4.8.6

$$A + X \xrightarrow{k_1} 2X$$

$$X + X \xrightarrow{k_2} Y$$

$$Y \xrightarrow{k_3} B$$

(i)

$\frac{d}{dt}x(t) = ax(t) + by(t)$

$\frac{d}{dt}y(t) = cx(t) + dy(t)$

$\frac{d}{dt}x(t) = k_1 A x(t) - k_2 x(t)^2$ — S_1

$\frac{d}{dt}y(t) = k_2 x(t)^2 - k_3 Y(t)$ — S_2

(ii) for the steady state

when $\frac{d}{dt}x(t) = 0$, $\frac{d}{dt}y(t) = 0$

$k_1 A \cancel{x(t)} = k_2 x(t)^{\cancel{2}}$, $x(t) = \frac{k_1 A}{k_2}$ or 0

$k_2 x(t)^2 = k_3 Y$ $Y = \frac{x^2(t)}{k_3} = \frac{A^2}{k_2^2 \cdot k_3}$ or 0

(iii)

$$\begin{bmatrix} k_1 A - 2k_2 x(t) & 0 \\ 2k_2 x(t) & -k_3 \end{bmatrix} \xrightarrow{\text{when } X=0,\ Y=0} \begin{bmatrix} \frac{\partial S_1}{\partial x(t)} & \frac{\partial S_1}{\partial y(t)} \\ \frac{\partial S_2}{\partial x(t)} & \frac{\partial S_2}{\partial y(t)} \end{bmatrix}$$

when $X = 0$, $Y = 0$

$$\Rightarrow \begin{bmatrix} k_1 A & 0 \\ 0 & -k_3 \end{bmatrix}$$

$$\lambda^2 - (k_1A - k_3) - k_1k_3A = 0$$

$$\lambda = \frac{k_1A - k_3 \pm \sqrt{k_1^2A^2 + k_3^2 - 2k_1k_3A + 4k_1k_3A}}{2} = \frac{k_1A - k_3 \pm (k_1A + k_3)}{2}$$

$$\lambda_1 = k_1A \quad \lambda_2 = -k_3$$

$\Rightarrow$ unstable

when $x = \frac{k_1A}{k_2}$

$$\begin{bmatrix} k_1A - 2k_1A & 0 \\ 2A & -k_3 \end{bmatrix} = \begin{bmatrix} -k_1A & 0 \\ 2A & -k_3 \end{bmatrix}$$

$$\lambda = -k_1A \text{ or } -k_3$$

$\Rightarrow$ stable

(iv)

$A + X \xrightarrow{k_1} 2X$

$X + X \xrightarrow{k_2} Y$

$Y \xrightarrow{k_3} B$

when $[X]=0$

$A \xrightarrow{k_1} 0 \; (X)$

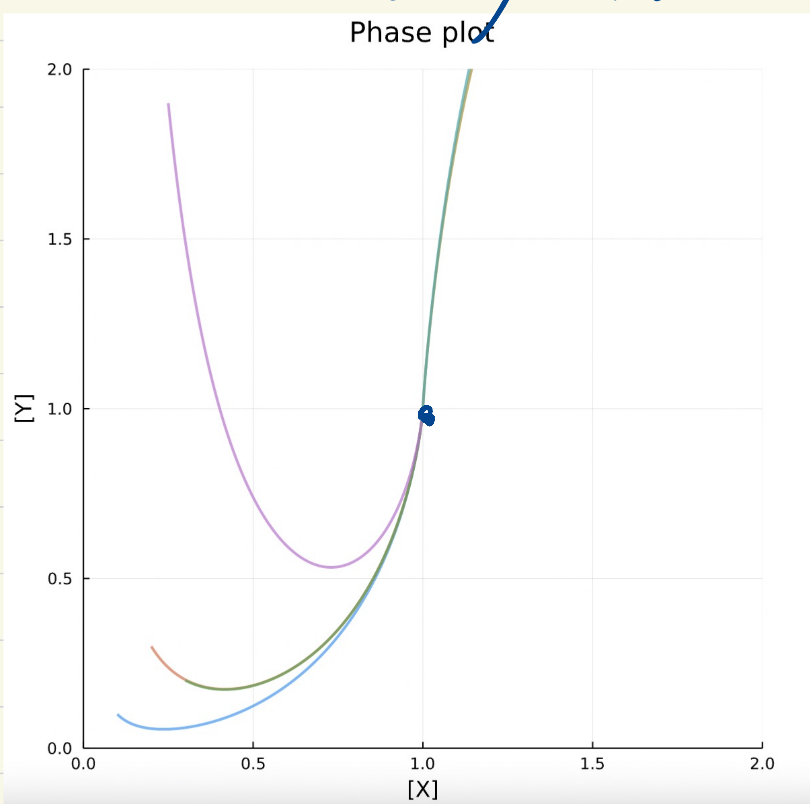
$0 \xrightarrow{k_2} Y \; (X)$

$Y \xrightarrow{k_3} B$

$$Y \xrightarrow{k_3} B$$

$\Rightarrow$ exponential decay,

steady state：交於一點



(B)

$$A + X \xrightarrow{k_1} 2X$$

$$X + Y \xrightarrow{k_2} 2Y$$

$$Y \xrightarrow{k_3} B$$

(i)

$$\frac{d}{dt}x(t) = k_1 A x(t) - k_2 x(t) Y(t)$$

$$\frac{d}{dt}Y(t) = k_2 x(t) Y(t) - k_3 Y(t)$$

(ii) They system has two steady states when

① $\frac{d}{dt}x(t) = 0$ and $\frac{d}{dt}Y(t) = 0$

$$k_1 A \cancel{x(t)} = k_2 \cancel{x(t)} Y(t) \quad , \quad k_2 x(t) \cancel{Y(t)} = k_3 \cancel{Y(t)}$$

$$Y(t) = \frac{k_1 A}{k_2} \text{ or } 0, \quad x(t) = \frac{k_3}{k_2}$$

$$x(t) = \frac{k_3}{k_2} \text{ or } 0$$

(iii)

$$\begin{bmatrix} k_1 A - k_2 Y(t) & -k_2 x(t) \\ k_2 Y(t) & k_2 x(t) - k_3 \end{bmatrix}$$

$x = 0, Y = 0$ $\Rightarrow \begin{bmatrix} k_1 A \cancel{k_1 A}^{0} & 0 \\ k_1 A & -k_3 \end{bmatrix}$

$$\lambda^2 - (k_1 A - k_3) - k_1 k_3 A = 0$$

$$\lambda = \frac{k_1 A - k_3 \pm \sqrt{k_1^2 A^2 + k_3^2 - 2k_1 k_3 A + 4k_1 k_3 A}}{2} = \frac{k_1 A - k_3 \pm (k_1 A + k_3)}{2}$$

$$\lambda_1 = k_1 A \quad \lambda_2 = -k_3 \quad \Rightarrow \text{unstable}$$

$$\begin{bmatrix} k_1A - k_2Y(t) & -k_2X(t) \\ k_2Y(t) & k_2X(t) - k_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -k_3 \\ k_1A & 0 \end{bmatrix}$$

$X = \dfrac{k_3}{k_2}$ $\quad Y = \dfrac{k_1A}{k_2}$

k_1k_3A

$\lambda^2 - (a+d)\lambda + (ad - cb) = 0$

$\lambda^2 + k_1k_3A = 0$

$\lambda^2 = -\underline{k_1k_3}A$

$\Rightarrow$ unstable

(iv)

$A + X \xrightarrow{k_1} 2X$

$X + Y \xrightarrow{k_2} 2Y$

$Y \xrightarrow{k_3} B$

for $X = 0$

$A + 0 \xrightarrow{k_1} 0$

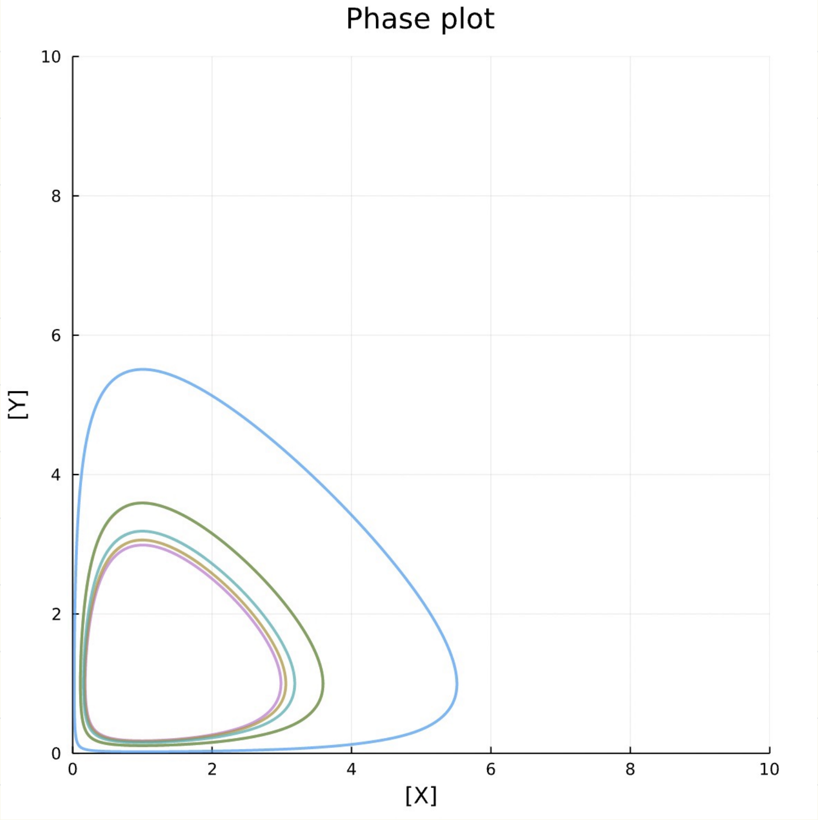
$Y \xrightarrow{k_2} 2Y$

$Y \xrightarrow{k_3} B$

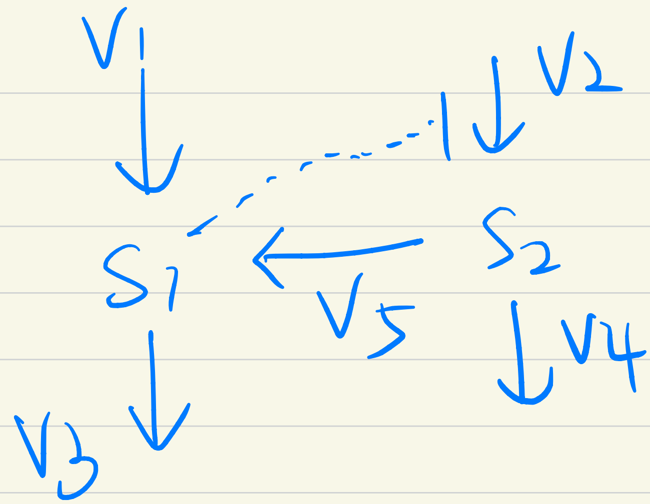
不能執行

$\Rightarrow$ exponential decay

(v)



4.8.1



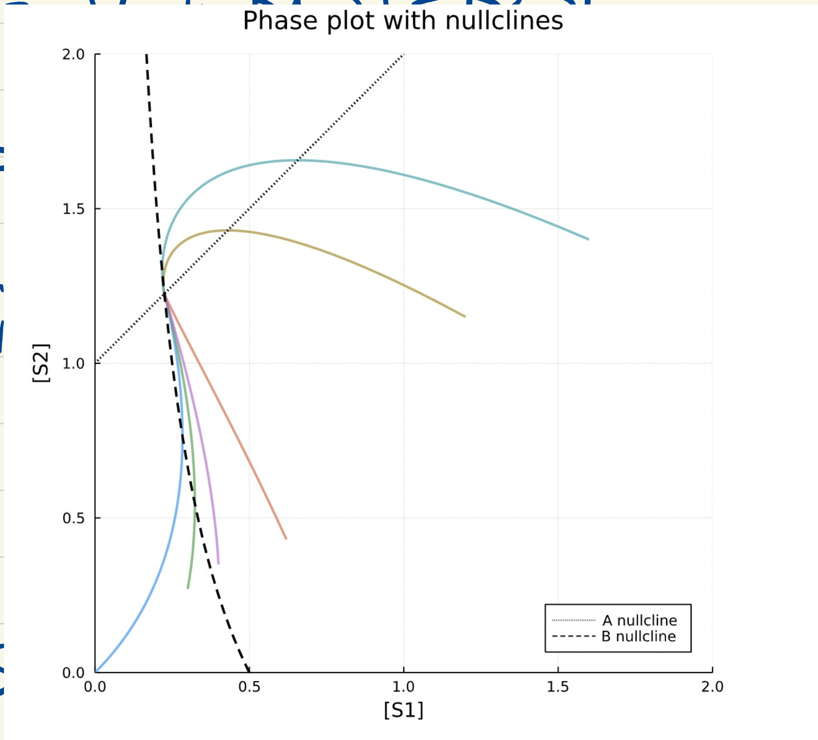
$V_1 = V \quad V_2 = f(S_1)$

$V_3 = k_3 S_1 \quad V_4 = k_4 S_2$

$V_5 = k_5 S_2$

$\frac{d}{dt} S_1(t) = V + k_5 S_2 - k_3 S_1$

$\frac{d}{dt} S_2(t) =$ … S_2



the nullcl…

$S_1 = \frac{V + k_5 S_2}{k_3}$

$S_2 =$ …