

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

DEPARTMENT OF COMPUTER SCIENCE

MIDSEMESTER EXAMINATION 2018/2019

CSM 491 GRAPH THEORY AND ITS APPLICATIONS

BSc. COMPUTER SCIENCE (Fourth Year)

December 2018

[40 Marks]

Time Allowed: 1 Hour

Instruction: Answer ALL THREE questions in the answer booklet provided.

- 1) a) Suppose that a connected planar simple graph with  $e$  edges and  $v$  vertices contains no simple circuits of length 4 or less. Show that  $e \leq (5/3)v - (10/3)$  if  $v \geq 4$ . [6 Marks]
- b) Show that if a simple graph  $G$  has  $k$  connected components and these components have  $n_1, n_2, \dots, n_k$  vertices, respectively, then the number of edges of  $G$  does not exceed

$$\sum_{i=1}^k C(n_i, 2).$$
 [5 Marks]

c) What is the chromatic number of;

- i)  $K_n$ ?    ii)  $K_{m,n}$ ?    iii)  $C_n$ ?    iv)  $W_n$ ? [8 Marks]

$n$     2    even = 2, odd = 3    even = 3, odd = 4

- 2.a) The complete **m-partite** graph  $K_{n_1, n_2, \dots, n_m}$  has vertices partitioned into  $m$  subsets of  $n_1, n_2, \dots, n_m$  elements each, and vertices are adjacent if and only if they are in different subsets in the partition.

i) Draw the following graphs. [6 Marks]

$\alpha$ )  $K_{1,2,3}$

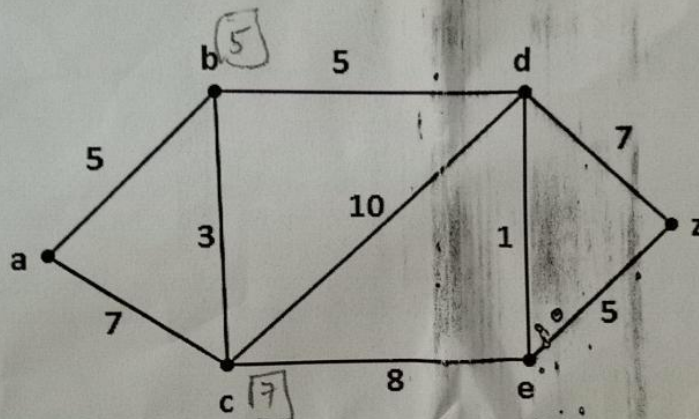
$\beta$ )  $K_{2,2,2}$

$\gamma$ )  $K_{1,2,2,3}$

b) How many vertices and how many edges does the complete  $m$ -partite graph  $K_{n_1, n_2, \dots, n_m}$  have? [4 Marks]

c) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have? [3 Marks]

3. Consider the following network.



Determine the shortest path from vertex  $a$  to every other vertex in the graph using the Dijkstra's algorithm [8 Marks]