TODO: (alex) think about proper decoration and styling of document!

1 Convex sets

Exercise 2.1 Proof.

We should prove generalization of convexity definition. We will use mathematical induction to prove this statement

1) Base case, where $n = 2$. It is obvious by definition of convex set.

2) Suppose, that statement is correct for any $n >= 2$

3) We will prove statement for $n + 1$. Let $\theta_i i \in R, i \in 1, ..., n + 1$, with $\theta_1 + ... + \theta_{n+1} = 1, \theta_i >= 0$. Also $x_i \in R^n$. Then we can use following fact (we suppose, that $\theta_{n+a} <> 1$):

$$\frac{\theta_1}{1 - \theta_{n+1}} + ... + \frac{\theta_n}{1 - \theta_{n+1}} = 1$$

Note, that this fact will lead from initial requirement for θ_i.

$$\theta_1 x_1 + ... + \theta_{n+1} x_{n+1} = (1 - \theta_{n+1})(\frac{\theta_1}{1 - \theta_{n+1}} x_1 + ... + \frac{\theta_n}{1 - \theta_{n+1}} x_n) + \theta_{n+1} x_{n+1} = (1 - \theta_{n+1}) x_c + \theta_{n+1} x_{n+1}$$

$x_c \in C due *$

Exercise 2.4 Proof

We should prove that $convC \subseteq \bigcup_{i=1}^{\infty} S_i$, where C is any set, S_i is any convex set, containing C. And $convC$ is convex hull of C.

Note that statement $A = B$, where A, B are any sets means that $A \subseteq B \subseteq A$. We will use this one for proof. Let $S = \bigcup_{i=1}^{\infty} S_i$.

1) Prove that $convC \subseteq S$. Suppose opposite. Let $\exists x_0 \in convC : x_0 \notin S$. It means that $\exists \theta_1, \theta_2 \in R : \theta_i >= 0, \theta_1 + theta_2 = 1$ and $\exists x_1, x_2 \in C | x_1 \theta_1 + x_2 \theta_2 = x_0 \in convC$. Set S is convex and $C \subseteq S$. It yields that $\theta_1 x_1 + \theta_2 x_2 = x_0 \in S$. It is contradiction.

2) Prove that $S \subseteq convC$. Suppose opposite. Let $\exists x_0 \in S : x0 \notin convC, x_0 \in S$. Thus $x0 \in S_i \forall i$. We know that $C \subseteq convC$ and $convC$ is convex. Hence $\exists j : S_j = convC$ and $x_0 \in S_j = convC$. It is contradiction.