KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

END OF FIRST SEMESTER EXAMINATION 2011-2012

CSM 491 GRAPH THEORY AND ITS APPLICATIONS

November 2011 [100 Marks] Time Allowed: 2 Hours

***Instructions:** Answer ALL the questions in the answer booklet provided. All questions carry equal marks!*

1. a) A simple graph is called *regular* if every vertex of this graph has the same degree. A regular graph is called *n-regular* if every vertex in this graph has degree n.

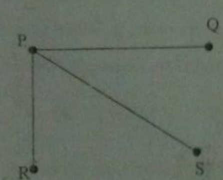
For which values of n are the following graphs regular?

i) K_n for all $n \geq 1$
ii) C_n $n \geq 3$
iii) W_n $n = 3$
iv) Q_n $n \geq 0$

b) How many vertices and how many edges do the following graphs have?

i) K_n n vertices $\frac{n^2-n}{2}$
ii) C_n
iii) W_n $(n+1)$ $2n$
iv) $K_{m,n}$ $m+n$ mn
v) Q_n 2^n vertices $n2^{n-1}$

c) Draw all subgraphs of the following graph.

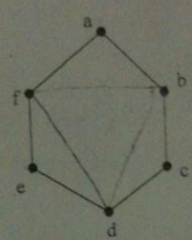


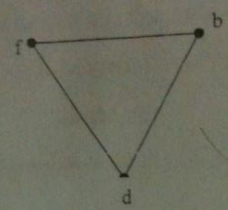
K_3 17 sub graphs

Pcu/cos/481-11

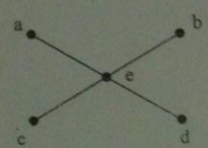
2. a) Find the union of each of the following pair of graphs.

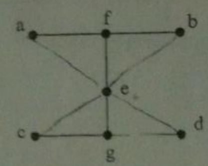
i)





ii)

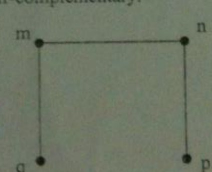




b) The complementary graph $\overline{H}$ of a simple graph H has the same vertices as H. Two vertices are adjacent in $\overline{H}$ if and only if they are not adjacent in H. Find the following.

i) $\overline{C_n}$
ii) $\overline{Q_n}$
iii) $\overline{K_n}$
iv) $\overline{K_{m,n}}$

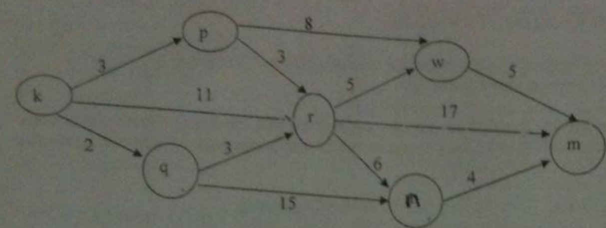
c) A simple graph G is called **self-complementary** if G and $\overline{G}$ are isomorphic. Show that the following graph is self-complementary.



3. a) Use the Dijkstra's algorithm to find the *shortest path* from the source node k to every

Ayisha G.

other node in the following directed graph.

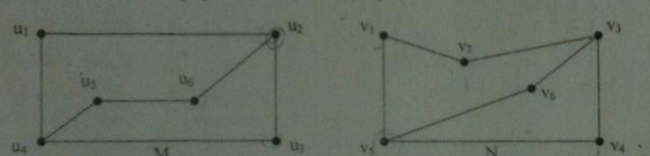


b) Indicate on the graph the shortest path from the source node k to the destination node m.

c) If the simple graph G has v vertices and e edges, how many edges does $\overline{G}$ have?

4. a) Show that every pair of processors in a mesh network of $n = m^2$ processors can communicate using $O(\sqrt{n}) = O(m)$ hops between directly connected processors.

b) Determine whether the graphs M and N are isomorphic.



Isomorphic

c) Determine whether the following graphs are planar. If so, draw the planar graph.

i) K_4
ii) Q_3
iii) $K_{3,3}$

d) Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

5. a) Define the following terms:

i) Chromatic number

$r = e - v + 2$
$v = 20$
$r = 30 - 20 + 2$
$= 12$

ii) The four color theorem
iii) Homeomorphic graphs
iv) Complete bipartite graph

b) Briefly describe the solution to this problem.

Scheduling Final Exam: How can the final exams at a university be scheduled so that no student has two exams at the same time?

c) The Computer Science Department has six committees that meet once a month. How many different meeting times must be used to ensure that no one is scheduled to be at two meetings at the same time if the committees are:

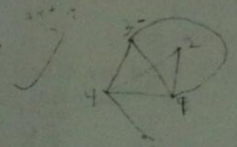
C_1 = {Oppong, Davis, Pabbi }, C_2 ={Davis, Panford, Agyepong },

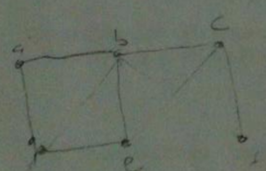
C_3 = {Oppong, Agyepong , Pabbi }, C_4 = { Panford, Agyepong, Pabbi },

C_5 = {Oppong, Davis }, and C_6 = { Davis, Agyepong, Pabbi }.

d) Does there exist a simple graph with five vertices of the following degrees? If so, draw such a graph.

i) 3, 3, 3, 3, 2
ii) 1, 2, 3, 4, 5
iii) 1, 2, 3, 4, 4
iv) 3, 4, 3, 4, 3
v) 0, 1, 2, 2, 3
vi) 1, 1, 1, 1, 1





GOOD LUCK! J. K. PANFORD

KWAME NKRUMAH UNIVERSITY OF SCIENCE & TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

DEPARTMENT OF COMPUTER SCIENCE

END OF FIRST SEMESTER EXAMINATION 2012-2013

BSc. COMPUTER SCIENCE IV

CSM 491 GRAPH THEORY AND ITS APPLICATIONS

December 2012 [100 Marks] Time Allowed: 2 Hours

Instructions: Answer FOUR questions in all; question ONE and ANY other THREE questions in the answer booklet provided. All questions carry equal marks!

1. a) i) Draw the pseudograph that has the following adjacency matrix: [3 Marks]

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

ii) Represent each of the following graphs with an adjacency matrix. [8 Marks]

α) K_4 β) $K_{1,4}$ γ) W_4 λ) Q_3

b) i) Is every zero-one square matrix that is symmetric and has zeros on the diagonal the adjacency matrix of a simple graph? Explain. [4 Marks]

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

ii) Show that the vertices of a bipartite graph with two or more vertices can be ordered so that its adjacency matrix has the form: [4 Marks]

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

where the four entries shown are rectangular blocks.

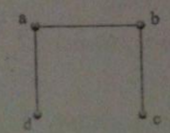
c) i) A **devil's pair** for a purported isomorphism test is a pair of nonisomorphic graphs that the test fails to show are not isomorphic.

Find a devil's pair for the test that checks the sequence of degrees of the vertices in the two graphs to make sure they agree. [3 Marks]

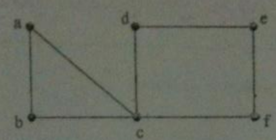
ii) A simple graph G is called **self-complementary** if G and $\overline{G}$ are isomorphic. Show that the following graph is self-complementary. [3 Marks]

0 1 0 0 0 1 1 0
1 0 1 0 0 0 0 1
0 1 0 1 0 0 1 0
0 0 1 1 0 0 0 1
0 0 ... 1 1 0

1 0 1 0 1 0 0 0
... 1 1 0 0 0



2. a) i) Find all the *cut vertices* of the following graph. [4 Marks]



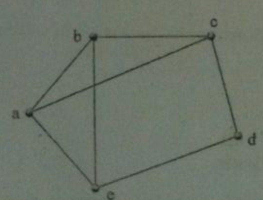
ii) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges. [5 Marks]

b) i) How many nonisomorphic connected simple graphs are there with n vertices when n is: [8 Marks]

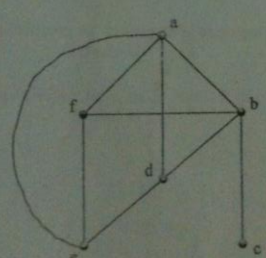
α) 2? β) 3? γ) 4? λ) 5?

c) i) Determine whether each of the following graphs has Euler circuit. Construct such a circuit when one exists. [8 Marks]

W:

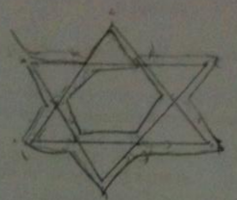


P:



3.a) Devise an algorithm for constructing Euler paths in directed graphs. [8 Marks]

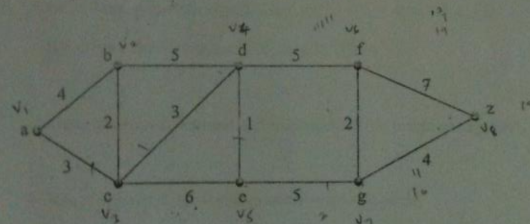
b) Determine whether the following picture can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture. [8 Marks]



c) Show that a directed multigraph having no isolated vertices has an Euler path but not an Euler circuit if and only if the graph is weakly connected and the in-degree and out-degree of each vertex are equal for all but two vertices, one that has in-degree 1 larger than its out-degree and the other that has out-degree 1 larger than its in-degree. [9 Marks]

4. a) i) For which values of m and n does the complete bipartite graph $K_{m,n}$ have a Hamilton circuit? [5 Marks]

Consider the following network:



Determine the shortest distance from node a to every other node using:

b) the Dijkstra's algorithm. [10 Marks]

c) the Floyd's algorithm. [10 Marks]

5.a) i) State *Kuratowski's theorem* [2 Marks]

ii) Can five houses be connected to two utilities without connections crossing? [5 Marks]

b) i) Show that K_5 is nonplanar. [5 Marks]

ii) Suppose that a connected planar simple graph with e edges and v vertices contain no simple circuits of length 4 or less. Show that

$$e \leq (5/3)v - (10/3) \quad \text{if } v \geq 4. \qquad [5 \text{ Marks}]$$

c) i) What is the chromatic number of the complete bipartite graph $K_{m,n}$, where m and n are positive integers. [3 Marks]

ii) Seven variables occur in a loop of a computer program. The variables and the steps during which they must be stored are:

t: steps 1 through 6; u: step 2; v: steps 2 through 4; w: steps 1, 3, and 5; x: steps 1 and 6;

y: steps 3 through 6; and z: steps 4 and 5.

How many different index registers are needed to store these variables during execution? [5 Marks]

J. K. PANFORD

KWAME NKRUMAH UNIVERSITY OF SCIENCE & TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL SCIENCES

DEPARTMENT OF COMPUTER SCIENCE

CSM 496 GRAPH THEORY AND ITS APPLICATIONS
Computer Science IV
END OF FIRST SEMESTER EXAMINATION, 2010

December 2010 Time Allowed: 2½ Hours

[100 Marks]

Instruction: Answer all questions in the answer booklet provided! All questions carry equal marks.

1. a) Determine whether the graphs shown in Figure 1 are isomorphic. [10 Marks]

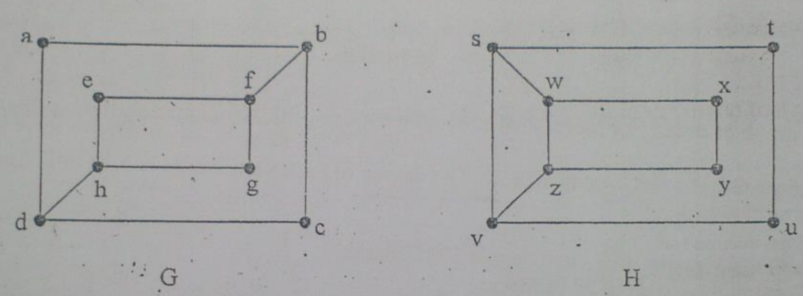


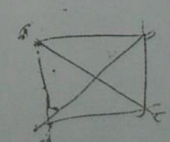
Figure 1: The Graphs G and H

b) Show whether a simple graph exist with 15 vertices each of degree 5? [5 Marks]

c) How many vertices and how many edges do the following graphs have? [10 Marks]

i) K_n n $\frac{n(n-1)}{2}$
ii) C_n n n
iii) W_n n+1 2n
iv) $K_{n,m}$ m+n mn
v) Q_n 2^n $n(2^{n-1})$

	vertices	edge
	n	n^2-n
	n	n
	n+1	2n
	n+m	nm



Pcu/cos /130-10

2. a) How many subgraphs with at least one vertex does K_3 have? [5 Marks]

b) The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Find the following. [10 Marks - 2½ each]

i) $\overline{K}_n$
ii) $\overline{K}_{m,n}$
iii) $\overline{C}_n$ $n \geq 3$
iv) $\overline{Q}_n$

c) Show that if G is a simple graph with n vertices, then the union of G and $\overline{G}$ is K_n. [5 Marks]

d) Prove that an undirected graph has an even number of vertices of odd degree. [5 Marks]

3. a) Draw the graph $3P_4 \cup 2C_4 \cup K_4$. [5 Marks]

b) Let G be a graph of order 5 or more. Prove that at most one of G and $\overline{G}$ is bipartite. [5 Marks]

c) A certain graph G has order 14 and size 27. The degree of each vertex of G is 3, 4, or 5. There are six vertices of degree 4.

i) How many vertices of G have degree 3? [5 Marks]
ii) How many have degree 5? [5 Marks]

d) How many edges does a graph have if it has vertices of degree 4, 3, 3, 2, 2? Draw such a graph. [5 Marks]

4. a) Consider the network shown in Figure 2.

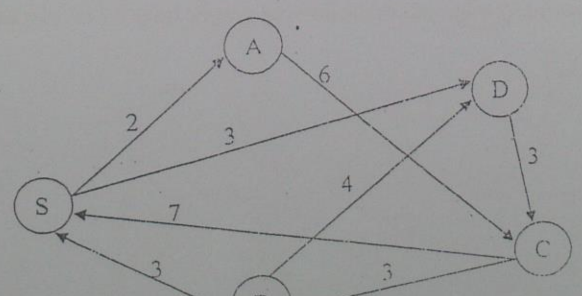


Figure 2

Determine the shortest distance from the source node, S, to every other node in the network using the Dijkstra's algorithm. [10 Marks]

b) Briefly explain the following and give an example under each. Illustrate your answer with a diagram. [10 Marks – 2 Marks each]

i) Walk

ii) Circuit

iii) Cycle

iv) Trail

v) Complete graph

c) Show that each of the following properties is an invariant that isomorphic simple graphs either both have or both do not have. [5 Marks - 2½ Marks each]

i) connectedness

ii) being bipartite

14 = v

27 = e

6×4 = 24

14 − 6 = 8

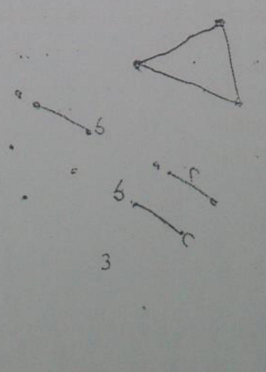
54 − 24 = 30

2e = E

2×27 = 54

x + y = 8

3x + 5y = 30



J. K. PANFORD