```
%matplotlib inline
 import numpy as np
 import matplotlib.pyplot as plt
def func(x):
     return np.exp(-2*x)*np.cos(10*x)
def func_integral(x):
     return (5*np.exp(-2*x)*np.sin(10*x))/52. - (np.exp(-2*x)*np.cos(10*x))/52.
def trapezoid core(f,x,h):
     return 0.5*h*(f(x+h) + f(x))
def trapezoid method(f,a,b,N):
     x = np.linspace(a,b,N)
    h = x[1] - x[0]
     Fint = 0.0
     for i in range (0, len(x) - 1, 1):
        Fint += trapezoid_core(f,x[i],h)
     return Fint
def simpson_core(f,x,h):
     return h*(f(x) + 4*f(x+h) + f(x+2*h))/3
def simpsons_method(f,a,b,N):
     x = np.linspace(a,b,N)
     h = x[1] - x[0]
     Fint = 0.0
     for i in range (0, len(x)-2, 2):
        Fint += simpson_core(f,x[i],h)
     if((N%2) == 0):
        Fint += simpson_core(f,x[-2],0.5*h)
     return Fint
def romberg_core(f,a,b,i):
     h = b-a
    dh = h/2.**(i)
    K = h/2.**(i+1)
    M = 0.0
     for j in range(2**i):
        M += f(a + 0.5*dh + j*dh)
     return K*M
def romberg_integration(f,a,b,tol):
    i = 0
    imax = 1000
    delta = 100.0*np.fabs(tol)
    I = np.zeros(imax,dtype=float)
     I[0] = 0.5*(b-a)*(f(a) + f(b))
     i += 1
     while(delta>tol):
         I[i] = 0.5*I[i-1] + romberg\_core(f,a,b,i)
         delta = np.fabs((I[i]-I[i-1])/I[i])
         print(i, I[i], I[i-1], delta)
         if (delta>tol):
             i+=1
             if(i>imax):
                 print("Max iterations reached.")
                 raise StopIteration('Stopping iterations after ',i)
     return I[i]
Answer = func integral(np.pi) - func integral(0)
print(Answer)
print("Trapezoid")
print(trapezoid method(func, 0, np.pi, 10))
print("Simpson's Method")
print(simpsons method(func,0,np.pi,10))
print("Romberg")
tolerance = 1.0e-6
RI = romberg integration(func, 0, 1, tolerance)
print(RI-Answer)/Answer, tolerance
23 0.014335207263496661 0.014335227461638966 1.408988508729909e-06
24 \ 0.014335197164426737 \ 0.014335207263496661 \ 7.044946650012679e-07
-0.0048596597061173405
TypeError
                                          Traceback (most recent call last)
<ipython-input-65-59f29568c514> in <module>
      8 tolerance = 1.0e-6
      9 RI = romberg_integration(func, 0, 1, tolerance)
---> 10 print (RI-Answer) / Answer, tolerance
TypeError: unsupported operand type(s) for /: 'NoneType' and 'float'
```