S - (c_F + c_S)$. In both cases, the traded volume is restricted to:

$$v = \min\big(100, \min(\mathrm{vol}(a_S), \mathrm{vol}(b_F))\big) \quad \text{or} \quad v = \min\big(100, \min(\mathrm{vol}(b_S), \mathrm{vol}(a_F))\big)\,,$$

respectively, to account for exchange position limits and available liquidity at the quoted prices.

(iii) In part (i), we opened a short position of 100 futures contracts at 19.70. Since trading in the underlying stock is not permitted, the only way to manage the position is to close it later in the futures market by buying back 100 contracts. In this question we differentiate between two scenarios (at $T = 1$):

- **Scenario A**: Repurchase 100 contracts at the best ask of 19.50. Since the position was opened at 19.70, the net profit is $100 \cdot (19.70 - 19.50) - 2 \cdot 100 \cdot c_F = 20 - 200 \cdot c_F$.
- **Scenario B**: Repurchase 100 contracts at the best ask of 20.10. Since the position was opened at 19.70, the net loss is $100 \cdot (19.70 - 20.10) - 2 \cdot 100 \cdot c_F = -40 - 200 \cdot c_F$.

Thus, without access to the stock, the strategy becomes speculative, carrying exposure to price movements in the futures.

(iv) In the above example, we held an uncovered short opened at time $T = 0$. Such a position is speculative and remains exposed to directional (delta) risk to the stock. If you are long the future and the stock price falls, the future price decreases On the other hand, if you are short and the stock rises, the future price increases. In both cases, we incur a loss, whose magnitude depends on the futures' spot delta. Under the no-arbitrage relation $F = Se^{r\tau}$:

$$\Delta = \frac{\partial F}{\partial S} = e^{r\tau} \approx 1.015,$$

where $r = 0.03$ and $\tau = 0.5$. Therefore, for an uncovered long (short) futures position, a \$1 decrease (increase) in the stock leads to an approximate loss in the futures position of \$1.015.

To offset this risk exactly, one must construct a delta-neutral hedge by holding Δ units of stock per futures contract in the opposite direction. Therefore, to hedge X long futures contracts, short $Xe^{0.03 \cdot 0.5}$ shares of the stock; to hedge Y short futures contracts, buy $Ye^{0.03 \cdot 0.5}$ shares of the stock. Note that as time passes and $\tau \to 0$ this hedge ratio converges to 1, matching the intuition that futures and spot coincide at expiry and indicating that the hedge should be rebalanced accordingly.

In A1, the stock and its dual listing are economically identical (same currency and identical ownership rights). As a result, the hedge ratio is always exactly 1:1: one unit of stock offsets one unit of the dual, with no adjustment required. On the other hand, in the case of futures, the contract price reflects both the time to expiry and the positive risk-free rate, implying a $\Delta > 1$ for $r > 0, \tau > 0$. Hence, the futures position is slightly more sensitive to changes in the underlying stock than the stock itself, requiring more than one unit of the underlying asset to hedge the position.

Question 2

(i) We trade the stock `ASML` and the future `ASML_202603_F`. As derived in question B1(ii), and in absence of transaction costs, the arbitrage signals simplify to:

$$b_F > a_S \cdot e^{r\tau} \;\Rightarrow\; \text{Sell futures (overvalued)}, \qquad a_F < b_S \cdot e^{r\tau} \;\Rightarrow\; \text{Buy futures (undervalued)}.$$

Before executing any order, feasibility is verified so that post-trade positions in both instruments remain within the exchange limits of ± 100. Once the futures trade is executed, the position is hedged entering an opposite stock trade. To do so, define the position's delta as $\Delta = p_F \cdot e^{r\tau} + p_S$, where p_F denotes the outstanding futures position. Then the hedging strategy can be formalized as follows:

$$\Delta < 0 \Rightarrow \text{Buy round}(\Delta) \text{ stocks}, \qquad \Delta > 0 \Rightarrow \text{Sell round}(\Delta) \text{ stocks},$$

where round(Δ) denotes the operator that rounds to the closest integer. For example, if $\Delta = -1.7$, then 2 stocks are bought. Although, rounding does not guarantee a delta of zero, it ensures that $\Delta \in (-\frac{1}{2}, \frac{1}{2}]$. The results of this strategy are reported in Table 5.

Table 5: Performance of stock-futures hedging strategy.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
697	23.22	57.45	0.40	126.71

Trading with futures and stocks appears to be more profitable than trading with stocks and their corresponding duals, with an observed increase of approximately 6 units of PnL per minute. However, this strategy also exhibits significantly higher volatility, nearly twice as much as the stocks-and-duals strategy. As a result, despite the higher returns, the Sharpe ratios are almost identical, with the futures-and-stocks strategy showing a slightly better performance of 0.02 units.

(ii) Given that the exchange position limit for each instrument is increased to 100 lots, we propose two hedging strategies, namely a Naive hedge and a Δ.minimizing hedge.

Firstly, the naive approach computes the full stock hedge implied by the rounded, subject to position limits. If the required stock hedge would breach limits, the order is truncated and a Δ-residual remain. For example, suppose $\Delta = 17.3$ before hedging and the current stock position is -85. The maximum feasible sell is 15 shares, leaving a Δ-residual $= 2.3$. This issue did not arise in the dual-market case, where both instruments had identical contract economies. In contrast, here the hedge spans stock and futures (which differ in behavior and constraints), where the futures' dollar delta equal $e^{r\tau} > 1$, complicating the hedge.

Alternatively, if the goal is minimizing $|\Delta|$, one can pre-adjust the futures order size so that, after applying the stock hedge, the resulting Δ remains within $\left(\frac{1}{2}, \frac{1}{2}\right]$, while still respecting position limits. Returning to the example, suppose the intended order is to buy 17 futures, which would give a pre-hedge $\Delta \approx 17.3$. Since only 15 additional stock shares can be sold, the futures long should be reduced so that the final Δ falls within the target interval. Reducing the futures long to 15 yields a final Δ-residual $= 0.23$

The naive strategy maximizes spread capture but leaves residual risk when limits bind. The Δ-minimizing strategy sacrifices some trading volume but ensures a tighter hedge, keeping exposure consistently small.

(iii) Now that we are no longer restricted to trading one lot per order, we adapt our stock–futures hedging approach by implementing the Δ-minimizing hedge strategy described above. The motivation for this choice is that Optibook enforces a soft delta risk limit, penalizing positions where Δ exceeds the ± 100 threshold.

As explained in part (ii), the main advantage of this strategy is that it directly targets delta-neutrality, leaving only a small residual exposure of magnitude $m \in \left(\frac{1}{2}, \frac{1}{2}\right]$. The trade-off, however, is reduced PnL. Because we deliberately cap futures trades to keep the hedge feasible, the strategy sometimes forgoes part of the available spread that a naive approach (trading larger futures sizes) would otherwise capture.

Beyond the scope of the specific question, it is important to highlight a flexibility issue of this overall approach. The main drawback is its tendency to saturate positions, thereby restricting the ability to exploit later opportunities. In practice, this occurs when the algorithm seizes early, low-margin arbitrages with maximum position sizes, exhausting capacity. For example, suppose that at $t = 0$ the algorithm identifies a small short-futures arbitrage with a margin of 0.01 and sells the full permitted volume, reaching a futures position of -100. At $t = 1$, a more profitable short opportunity emerges with a margin of 0.02. However, the position limit prevents any further trading, and the algorithm is unable to exploit the superior opportunity. In effect, the strategy sacrifices long-term flexibility for immediate but limited profit.

The performance of this strategy is reported in Table 6.

Table 6: Performance of stock-futures hedging strategy with unfixed volume.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
5629	187.64	181.42	1.03	322.90

When comparing the results in Table 6 with those in Table Table 5, it becomes evident that trading with variable volumes is a more effective strategy than trading with fixed one-unit volumes. This approach results in a traded volume per minute that is approximately eight times higher, accompanied by a correspondingly higher standard deviation. Nevertheless, the strategy yields to a Sharpe ratio that is roughly twice as high, exceeding 1 unit per minute.

Question 3

In this section we trade the futures contracts `ASML_202603_F` and `ASML_202606_F`, with the underlying stock `ASML`.

(i) For two futures on the same stock, the no-arbitrage condition requires that the present value of each futures contract equals the spot price. Hence, the discounted futures prices must coincide. Using the no-arbitrage relation $F_i = Se^{r\tau_i}$, this can be formalized as:

$$e^{-r\tau_1}F_1 = e^{-r\tau_2}F_2 = S$$

where F_1, F_2 denote two future contracts and τ_1, τ_2 the time of maturity of each contract, respectively. Any deviation from equality indicates a potential arbitrage opportunity, which can be of the following two forms:

- **Stock-Future arbitrage:** By the no-arbitrage relation, the discounted order book of the futures contracts should match the order book of the stock. As formally described in B1(ii), if the best discounted futures (i.e. synthetic stock) bid exceeds the best stock ask (plus transaction costs), one can conduct arbitrage by shorting the future and buying the stock. Conversely, if the best stock bid exceeds the best discounted futures ask, then the future is undervalued: one profits from buying the future and shorting the stock.
- **Future-Future arbitrage:** In theory, the discounted order book of both contracts (regardless of expiration date) should be the same. If the best discounted bid of one future exceeds the best discounted ask of the other, then the former is overvalued relative to the latter. Therefore, arbitrage consists of longing the undervalued contract and shorting the overvalued one simultaneously, neutralizing immediate exposure to the underlying and locking in the spread.

(ii) We extend the stock–futures arbitrage algorithm described in question B2(iii) to a two-futures setting. Consider two futures contracts, denoted F_1 and F_2. Without loss of generality, assume that F_2 has a longer maturity than F_1. Define the actualization factor:

$$A_F = e^{r(\tau_2 - \tau_1)},$$

Here, F_1 plays the role of the stock (as in the stock–future case), while F_2 plays the role of the future. Thus, in our trading rule, the discount factor $e^{r\tau}$ is replaced by A_F. The strategy detects deviations between F_1 and F_2, and executes trades to capture the spread. Table 7 reports the performance of this Future-Future strategy.

Table 7: Performance Future-Future strategy with unfixed volume.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
45	1.49	55.03	0.027	8.46

When trading futures against futures, the results deteriorate significantly, representing the worst performance observed so far — even when using the variable-volume strategy. This is reflected in Table 7, which shows the lowest Sharpe ratio among all strategies tested. A possible explanation for this underperformance is the lack of liquidity in both futures markets. This low liquidity makes it difficult to promptly execute hedging trades, often resulting in uncovered positions that can take considerable time to close, thereby increasing exposure and volatility.

(iii) We next enhance the algorithm by hedging any residual risk using the underlying stock. When an arbitrage signal is detected, the algorithm simultaneously places IOC orders to go long on the undervalued future and short the overvalued future. Once these trades are executed, the combined futures position creates a net exposure to the underlying stock, measured by the combined future's Δ. Formally, let Δ_{F_i} denote the delta of future i. As shown in B1(iv), for a single futures contract:

$$\Delta_{F_i} = e^{r\tau_i}, \qquad \Delta^{(i)} = p_{F_i} e^{r\tau_i},$$

where p_{F_i} denotes the outstanding position in future i, Δ_{F_i} the delta of a single contract i, and $\Delta^{(i)}$ the total delta contribution from that position. Then, the total futures delta after execution is:

$$\Delta_{futures} = p_{F_1} e^{r\tau_1} + p_{F_2} e^{r\tau_2}$$

After computing the combined future's Δ, the total portfolio delta $\Delta_{\text{tot}} = \Delta_{\text{fut}} + p_S$ is used to execute stock hedge. If $|\Delta_{\text{tot}}| > \frac{1}{2}$, the remaining risk is then neutralized by trading the underlying stock until the residual delta is within $\left(-\frac{1}{2}, \frac{1}{2}\right]$. In other words, the hedge volume (number of traded stocks for hedging purposes) is:

$$p_S^{\text{hedge}} = -\text{round}(\Delta_{\text{tot}}).$$

Summarizing, the stock is used as the final adjustment instrument to ensure the portfolio remains delta-neutral within tolerance. Table 8 reports the performance of the hedged Future-Future strategy.

Table 8: Performance Future-Future strategy with stock hedging and unfixed volume.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
669	22.48	95.38	0.24	75.24

When incorporating the underlying stock to hedge the position, performance improves significantly. Specifically, the PnL per minute increases by a factor of 20, while the standard deviation only doubles. This leads to a substantially higher Sharpe ratio, indicating a much more favorable risk-adjusted strategy.

Therefore, hedging with the stock appears to be a highly effective strategy. However, it's important to note that even with stock-based hedging, full neutralization of the position may not always be achievable in practice. For example, consider a situation where the algorithm attempts to go short on the first future and long on the second, but the second operation of the trade fails to execute. If a similar issue occurs in reverse shortly after, the resulting net delta could fall outside the expected interval of $\Delta \in \left(-\frac{1}{2}, \frac{1}{2}\right]$, despite using the stock for hedging.

(iv) Finally, we generalize the hedging step. Since total delta depends on both futures positions, it is not always optimal to hedge exclusively with the stock. Instead, we are able to hedge one future by using the other. Concretely, after each single futures trade, we compare the order books of both the stock and the other future and use the better suiting (cheaper) instrument. If buying is required, we choose the instrument with the lowest ask; if selling, the one with the highest bid. To ensure comparability, the futures order book is adjusted by the appropriate discount factors (i.e., create synthetic stock order book). As it is not always guaranteed that the volume offered in the best instrument is enough to fully hedge our position, this selection process repeats until the residual delta has an absolute value below 1, at which point we consider the position effectively hedged. Table 9 reports the performance of the Future-Future strategy with modified hedging.

Table 9: Performance Future-Future strategy with modified hedging.

Total PnL	Mean (increments)	Std. Dev. (increments)	Sharpe (1 minute)	Sharpe (annual)
1076.3	35.9	49.0	0.73	228.85

Compared to hedging exclusively with stocks (Table 8), our modified hedging algorithm yields a substantial increase in mean PnL, since it allows us to always hedge at the most favorable available price. At the same time, the reduction in PnL volatility leads to a significantly higher Sharpe ratio. However, when comparing the results of Part B3 with those of Part B2 (e.g.,Table 6), we observe that the Sharpe ratio obtained from arbitraging between two futures remains lower than that from arbitraging between a stock and a future. A plausible explanation lies in two factors. First, the assignment assumes that the stock price leads and the futures follow, meaning arbitrage in B2 works better because the stock leads, whereas in B3 one is only arbitraging between two lagging instruments. Second, the stock order books are typically much more liquid, which not only ensures a more accurate reflection of the theoretical fair price but also increases the likelihood of successful order execution. This higher fill rate directly translates into a greater number of realized arbitrage opportunities and therefore a higher cumulative PnL.