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Elastic electron scattering from argon at low incident energies

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Abstract. Absolute differential (12°–130°) cross sections for low energy (1–10 eV) electron scattering from argon have been measured on two separate and different spectrometers using the relative flow technique. The data have also been analysed using phaseshift techniques. Comparisons are made between the present cross sections and phaseshifts and similar data from previous experiments and theory. The results are particularly encouraging, with recent experiments and scattering theories being in good general agreement.

1. Introduction

Although the noble gases are among the most often studied systems in low energy electron–atom collision experiments and the subject of extensive theoretical investigation, only helium (e.g. Register *et al* 1980, Brunger *et al* 1992, Fursa and Bray 1995) and neon (Shi and Burrow 1992, Gulley *et al* 1994) are regarded as systems for which the elastic differential cross sections are well established in that recent experimental and theoretical studies result in cross sections which agree to within ±5% over a wide range of incident energies and scattering angles.

Electron scattering from argon is a case in point. At energies of 10 eV and below, despite several recent experiments (Williams 1979, Srivastava *et al* 1981, Zhou Qing *et al* 1982, Weyhreter *et al* 1988, Furst *et al* 1989) and extensive recent theoretical investigation (Berg 1983, Fon *et al* 1983, Kemper *et al* 1983, McEachran and Stauffer 1983, Bell *et al* 1984, Dasgupta and Bhatia 1985, Bloor and Sherrod 1986, Haberland *et al* 1986, Yousif and Matthew 1986, Nahar and Wadehra 1987, Sienkiewicz and Baylis 1987, Saha 1991, 1993b, 1995, Mimnagh *et al* 1993, McEachran and Stauffer 1996) the experimentally determined cross sections, whilst agreeing reasonably well with each other, do not agree with any of the theoretical calculations to within ±5% over the whole energy and angular range. The main goal of the present work is to provide some further accurate absolute cross sections in the low energy region for comparison and to closely investigate these discrepancies in an effort to determine whether the most recent theories and experiments are converging.

As touched upon above, in the recent work by Shi and Burrow (1992) and Gulley *et al* (1994), it was suggested that neon could be used as a 'secondary standard' in relative flow experiments in order to provide a check on the operation of a crossed-beam apparatus.

A logical extension to this suggestion is to determine whether the elastic differential cross sections for the other, heavier noble gases are well enough understood by both experiment and theory at low energies to also qualify as secondary standards for the relative flow technique. Given that the technique is predicated upon the assumption that the two gases under study exhibit equivalent behaviour when formed into beams under well controlled conditions of pressure and mean free path and that any departures from this assumption may be due to disparities in molecular weight (see e.g. Buckman *et al* 1993), 'standard' gases with substantially larger molecular weight may be useful for studies of 'heavy' atomic and molecular targets.

In the present work we have also undertaken a joint programme of measurements on two similar but different electron spectrometers, located at the Australian National University and the University of Nebraska, using similar but slightly different implementations of the relative flow technique. In part this is an attempt to undertake a critical assessment of the techniques employed in our two laboratories in order to better understand the small differences that still persist amongst scattering cross section measurements in general.

The experimental apparatus and techniques for the present measurements are briefly described in section 2. The measured differential scattering cross sections are presented and compared with other results in section 3. The phaseshift analysis procedure is outlined in section 4 where the derived phaseshifts are also presented and compared with those of other workers. Finally, a general discussion and conclusions are presented in section 5.

2. Experimental apparatus and techniques

The majority of the measurements presented here have been carried out on the crossed-beam apparatus at the ANU. This apparatus has been previously described in detail (Brunger *et al* 1991, Gulley *et al* 1993) and that detailed description will not be repeated here. Briefly, a beam of argon effusing from a multichannel capillary array is crossed with a beam of monoenergetic electrons produced by an electrostatic hemispherical monochromator. Electrons scattered at a particular angle are then energy analysed by a similar hemispherical device and detected in a channel electron multiplier. The absolute scale of the measured cross section is determined using the relative flow technique (Srivastava *et al* 1975) in conjunction with the 'known' helium cross section (Nesbet 1979). The relative flow rates of both gases were measured for a range of driving pressures and those driving pressures were maintained at a sufficiently low level to ensure (i) that the effect of collisions on the size of the atomic beams exiting the multichannel capillary array (Buckman *et al* 1993) was negligible and (ii) near molecular flow took place through the array (Nickel *et al* 1989). The analyser rotates about an axis coincident with the atomic beam, allowing access to an angular range of -15° to $+130^\circ$. The true 0° position was determined as the position about which the intensity of elastically scattered electrons was symmetric. The estimated error in this determination is $\pm 1^\circ$. The energy of the incident electron beam is calibrated by using (i) the position of the second peak of the N_2^- shape resonance, which occurs at an energy of 2.198 eV in the elastic channel at an angle of 60° (Rohr 1977) and (ii) the position of the pair of Ar^- resonances $((^2P_{3/2,1/2})4s^2\ ^1S)$ at 11.098 and 11.270 eV (Brunt *et al* 1977). The overall energy resolution of the apparatus was typically 50 meV.

The apparatus used for the measurements at the University of Nebraska has also been described in detail in previous publications (Shi and Burrow 1992, Shi *et al* 1993) so only a brief discussion will be given here. In many respects it is similar to the ANU apparatus in that the electron beam is formed, and the scattered electrons are analysed, by a combination of electrostatic electron optics and hemispherical deflectors. However, in some aspects there are important differences between the two spectrometers and the experimental techniques employed, which are relevant to the present measurements. The most significant differences occur in the way in which the interaction volume is defined in the two spectrometers. The Nebraska apparatus employs a single capillary tube of 0.5 mm diameter and 20 mm length to form the atomic beam with the interaction volume a few mm above the exit of the tube. The interaction volume lies at the object position of the analyser entrance zoom optics and as a result the effective size of the interaction volume, i.e. the region from which scattered electrons are detected, is determined by the focal properties of the zoom lens. This differs from the ANU apparatus which uses real apertures at the entrance to the analyser to determine the effective size of the interaction region. In both cases the rationale is to ensure that the analyser views the entire volume formed by the overlap of the electron and atomic beams. The energy resolution of the Nebraska apparatus is typically 30 meV and the energy scale is calibrated in a similar fashion to that outlined above for the ANU measurements. The relative angular distributions are placed on an absolute scale by use of the relative flow technique in a similar fashion to that employed in the ANU apparatus.

3. Experimental results

Differential cross sections for elastic scattering of electrons from argon at incident electron energies of 1, 1.5, 2, 3, 4, 5, 7.5 and 10 eV are presented in table 1. The absolute uncertainties expressed as percentages are presented in parentheses next to the values. In most cases the cross sections have been measured at 5° intervals over the available angular range to better facilitate phaseshift analysis. Measurements at smaller intervals have been made where necessary to accurately determine the position of minima in the cross sections. The 4 eV data has been measured on the Nebraska apparatus and the measurements from the two laboratories also overlap at 5 eV.

The current measurements were carried out at energies which did not always correspond to the published results of other workers. In order to enable a comparison in these cases we fit, where possible, the low energy published phaseshifts and generate the cross sections at the energies required from interpolated values obtained from the fit. Usually a fourth- or fifth-order polynomial was adequate for the fit. This process has been applied to the phaseshifts of Williams (1979), Bell *et al* (1984), Dasgupta and Bhatia (1985), McEachran and Stauffer (1983), Nahar and Wadehra (1987), Saha (1991, 1993b) and Sienkiewicz and Baylis (1987). Higher-order phaseshifts were generated from the Born approximation. The most recent calculation of Saha (Saha 1995) is an extension of earlier work in that quadrupole and higher multipole polarization terms are included. These, it transpires, have a substantial effect on the low energy phaseshifts, particularly the s- and p-wave and cross sections. Similarly the calculation of McEachran and Stauffer (1996) extends their earlier work, which included the effects of dynamic distortion (Mimnagh *et al* 1993), by the further inclusion of relativistic effects using the Dirac formalism for the scattering equation.

At 1.0 eV (figure 1(*a*)) the present measured DCS and a curve generated from a phaseshift analysis of this data (see section 4) can be compared with the experimental results of Weyhreter *et al* (1988) and the cross section calculated from the interpolated phaseshifts of Williams (1979). The present results and those of Weyhreter *et al* are in excellent agreement (within the combined experimental error) at all common scattering angles while the values of Williams are larger in magnitude than the present values between scattering angles of 35° and 45° and lower above about 115°. At forward angles ($< 40^\circ$) the present measurements suffer from large uncertainties, a combination of the effects of a very small and decreasing cross section and an increasing background due to the primary

Table 1. Absolute differential cross sections for elastic electron scattering from argon at energies between 1.0 and 10.0 eV. Figures in parentheses indicate the absolute uncertainty expressed as a percentage. Units are 10^{-16} cm^2 sr^{-1}.

Angle	Energy (eV)								
(deg)	1.0	1.5	2.0	3.0	4.0 (Nebraska)	5.0	5.0 (Nebraska)	7.5	10.0
12	—	—	—	—	—	0.750 (7.1)	—	—	—
15	—	—	0.0486 (21.5)	0.122 (10.8)	—	0.686 (8.1)	—	2.996 (7.6)	6.916 (7.4)
20	—	0.0213 (20.9)	0.0619 (15.0)	0.157 (6.7)	—	0.603 (7.9)	—	2.424 (6.5)	5.959 (8.0)
25	—	0.0441 (17.7)	0.0981 (7.3)	0.211 (7.3)	0.427 (8)	0.595 (7.2)	0.612 (8)	2.0259 (6.8)	4.676 (7.1)
30	0.0150 (96.2)	0.0705 (7.2)	0.144 (7.7)	0.293 (7.0)	—	0.638 (7.4)	—	1.684 (6.7)	3.841 (7.7)
35	0.0272 (50.8)	0.107 (9.1)	0.201 (8.4)	0.375 (6.5)	0.616 (8)	0.701 (7.0)	—	1.465 (6.4)	2.999 (6.9)
40	0.0489 (11.2)	0.146 (6.9)	0.267 (6.7)	0.470 (7.1)	—	0.800 (7.6)	—	1.335 (6.8)	2.392 (6.7)
45	0.0718 (7.5)	0.188 (6.7)	0.314 (7.8)	0.558 (6.6)	0.877 (8)	0.902 (6.8)	—	1.290 (6.7)	1.929 (7.2)
50	0.0926 (8.0)	0.228 (6.7)	0.379 (6.5)	0.636 (6.9)	—	1.0305 (7.3)	—	1.301 (6.9)	1.629 (6.9)
55	0.111 (7.8)	0.256 (7.1)	0.409 (6.7)	0.711 (7.1)	1.097 (8)	1.156 (7.1)	1.193 (8)	1.336 (6.9)	1.4041 (6.5)
60	0.130 (9.1)	0.282 (7.2)	0.456 (6.3)	0.751 (6.8)	—	1.214 (6.4)	—	1.393 (6.6)	1.251 (7.1)
65	0.152 (9.2)	0.298 (6.6)	0.473 (6.5)	0.762 (7.2)	1.191 (8)	1.274 (6.3)	1.311 (8)	1.446 (6.4)	1.266 (7.8)
70	0.162 (7.5)	0.312 (6.7)	0.491 (6.4)	0.788 (6.9)	—	1.264 (6.5)	—	1.476 (6.3)	1.244 (6.7)
75	0.166 (11.0)	0.318 (6.6)	0.486 (6.4)	0.753 (7.2)	1.163 (8)	1.233 (6.6)	—	1.451 (6.4)	1.229 (8.4)
80	0.179 (8.6)	0.317 (6.8)	0.470 (6.5)	0.737 (7.1)	—	1.154 (6.4)	—	1.383 (6.5)	1.172 (6.6)
85	0.181 (7.9)	0.307 (7.2)	0.445 (6.5)	0.669 (6.9)	0.989 (8)	1.0616 (6.4)	1.185 (8)	1.264 (6.6)	1.125 (6.8)

Table 1. Continued.

Angle (deg)	Energy (eV)								
	1.0	1.5	2.0	3.0	4.0 (Nebraska)	5.0	5.0 (Nebraska)	7.5	10.0
90	0.181 (11.7)	0.290 (6.8)	0.413 (6.5)	0.602 (6.7)	—	0.9056 (6.4)	—	1.089 (6.6)	0.910 (6.9)
95	0.170 (7.0)	0.268 (6.5)	0.370 (6.7)	0.516 (6.4)	0.738 (8)	0.746 (6.4)	—	0.875 (7.3)	0.726 (6.8)
100	0.166 (7.1)	0.250 (6.4)	0.330 (6.8)	0.431 (9.0)	—	0.614 (6.6)	—	0.672 (7.0)	0.558 (6.9)
105	0.155 (7.4)	0.219 (6.5)	0.291 (7.1)	0.369 (7.8)	0.474 (8)	0.459 (6.4)	0.515 (8)	0.446 (7.6)	0.371 (7.6)
110	0.140 (8.6)	0.187 (6.5)	0.235 (7.3)	0.277 (7.7)	0.324 (8)	0.314 (6.5)	—	0.259 (6.9)	0.179 (6.8)
115	0.129 (8.1)	0.158 (7.0)	0.194 (7.2)	0.209 (9.4)	—	0.199 (7.0)	—	0.119 (7.0)	0.0531 (10.1)
117	—	—	—	—	—	—	—	—	0.0144 (13.4)
118	—	—	—	—	—	—	—	0.0634 (7.1)	0.0137 (16.2)
119	—	—	—	—	—	—	—	—	0.0195 (20.0)
120	0.113 (8.0)	0.135 (7.2)	0.148 (6.8)	0.144 (9.6)	—	0.120 (11.1)	—	0.0518 (13.7)	0.0284 (14.9)
121	—	—	—	—	—	—	—	—	0.0345 (11.2)
122	—	—	—	—	—	0.0961 (6.9)	—	0.0471 (14.3)	0.0564 (7.7)
124	—	—	—	—	—	0.0832 (6.6)	—	0.0518 (9.7)	—
125	0.106 (9.7)	0.112 (6.5)	0.118 (9.2)	0.0981 (9.7)	—	0.0820 (8.4)	—	0.0614 (9.2)	0.0848 (7.6)
126	—	—	—	—	—	0.0772 (6.8)	—	—	—
128	—	—	—	—	—	0.0812 (6.6)	—	—	—
130	0.0981 (14.0)	0.0923 (8.6)	0.0826 (7.3)	0.0676 (13.1)	—	0.0918 (6.7)	—	0.173 (8.1)	0.281 (6.3)

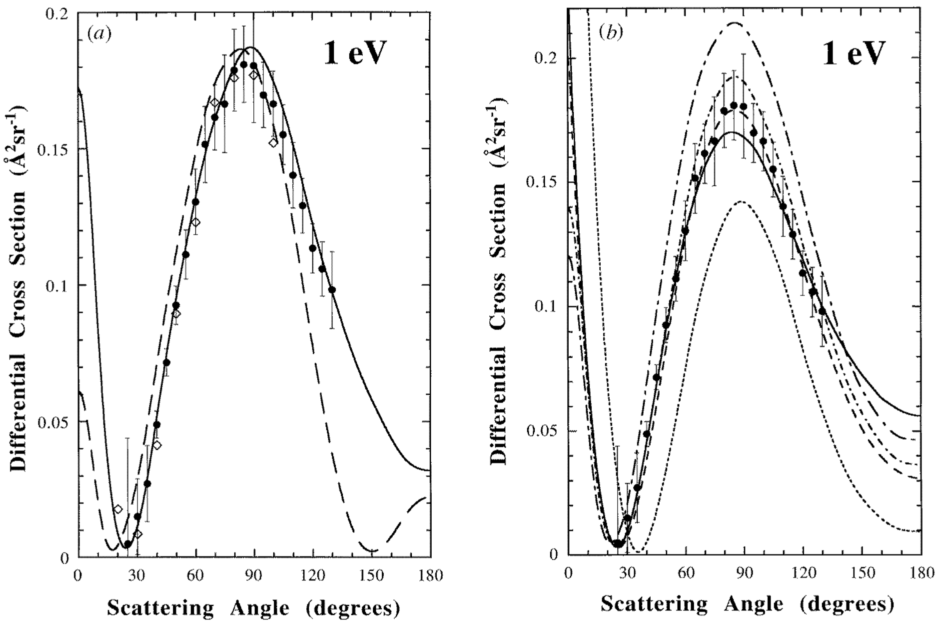


Figure 1. Absolute DCS for elastic scattering from argon at 1 eV. (*a*) Comparison of experimental data: (— — —) Williams, (◇) Weyhreter *et al*, (●) present results, (——) present phaseshift analysis; (*b*) comparison of present experiment with theory (— - —) Sienkewicz and Baylis, (— · —) Mimnagh *et al*, (- - - -) Saha (1991), (——) Saha (1995), (– – –) McEachran and Stauffer (1996), (●) present results.

electron beam. Nonetheless, the agreement with the measurements of Weyhreter *et al* is still extremely good.

At this energy there are also the theoretical results of Bell *et al* (1984), Bloor and Sherrod (1986), Dasgupta and Bhatia (1985), McEachran and Stauffer (1983), Sienkiewicz and Baylis (1987), Mimnagh *et al* (1993), Saha (1991, 1993b, 1995) and McEachran and Stauffer (1996). For the sake of clarity at this and at all other energies, we show only a few of the calculated cross sections which demonstrate the best accord with experiment. Whilst this is an undeniably subjective judgement it nonetheless remains the main goal of the present work to test the convergence of experiment and theory and, given the large number of calculations available, to do otherwise only serves to confuse the situation. Included in this selection at each energy are the calculations of Saha (1991, 1995) which have proved so successful in the description of electron–helium (Saha 1989b, 1993a) and electron–neon (Saha 1989a, 1990) elastic scattering. In figure 1(*b*) we compare the present data with the dynamic distortion polarized orbital calculation of Mimnagh *et al*, the multiconfiguration Hartree–Fock (MCHF) calculation of Saha (1991, 1995), the fully relativistic calculation of Sienkiewicz and Baylis and the recent model of McEachran and Stauffer (1996) which includes relativistic and dynamic distortion effects. Clearly the best agreement exists with the calculations of McEachran and Stauffer (1996) and Saha (1995), with both cross sections lying within the experimental uncertainties of the present data (and that of Weyhreter *et al*) at all angles. At this energy we also show the previous MCHF calculation of Saha (Saha 1991), which is about 25% lower than the present results at most scattering angles, to highlight the effects that the inclusion of higher-order polarization terms has had on the calculated DCS. The calculation of Sienkiewicz and Baylis, whilst displaying the same shape as the present cross section, is uniformly higher in magnitude. It is interesting to note the significant differences ($\simeq$ 10%) between the calculation of Mimnagh *et al* and McEachran and Stauffer (1996) around the maximum in the cross section. The only substantive difference in the two calculations is the relativistic nature of the latter and the differences in the calculated cross sections reflect a surprising difference of about 20% in the $p_{1/2}$ and $p_{3/2}$ phaseshifts at this energy (McEachran 1996).

At 1.5 eV (figure 2(*a*)) we can again make a comparison with the data of Weyhreter *et al* and that calculated with the interpolated phaseshifts of Williams. The present DCS results and those of Weyhreter *et al* agree to within the combined experimental error at all angles, while the values of Williams are again lower above 120° but are within the combined errors at all other angles. We compare our DCS with the calculations of McEachran and Stauffer (1996), Sienkiewicz and Baylis and also Saha (1995) in figure 2(*b*). Best overall agreement is once again found with the cross sections of McEachran and Stauffer and those of Saha, although the former is slightly larger than the experimental cross section in the region of the maximum at 60–90° and the latter once again shows the cross section maximum occurring at a slightly smaller angle than the experiment. The cross section of Sienkiewicz and Baylis is in good agreement with experiment at forward and backward angles but is about 25% larger in magnitude near the cross section maximum at mid angles.

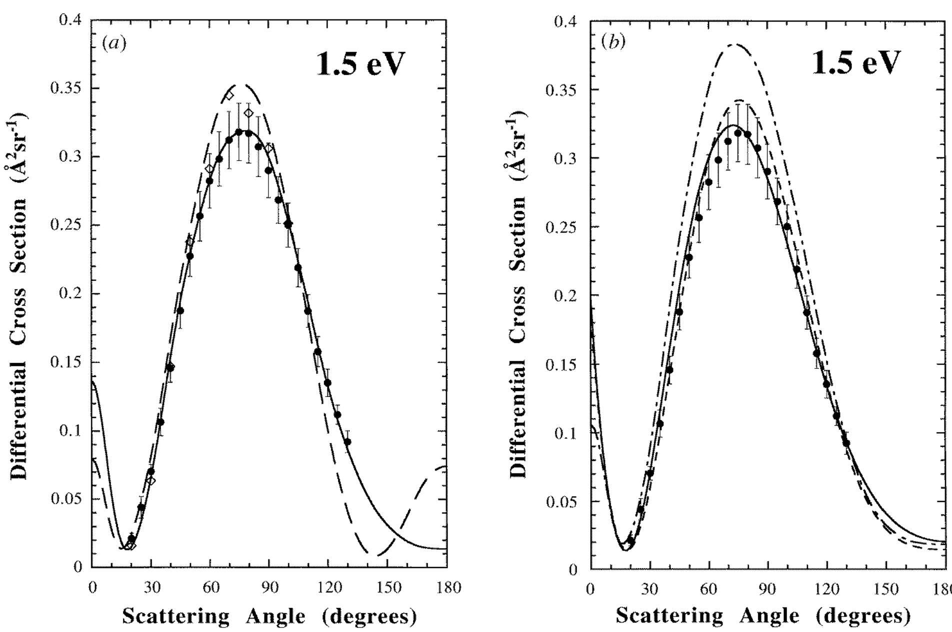


Figure 2. Absolute DCS for elastic scattering from argon at 1.5 eV. (*a*) Comparison of experimental data: (— — —) Williams, (◇) Weyhreter *et al*, (●) present results, (——) present phaseshift analysis; (*b*) comparison of present experiment with theory (— - —) Sienkewicz and Baylis, (——) Saha (1995), (– – –) McEachran and Stauffer (1996), (●) present results.

At 2.0 eV (figure 3(*a*)) the only other experimental result is derived from the interpolated phaseshifts of Williams. It shows similar small differences to the present measurement as were evident at the lower energies but, with the exception of the largest scattering angles ($> 120°$) where it lies about 50% lower than the present measurement, the agreement is generally very good. Comparison with several theoretical calculations is made in figure 3(*b*).

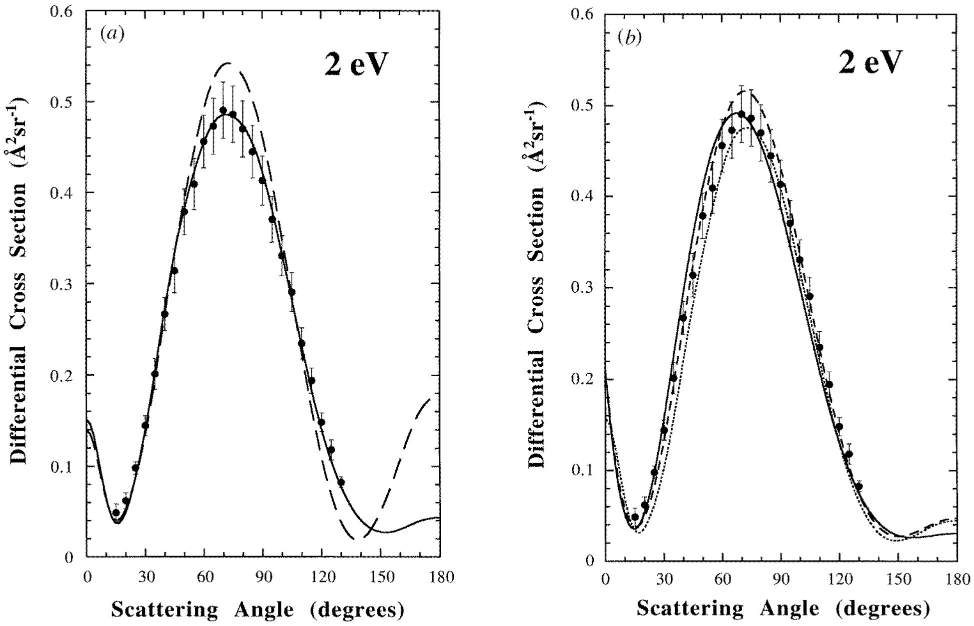


Figure 3. Absolute DCS for elastic scattering from argon at 2.0 eV. (*a*) Comparison of experimental data: (— — —) Williams, (●) present results, (——) present phaseshift analysis. (*b*) Comparison of present experiment with theory (· · · · · ·) McEachran and Stauffer (1983), (——) Saha (1995), (– – –) McEachran and Stauffer (1996), (●) present results.

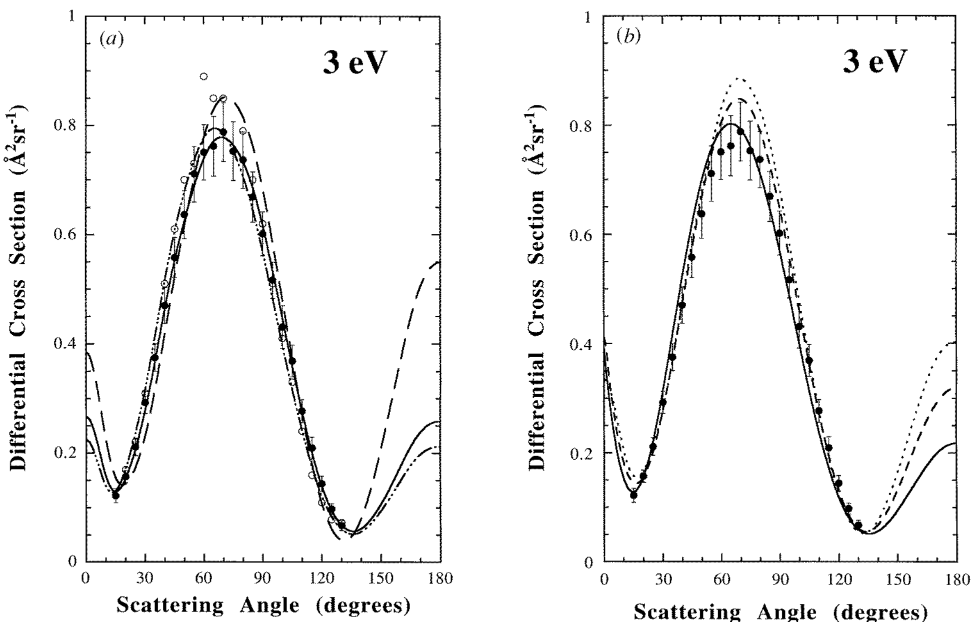


Figure 4. Absolute DCS for elastic scattering from argon at 3.0 eV. (*a*) Comparison of experimental data: (— — —) Williams, (○) Srivastava *et al*, (— · · · —) Furst *et al*, (●) present results, (——) present phaseshift analysis. (*b*) Comparison of present experiment with theory (· · · · · ·) Nahar and Wadehra, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (●) present results.

Here we see reasonably good agreement with the polarized orbital calculation of McEachran and Stauffer (1983) over most of the angular range and also the recent calculations of Saha (1995) and McEachran and Stauffer (1996), although in all cases there are some slight angular discrepancies with the experimental cross section.

At 3 eV comparison with the lowest energy measurements of both Srivastava *et al* (1981) and the cross section derived from the phaseshifts of Furst *et al* (1989) can be made, as well as with the cross section of Williams (figure 4(*a*)). The present cross section is in good agreement with those of Srivastava *et al* and Furst *et al* within the combined errors at all angles, with the largest differences occurring at the peak of the cross section ($\simeq 60^\circ$) in the former case. The agreement with the cross section of Williams is also reasonably good, although the differences are marginally outside combined experimental errors at very forward and backward angles. At this energy we show comparison with the calculations of Nahar and Wadehra (1987), Saha (1995) and McEachran and Stauffer (1996) in figure 4(*b*). Again, with the exception of the region of the cross section maximum, all of these calculations are in good agreement with the present data. The theory of Saha is also in very good agreement with the shape and relative magnitude of the features in the experimental cross section, although these features are shifted by a few degrees ($\simeq 5^\circ$) to lower angles relative to the experiment. On the other hand, the McEachran and Stauffer cross section is in excellent agreement with the shape of the present experimental cross section. It is also interesting to note that the relativistic calculation of McEachran and Stauffer (1996) is essentially identical to the earlier non-relativistic version of this model (Mimnagh *et al*—not shown in figure 4(*b*)) at this energy.

At 4 eV (figure 5(*a*)) we compare the results from the Nebraska spectrometer (and the

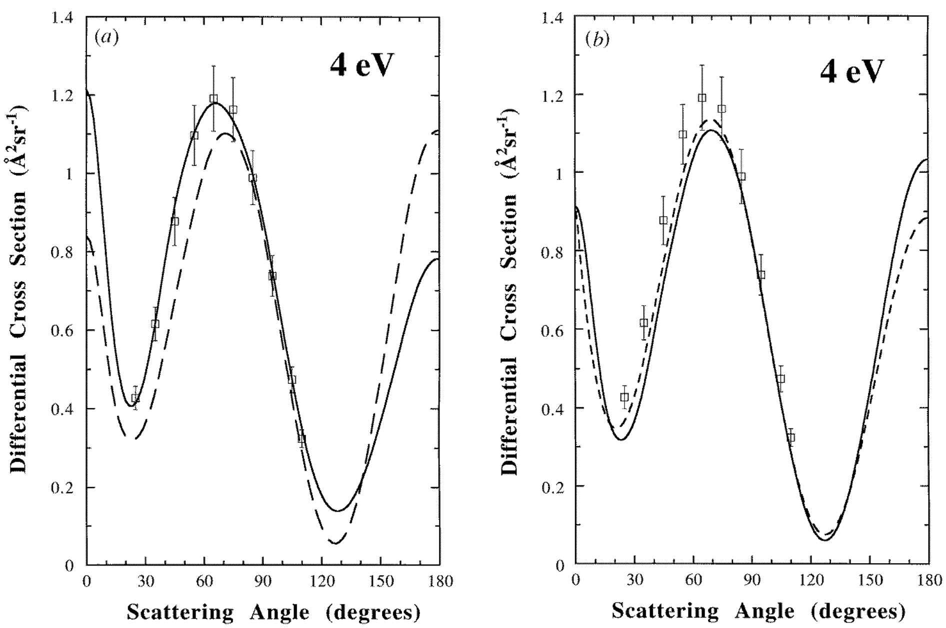


Figure 5. Absolute DCS for elastic scattering from argon at 4.0 eV. (*a*) Comparison of experimental data: (— — —) Williams, (□) present results (Nebraska), (——) present phaseshift analysis. (*b*) Comparison of present experiment with theory (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (□) present results (Nebraska).

phaseshift analysed curve) with the experimental DCS of Williams. They show a similar trend to the ANU results at lower energies in that the structure in the DCS is shifted to lower angles below about 90° than that of Williams. However, it is also interesting to note that the magnitude of the cross section peak around 60–70° is larger than that of Williams by about 10%, which is different to all of the previous comparisons at lower energies between the Williams and ANU data. At angles above 90° the agreement between the two cross sections is excellent. In figure 5(*b*) we compare the present Nebraska measurements with the calculations of McEachran and Stauffer (1996) and Saha (1995). The experimental DCS is larger than both of the calculated values at all scattering angles, particularly for those less than 90°. The best agreement is found with the relativistic plus dynamic distortion calculation of McEachran and Stauffer. The agreement at larger scattering angles between each of these theories and the experiment is very good.

There is a substantial change in the shape of the differential cross section between 4 and 5 eV. At 5 eV the two minima of the DCS fall within the angular range of our spectrometer and the relative magnitude of the cross section in these two regions changes dramatically from that observed at the lower energies. Comparison of both the present ANU and Nebraska measurements with three previous experiments is available and is given in figure 6(*a*). Firstly there is excellent agreement between the present measurements from the two different spectrometers, with the DCS values lying well within combined experimental uncertainties at all common angles. In comparison with the other experimental values, whilst the overall features of the cross sections are similar there are some substantial differences between them when one considers the fine detail. The present cross section exhibits minima at 25° and 126° and a broad maximum at around 70°. Whilst the cross section of Williams

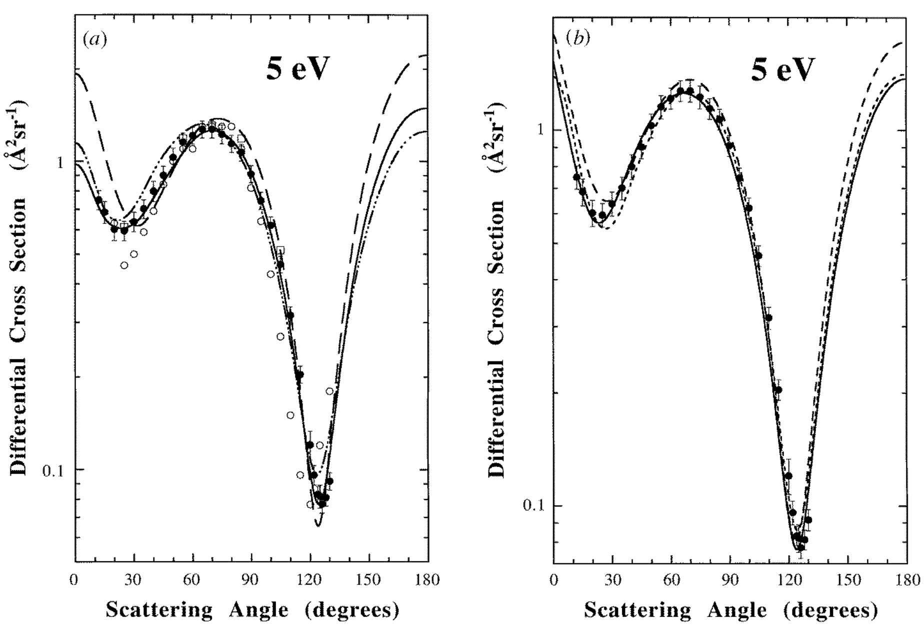


Figure 6. Absolute DCS for elastic scattering from argon at 5.0 eV. (*a*) Comparison of experimental data: (— — —) Williams, (○) Srivastava *et al*, (— · · · —) Furst *et al*, (●) present results ANU, (□) present results Nebraska, (——) present phaseshift analysis ANU data. (*b*) Comparison of present experiment with theory (- - - -) Dasgupta and Bhatia, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (●) present results.

is in good agreement with the present data in terms of the relative magnitudes of these features, there are substantial differences in the magnitude of the cross section at forward angles (it is 60% higher than the present results at 12°) and the position (32°) of the first minima in his cross section. Both cross sections are in reasonably good accord in the region of the deep second minimum. The cross section of Srivastava *et al* shows better agreement at angles less than about 80° but drops more rapidly than the present measurement to a minimum at about 120°. The data of Furst *et al* agree with the present results (within the combined error bars) at all but the most backward angles, although both minima in their cross section occur at angles about 5° lower than in the present measurement. In figure 6(*b*) we make a comparison between the present ANU cross section and the calculations of Dasgupta and Bhatia (1985), McEachran and Stauffer (1996) and Saha (1995). All three calculations show a good level of agreement with the experiment, although the best overall accord appears to be with the cross section of Saha.

At 7.5 eV there are experimental results from Williams and Srivastava *et al* with which to compare (figure 7(*a*)). The agreement with the cross section of Williams is slightly better at this energy than that observed at 5 eV, although the same general differences at forward and backward angles occur. The experimental data of Srivastava *et al* are generally smaller in magnitude than the present measurements and the deep second minimum in their cross section occurs at a smaller angle. The present experiments are compared with the calculations of Bell *et al* (1984), Dasgupta and Bhatia (1985), Saha (1995) and McEachran and Stauffer (1996) in figure 7(*b*). The agreement with the DCS of Dasgupta and Bhatia is excellent at all but the most backward angles—once again they find the minimum at a

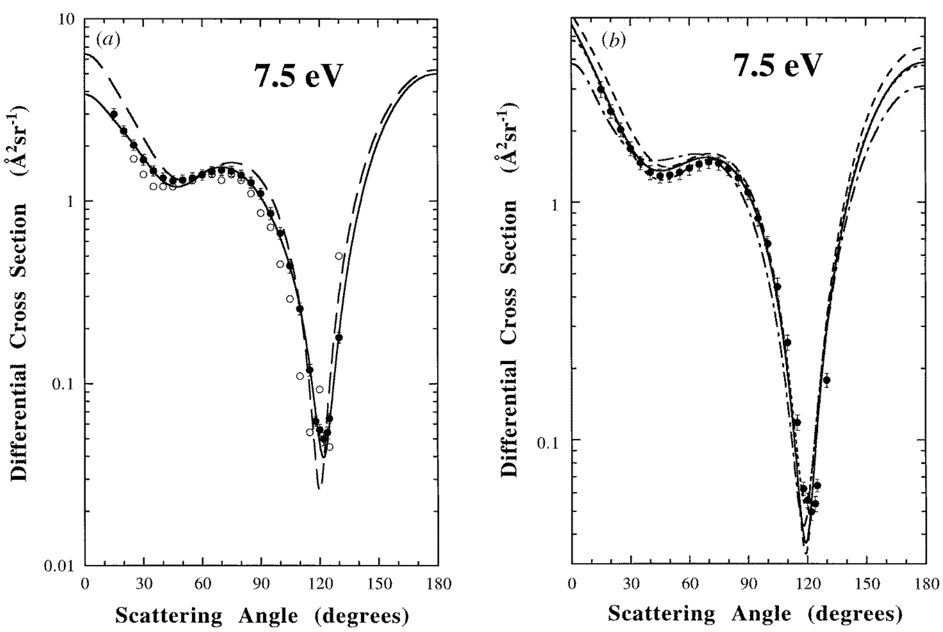
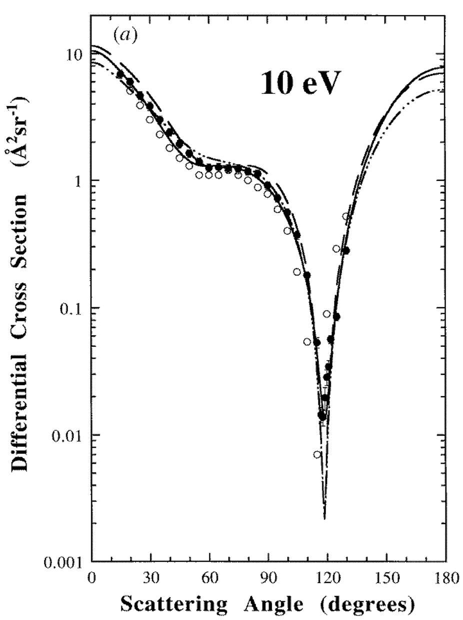


Figure 7. Absolute DCS for elastic scattering from argon at 7.5 eV. (a) Comparison of experimental data: (— — —) Williams, (○) Srivastava *et al*, (●) present results, (——) present phaseshift analysis. (*b*) Comparison of present experiment with theory (- - - -) Dasgupta and Bhatia, (— - —) Bell *et al*, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (●) present results.

slightly smaller angle. Similar comments apply to the comparison with the DCS of Saha, which is very similar to that of Dasgupta and Bhatia. The DCS of Bell *et al* is also in good agreement concerning the relative magnitudes of the principal features of the cross section but these features occur at smaller scattering angles than in the experiment. The cross section of McEachran and Stauffer is a little higher than the experiment at forward angles but is in good accord otherwise.

Finally, we show the present experimental DCS and those of Williams, Srivastava *et al* and Furst *et al* at an incident energy of 10 eV in figure 8(*a*). The present cross section is in best agreement with that of Williams, the only substantive difference occurring at the position of the deep second minimum, which both find at around 118°, but which is much deeper in the Williams DCS, possibly as a consequence of the higher angular resolution in the latter measurements. The DCS of Srivastava *et al* is in good agreement with the shape of the present cross section but it appears that there is a consistent angular offset of about 3–5° between the two. The cross section of Furst *et al* is in excellent agreement with the present except between about 40–70° where the former does not show the shallow minimum evident in the present cross section and those arising from the other measurements. The experimental cross section of Zhou Qing *et al* (1982) is not shown since the values are significantly lower than those of Srivastava *et al*, at both forward and backward angles. Comparison with a group of theoretical cross sections is shown in figure 8(*b*). In this case there is little to differentiate between those calculations which show the best agreement with the present experiment, namely the *R*-matrix calculation of Fon *et al* (1983), the polarized orbital DCS of Dasgupta and Bhatia, the multiconfiguration Hartree–Fock treatment of Saha (1995) and the relativistic plus dynamic distortion calculation of McEachran and Stauffer (1996).



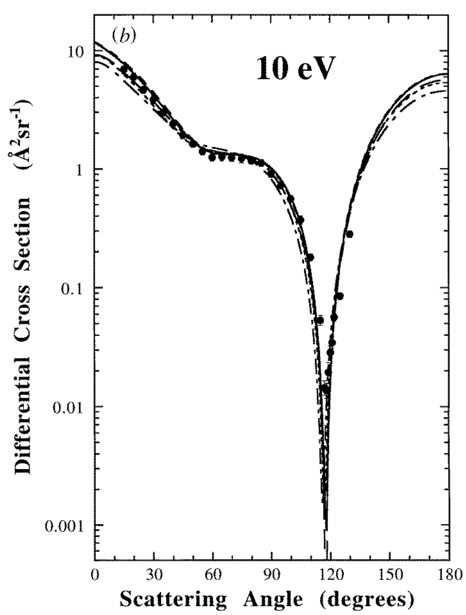


Figure 8. Absolute DCS for elastic scattering from argon at 10 eV. (*a*) Comparison of experimental data: (— — —) Williams, (○) Srivastava *et al*, (— · · · —) Furst *et al*, (●) present results, (——) present phaseshift analysis. (*b*) Comparison of present experiment with theory (- - - -) Dasgupta and Bhatia, (— — —) Fon *et al*, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (●) present results.

4. Phaseshift analysis

The measured differential cross sections have been analysed to extract scattering phaseshifts which can be compared with the published experimental and theoretical phaseshifts of other workers. The technique that is used is the regularized phaseshift analysis of Allen (1986) and Allen and McCarthy (1987) which has also been used recently by Brunger *et al* (1992) and Gulley *et al* (1994) for elastic electron scattering from helium and neon, respectively. Details of the procedure can be found in these references and in earlier work (e.g. Andrick and Bitsch 1975, Steph *et al* 1979), however the essential points follow.

The differential cross section is expanded as

$$\sigma(\theta, \boldsymbol{a}) = \left|(2k)^{-1}\sum_{l=0}^{\infty}(2l+1)[S_l(\boldsymbol{a})-1]P_l(\cos\theta)\right|^2 \tag{1}$$

where

$$S_l(\boldsymbol{a}) = \exp(2\mathrm{i}\delta_l(\boldsymbol{a})) = \prod_{n=1}^{N}\frac{\lambda^2-\beta_n^2}{\lambda^2-\alpha_n^2} \tag{2}$$

and $\delta_l(\boldsymbol{a})$ are the phaseshifts for each partial wave, $\lambda = l + \frac{1}{2}$ and $\boldsymbol{a} = \{a_n\}$ is the set of all the real and imaginary parts of the $2N$ complex parameters α_n and β_n $(n = 1, 2, \ldots, 2N)$. During the analysis, the function

$$\chi_1^2 = \frac{1}{M-P}\sum_{i=1}^{M}\frac{\sigma_i - \sigma(\theta_i, \boldsymbol{a})}{(\Delta\sigma_i)^2} \tag{3}$$

is minimized and should be close to unity if the parametrization of (1) is satisfactory and the non-statistical errors are small. M is the number of data points in the experiment, $P = 2N$ for real phaseshifts, σ_i is the measured differential cross section at the scattering angle θ_i and $\Delta\sigma_i$ is the *statistical* error in σ_i. The procedure is regularized by actually minimizing

$$\chi^2 = \chi_1^2 + \gamma\sum_{n=1}^{P}\left(a_n - a_n^{(0)}\right)^2 \tag{4}$$

where the *a priori* information for the set $\{a_n^{(0)}\}$ is taken from the theory which best reproduces the present data at that energy and which will produce positive error matrices. In the present analysis we have used the phaseshifts of Saha (1995) and McEachran and Stauffer (1996) to regularize the fit at various energies. γ is the regularization parameter used to vary the weighting given to the *a priori* phaseshifts and it was chosen such that χ^2 in equation (4) was minimized. The static dipole polarizability of argon, used for the calculation of higher-order phases, was taken to be 11.08 a_0^3 (Miller and Bederson 1977). Further details of the procedure are given by Allen (1986) and Allen and McCarthy (1987).

The results of the present analysis, namely the first four phaseshifts, the χ^2 values and the total elastic and momentum transfer cross sections generated from the derived phaseshifts, are shown in table 2, along with a selected summary of other available values. There has been some more general uncertainty concerning the application of such a phaseshift analysis to an atom such as argon. In fact, it has been pointed out by Furst *et al* (1989), that the applicability of the technique to heavier atoms is not generally accepted. Furst *et al* (1989) also question the use of the Born approximation based on a $-\alpha/2r^4$ interaction potential to generate high-order phaseshifts in these circumstances, even at low energies. Its use in the present case may also be questionable in that departures from LS coupling, as a result of spin–orbit interaction and other relativistic effects, become increasingly important as one

Table 2. A comparison of the first four elastic scattering phaseshifts derived from the present analysis with those from a selection of previous studies. Also shown are the values obtained for the goodness of fit indicator χ^2 and the total and momentum transfer cross sections (in Å^2) obtained from the present DCS via the phaseshift analysis.

E (eV)	Reference	η_0	η_1	η_2	η_3	χ^2	Q_{el}	Q_{mt}
1.0	Present	−0.162 5	0.011 4	0.027 4	0.007 88	1.006	1.49	1.51
	Williams[a]	−0.144	−0.014 8	0.033 9	0.026 4		1.30[b]	1.16[b]
	Mimnagh *et al*	−0.164 2	0.011 3	0.027 9	0.008 00		1.51[b]	1.55[b]
	McEachran and Stauffer (1996)	−0.156 6	0.010 9	0.027 8	0.008 01		1.40	1.43
			0.012 9[c]					
	Saha (1991)	−0.117 2	0.019 01	0.034 5	0.009 3		1.03[b]	1.00[b]
	Saha (1995)	−0.160 2	0.013 9	0.024 5	0.009 25		1.43	1.49
	Sienkiewicz and Baylis	−0.179 9	0.008 873	0.027 31	0.008 022		1.76[b]	1.78[b]
1.5	Present	−0.251 3	−0.015 4	0.043 9	0.011 6	1.009	2.36	2.07
	Williams[a]	−0.235	−0.040 7	0.056	0.006 45		2.41[b]	2.02[b]
	McEachran and Stauffer (1996)	−0.254 5	−0.016 8	0.046 0	0.012 1		2.43	2.14
			−0.013 9[c]					
	Saha (1995)	−0.255 6	−0.014 8	0.040 3	0.013 5		2.38	2.09
	Sienkiewicz and Baylis[a]	−0.274 6	−0.022 0	0.046 6	0.012 1		2.79[b]	2.41[b]
2.0	Present	−0.343 4	−0.042 5	0.062 7	0.018 0	1.002	3.41	2.78
	Williams[a]	−0.319	−0.071 6	0.082 5	0.011 7		3.58[b]	2.93[b]
	McEachran and Stauffer (1983)	−0.322 8	−0.045 3	0.068 4	0.015 8		3.17	2.62
	McEachran and Stauffer (1996)	−0.341 5	−0.050 9	0.067 5	0.016 3		3.47	2.83
			−0.047 0[c]					
	Saha (1995)	−0.340 4	−0.050 0	0.058 3	0.018 4		3.34	2.65
3.0	Present	−0.490 4	−0.114 9	0.111 7	0.022 0	0.999	5.25	4.15
	Williams	−0.457	−0.134	0.142	0.021		5.64[b]	4.69[b]
	Srivastava *et al*	−0.548	−0.140	0.125	0.035		5.5	4.1
	Furst *et al*	−0.488	−0.124	0.102	0.025		5.20	
	Nahar and Wadehra	−0.490	−0.132	0.130	0.024 1		5.80	4.64
	McEachran and Stauffer (1996)	−0.492 1	−0.125 3	0.121 5	0.025 1		5.56	4.41
			−0.120 6[c]					
	Saha (1995)	−0.486 9	−0.123 5	0.104 0	0.027 6		5.20	3.94
4.0	Present (Nebraska)	−0.704 5	−0.l78	0.186	0.044 7	1.272	8.46	6.80
	Williams[a]	0.574	−0.197	0.208 5	0.029 9		7.58[b]	6.50[b]
	McEachran and Stauffer (1996)	−0.620 5	−0.200 6	0.191 1	0.034 3		7.73	6.35
	Saha[a] (1995)	−0.609 4	−0.198 8	0.205 3	0.034 9		7.96[b]	6.67[b]
5.0	Present	−0.725 3	−0.277	0.267	0.014 9	1.272	9.74	8.73
	Williams[a]	−0.692	−0.257	0.319	0.038 9		10.61[b]	9.59[b]
	Srivastava *et al*	−0.747	−0.256	0.254	0.102		8.4	6.4
	Furst *et al*	−0.770	−0.277	0.228	0.044		9.42	
	Dasgupta and Bhatia	−0.720 9	−0.245 9	0.258 0	0.044 2		9.17	7.80
	McEachran and Stauffer (1996)	−0.733 3	−0.272 0	0.276 8	0.044 1		10.07	8.63
	Saha (1995)	−0.720 4	−0.266 2	0.239 9	0.042 8		9.04	7.47
7.5	Present	−0.906 1	−0.402 7	0.511 8	0.058	0.990	14.97	13.22
	Williams[a]	−0.918	−0.404	0.617	0.065		17.95[b]	16.15[b]
	Srivastava *et al*	−1.051	−0.398	0.491	0.125		13.0	11.0
	Bell *et al*[a]	−0.996 2	−0.467 7	0.425 6	0.070 18		14.13	11.75
	Dasgupta and Bhatia[a]	−0.967 1	−0.390 7	0.509 2	0.070 8		14.98[b]	13.24[b]
	McEachran and Stauffer (1996)	−0.969 2	−0.433 0	0.552 0	0.070 5		16.72	14.65
	Saha (1995)	−0.945 5	−0.421 9	0.505 7	0.072 2		15.22	13.20

Table 2. Continued.

E (eV)	Reference	η_0	η_1	η_2	η_3	χ^2	$Q_{\rm el}$	$Q_{\rm mt}$
10.0	Present	−1.1031	−0.4245	0.8638	0.091	4.62	20.66	17.85
	Williams	−1.098	−0.528	0.936	0.093		23.35[b]	18.94[b]
	Srivastava *et al*	−1.243	−0.430	0.805	0.171		18.0	15.0
	Furst *et al*	−1.08	−0.650	0.720	0.071		19.95	
	Dasgupta and Bhatia	−1.1438	−0.5376	0.7539	0.0999		19.38	15.80
	Fon *et al*	−1.1554	−0.5865	0.7568	0.0849		20.05	16.09
	McEachran and Stauffer (1996)	−1.1607	−0.5694	0.8633	0.0995		22.41	17.83
	Saha (1995)	−1.1302	−0.5473	0.8532	0.1001		21.86	17.55

[a] Indicates interpolated values.
[b] Indicates values generated from interpolated or published values.
[c] $p_{3/2}$ phaseshift.

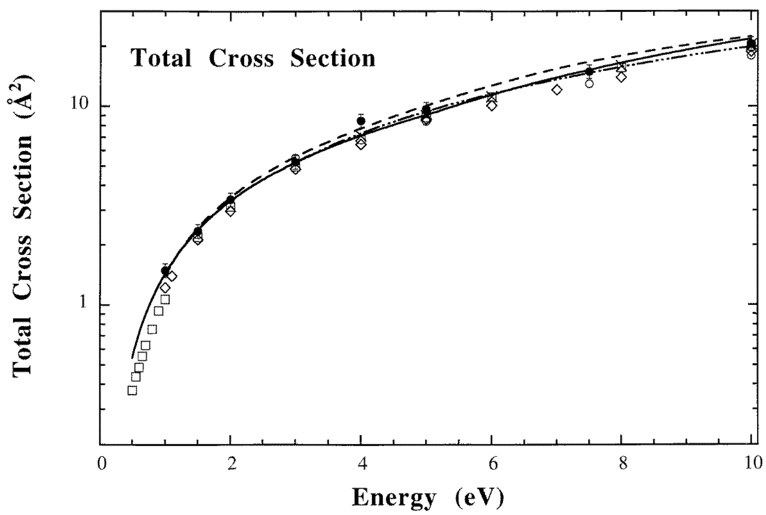


Figure 9. Total elastic cross section for electron scattering from argon. (○) Srivastava *et al*, (— · · · —) Furst *et al*, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (◇) Ferch *et al* (1985), (□) Buckman and Lohmann (1986), (×) Nickel *et al*, (●) Present results.

descends the rare gas column of the periodic table. A recent low energy spin-polarized scattering study by Dümmler *et al* (1995) suggests that there is only a weak departure from LS coupling in argon with the two fine-structure states having approximately the same radial wavefunctions and that the spin–orbit interaction of the continuum electron can be neglected. On the other hand the calculations of McEachran and Stauffer (1996) indicate that relativistic effects are significant at energies below 5 eV, giving rise to $p_{1/2}$ and $p_{3/2}$ phaseshifts which differ by 20% at 1 eV and 3% at 3 eV. The extent to which these differences can be approximated by a single non-relativistic p-wave phaseshift is not known.

The total elastic and elastic momentum transfer cross sections generated from the presently derived phaseshift are also shown in figures 9 and 10, respectively, where they are compared with other experimental and theoretical values. In figure 9 the present total cross section is compared with a selection of other values, including those derived from the

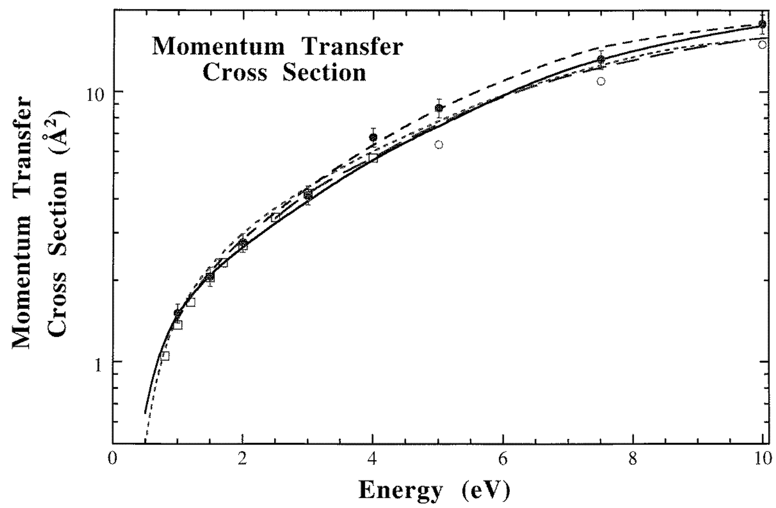


Figure 10. Momentum transfer cross section for argon. (○) Srivastava *et al*, (- - - -) Dasgupta and Bhatia, (– – –) McEachran and Stauffer (1996), (——) Saha (1995), (□) Milloy *et al*, (— — —) Nakamura and Kurachi, (●) present results.

DCS measurements of Srivastava *et al* and Furst *et al* and with the direct total cross section measurements of Nickel *et al* (1985), Ferch *et al* (1985) and Buckman and Lohmann (1986). We also compare in this figure with the theoretical values of Saha (1995) and McEachran and Stauffer (1996). The present results are in excellent agreement with the previous time-of-flight attenuation measurements from this laboratory (Buckman and Lohmann 1986), with the exception of the 1 eV point, where the DCS-derived cross section is about 30% higher than this attenuation measurement and about 20% higher than that of a similar attenuation measurement of Ferch et al. (1985). The present results also show excellent agreement with the phaseshift-derived measurements of Furst *et al* and Srivastava *et al* and the calculation of Saha (1995) across the whole energy range of comparison. Below about 5 eV the same is true of the calculation of McEachran and Stauffer, whilst above this energy this cross section is about 10% higher than the present, reflecting in part the differences observed in the DCS at forward scattering angles.

In figure 10 the elastic momentum transfer cross section is compared with the swarm-derived results of Milloy *et al* (1977) and Nakamura and Kurachi (1988), the phaseshift derived results of Srivastava *et al* and the calculations of Dasgupta and Bhatia, Saha (1995) and McEachran and Stauffer (1996). The agreement with the swarm result is excellent at all common energies up to their maximum value of 4 eV. There is also reasonably good agreement with the calculations of Dasgupta and Bhatia, Saha, and McEachran and Stauffer.

5. Discussion and conclusions

The present experimental investigation provides a further set of accurate DCS data and scattering phaseshifts for elastic electron scattering from argon for comparison with previous experiment and theory, particularly at energies below 5 eV, where the number of absolute determinations is limited. In terms of previous experiments the present data are in excellent agreement with the data of Weyhreter *et al* within the combined experimental error at all common energies. The differential cross sections generated from the Williams phaseshifts are generally higher than our measured values at both forward and backward angles but agree well in the central region (40°–100°). At the phaseshift level the differences, for example, between the present s-wave phaseshift and that of Williams varies from about 1% at 7.5 eV to about 10% at 5 eV. Agreement with the measurements of Srivastava *et al* is quite good at 3.0 eV but at higher energies it is clear that there are some fairly consistent angular differences between the two data sets, in particular, the deep second minimum in their data set occurs at a lower angle (by $\simeq 5^\circ$) than in the present measurements and these differences are larger than the combination of the angular uncertainties. The data of Furst *et al* generally agree well in magnitude with the present data, lying within combined errors at almost all energies, but small differences in shape result in cross sections which are persistently higher or lower than the present values in certain angular regions.

Part of the motivation for the present work was provided by the excellent level of agreement with experiment that has been shown in recent applications of the multiconfiguration Hartree–Fock calculation of Saha for low energy elastic electron scattering from helium (Saha 1989b, 1993a) and neon (Saha 1989a, 1990). This method has been shown to account for electron correlation and polarization effects very accurately in an *ab initio* fashion and it assumes that spin–orbit and other relativistic effects are not significant. As this good agreement was also part of the rationale behind the proposal that the neon cross section was appropriate for use as a 'secondary standard' it was only appropriate that a thorough comparison be made in the present case with the multiconfiguration Hartree–Fock calculation for argon (Saha 1991, 1993b, 1995). In general, the level of agreement between the present data and Saha's most recent calculated DCS at all energies is very good, with the theoretical cross section lying within the experimental uncertainties at most energies and scattering angles.

There is also excellent overall agreement between the present experimental cross sections and the dynamic distortion calculations of Mimnagh *et al* (1993) and the relativistic version of this model (McEachran and Stauffer 1996). The inclusion of relativistic effects in the latter makes a significant difference at energies below about 3 eV. The calculation of Sienkiewicz and Baylis, which uses a fully relativistic Hartree–Fock method consistently overestimates the magnitude of the cross section compared to the present data but these differences diminish as the energy increases. The calculation of Dasgupta and Bhatia also provides a very close comparison with the present results, particularly as the energy is increased. The *R*-matrix calculations of Bell *et al* and Fon *et al* are generally lower than the present experiments at forward scattering angles and higher at backward, in part reflecting a slight difference in the shape of the DCS. The agreement with experiment for both calculations improves as the energy is increased although the DCS of Fon *et al* are slightly closer to the present values. Fon *et al* note that their calculation may be unreliable at energies lower than about 5 eV because contributions to the polarizability from higher multipoles, which may be playing an important part in the scattering, are not included. Such an observation is consistent with the differences seen in the two calculations by Saha (1991, 1995). Nahar and Wadehra used model static and polarization potentials with an exchange potential, their aim being to produce a single potential to predict both positron and electron scattering from argon. Their theory produces differential cross sections which are a bit higher than the present experiment in the forward and central angular ranges.

For the total and momentum transfer cross sections agreement between the present cross sections derived from the fitted phaseshift and direct measurements and swarm derivations is very good, generally within the combined experimental errors. Most of the calculations also show much better agreement with the present experiment across the entire range of energies, though in some cases this is a function of differing relative magnitudes in the DCS in different angular regions cancelling one another in the integral cross sections. Once again this highlights the value of angular information in assessing agreement between experiment and theory.

The present experimental data set also provides for a critical comparison of results obtained with two quite different electron spectrometers in which the relative flow technique is applied in a slightly different fashion. These results are generally in very good accord although there are some small (5–10%) but systematic differences which are apparent when one looks carefully at the energy dependence of the cross sections between 2–5 eV.

In the light of the forgoing discussion of the various calculations and experimental results one would have to conclude that the situation regarding the agreement between experiment and theory for low energy electron scattering from argon is extremely encouraging. This is particularly true at energies below about 5 eV, where the two most recent theoretical calculations are generally within 5–10% of the experimental values at all scattering angles.

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