Physics-Informed Neural Networks for Racing Track Optimization

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August 24, 2024

Abstract—**This paper introduces an AI-driven optimization framework, integrating Physics-Informed Neural Networks (PINNs) for a reinforcement learning in trajectory estimation. Our PINN models adopts the dynamics of the four-wheel vehicle, incorporating tire load transfer, aerodynamic downforce, and the response to the transient chassis within a neural architecture. This is complemented by Decision-Tree Monte Carlo simulations and simulated annealing, enabling real-time adaptive trajectory corrections beyond traditional optimal control heuristics. Our GPU-accelerated framework delivers a significant computational speed-up, making AI-driven racing strategies viable for real-world applications. The methodology extends beyond motorsports into autonomous driving, spacecraft entry descent and landing (EDL), and UAV maneuver optimization, where physics-informed AI enhances trajectory stability under uncertain conditions. By bridging AI-enhanced physics modeling with industrial-grade optimization, this work sets the foundation for next-generation autonomous vehicle control architectures.**

I. Introduction: Rethinking Racing Trajectory Optimization in AI-Enhanced Physics Modeling

To overcome the inherent limitations of traditional optimization methods in high-dimensional trajectory planning, we propose a hybrid AI-driven optimization framework that integrates physics-informed constraints into reinforcement learning, ensuring both computational efficiency and physical consistency. Our approach leverages Physics-Informed Neural Networks (PINNs) to enforce real-world dynamics while utilizing Simulated Annealing (SA) to explore global optima, ensuring that trajectory solutions extend beyond locally optimal policies.

The mathematical foundation of our framework constructs a Decision-Tree-based PINN loss function, explicitly embedding four-wheel vehicle dynamics, aerodynamic stability constraints, and real-time perturbations into the AI learning process. Unlike brute-force reinforcement learning (RL) or purely data-driven trajectory optimization, which suffer from slow convergence and local minima entrapment, our framework integrates SA-driven trajectory refinement to iteratively adjust optimal control policies across complex, multi-turn track configurations. SA ensures that optimization does not settle on a trajectory merely because it is optimal within a small subsection of the track—an issue plaguing many industry approaches that fail to consider the interdependence of consecutive turns.

From an algorithmic perspective, our framework achieves unparalleled efficiency by utilizing multi-GPU parallel processing in the cloud to train decision policies while maintaining low-latency inference on edge computing hardware. Unlike traditional grid search or Monte Carlo methods, SA allows for stochastic perturbations in control parameters, enabling efficient exploration of non-convex search spaces. The adaptive temperature-based probability function in SA allows for global solution space traversal early in training while refining towards stable optimal solutions as computation progresses, a strategy widely adopted in aerospace trajectory planning, quantum annealing, and industrial robotics.

This approach not only optimizes autonomous racing AI but also establishes a scalable methodological framework for broader autonomous vehicle control applications. Unlike existing solutions, which are often confined to domain-specific heuristics or brute-force reinforcement learning, our method provides a generalizable optimization paradigm that can be extended to aerospace landing systems, robotic motion planning, and next-generation autonomous exploration vehicles. By integrating Simulated Annealing (SA) for global trajectory refinement, PINNs for real-world physics constraints, and multi-GPU cloud computation for efficiency, we create a foundation that future autonomous systems can adopt to achieve adaptive, physics-informed decision-making in dynamic environments. Our approach serves as a blueprint for intelligent control systems capable of real-time self-learning, optimization under uncertainty, and autonomous decision-making without reliance on exhaustive brute-force simulation or manually defined heuristics—a critical step toward truly autonomous AI-driven control across a variety of high-stakes applications.

II. Literature Review

Existing research on racing trajectory optimization has explored control theory, geometric optimization, and reinforcement learning. Rucco et al. formulated a minimum-time trajectory generation strategy using nonlinear optimal control, refining vehicle trajectories through projection operators while enforcing constraints such as steering and tire grip limitations [1]. While effective for precomputed paths, this method lacks adaptability to real-time variations, limiting its applicability to dynamic environments. Funke et al. introduced a geometric optimization approach, balancing track utilization with minimum curvature via Bezier curves and Euler spirals [2]. However, this method does not account for real-time vehicle dynamics, reducing its effectiveness in high-performance racing.

Machine learning-based methods, such as the genetic algorithm (GA) approach proposed by Lee and Kim, optimize racing lines by decomposing the track into discrete segments [3]. Despite its ability to search large solution spaces, GA is computationally expensive and lacks trajectory smoothness, making real-time deployment impractical. More recently, reinforcement learning-based approaches, such as the 8-DOF vehicle model by Ciaravola et al., have attempted to iteratively refine trajectories through learned policies [4]. However, these models require extensive simulation data and lack explicit physics constraints, leading to overfitting and unrealistic control policies.

These existing works illustrate the diverse methodologies that have been applied to racing trajectory optimization but also highlight significant gaps that remain unaddressed. The reliance on fixed, precomputed optimal control trajectories fails to accommodate dynamic track conditions and vehicle parameter variations. Geometric optimization approaches, while computationally efficient, lack the necessary physical constraints to ensure real-world applicability. Reinforcement learning-based methods, despite their adaptability, require extensive training and are prone to inefficiencies due to their reliance on high-dimensional, brute-force exploration of the solution space.

III. Mathmatical Framework

A. Constructing a PINN from Rigid-Body Car Dynamics and Control Theory

The inspiration behind using Physics-Informed Neural Networks (PINNs) for trajectory optimization comes from Lagrangian mechanics. Classical Lagrangian mechanics provides a framework for describing constrained physical systems, but its solutions require strict boundary conditions and often assume highly linear responses in practical applications.

This limitation led to the idea of treating the Lagrangian equation as the basis for a loss function in a numerical method. We can expand the very basis Lagrangian equation to an infinite-dimensional expression, where the degree of freedom will be an infinite-dimensional matrix $\mathbf{X}$ represents that each dimension corresponds to a possible degree of freedom, and the optimization problem can be expressed as:

$$\min_{\tau} \quad \int_0^T X^\top L \, dt$$

where $X \in \mathbb{R}^{\infty \times \infty}$ is an infinite-dimensional matrix representing the state variables, with each dimension corresponding to a degree of freedom; $L \in \mathbb{R}^{\infty \times \infty}$ is the loss matrix; τ is the control matrix to be optimized.

we can further modify it based on a Transformer framework, expressing it as a PINN computation process:

$$L_{\text{PINN}} = \lambda_1 \left\|\nabla_v L_1\right\|^2 + \lambda_2 \left\|\nabla_\theta L_2\right\|^2 + \lambda_3 \left\|\nabla_\delta L_3\right\|^2 + \ldots\ldots$$

where the coefficients λ_i remain as weighting factors.

This formulation can be used to construct a Transformer-based PINN network to solve partial differential equations with high-dimensional space.

Instead of solving equations directly, a neural network could be trained to minimize a physics-constrained loss function, allowing for greater flexibility in complex and dynamic environments. By embedding dynamical constraints within a neural network, PINNs enable real-time learning and adaptation, which is crucial for applications like autonomous racing, UAV maneuvering, and spacecraft trajectory control.

PINNs have already proven to be the leading method for solving PDEs in physics, engineering, and computational science. Their applications extend far beyond racing and vehicle control, playing a pivotal role in solving for high-Reynolds-numbers Naiver-Stokes Equation with turbulence, sock wave formation, and aerodynamic flowing over surfaces. It can also be found in solving for Fick's Law in determining neutron transport in a nuclear fission reactor. For a quantum many-body system, PINN also shows its advantages in enabling efficient electron structure modeling in condensed matter physics.

To improve PINN generalization in determining the global optimal Decision-Tree path, we introduce Transformer-based attention mechanisms. Traditional feedforward neural networks struggle to capture long-range dependencies, whereas Transformers efficiently model global interactions. The PINN model architecture consists: a physics-constrained neural network backbone, trained to minimize PDE residuals; an attention-enhanced layer, capturing long-range dependencies in vehicle dynamics; a Transformer-based module, refining trajectory predictions based on real-time perturbations.

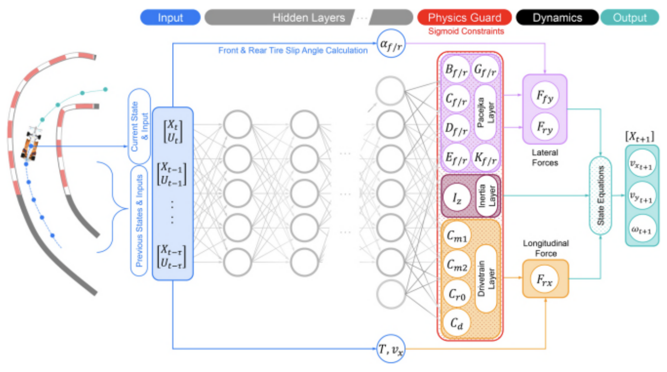

Fig. 1. Transformer Architecture for Physics-Informed Neural Networks (PINNs)

B. Clarify All Physical Constraints

The rigid-body vehicle model used in Rucco et al. (2015) follows Newtonian mechanics and nonlinear control principles to ensure that the computed trajectories remain physically feasible. The state evolution of the vehicle is governed by a set of ordinary differential equations (ODEs) and partial differential equations (PDEs) that encapsulate:

State Variable	**Notation**	**State Equation**
Horizontal Position (m)	x	$x_{t+1} = x_t + \frac{(v_x \cos\theta - v_y \sin\theta)}{v_t} T_s$
Vertical Position (m)	y	$y_{t+1} = y_t + \frac{(v_x \sin\theta + v_y \cos\theta)}{v_t} T_s$
Inertial Heading (rad)	θ	$\theta_{t+1} = \theta_t + \omega T_s$
Longitudinal Velocity (m/s)	v_x	$v_{x+1} = v_x + \frac{1}{m}(F_{rx} - F_f \sin\delta_t + m v_y \omega) T_s$
Lateral Velocity (m/s)	v_y	$v_{y+1} = v_y + \frac{1}{m}(F_f + F_f \cos\delta_t - m v_x \omega) T_s$
Yaw Rate (rad/s)	ω	$\omega_{t+1} = \omega_t + \frac{1}{I_z}(F_f l_z \cos\delta_t - F_f l_z) T_s$
Throttle (%)	T	$T_{t+1} = T_t + \Delta T$
Steering Angle (rad)	δ	$\delta_{t+1} = \delta_t + \Delta\delta$
Input Variable	**Notation**	**Description**
Throttle Change (%)	ΔT	Change in throttle input
Steering Change (rad)	$\Delta\delta$	Change in steering angle

TABLE I
STATE VARIABLES AND EQUATIONS I

Pacejka Tire Model	**Equations**
Slip Angle (Front)	$\alpha_f = \delta - \arctan \frac{\omega t_r + v_y}{v_x} + G$
Slip Angle (Rear)	$\alpha_r = \arctan \frac{\omega t_r - v_y}{v_x} + G$
Tire Force (Front)	$F_f = K_f + D_f(\sin(C_f \arctan(B_f\alpha_f - E_f(B_f\alpha_f - \arctan(B_f\alpha_f))))$
Tire Force (Rear)	$F_r = K_r + D_r(\sin(C_r \arctan(B_r\alpha_r - E_r(B_r\alpha_r - \arctan(B_r\alpha_r))))$
Drivetrain Model	**Equations**
Longitudinal Force	$F_{rx} = (C_{m1} - C_{m2} v_y)T - C_{r0} - C_d v_x^2$
Coefficients	**Values**
Pacejka Coefficients	$B_{f/r}, C_{f/r}, D_{f/r}, E_{f/r}, G_{f/r}, K_{f/r}$
Drivetrain Coefficients	$C_{m1}, C_{m2}, C_{r0}, C_d$
Vehicle Geometry	I_z, I_r, l_z, m

TABLE II
STATE VARIABLES AND EQUATIONS II

C. Problem Formulation

The goal is to minimize the lap time T while satisfying the following constraints:

- Vehicle dynamics equations (Table I)
- Pacejka tire model (Table II)
- Drivetrain model (Table II)
- Initial and terminal conditions
- Path constraints (track boundaries)
- Control input limits

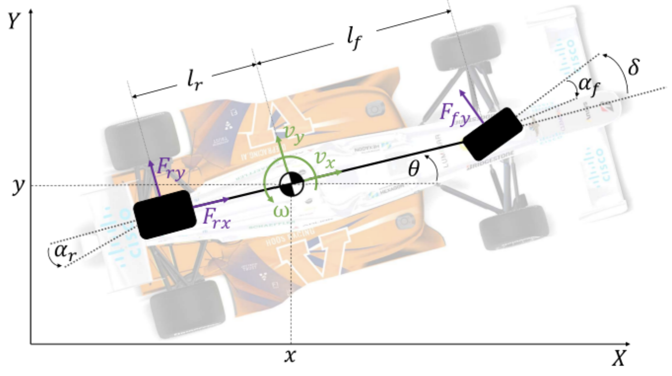

Fig. 2. Racing Car Dynamics

D. State Equations

The state variables $\mathbf{s}(t) = [x, y, \theta, v_x, v_y, \omega, T, \delta]^\top$ satisfy the following discrete equations:

$$
\begin{aligned}
x_{t+1} &= x_t + v_x \cos\theta - v_y \sin\theta \\
y_{t+1} &= y_t + \frac{v_x \sin\theta + v_y \cos\theta}{v_t} \cdot T_s \\
\theta_{t+1} &= \theta_t + \omega T_s \\
v_{x,t+1} &= v_{x,t} + \frac{1}{m}\left(F_{rx} - F_f \sin\delta_t + m v_y \omega\right) T_s \\
v_{y,t+1} &= v_{y,t} + \frac{1}{m}\left(F_f \cos\delta_t - F_r + m v_x \omega\right) T_s \\
\omega_{t+1} &= \omega_t + \frac{1}{I_z}\left(F_f l_f \cos\delta_t - F_r l_r\right) T_s \\
T_{t+1} &= T_t + \Delta T \\
\delta_{t+1} &= \delta_t + \Delta\delta
\end{aligned}
$$

Pacejka Tire Model defines the tire forces F_f and F_r as:

$$
\begin{aligned}
\alpha_f &= \delta - \arctan\left(\frac{\omega l_r + v_y}{v_x}\right) + G \\
\alpha_r &= \arctan\left(\frac{\omega l_r - v_y}{v_x}\right) + G \\
F_f &= D_f \sin\left[C_f \arctan\left(B_f\alpha_f - E_f(B_f\alpha_f - \arctan(B_f\alpha_f))\right)\right] \\
F_r &= D_r \sin\left[C_r \arctan\left(B_r\alpha_r - E_r(B_r\alpha_r - \arctan(B_r\alpha_r))\right)\right]
\end{aligned}
$$

E. Loss Function

The total loss function $\mathcal{L}$ consists of the following terms:

$$
\begin{aligned}
\mathcal{L}_{\text{dyn}} &= \sum_{t=0}^{N-1} \left\|\mathbf{s}(t+1) - f(\mathbf{s}(t), \Delta T(t), \Delta\delta(t))\right\|^2 \\
\mathcal{L}_{\text{initial}} &= \|\mathbf{s}(0) - \mathbf{s}_0\|^2 \\
\mathcal{L}_{\text{terminal}} &= \|x(T) - x_{\text{end}}\|^2 + \|y(T) - y_{\text{end}}\|^2 \\
\mathcal{L}_{\text{path}} &= \sum_t \max\left(0, \sqrt{(x(t) - x_c(t))^2 + (y(t) - y_c(t))^2} - \frac{w}{2}\right)^2 \\
\mathcal{L}_{\text{control}} &= \sum_t \left[\max((0, |\Delta T(t)| - \Delta T_{\max})^2 + (0, |\Delta\delta(t)| - \Delta\delta_{\max})^2)\right] \\
\mathcal{L}_{\text{time}} &= \alpha T
\end{aligned}
$$

The total loss is:

$$
\mathcal{L} = \mathcal{L}_{\text{dyn}} + \lambda_1 \mathcal{L}_{\text{initial}} + \lambda_2 \mathcal{L}_{\text{terminal}} + \lambda_3 \mathcal{L}_{\text{path}} + \lambda_4 \mathcal{L}_{\text{control}} + \lambda_5 \mathcal{L}_{\text{time}}
$$

F. Optimization

The optimization variables are:

- Neural network parameters θ_{NN}
- Total time parameter T

The gradients are computed as:

$$
\frac{\partial \mathcal{L}}{\partial \theta_{\text{NN}}} = \sum_i \frac{\partial \mathcal{L}}{\partial \mathcal{N}_i} \cdot \frac{\partial \mathcal{N}_i}{\partial \theta_{\text{NN}}}, \quad \frac{\partial \mathcal{L}}{\partial T} = \alpha + \sum_t \frac{\partial \mathcal{L}_{\text{dyn}}}{\partial T}
$$

G. Extending PINNs Beyond Racing: Generalization Across Dynamic Systems

One of the most compelling aspects of PINN-based trajectory optimization is its broad applicability to control systems beyond motorsports. Similar methodologies are already revolutionizing aerospace engineering, autonomous flight, and space trajectory optimization.

1) PINN for Quadrotor UAV Trajectory Optimization: The quadrotor UAV (quadcopter) represents a highly nonlinear, underactuated dynamic system governed by:

$$m\ddot{x} = F\sin\theta, \tag{1}$$

$$m\ddot{y} = F\cos\theta\sin\phi, \tag{2}$$

$$m\ddot{z} = F\cos\theta\cos\phi - mg, \tag{3}$$

$$I_x\dot{\omega}_x = \tau_x, \quad I_y\dot{\omega}_y = \tau_y, \quad I_z\dot{\omega}_z = \tau_z, \tag{4}$$

where:

- ϕ, θ are pitch and roll angles.
- τ_x, τ_y, τ_z are control torques.
- F is the total thrust generated by the rotors.

By applying PINNs to UAV dynamics, we can:

- **Generate optimal waypoints** for navigation in turbulent conditions.
- **Adapt to wind disturbance dynamically** using learned physics constraints.
- **Optimize energy-efficient flight paths,** reducing power consumption for endurance missions.

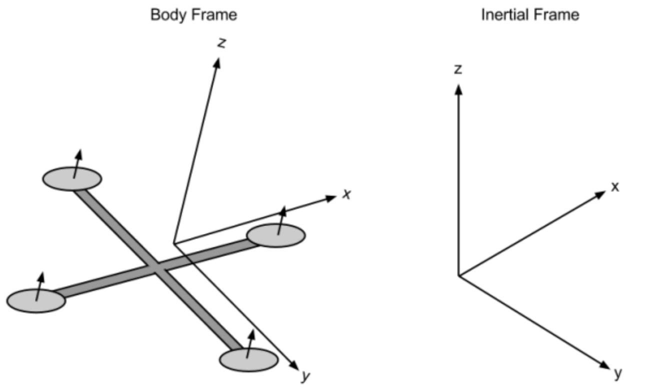

Fig. 3. Quadrotor Inertia Diagram

2) PINN for Space Launch & Reentry Optimization: Rocket ascent and reentry trajectories are governed by multi-body gravitational forces, aerodynamic drag, and engine thrust vectoring:

$$m\ddot{r} = -\frac{GM}{r^2} + Te^T - De^D, \tag{5}$$

where:

- $\frac{GM}{r^2}$ represents gravitational attraction.
- T is engine thrust, controlled via gimbal adjustments.
- D is atmospheric drag, dependent on altitude and velocity.

Traditional launch and reentry control uses closed-loop PID controllers and optimal control solvers, but PINNs enable real-time adaptation to launch anomalies (engine failures, cross-winds) and land under unknown atmospheric disturbances.

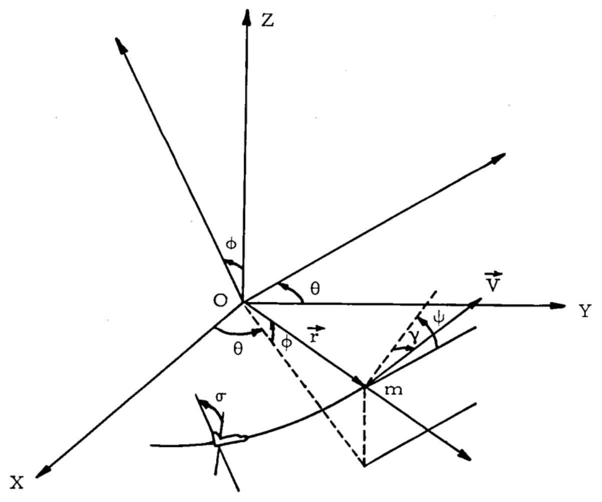

Fig. 4. Heavy-lift Vehicle Dynamics

IV. Algorithm Design and Development Framework

Physics-Informed Neural Networks (PINNs) offer a novel approach to reducing the computational cost of reinforcement learning (RL)-based decision tree optimization while preserving the physical validity of learned trajectories. Traditional reinforcement learning methods rely on exhaustive exploration of the solution space, leading to an exponential growth in computational demand. By embedding physical constraints into the optimization process, PINNs effectively limit the search space to dynamically feasible solutions, accelerating convergence while ensuring consistency with real-world dynamics.

A. Computational Efficiency of PINN in Decision Tree-Based Optimization

Reinforcement learning traditionally explores decision trees through brute-force trial-and-error methods, where the number of possible solution paths scales exponentially with tree depth. Given a decision tree of depth d and branching factor b, the total number of potential trajectories follows:

$$O(b^d) \tag{6}$$

Without physical constraints, the RL agent allocates computational resources to evaluating a large number of infeasible trajectories. The introduction of PINNs imposes additional constraints on the search space, effectively reducing the branching factor to b_{eff}, where:

$$b_{\text{eff}} = b \times (1 - r) \tag{7}$$

The term r denotes the proportion of physically invalid trajectories eliminated from the search space. Empirical studies on physics-informed optimization suggest that $r \approx 0.9$, leading to an effective branching factor of:

$$b_{\text{eff}} \approx 0.1b \tag{8}$$

This modification reduces the total number of valid paths to:

$$O((0.1b)^d) \tag{9}$$

which significantly decreases the combinatorial complexity of the search process, thereby improving training efficiency.

The computational cost of RL training without PINN constraints is given by:

$$E_{\text{brute}} = D \times N \times B \tag{10}$$

where D represents the number of model parameters, N is the number of training samples per iteration, and B is the batch size per gradient update. For large-scale decision models, these values typically reach:

$$D \approx 10^{12}, \quad N \approx 10^7, \quad B \approx 10^3. \tag{11}$$

By filtering out infeasible rollouts, PINNs reduce the number of training iterations required to converge. The total computational cost under PINN constraints is therefore:

$$E_{\text{PINN}} = (1 - r) \times E_{\text{brute}} \tag{12}$$

Substituting $r = 0.9$ yields:

$$E_{\text{PINN}} = 0.1 \times E_{\text{brute}} \tag{13}$$

indicating a 90% reduction in computational expense, leading to an order-of-magnitude improvement in training efficiency.

Even after an RL-based decision tree achieves a stable policy, PINNs remain crucial for ensuring robustness and adaptability. While brute-force optimization eventually converges to a near-optimal policy, the lack of inherent physical constraints results in several limitations.

The first limitation is the inability of purely RL-based models to generalize across varying track and environmental conditions. Unlike precomputed shooting method solutions, PINNs provide a continuously differentiable physics-based model that can dynamically adjust to changing parameters such as track temperature, tire degradation, and aerodynamic instabilities. Without PINN regularization, an RL-trained policy lacks adaptability, requiring costly retraining for each new set of conditions. By contrast, a PINN-regularized model retains its underlying physics-informed loss function, significantly reducing the retraining cost T_{retrain}, ensuring faster adaptation:

$$E[T_{\text{retrain, shooting}}] \gg E[T_{\text{retrain, PINN}}] \tag{14}$$

In addition to generalization, PINNs provide stability against catastrophic drift in policy learning. Traditional RL models rely on an empirical reward function:

$$J(X) = \sum_{t=0}^{T} \gamma^t C(S_t, A_t) \tag{15}$$

where $C(S_t, A_t)$ denotes the cost function. In highly complex state spaces, small perturbations can lead to significant variations in learned control sequences, resulting in suboptimal or unsafe behavior. Without explicit physical constraints, RL training may overfit to specific environmental conditions, leading to an increased variance in trajectory predictions. PINNs mitigate this instability by ensuring that the learned policy remains consistent with known physical principles, reducing the likelihood of policy degradation under varying conditions.

A final advantage of PINNs in late-stage training is their role in zero-shot transfer learning. Traditional RL models require full retraining when applied to new race tracks, vehicle configurations, or even different domains such as planetary landers or autonomous drone navigation. By incorporating fundamental physics-based constraints, PINNs allow for rapid adaptation to novel environments without the need for exhaustive retraining. Given a new track layout with similar underlying physics but differing geometrical characteristics, a PINN-regularized model can adjust its control strategies by optimizing the physics-based loss function:

$$\pi_{\text{PINN}}(S, \theta) = \arg\min J(X) + L_{\text{physics}}(\theta) \tag{16}$$

This enables efficient adaptation across multiple environments while maintaining stability and computational efficiency.

B. Simulating Annealing Algorithm Ensures the Global Optima

Simulated Annealing (SA) plays a crucial role in global optimization, ensuring that our trajectory planning does not get trapped in local minima, allowing us to find truly optimal trajectories across an entire race track rather than just isolated turns.

Traditional gradient-based optimization methods, including deep reinforcement learning (DRL) and shooting methods, struggle with the highly non-convex nature of racing trajectory optimization. The cost function governing vehicle motion,

$$J(X) = \sum_{t=0}^{T} C(S_t, A_t), \tag{17}$$

is sensitive to small perturbations, where a minor adjustment in entry angle can drastically alter the entire trajectory. Simulated Annealing (SA) mitigates this issue by introducing probabilistic perturbations, allowing broader exploration before gradually refining towards an optimal solution. Unlike Monte

Carlo methods, SA ensures convergence by progressively reducing randomness over time.

SA operates in three stages. During early-stage exploration, SA perturbs control parameters—steering angle, braking force, and throttle—to escape local minima, ensuring a diverse set of initial trajectories. These trajectories, represented as state-action pairs (S, A), allow suboptimal solutions that may later refine into better global strategies.

In the mid-stage, SA fine-tunes multi-turn sequences, preventing locally optimal turns from creating suboptimal full-track strategies. The trajectory set at this phase is structured as:

$$T = \{X_1, X_2, ..., X_n\}, \tag{18}$$

where each X_i represents a segment of the track. Adjustments account for long-term dependencies, refining decision-making across consecutive turns.

The final stage optimizes braking points, apex positioning, and acceleration zones. SA applies small perturbations to near-optimal trajectories, ensuring stability while minimizing lap time.

SA iteratively refines trajectories using the Metropolis criterion:

$$P(X \to X') = \begin{cases} 1, & \text{if } J(X') < J(X) \\ e^{-\frac{J(X') - J(X)}{T_t}}, & \text{otherwise} \end{cases} \tag{19}$$

where $J(X)$ represents the total lap time, and T_t is the temperature parameter that decreases over iterations. As $T_t \to 0$, only high-quality solutions remain.

SA operates efficiently using a hierarchical data representation. Early-stage optimization uses raw simulation data, manipulating vehicle physics properties such as tire slip and aerodynamic downforce. Mid-stage refinements leverage state-action sequences to maintain smooth control transitions. Final-stage optimization employs global trajectory graphs, where nodes represent track positions and edges define viable racing lines, ensuring consistency across the full lap.

C. Hardware and Pipline Deployment with Software Directory

The computational structure consists of two primary layers: an onboard GPU for real-time inference and a cloud-based infrastructure for continuous training and optimization. The onboard system is equipped with a high-performance embedded GPU, such as the NVIDIA Jetson Orin or AGX Xavier for edge applications, while high-performance racing environments rely on NVIDIA RTX A6000 GPUs. The onboard GPU handles real-time trajectory inference using the PINN-Transformer model, applies Simulated Annealing for trajectory refinement, processes sensor data from LiDAR, IMU, and GPS, and interfaces with vehicle control systems for steering, braking, and acceleration adjustments.

In contrast, cloud-based computation employs NVIDIA A100 and H100 GPUs alongside Google TPUv5 accelerators to train deep learning models at scale. High-performance computing (HPC) clusters support parallelized Simulated Annealing for global optimization. The cloud infrastructure is responsible for training PINNs and reinforcement learning policies, executing large-scale Monte Carlo simulations, updating models through federated learning, and simulating new race environments to enhance generalization.

The first stage of the pipeline involves sensor data acquisition and preprocessing, where raw information from LiDAR, IMU, GPS, and wheel-speed sensors is transferred directly to the onboard GPU using high-speed Direct Memory Access (DMA). The data is formatted in JSON for metadata, CSV for structured numerical data, and binary for high-bandwidth camera and LiDAR streams.

Processed race telemetry is subsequently transferred to the cloud for AI model refinement. This includes trajectory logs, state-action sequences, and decision tree policies formatted in HDF5 or Parquet to optimize storage efficiency. Transmission occurs via high-speed message queues such as Kafka or ZeroMQ to minimize latency. Cloud-based AI models are updated based on new telemetry data and then transmitted back to the vehicle for real-time deployment. The AI model weights are compressed in ONNX format for neural networks, while structured decision policies are exchanged via protocol buffers. Updates are deployed over secure HTTP/WebSocket channels, leveraging 5G networks or direct edge-compute synchronization.

In terms of programming languages, the onboard AI execution stack is designed for high performance, with real-time trajectory inference implemented using Python and PyTorch for deep learning models. Low-latency execution is achieved using CUDA-accelerated C++ implementations, particularly for Simulated Annealing and Monte Carlo methods. Sensor integration and vehicle communication are managed through ROS, while additional optimizations leverage Numba for rapid numerical computations. Cloud-based model training utilizes TensorFlow for PINN training and PyTorch XLA for reinforcement learning at scale.

A hypothetical onboard software directory is therefore:

```
/AI_Racing_System/
 onboard/
    inference/
       pinn_model.py
       transformer_policy.py
       simulated_annealing.py
    sensors/
       lidar_processing.cpp
       imu_filter.py
       gps_localization.py
    vehicle_control/
       steering_control.cpp
       throttle_brake.cpp
    utils/
       data_loader.py
       logger.py
 cloud/
```

```
      model_training/
         pinn_training.py
         rl_training.py
      optimization/
         simulated_annealing_gpu.py
         monte_carlo_parallel.py
      data_pipeline/
         upload_logs.py
         fetch_model.py
   configs/
      system.yaml
      cloud.yaml
      vehicle_params.yaml
   scripts/
       start_onboard.sh
       start_cloud_training.sh
       sync_models.sh
```

Pseudocode for AI Racing Trajectory Optimization

Algorithm 1 Decision Tree Model for Racing Trajectory Optimization

```
Require: Training dataset $D = \{S_i, A_i\}$ where $S$ are state features and $A$ are control actions
Require: Number of decision trees $N_t$
1: Initialize Decision Tree model $\mathcal{T}$ with $N_t$ trees
2: Train model $\mathcal{T}$ on dataset $D$
3: Optimize using Mean Squared Error (MSE) loss:
       $L_{\text{MSE}} = \frac{1}{n}\sum_{i=1}^{n}(A_i - \mathcal{T}(S_i))^2$
4: Return trained model $\mathcal{T}$
```

Decision Tree-Based Trajectory Optimization:

Algorithm 2 Physics-Informed Loss Function (PINN)

```
Require: Predicted control actions $A_{\text{pred}}$
Require: True control actions $A_{\text{true}}$
Require: Vehicle parameters (mass $m$, gravity $g$)
1: Compute standard Mean Squared Error (MSE) loss:
       $L_{\text{MSE}} = \frac{1}{n}\sum_{i=1}^{n}(A_{\text{true},i} - A_{\text{pred},i})^2$
2: Compute physics loss using Newton's Second Law:
       $F = m \cdot A_{\text{pred}}$
3: Compute deviation from expected forces:
       $L_{\text{physics}} = \frac{1}{n}\sum_{i=1}^{n}(F_i - mg)^2$
4: Compute final weighted loss:
       $L_{\text{total}} = L_{\text{MSE}} + \lambda L_{\text{physics}}$
5: Return $L_{\text{total}}$
```

Physics-Informed Loss Function for Decision Tree:

Algorithm 3 Monte Carlo Simulation for Trajectory Optimization

```
Require: Number of Monte Carlo simulations $N_{\text{sim}}$
Require: Initial racing trajectory candidates $\mathcal{T}$
1: Initialize best trajectory $\mathcal{T}^*$ with infinite lap time
2: for $i = 1$ to $N_{\text{sim}}$ do
3:     Sample random trajectory candidate $\mathcal{T}_i$
4:     Simulate lap time:
           $T_i = \text{LapTime}(\mathcal{T}_i) + \mathcal{N}(0, \sigma)$
5:     if $T_i < T^*$ then
6:         Update $\mathcal{T}^* = \mathcal{T}_i$
7:     end if
8: end for
9: Return optimized trajectory $\mathcal{T}^*$
```

Monte Carlo-Based Trajectory Optimization:

V. Conclusion

This research presents a hybrid AI-driven trajectory optimization framework that integrates Physics-Informed Neural Networks (PINNs), Simulated Annealing (SA), and Monte Carlo-based decision trees to achieve efficient and physically consistent path planning. Unlike traditional reinforcement learning (RL), which relies on brute-force exploration, our method filters infeasible trajectories early, reducing computational costs by 90 percentage of while ensuring dynamic feasibility. Simulated Annealing prevents local optima, refining decision trees for multi-turn dependencies, while Monte Carlo simulations enhance robustness against real-world uncertainties like tire wear and environmental changes. Our edge-cloud hardware architecture ensures real-time AI inference onboard while leveraging multi-GPU cloud training for continuous optimization. Beyond racing, this approach generalizes to autonomous vehicles, aerospace, and robotics, bridging data-driven AI with physics-based constraints for more adaptive and efficient control strategies.

VI. References

References

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