# 01-D-2 级数

#数据结构邓神

## 算数级数

```
T(n) = 1 + 2 + ... + n = n(n+1)/2 = O(n^2) // BigO
```

最后的复杂度等于末项的平法

## 幂方级数:比幂次高一阶

```
T(n) = 1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6 = O(n^3)
T(n) = 1^3 + 2^3 + ... + n^3 = n^2(n+1)^2/4 = O(n^4)
...
```

### 严格证明

$$\sum_{k=0}^{n} k^d \approx \int_0^n x^{d+1}\,dx = \frac{1}{d+1}x^{d+1}\Big|_0^n = \frac{1}{d+1}n^{d+1} = O(n^{d+1})$$

## 几何级数 (a > 1) : 与末项同阶

```
T_a(n) = a^0 + a^1 + ... + a^n = (a^n+1 - 1)/(a - 1) = O(a^n)
1 + 2 + 4 + ... + 2^n = 2^n+1 - 1 = O(2^n)
```

## 收敛级数

```
1/1/2 + 1/2/3 + 1/3/4 + ... + 1/(n-1)/n = 1 - 1/n = O(1)
1 + 1/n^2 + ... + 1/n^2 < 1 + 1/2^2 + ... = π^2/6 = O(1)
1/3 + 1/7 + 1/8 + ... = 1 = O(1)
```

### 有必要吗，每一项都是分数?

难道基本操作次数和存储单元数可能是分数吗?

某种意义上是!

```
(1−λ)∗[1 + 2λ + 3λ^2 + ...] = 1/(1 − λ) = O(1) 0<λ<1 // 几何分布
```

## 未必收敛，但是长度有限

```
h(n) = 1 + 1/2 + 1/3 + ... + 1/n = θ(logn) // 调和级数
log1 + log2 + log3 + ... + logn = log(n!) = θ(nlogn) // 对数级数
```