DEPARTMENT OF COMPUTER SCIENCE

CSM 491 GRAPH THEORY AND ITS APPLICATIONS

MIDSEMESTER EXAMINATION 2014 Time Allowed: 45 Minutes

1.a) The complete m-partite graph $K_{n1,n2,...,nm}$ has vertices partitioned into m subsets of $n_1, n_2, ..., n_m$ elements each, and vertices are adjacent if and only if they are in different subsets in the partition. Draw the following graphs.

i) $K_{1,2,3}$ ii) $K_{2,2,2}$ iii) $K_{1,2,2,3}$

b) The thickness of a simple graph G is the smallest number of planar subgraphs of G that have G as their union. Show that $K_{3,3}$ has 2 as its thickness.

2. a) Show that the vertices of a bipartite graph with two or more vertices can be ordered so that its adjacency matrix has the form

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

b) How many nonisomorphic simple graphs are there with n vertices, when n is:

i) 2? ii) 3? iii) 4?

KWAME NKRUMAH UNIVERSITY OF SCIENCE & TECHNOLOGY, KUMASI

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL SCIENCES

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COMPUTER SCIENCE IV

November 2010 MID-SEMESTER EXAMINATION, 2010 Time Allowed: 1½ Hour

[30 Marks]

Instruction: Answer all questions in the answer booklet provided! ALL questions carry equal marks.

1. a) i) How many edges does a graph have if it has vertices of degree 4, 3, 3, 2, 2? Draw such a graph.
   ii) Prove that an undirected graph has an even number of vertices of odd degree.

   b) i) Construct a precedence graph for the following program:

$S_1 : x = 0$
$S_2 : x = x + 1$
$S_3 : y = 2$
$S_4 : z = y$
$S_5 : x = x + 2$
$S_6 : y = x + z$
$S_7 : z = 4$

   ii) Can a simple graph exist with 15 vertices each of degree 5? Explain your answer.

2. a) A simple graph is called *regular* if every vertex of this graph has the same degree. A regular graph is called *n-regular* if every vertex in this graph has degree n. For which values of n are the following graphs regular?

   i) K_n ii) C_n iii) W_n iv) Q_n

   b) Does there exist a simple graph with five vertices of the following degrees? If so, draw such a graph.

   i) 3, 3, 3, 3, 2 ii) 1, 2, 3, 4, 5 iii) 1, 2, 3, 4, 4

   iv) 3, 4, 3, 4, 3 v) 0, 1, 2, 2, 3 vi) 1, 1, 1, 1, 1

J. K. PANFORD

DEPARTMENT OF COMPUTER SCIENCE

CSM 491 GRAPH THEORY & ITS APPLICATIONS Time Allowed: 1 Hour

Answer ALL questions

1. How many nonisomorphic connected simple graphs are there with n vertices when n is

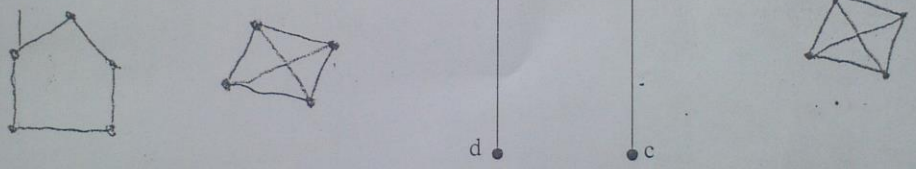
   a) 2? b) 3? c) 4? d) 5?

2. Show that the vertices of a bipartite graph with two or more vertices can be ordered so that its adjacency matrix has the form

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

where the four entries shown are rectangular blocks.

3. A simple graph G is called self-complementary if G and $\overline{G}$ are isomorphic. Show that the following graph is self-complementary.



4. Show that every pair of processors in a mesh network of $n = m^2$ processors can communicate using $O(m)$ hops between directly connected processors.