GRAPH THEORY ASSIGNMENT 2

1. **The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two (2) vertices are adjacent in $\overline{G}$ if and only if (iff) they are not adjacent in G. find the ff.**
   - **i.** $\overline{Kn}$
   - **ii.** $\overline{Km, n}$
   - **iii.** $\overline{Cn}$
   - **iv.** $\overline{Qn}$

   Solution

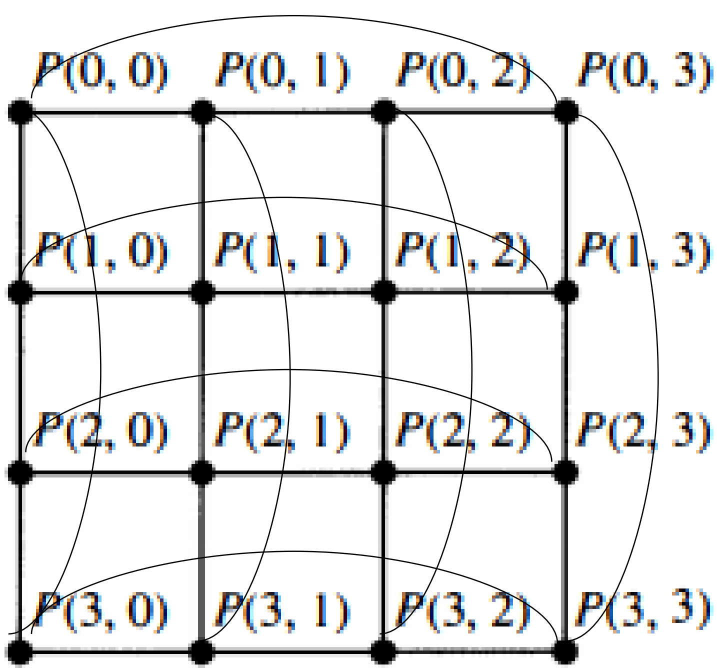
   i. The graph with n vertices and no edges.

   ii. The disjoint union of Km and Kn

   iii. The graph with vertices (v1, v2, v3, …Vn) with an edge between Vi and Vj unless $i \equiv j \pm 1$ (mod n)

   iv. The graph whose vertices are represented by bit strings of length n with an edge between two vertices if the associated bit strings differ in more than one bit.

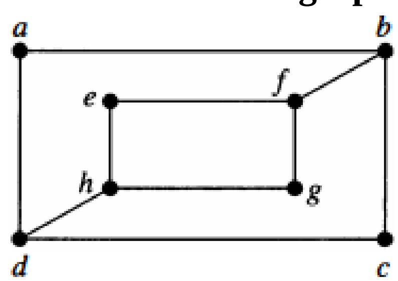
2. In a variant of a mesh network for interconnecting $n = m^2$ processors, processor $P(i, j)$ is connected to the four processors $P((i \pm 1) \textbf{ mod } m, j)$ and $P(i, (j \pm 1) \textbf{ mod } m)$, so that connections wrap around the edges of the mesh. Draw this variant of the mesh network for 16 processors.



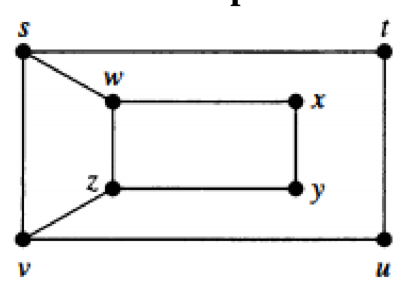
3. Show that every pair of processors in a mesh network of $n = m^2$ processors can communicate using $O(\sqrt{n}) = O(m)$ hops between directly connected processors.

We can connect P(I,j) and P(k,l) by using |i – k| hops to connect P(i,j) and P(k,j) and |j – i| hops to connect P(k,j) and P(k,l). Hence, the total number of hops required to connect P(i,j) and P(k,l) does not exceed |i – k| + |j – l|. This is less than or equal to m + m = 2m, which is O(m).

4. **Determine whether graph G and H are isomorphic.**



G



H

1. Number of edges in G = 10
   Number of vertices in G = 8

   Number of edges in H = 10
   Number of vertices in H = 8

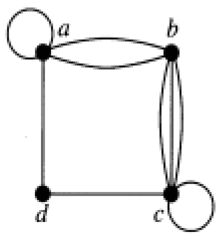
Number of degree G	Number of degree H
Deg(a) = 2	deg(s) = 3
Deg(b) = 3	deg(t) = 2
Deg(c) = 2	deg(u) = 2
Deg(d) = 3	deg(v) = 3
Deg(e) = 2	deg(w) = 3
Deg(f) = 3	deg(x) =2
Deg(g) = 2	deg(y) = 2
Deg(h) = 3	deg(z) = 3

G and H have the same number of edges and vertices and the same degree sequence of 3,3,3,3,2,2,2,2 but since the deg(a) = 2 does not correspond to any other vertex of degree 2 in H. Vertex a is adjacent to b of deg(b) = 3 and d of deg(d) = 3 but there is no vertex in H with the same similarities.

So therefore G and H is not isomorphic.

5. **Draw the graph with adjacency matrix.**

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$



6. Represent each of these graphs with an adjacency matrix.

a) K_4 b) $K_{1,4}$ c) $K_{2,3}$

d) C_4 e) W_4 f) Q_3

Draw the graph for each of them before you find the adjacency matrix.

a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

7. Show that the vertices of a bipartite graph with two or more vertices can be ordered so that its adjacency matrix has the form

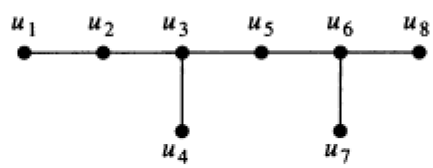
$$\begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{B} & \mathbf{0} \end{bmatrix},$$

where the four entries shown are rectangular blocks.

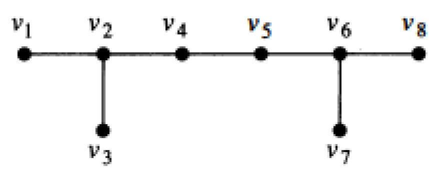
Draw a bipartite graph with two or more vertices and find the adjacency matrix.

8. The devil's pair for a purported isomorphism test is a pair of nonisomorphic graphs that the test fails to show are not isomorphic.
   Find the devil's pair for the test that checks the sequence of degrees of vertices in the two graphs to make sure they agree.

A



B



A			B		
Deg(u1)	=	1	deg(v1)	=	1
Deg(u2)	=	2	deg(v2)	=	3
Deg(u3)	=	3	deg(v3)	=	1
Deg(u4)	=	1	deg(v4)	=	2
Deg(u5)	=	2	deg(v5)	=	2

Deg(u6)	=	3	deg(v6)	=	3
Deg(u7)	=	1	deg(v7)	=	1
Deg(u8)	=	1	deg(v8)	=	1

Degree sequence = 3, 3, 2, 2, 1, 1, 1, 1

Degree sequence = 3, 3, 2, 2, 1, 1, 1, 1

Draw the graph for Q4.

Solution

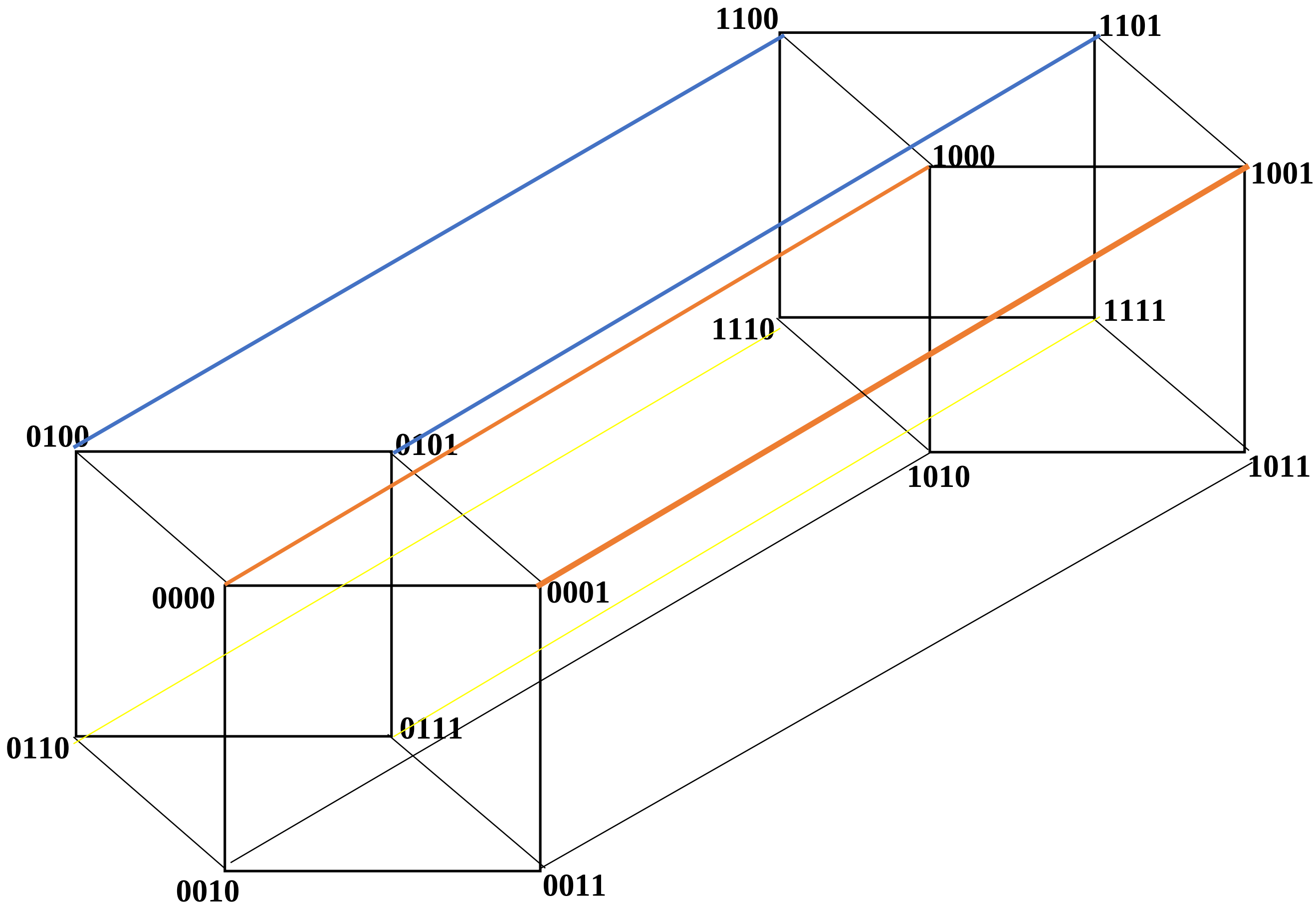
n = 4 => Q4

Number of bit strings = n = 4

Number of vertices $= 2^n = 2^4 = 16$

$=> 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15$

	Bit string
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111



1100
1101
1000
1001
1110
1111
0100
0101
1010
1011
0000
0001
0110
0111
0010
0011