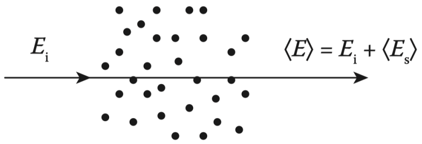
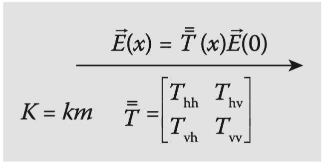
# Z-PHI method

## Notation

- upper case for dB scale and lower case for linear scale
- $Z_H = 10 \cdot \log_{10} Z_h$ (dBZ), $Z_h = 10^{Z_H/10}$ ($\mathrm{mm^6 m^{-3}}$)




(a)




(b)

## Assumption

- **First-order multiple scattering** (See [1], *4.3 Coherent Wave Propagation* for details)

$$A = 8.686 \times \mathrm{Im}(K)$$

where $A$ is the **specific attenuation** which is the attenuation of a wave during its propagation in a unit distance and $K$ is the **effective propagation constant**.

- **Power law between $A_h$ and $Z_h$**

$$A_h = a[Z_h(r)]^b$$

where $a$ is varies with temperature as well as a parameter $N_w$ from Raindrop Size Distribution (see [2], *7.4.2 Attenuation Correction Using Constraints* for details) and $b$ is nearly constant.

## Z-PHI method

### Derivation

$$Z_H^M = Z_H - 2\int_{r_0}^{r} A_H(l)\mathrm{d}l = Z_H - \mathrm{PIA}(r_0, r)$$

or in linear form $Z_h^M = Z_h \exp\left[-0.46\int_{r_0}^{r} A_H(l)\mathrm{d}l\right]$

Let $g(r) = \exp\left[-0.46b\int_{r_0}^{r} A_H(l)\mathrm{d}l\right]$

We have $Z_h^M = Z_h \cdot [g(r)]^{1/b}$

$$Z_h(r) = \frac{Z_h^M(r)}{\left[1 - 0.46ab\int_{r_0}^{r}[Z_h^M(l)]^b\mathrm{d}l\right]^{1/b}}$$

$$\frac{\mathrm{d}g(r)}{\mathrm{d}r} = -0.46ab\left[Z_h^M(r)\right]^b$$

integrate between $[r_0, r]$

$$g(r) - 1 = -0.46ab\int_{r_0}^{r}\left[Z_h^M(l)\right]^b\mathrm{d}l$$

## Z-PHI method

- **Question: How to determine $a$**

By definition, we have $g(r_m) = \exp\left[-0.46b\int_{r_0}^{r_m} A_H(l)\mathrm{d}l\right] = \exp[-0.23b \cdot \mathrm{PIA}(r_0, r_m)]$

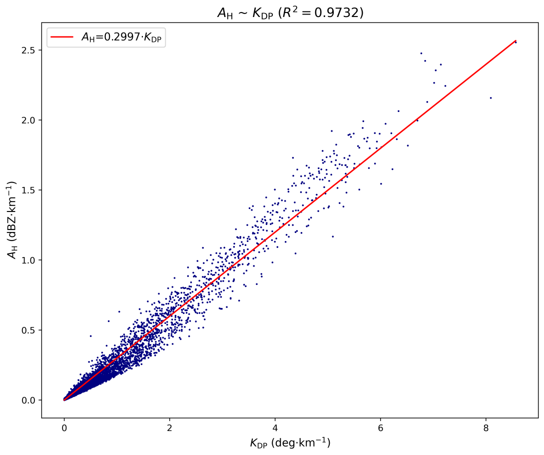
we can estimate $\mathrm{PIA}(r_0, r_m)$ by $\mathrm{PIA}(r_0, r_m) = c[\phi_{DP}(r_m) - \phi_{DP}(r_0)] = c\Delta\phi(r_0, r_m)$

which comes from the assumption $A_H(r) = cK_{DP}$

(See [1], *6.5.1.1 DP Method* for details)

Combined with $g(r_m) - 1 = -0.46ab\int_{r_0}^{r_m}\left[Z_h^M(l)\right]^b \mathrm{d}l$

we have $a = \dfrac{1 - \exp[-0.23b \cdot c\Delta\phi(r_0, r_m)]}{0.46b\int_{r_0}^{r_m}[Z_h^M(l)]^b \mathrm{d}l}$



## Z-PHI method

- **Question: How to find an optimal *c***
  - Self-consistent method [3]

$$\left.\begin{aligned}\mathrm{PIA}(r_0, r_i) &= c\Delta\phi(r_0, r_i)\\ \mathrm{PIA}(r_0, r_i) &= 2\int_{r_0}^{r_i} A_H(l)\mathrm{d}l\end{aligned}\right\} \Rightarrow \phi_{DP}^{\mathrm{rec}}(r_i) = \phi_{DP}(r_0) + \frac{2}{c}\int_{r_0}^{r_i} A_H(l)\mathrm{d}l$$

where $A_H(r) = a[Z_h(r)]^b = \dfrac{a\left[Z_h^M(r)\right]^b}{1 - 0.46ab\int_{r_0}^{r}[Z_h^M(l)]^b \mathrm{d}l}$

$$c^* = \operatorname*{argmin}_c \frac{1}{N}\sum_i [\phi_{DP}^{\mathrm{rec}}(r_i; c) - \phi_{DP}(r_i)]^2$$

## Z-PHI method

### Some practical issues

- NoData points: leave them as zero?
- How do we select a proper interval to estimate $a$? (not necessarily $r_0, r_m$)
- Magnitude problem: $a \sim 10^{-5} - 10^{-4}, b \sim 0.7, c \sim 10^{-2} - 0.3, Z_h^M \sim 10^1 - 10^5$

$$a = \frac{1 - \exp[-0.23b \cdot c\Delta\phi(r_0, r_m)]}{0.46b \int_{r_0}^{r_m} [Z_h^M(l)]^b \mathrm{d}l} \qquad A_H(r) = a[Z_h(r)]^b = \frac{a\left[Z_h^M(r)\right]^b}{1 - 0.46ab \int_{r_0}^{r} [Z_h^M(l)]^b \mathrm{d}l}$$

- The optimal $c^*$ may not exist when $\Delta\phi(r_0, r_m)$ is small [3]
- How can we tell whether the target $\phi_{DP}(r_i)$ are correct/reasonable?
- ...

# ◆ Reference

[1] Guifu Zhang, Weather Radar Polarimetry. Florida, United States: CRC Press, 2017.

[2] V. Bringi and V. Chandrasekar, Polarimetric Doppler Weather Radar: Principles and Applications. Cambridge, U.K.: Cambridge Univ. Press, 2001.

[3] Bringi, V. N., et al. "Correcting C-Band Radar Reflectivity and Differential Reflectivity Data for Rain Attenuation: A Self-Consistent Method with Constraints." IEEE Transactions on Geoscience and Remote Sensing, vol. 39, no. 9, 2001, pp. 1906–1915.