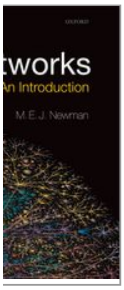

Networks: An Introduction
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Introduction

A short introduction to networks and why we study them

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Abstract and Keywords

A network is, in its simplest form, a collection of points joined together in pairs by lines. In the jargon of the field the points are referred to as *vertices* or *nodes* and the lines are referred to as *edges*. Many objects of interest in the physical, biological, and social sciences can be thought of as networks and, as this book aims to show, thinking of them in this way can often lead to new and useful insights. This introductory chapter discusses why we are interested in networks and describes some specific networks of note. An overview of the subsequent chapters is presented.

Keywords: networks structure, Internet, computer networks, biological networks, social networks

A NETWORK is, in its simplest form, a collection of points joined together in pairs by lines. In the jargon of the field the points are referred to as *vertices*[1] or *nodes* and the lines are referred to as *edges*. Many objects of interest in the physical, biological, and social sciences can be thought of as networks and, as this book aims to show, thinking of them in this way can often lead to new and useful insights.

We begin, in this introductory chapter, with a discussion of why we are interested in networks and a brief description of some specific networks of note. All the topics in this chapter are covered in greater depth elsewhere in the book.

Why Are We Interested in Networks?

There are many systems of interest to scientists that are composed of individual parts or components linked together in some way. Examples include the Internet, a collection of computers linked by data connections, and human societies, which are collections of people linked by acquaintance or social interaction.

Many aspects of these systems are worthy of study. Some people study the nature of the individual components—how a computer works, for instance, or how a human being feels or acts—while others study the nature of the connections or interactions—the communication protocols used on the Internet or the dynamics of human friendships. But there is a third aspect to these interacting (p.2) systems, sometimes neglected but almost always crucial to the behavior of the system, which is the *pattern* of connections between components.

The pattern of connections in a given system can be represented as a network, the components of the system being the network vertices and the connections the edges. Upon reflection it should come as no surprise (although in some fields it is a relatively recent realization) that the structure of such networks, the particular pattern of interactions, can have a big effect on the behavior of the system. The pattern of connections between computers on the Internet, for instance, affects the routes that data take over the network and the efficiency with which the network transports those data. The connections in a social network affect how people learn, form opinions, and gather news, as well as affecting other less obvious phenomena, such as the spread of disease. Unless we know something about the structure of these networks, we cannot hope to understand fully how the corresponding systems work.

A network is a simplified representation that reduces a system to an abstract structure capturing only the basics of connection patterns and little else. Vertices and edges in a network can be labeled with additional information, such as names or strengths, to capture more details of the system, but even so a lot of information is usually lost in the process of reducing a full system to a network representation. This certainly has its disadvantages but it has advantages as well.

Scientists in a wide variety of fields have, over the years, developed an extensive set of tools-mathematical, computational, and statistical-for analyzing, modeling, and understanding networks. Many of these tools start from a simple network representation, a set of vertices and edges, and after suitable calculations tell you something about the network that might well be useful to you: which is the best connected vertex, say, or the length of a path from one vertex to another. Other tools take the form of network models that can make mathematical predictions about processes taking place on networks, such as the way traffic will flow over the Internet or the way a disease will spread through a community. Because they work with networks in their abstract form, these tools can in theory be applied to almost any system represented as a network. Thus if there is a system you are interested in, and it can usefully be represented as a network, then there are hundreds of different tools out there, already developed and well understood, that you can immediately apply to the analysis of your system. Certainly not all of them will give useful results—which measurements or calculations are useful for a particular system depends on what the system is and does and on what specific questions you are trying to answer about it. Still, if you have a well-posed question about a networked system there will, in many cases, already be a tool available that (p.3) will help you address it.

Networks are thus a general yet powerful means of representing patterns of connections or interactions between the parts of a system. In this book, we discuss many examples of specific networks in different fields, along with techniques for their analysis drawn from mathematics, physics, the computer and information sciences, the social sciences, biology, and elsewhere. In doing so, we bring together a wide range of ideas and expertise from many disciplines to give a comprehensive introduction to the science of networks.

Some Examples of Networks

One of the best known and most widely studied examples of a network is the Internet, the computer data network in which the vertices are computers and the edges are physical data connections between them, such as optical fiber cables or telephone lines. Figure 1.1 shows a picture of the structure of the Internet, a snapshot of the network as it was in 2003, reconstructed by observing the paths taken across the network by a large number of Internet data packets traveling between different sources and destinations. It is a curious fact that although the Internet is a man-made and carefully engineered network we don't know exactly what its structure is, since it was built by many different groups of people with only limited knowledge of each other's actions and little centralized control. Our best current data on its structure are derived from experimental studies, such as the one that produced this figure, rather than from any central repository of knowledge or coordinating authority.

There are a number of excellent practical reasons why we might want to study the network structure of the Internet. The function of the Internet is to transport data between computers (and other devices) in different parts of the world, which it does by dividing the data into pieces or *packets* and shipping them from vertex to vertex across the network until they reach their intended destination. Certainly the structure of the network will affect how efficiently it accomplishes this function and if we know the network structure we can address many questions of practical relevance. How should we choose the route by which data are transported? Is the shortest route always necessarily the fastest? If not, then what is, and how can we find it? How can we avoid bottlenecks in the traffic flow that might slow things down? What happens when a vertex or an edge fails (which they do with some regularity)? How can we devise schemes to route around such failures? If we have the opportunity to add new capacity to the network, where should it be added?

Knowledge of Internet structure also plays a central role in the development of new communications standards. New standards and protocols are (p.4)

(p.5) continually being devised for communication over the Internet, and old ones are revised. The parameters of these protocols are tuned for optimal performance with the structure of the Internet in mind. In the early days of the network, rather primitive models of network structure were employed in the tuning process, but as better structural data become available it becomes possible to better understand and improve performance.

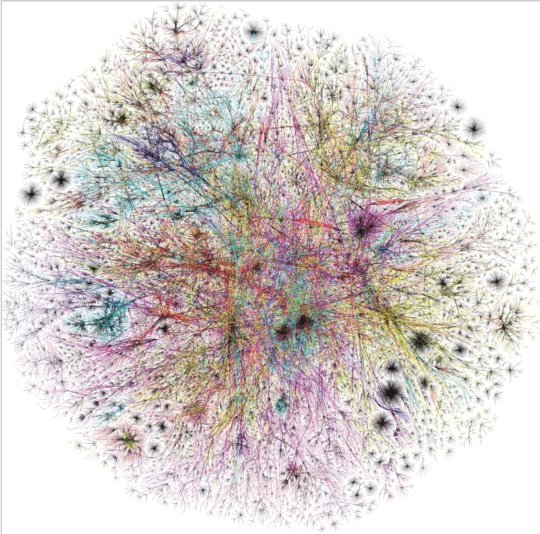

Figure 1.1: **The network structure of the Internet.** The vertices in this representation of the Internet are "class C subnets"—groups of computers with similar Internet addresses that are usually under the management of a single organization—and the connections between them represent the routes taken by Internet data packets as they hop between subnets. The geometric positions of the vertices in the picture have no special meaning; they are chosen simply to give a pleasing layout and are not related, for instance, to geographic position of the vertices. The structure of the Internet is discussed in detail in Section 2.1. Figure created by the Opte Project (www.opte.org). Reproduced with permission.

A more abstract example of a network is the World Wide Web. In common parlance the words "Web" and "Internet" are often used interchangeably, but technically the two are quite distinct. The Internet is a physical network of computers linked by actual cables (or sometimes radio links) running between them. The Web, on the other hand, is a network of information stored on web pages. The vertices of the World Wide Web are web pages and the edges are "hyperlinks," the highlighted snippets of text or push-buttons on web pages that we click on to navigate from one page to another. A hyperlink is purely a software construct; you can link from your web page to a page that lives on a computer on the other side of the world just as easily as you can link to a friend down the hall. There is no physical structure, like an optical fiber, that needs to be built when you make a new link. The link is merely an address that tells the computer where to look next when you click on it.

Abstract though it may be, the World Wide Web, with its billions of pages and links, has proved enormously useful, not to mention profitable, to many people, and the structure of the network of links is of substantial interest. Since people tend to add hyperlinks between pages with related content, the link structure of the Web reveals something about the content structure. What's more, people tend to link more often to pages that they find useful than to those they do not, so that the number of links pointing to a page can be used as a measure of its usefulness. A more sophisticated version of this idea lies behind the operation of the popular Web search engine *Google*, as well as some others.

The Web also illustrates another concept of network theory, the *directed network*. Hyperlinks on the Web run in one specific direction, from one web page to another. Given an appropriate link on page A, you can click and arrive at page B. But there is no requirement that B contains a link back to A again. (It may contain such a link, but there is no law that says that it must and much of the time it will not.) One says that the edges in the World Wide Web are *directed*, running from the linking page to the linked.

Moving away from the technological realm, another type of network of scientific interest is the social network. A social network is, usually, a network of people, although it may sometimes be a network of groups of people, such as companies. The people or groups form the vertices of the network and the (p.6) edges represent connections of some kind between them, such as friendship between individuals or business relationships between companies. The field of sociology has perhaps the longest and best developed tradition of the empirical study of networks as they occur in the real world, and many of the mathematical and statistical tools that are used in the study of networks are borrowed, directly or indirectly, from sociologists.

Figure 1.2 shows a famous example of a social network from the sociology literature, Wayne Zachary's “karate club” network. This network represents the pattern of friendships among members of a karate club at a north American university. The network was constructed by direct observation of interactions between the club's members. As is typical of such studies the network is small, having, in this case, only 34 vertices. Network representations of the Internet or the World Wide Web, by contrast, can have thousands ormillions of vertices. In principle there is no reason why social networks cannot be similarly large. The entire population of the world, for example, can be regarded as a very large social network. But in practice social network data are limited to relatively small groups because of the effort involved in compiling them. The network of Fig. 1.2, for instance, was the product of two years of observations by one experimenter. In recent years a few larger social networks have been constructed by dint of enormous effort on the part of large groups of researchers. And online social networking services, such as Facebook or instant message “buddy lists,” can provide network data on a previously unreachable scale. Studies are just beginning to emerge of the structure and properties of these larger networks.

A third realm in which networks have become important in recent years is biology. Networks occur in a number of situations in biology. Some are concrete physical networks like neural networks—the networks of connections between neurons in the brain—while others are more abstract. In Fig. 1.3 we show a picture of a "food web," an ecological network in which the vertices are species in an ecosystem and the edges represent predator-prey relationships between them. That is, pairs of species are connected by edges in this network if one species eats the other. The study of food webs forms a substantial branch of ecology and helps us to understand and quantify many ecological phenomena, particularly concerning energy and carbon flows in ecosystems. Food webs also provide us with another example of a directed network, like the World Wide Web discussed previously. The edges in a food web are

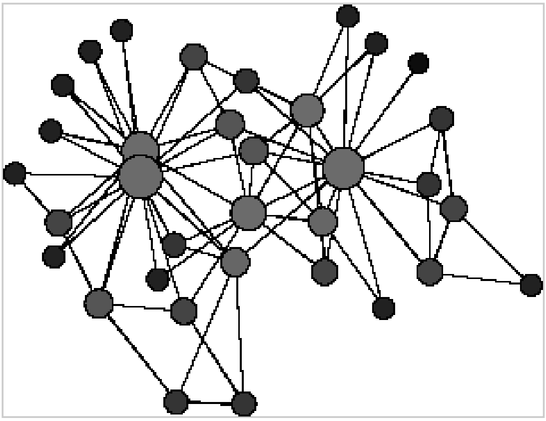

Figure 1.2: **Friendship network between members of a club.** This social network from a study conducted in the 1970s shows the pattern of friendships between the members of a karate club at an American university. The data were collected and published by Zachary [334].

(p.7) asymmetric and are conventionally thought of as pointing from the prey to the predator, indicating the direction of the flow of energy when the prey is eaten. (This choice of direction is only a convention and one could certainly make the reverse choice. The important point is the asymmetry of the predator-prey interaction.)

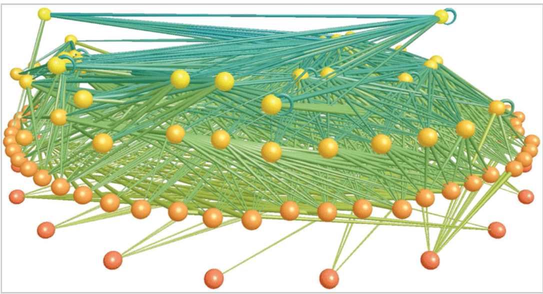

Figure 1.3: **The food web of Little Rock Lake, Wisconsin.** This elegant picture summarizes the known predatory interactions between species in a freshwater lake in the northern United States. The vertices represent the species and the edges run between predator-prey species pairs. The vertical position of the vertices represents, roughly speaking, the trophic level of the corresponding species. The figure was created by Richard Williams and Neo Martinez [209].

Another class of biological networks is that of biochemical networks, such as metabolic networks, protein-protein interaction networks, and genetic regulatory networks. A metabolic network, for instance, is a representation of the chemical reactions that fuel cells and organisms. The reader may have seen the wallcharts of metabolic reactions that adorn the offices of some biochemists, incredibly detailed maps with hundreds of tiny inscriptions linked by a maze of arrows.[2] The inscriptions—the vertices in this network—are metabolites, the substrates and products of metabolism, and the arrows—directed edges—are reactions that turn one metabolite into another. The depiction of reactions as a (p.8) network is one of the first steps towards making sense of the bewildering array of biochemical data generated by recent and ongoing experiments inmolecular genetics.

These are just a few examples of the types of network whose study is the focus of this book. There are many others that we will come across in later pages. Among them some of the best known are telephone networks, road, rail, and air networks, the power grid, citation networks, recommender networks, peer-to-peer networks, email networks, collaboration networks, disease transmission networks, river networks, and word networks.

Properties of Networks

We have seen that a variety of systems can be represented as networks. If we can gather data on the structure of one of these networks, what then can we do with those data? What can they tell us about the form and function of the system the network represents? What properties of networked systems can we measure or model and how are those properties related to the practical issues we care about? This, essentially, is the topic of this entire book, and we are not going to answer it in this chapter alone. Let us, however, look briefly here at a few representative concepts, to get a feel for the kinds of ideas we will be dealing with.

A first step in analyzing the structure of a network is often to make a picture of it. Figures 1.1, 1.2, and 1.3 are typical examples. Each of these was generated by a specialized computer program designed for network visualization and there are many such programs available, both commercially and for free, if you want to produce pictures like these for yourself. Visualization can be an extraordinarily useful tool in the analysis of network data, allowing one to see instantly important structural features of a network that would otherwise be difficult to pick out of the raw data. The human eye is enormously gifted at picking out patterns, and visualizations allow us to put this gift to work on our network problems. On the other hand, direct visualization of networks is only really useful for networks up to a few hundreds or thousands of vertices, and for networks that are relatively sparse, meaning that the number of edges is quite small. If there are too many vertices or edges in a network then pictures of the network will be too complicated for the eye to comprehend and their usefulness becomes limited. Many of the networks that scientists are interested in today have hundreds of thousands or even millions of vertices, which means that visualization is not of much help in their analysis and we need to employ other techniques to determine their structural features. In response to this need, network theory has developed a large toolchest of measures and (p.9) metrics that can help us understand what our network data are telling us, even in cases where useful visualization is impossible.

An example of an important and useful class of network measures is that of measures of *centrality*. Centrality quantifies how important vertices (or edges) are in a networked system, and social network analysts in particular have expended considerable effort studying it. There are a wide variety of mathematical measures of vertex centrality that focus on different concepts and definitions of what it means to be central in a network. A simple but very useful example is the measure called *degree*. The degree of a vertex in a network is the number of edges attached to it. In a social network of friendships between individuals, for instance, such as the network of Fig. 1.2, the degree of an individual is the number of friends he or she has within the network. In the Internet degree would be the number of data connections a computer, router, or other device has. In many cases the vertices with the highest degrees in a network, those with the most connections, also play important roles in the functioning of the system, and hence degree can be a useful guide for focusing our attention on the system's most crucial elements.

In undirected networks degree is just a single number, but in directed networks vertices have two different degrees, *in-degree* and *out-degree*, corresponding to the number of edges pointing inward to and outward from those vertices. For example, the in-degree of a web page is the number of other pages that link to it and the out-degree is the number of pages to which it links. We have already mentioned one example of how centrality can be put to use on the Web to answer an important practical question: by counting the number of links a web page gets—the in-degree of the page—we (or a search engine operating on our behalf) can make a guess about which pages are most likely to contain information that might be of use to us.

It is an interesting observation that many networks are found to contain a small but significant number of "hubs"—vertices with unusually high degree. Social networks often contain a few central individuals with very many acquaintances; there are a few websites with an extraordinarily large number of links; there are a few metabolites that take part in almost all metabolic processes. A major topic of research in recent years has been the investigation of the effects of hubs on the performance and behavior of networked systems. Both empirical and theoretical results indicate that hubs can have a quite disproportionate effect, playing a central role particularly in network transport phenomena and resilience, despite being few in number.

Another example of a network concept that arises repeatedly and has real practical implications is the so-called *small-world effect*. One can define a distance, called the *geodesic distance*, between two vertices in a network to be the (p.10) minimum number of edges one would have to traverse in order to get from one vertex to the other. For instance, two friends would have geodesic distance 1 in a friendship network because there is a single edge connecting them directly, while the friend of your friend would have distance 2 from you. As discussed in Sections 3.6 and 8.2, it is found empirically (and can be proven mathematically in some cases) that the mean geodesic distance, appropriately defined,[3] between vertex pairs is very short, typically increasing only as the logarithm of the number of vertices in the network. Although first studied in the context of friendship networks, this small-world effect appears to be very widespread, occurring in essentially all types of networks. In popular culture it is referred to as the "six degrees of separation," after a successful stage play and film of the same name. The semi-mythological claim is that you can get from anyone in the world to anyone else via a sequence of no more than five intermediate acquaintances—six steps in all.

The small-world effect can have interesting repercussions. For example, news and gossip spread over social networks. If you hear an interesting rumor from a friend, you may pass it on to your other friends, and they in turn pass it on to theirs, and so forth. Clearly the rumor will spread further and faster if it only takes six steps to reach anyone in the world than if it takes a hundred, or a million. It is a matter of common experience that indeed a suitably scandalous rumor can reach the ears of an entire community in what seems like the blink of an eye, and the structure of social networks has a lot to do with it.

And consider the Internet. One of the reasons the Internet functions at all is because any computer on the network is only a few "hops" over optical and other data lines from any other. In practice the paths taken by packets over the Internet are typically in the range of about ten to twenty hops long. Certainly the performance of the network would be much worse if packets had to make a thousand hops instead.

A third example of a network concept of practical importance is provided by clusters or communities in networks. We are most of us familiar with the idea that social networks break up into subcommunities—tightly knit groups of friends or acquaintances within the larger, looser network. Friendship networks, for instance, tend to contain cliques, circles, and gangs of friends within which connections are strong and frequent but between which they are weaker or rarer. The same is true of other kinds of social network also. For instance, in a network of business relationships between companies one often finds clusters formed of sets of companies that operate in particular sections of the economy. (p.11) Connections might be stronger, for instance, between a pair of computer companies or a pair of biotech companies than between a computer company and a biotech company. And if it is the case that communities correspond to genuine divisions of interest or purpose in this way, then we may well learn something by taking a network and examining it to determine what communities it contains. The way a network breaks down into communities can reveal levels and concepts of organization that are not easy to see without network data, and can help us to understand how a system is structured. There is a substantial research literature in social network analysis as well as in other fields concerned with precisely these kinds of questions, and a large number of techniques have been developed to help us extract and analyze subcommunities within larger networks. These are highly active topics of research at present, and hold promise for exciting applications in the future.

Outline of This Book

This book is divided into five parts. In the first part, consisting of Chapters 2 to 5, we introduce the various types of network encountered in the real world, including technological, social, and biological networks, and the empirical techniques used to discover their structure. Although it is not the purpose of this book to describe any one particular network in great detail, the study of networks is nonetheless firmly founded on empirical observations and a good understanding of what data are available and how they are obtained is immensely helpful in understanding the science of networks as it is practiced today.

The second part of the book, Chapters 6 to 8, introduces the fundamental theoretical ideas on which our current understanding of networks is based. Chapter 6 describes the basic mathematics used to capture network ideas, Chapter 7 describes the measures and metrics we use to quantify network structure, and Chapter 8 describes some of the intriguing patterns and principles that emerge when we apply our mathematics and our metrics to realworld network data.

In the third part of the book, Chapters 9 to 11, we discuss computer algorithms for analyzing and understanding network data. Measurements of network properties, such as those described in Chapter 7, are typically only possible with the help of fast computers and much effort has been devoted over the years to the development of efficient algorithms for analyzing network data. This part of the book describes in detail some of the most important of these algorithms. A knowledge of this material will be of use to anyone who wants to work with network data.

(p.12) In the fourth part of the book, Chapters 12 to 15, we look at mathematical models of networks. The material in these chapters forms a central part of the canon of the field and has been the subject of a vast amount of published scientific research. We study both traditional models, such as random graphs and their extensions, and newer models, such as models of growing networks and the "small-world model."

Finally, in the fifth and last part of the book, Chapters 16 to 19, we look at processes taking place on networks, including failure processes and resilience, network epidemiology, dynamical systems, and network search processes. The theory of these processes is less well developed than other aspects of the theory of networks and there is much work still to be done. The last chapters of the book probably raise at least as many questions as they answer, but this, surely, is a good thing. With luck readers will feel inspired to answer some of those questions themselves and the author looks forward to the new and exciting results they generate when they do. (p.13) (p.14)

Notes:

(1) Singular: vertex.

(2) An example appears as Fig. 5.2 on page 83.

(3) One must be careful when there are vertex pairs in the network that are connected by no path at all. Such issues are dealt with in Section 8.2.