Anaconda

The bulge wave sea energy converter

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Introduction

This note describes a completely new way of extracting energy from ocean waves, based on bulge waves traveling along a distensible rubber tube[1]. The tube, typically 7 m diameter and 150 m long and filled with water, is oriented in the direction of wave travel. The waves excite a bulge in the tube which travels just in front of the wave rather like a surf-board, picking up energy and increasing progressively in size. The traveling bulge concentrates the energy from the sea and at the end of the tube the energy can be extracted to drive a turbine. Calculations suggest that a tube of this size would pick up about 1 megawatt average power from the Atlantic waves.

Figure 1. Bulge wave launched by hand; velocity about 3 m/s.

The system is very simple; just a rubber tube in the sea with some inlet valves admitting water and a turbine-generator at the bow. The tube is soft and flexible, it can bend with the waves and there is nothing to break. It can survive in the wildest sea.

Bulge waves in tubes with elastic walls have been described by Lighthill[2]. He shows that the velocity of bulge waves in the tube is related to the distensibility of the tube (see equations (4) and (8) in the Appendix). By correctly choosing the elasticity of the

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walls, the velocity of bulges in the tube is set equal to the velocity of the waves in the sea. In this case there is a resonant interaction between the sea and the tube, and energy is transfered progressively from the ocean waves to the tube.

A photograph of a bulge wave traveling along a small water-filled tube is shown in fig. 1.

Theory

The detailed mathematical theory is set out in the appendix. This shows that on resonance, when the velocity of the bulge in the tube matches the velocity of the sea wave, the bulge grows linearly along the tube. The energy in the bulge grows as the square of the distance from the bow. Off resonance, the bulge grows initially, reaches a maximum and then decreases; this gives rise to the oscillations in the power output curve seen in fig. 2.

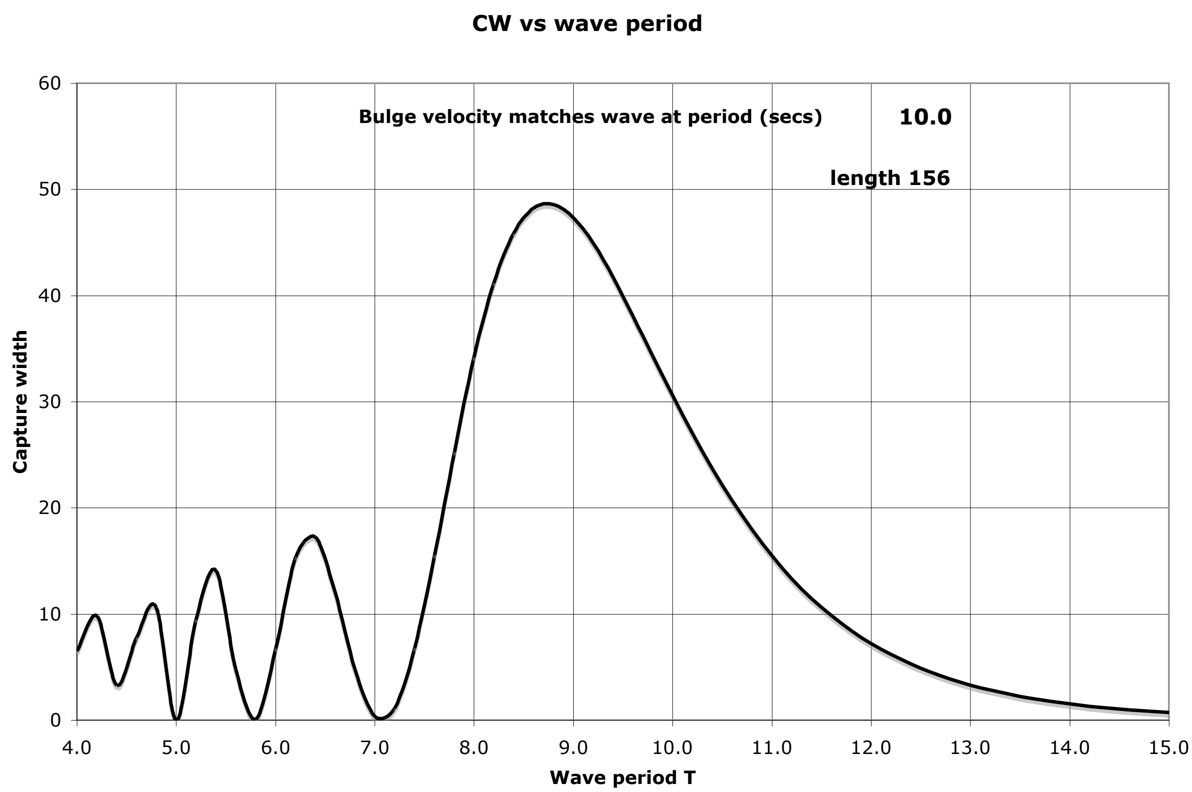

Figure 2. Theoretical capture width vs wave period for tube 7 m diameter, 156 m long. The zeroes at small T can be filled in by modifying the design.

The bulge wave in the tube is a wave of oscillating pressure, positive and negative, associated with a to-and-fro longitudinal movement of the water in the tube. At the end of a tube one wavelength long, the pressure oscillation is three times the incoming sea wave amplitude and the movement of the water in the tube is three times the water movement in the sea. The energy in the tube is proportional to its area. The energy captured is related to the energy per metre of wave front in the sea by a parameter called the "capture width" (CW). In effect, the device collects all the energy in the sea from a wave frontage equal to the capture width. The predicted capture width is plotted in fig. 2 as a function of wave period for a tube 7 m diameter and 150 m long. We see that the maximum capture width is nearly 50 m and the response is quite broad. Averaging over the wave spectrum in the sea, one expects the mean capture width to be about 20 m. In typical Atlantic conditions, the wave energy in the sea is 50 kW/m, so the energy captured on average would be about 1 MW.

The capture width of existing wave energy converters is typically only a fraction of this, and they are relatively elaborate floating structures. It therefore seems that the bulge wave tube may have some future.

Figure 3. Bulge tube in wave tank with manometer. The waves are coming from the left. Oscillations in the manometer were about 5 times the wave height.

Experimental verification

The concept has been confirmed in a wave tank, see fig. 3. A rubber tube 12 cm diameter and 2.2 m long was equipped with manometers at each end to measure the internal water pressure. With waves of 2 s period and 1 cm amplitude, the pressure oscillations at the stern of the tube were about ± 5 cm ; we observed a pressure amplification factor of 5, very much as predicted by the theory: for a tube closed at the stern, the pressure is doubled by total reflection. As expected, the pressure oscillations at the bow were much smaller. This verified the prediction that the bulge wave inside the tube grows progressively.

Full scale design

The detail of a full scale bulge tube is still evolving and we are open to expert suggestions, both as to the structure itself and for the power output. A preliminary design for a tube 7 m in diameter and 150 m long, which would capture 1 MW average power, is shown in fig. 4.

A mean positive water pressure inside the tube is established by water entering through the one-way “duck bill valve”(DBV) at the stern. At the end of the tube there is a large oscillating pressure: when this oscillation, combined with the positive bias, swings negative, water is sucked in through the DBV. The result is that the mean pressure builds

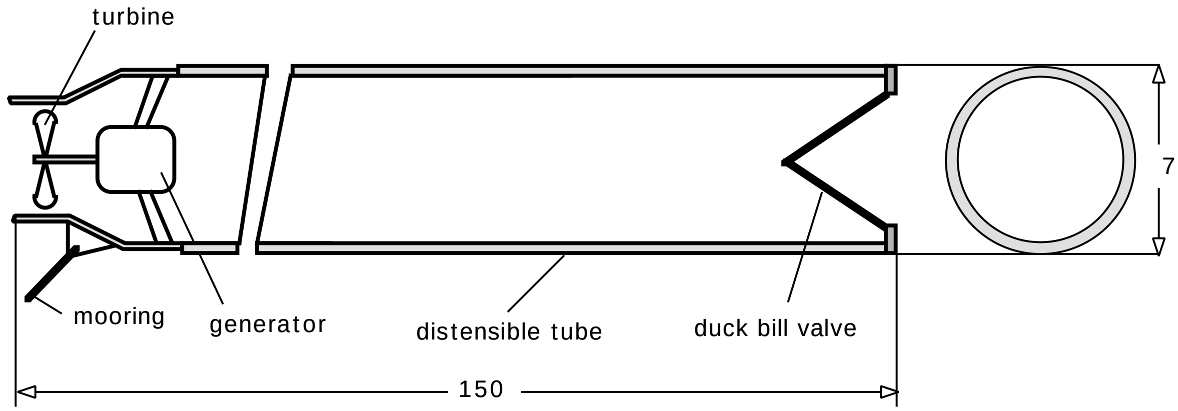

Figure 4. Full scale design. When the oscillating bulge pressure at the end of the tube swings negative, water is sucked in through the duck bill valve, maintaining a generally positive pressure in the tube. A steady outflow at the bow drives the turbine generating electricity.

up to equal the amplitude of the pressure oscillation, typically about 4 m water head. The water entering through the stern flows along the tube and exits through a standard low-head turbine located at the bow. The distensibility of the tube smooths the flow. Thus the power in the wave is converted into a smooth flow of water through the turbine at the bow, generating electricity. A more elaborate arrangement ("pressure amplifier") can be used at the stern to inflate the tube, ensuring that the pressure never swings negative.

When the tube bulges, energy is stored in the rubber walls, see appendix eq(21). For 1 MW average output the peak power would be about 3 MW. With wave velocity 15 m/s corresponding to 10 s waves, the energy to be stored per metre length is 800 kJ/m. Comparing this with 250 kJ/m^3 stored in rubber with $E = 2MPa$ at strain 50%, the wall thickness required for 7 m diameter tube comes to 15 cm. But this is only needed at the end of the tube: half way along the bulge energy is four times smaller, so less rubber could be used. Less rubber would also be required if it had a higher modulus.

To get the correct bulge velocity requires the right combination of modulus E and wall thickness h, as specified in the appendix eq(19). This applies to the simplest case of a uniform tube. To use the rubber most effectively the tube will have a composite wall, partly rubber and partly inextensible polymer-coated fabric, The weight of rubber required for a 1 MW installation will then be only a few hundred tonnes and, on preliminary estimates, the cost of the structure less than £1 M. Our target for a complete system is £ 2 k per average kilowatt out.

Note that the power output is proportional to the cross-sectional area of the tube, see eqn(15) in the appendix. The wall thickness scales as the radius (eqn(19)), so the volume of rubber required is also proportional to the cross-section. This means that the amount of rubber needed to capture a given amount of power is the same, whether there are many small tubes or one large one. A single large tube with one large generator could ultimately be the best solution. But with little increase in cost one can start with small tubes, say 1.5 m in diameter, which would each capture about 50 kW on average.

As well as capital cost, survivability and operational costare important for wave energy conveters. Being flexible, Anaconda should be a good survivor and simple enough to require little maintenance. Rubber is a supremely durable material, used at sea in the rubber dinghy and dracones for transporting fluids: also in the car tyre, which requires virtually no maintenance and is unaffected by fatigue.

Competition

The front runner in wave power converters is Pelamis made by Ocean Power Delivery, see the web site www.oceanpd.com. This is a long articulated structure of 4 sections, which oscillates laterally in the waves. The average capture width in Atlantic seas is 5.5 m, so in a 50 kW/m sea the mean power collected is about 280 kW. Some of these machines have been sold to Portugal at a cost of £ 1.8 M each. The capital cost of generating on average one kilowatt of electricity from Pelamis is therefore £6,500. Other wave enegy convereters have all proved more expensive than this, though all should eventually benefit from economies of scale. Maintenance costs remain to be determined; they could be significant, as experience is demonstrating in the case of offshore wind turbines. For wave power to be really competitive the cost needs to come down by a factor of two or three, to say £ 2 k per average kilowatt delivered with small maintenance cost.

Market potential

Figure 5. European Wave Resource Chart The chart shows annual average wave power in kilowatts per metre of crest width for various European sites. (*from the OPD website*)

The size of the wave energy resource is well set out in the website of Ocean Power Delivery (*http://www.oceanpd.com/Resource/default.html*). The economically recoverable wave power for the UK alone has been estimated to be 10,000 megawatts, that is 25% of current UK demand. At a capital cost of £ 2 M per average megawatt delivered, the UK market alone would be worth 20 billion pounds.

The wave power resource for western Europe is shown in figure 5 (taken from Ocean Power Delivery). It is clear that many countries could purchase converters. There are also potential users in South Africa, India, Australia, the USA, not to speak of the numerous isolated communities all over the major oceans who could benefit from small machines.

Advantages

In comparison with other wave power converters the Anaconda appears to have the following advantages:

- Simple system
- Low maintenance
- Good capture width
- Good survivor
- On our preliminary estimates, it will be cheaper per kilowatt delivered.

Present status

The concept is protected by a UK patent application[3] and there is plenty of time for international applications. The official search by the Patent Office has revealed no significant prior art.

We are confident that our research programme with Prof. Chaplin will resolve all the remaining scientific issues regarding power capture and power take-off. We are seeking to sell or license the patent rights to a company interested in developing the concept. A suitable development programme could be:

(a) develop a bottom-mounted version of about 1.5 m diameter, suitable for installation on a beach, average power output about 50 kW. The power take-off is simpler in this case, because the seabed provides a reference, and such a device could pump water to the shore. As well as a demonstrator of the technology, the device could have a niche market, perhaps in desalination.

(b) develop a floating version of (a), designed to produce electricity. This means solving the problem of a power take-off without a seabed reference, and designing an electrical and a mooring system. The device would be primarily a demonstrator, but again could have a niche market.

(c) full-scale device as described above.

Acknowledgement

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References

[1] F.J.M. Farley and R.C.T. Rainey, *Radical design options for wave profiling wave energy converters*, 21st International Workshop on Water Waves and Floating Bodies, Loughborough, UK, 2-5 April 2006.

[2] J. Lighthill, *Waves in fluids*, Cambridge (1978), p.93 *ff*

[3] British patent application GB 0602278.4, 4 Feb 2006.

Let p_b be the pressure inside the bulge tube due to the distension of the tube (called the *bulge pressure*) and p_w the pressure in the wave outside. The total pressure inside is then $p = p_b + p_w$. Let u be the particle velocity in the bulge wave at longitudinal coordinate x. Then following Lighthill [2] with an incompressible fluid of denisty ρ inside the tube the acceleration is driven by the gradient of the total pressure

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1}$$

while the change in tube cross section S is

$$\frac{\partial S}{\partial t} = -S\frac{\partial u}{\partial x} \tag{2}$$

Differentiating again to eliminate u

$$\frac{\partial^2 S}{\partial t^2} = \frac{S}{\rho}\frac{\partial^2 p}{\partial x^2} \tag{3}$$

But S is linearly related to the bulge pressure p_b by

$$\frac{dS}{dp_b} = DS \tag{4}$$

(This equation defines the distensibility D). Therefore

$$\frac{\partial^2 S}{\partial t^2} = DS\frac{\partial^2 p_b}{\partial t^2} \tag{5}$$

Combining (3) and (5) gives

$$\frac{\partial^2 p_b}{\partial t^2} = \frac{1}{D\rho}\frac{\partial^2}{\partial x^2}\{p_b + p_w\} \tag{6}$$

With no external wave, this is the wave equation for the bulge pressure p_b, with the solution

$$p_b = \rho g B\cos(\omega t - k_2 x) \tag{7}$$

a wave of arbitrary amplitude $\rho g B$ and velocity

$$c_2 = \omega/k_2 \;=\; 1/\sqrt{D\rho} \tag{8}$$

B is the height of a water column corresponding to the peak bulge pressure.

(7) is the expression for the bulge wave with the tube in air or calm water. Following the usual convention for differential equations, we call it the complementary function, CF.

In the presence of an external wave with wave number k_1 and velocity $c_1 = \omega/k_1$ and amplitude A, the wave pressure is $p_w = \rho g A\cos(\omega t - k_1 x)$ and we can solve (6) to get a solution, the particular integral PI, by substituting $p_b = \rho g B\cos(\omega t - k_1 x)$. Using (8) this satisfies (6) if

$$B = \frac{A}{k_2^2/k_1^2 - 1} = \frac{A}{c_1^2/c_2^2 - 1} \tag{9}$$

in perfect agreement with equation (6) of reference [1]. (In this paper we presented an alternative way of deriving the theory in a frame of reference moving with the waves).

Note that the particular integral has the same wave number k_1 (and velocity) as the external wave. The complementary function has the same frequency, but its wave number k_2 (and velocity) is that of a free bulge wave.

To obtain the general solution of (6) we can add a CF (7) of any amplitude to the PI (9), because the combination still satisfies (6). For a distensible tube of finite length we expect the bulge wave to be zero at the bow, $x = 0$, so the desired solution is

$$p_b = \rho g B \left\{ \cos(\omega t - k_1 x) - \cos(\omega t - k_2 x) \right\} \tag{10}$$

with B given by (9) This can be written

$$p_b = -2\rho g B \sin(\omega t - \bar{k}x) . \sin(x\Delta k/2) \tag{11}$$

where $\bar{k} = (k_1 + k_2)/2$ and $\Delta k = k_2 - k_1$. This is a traveling wave with the mean wave number, and amplitude $B = AF$ where the pressure amplification factor F is

$$F = \frac{k_1^2}{\bar{k}\Delta k} \sin(x\Delta k/2) \approx \frac{1}{2} k_1 x \tag{12}$$

The approximation is valid if $x\Delta k$ is small; note that it does not matter if Δk is positive or negative as this term cancels.

We see from (12) that for small Δk the bulge amplitude grows linearly along the tube: bulge energy (see below) rises as x^2. But when the bulge-wave velocity differs appreciably from the sea-wave velocity, the beating between the CF and the PI, which have different wave numbers, causes the bulge amplitude to rise and fall along the tube.

Discussion

If one takes a snapshot of the system at a moment when $\omega t = 2n\pi$, the bulge wave in (11) will look like $\sin(\bar{k}x)$ while the water wave looks like $\cos(k_1 x)$. The wave is propagating towards larger x, so we see that the bulge leads the wave by 90^o: maximum bulge occurs where the water outside the tube is rising fastest. Thus the wave does work in lifting the bulge and this is not completely refunded where the wave is falling, because the bulge is then at its minimum. A net amount of energy is transfered to the bulge. In another sense, the bulge is surf-riding on the front of the wave, picking up energy as it does so.

According to (9) the PI is either in phase with the water wave or in antiphase depending on which side of the resonance the bulge velocity falls. But (11) shows that the initial bulge response is in quadrature with the water wave. This apparent contradiction is resolved when we include damping due to hysteresis or wave radiation. In this case the CF will slowly die away and at the end of a long tube we will be left with the PI only.

Also, if there is damping, the PI response on resonance will not be infinite, as suggested by (9), but will have the shape of a typical resonance curve. The phase variation of 180 degrees on going through resonance is typical. At the peak of the resonance the phase is exactly half way, so the bulge will lead the crest of the wave by 90 degrees even at large

x. This has been confirmed by more detailed mathematics which includes damping due to hysteresis in the rubber.

(11) shows that if the velocities do not match the bulge amplitude will oscillate as we progress along the tube. But this oscillation will die out as the CF is attenuated.

Capture width

Using (1) and (7) one finds that the horizontal velocity amplitude in the bulge wave is

$$u_{max} = \frac{p_{bmax}}{\rho c_2} \tag{13}$$

If $c_2 = c_1 = g/\omega$, then $u_{max} = \omega B$, implying that the horizontal excursion of water particles in the bulge wave is $\pm B$.

The power in the bulge wave for a tube of area S is

$$P_b = S\ u_{max} p_{bmax}/2 = \frac{S p_{bmax}^2}{2\rho c_2} = \frac{S(\rho g A F)^2}{2\rho c_2} \tag{14}$$

The power in the sea wave per unit width of wavefront is $P_w = \rho g^2 A^2/4\omega$. Dividing, we find the capture width

$$W = 2k_2 S F^2 \tag{15}$$

with the amplification factor F is given by (12).

For example a tube 7 m diameter and 150 m long in 10 s waves would have a capture width of 32 m. But owing to the effect of the factors k in (12) and (15) the peak capture is at smaller wave periods, see fig. 2.

Combining (13) and (12), if the wave propagates at the same velocity as the bulge, ($c_2 = c_1$), the particle velocity in the bulge is just F times the particle velocity ωA in the wave, and the particle amplitude in the bulge is a factor F times the particle amplitude in the wave.

The capture width is plotted vs wave period in fig. 2 for a tube 7 m diameter and 150 m long. The multiple zeroes are due to the beating of the complimentary function with the particular integral giving the $sin(\Delta k)$ term in (12). Radiation damping, not included here, reduces the peak capture width and smoothes out the zeroes.

Bulge size

Combining (4) and (8)

$$\frac{dS_{max}}{S} = \frac{dp_{bmax}}{\rho c_2^2} \tag{16}$$

the strain at the end of a bulge tube of radius r with an incident wave of amplitude A is

$$\frac{dr}{r} = \frac{dS_{max}}{2S} = \frac{1}{2} k_2 A F \tag{17}$$

For example, on resonance with a tube one wavelength long, (12) shows that $F = \pi$, so a wave of 2 m amplitude would give a pressure amplitude of $AF \approx 6$ m water pressure at the end of the tube, while the wave number k_2 is typically $1/24\ m^{-1}$, so the strain in the tube would be about $\pm 12\%$; quite small for natural rubber in these rather extreme conditions, corresponding to 5 MW at the end of a 7 m diameter tube. If the tube is pressurised to avoid negative pressure swings, the maximum strain in the wall would be 25%.

Design of the bulge tube

For a tube of radius r with wall thickness h made of rubber with Young's modulus E, the distensibility D defined in eqn (4) is

$$D = \frac{2r}{Eh} \tag{18}$$

Combining with eqn (8), the condition for the bulge wave to have velocity c is

$$E\ h = 2\ r\rho\ c^2 \tag{19}$$

This shows that the wall thickness h, and therefore the total volume of the rubber, can be reduced if the elastic modulus E of the material is high. How much E can be increased without unacceptable hysteresis and limits on the working strain depends on the available rubbers and a judicious choice of filler.

Another consequence of eqn(19) is that for a given value of E, the ratio h/r is independent of the radius r of the machine. This means that the volume of rubber required for a given power output (see eqn(15)) is the same for one large tube or a multiplicity of smaller tubes.

Stored energy

In all waves and oscillating systems there is a rhythmic interchange of energy, usually between kinetic and potential energy. In the bulge wave the potential energy is stored in the elastic wall of the tube. From (16) and (14) the energy to be stored per unit length is

$$J_b = \frac{1}{2} p_{bmax} dS_{max} = \frac{S p_{bmax}^2}{2\rho c_2^2} = \frac{P_b}{c_2} \tag{20}$$

If the tube is inflated to a mean pressure equal to p_{bmax} so that the internal pressure never swings negative, then the energy stored in the rubber is four times larger,

$$J_b = \frac{4\ P_b}{c_2} \tag{21}$$

Let us compare this with the energy that can be stored in rubber. With modulus E at strain η the energy stored per cubic metre is $E\eta^2/2$. So for tube radius r with wall thickness h, the energy stored per metre length would be

$$J_r = \pi r E h \eta^2 = 2\pi r^2 \rho\ c^2 \eta^2 \tag{22}$$

where we have taken the value of Eh from (19). Equating this to the requirement (20) gives the power capability of the rubber tube

$$P_b = \pi r^2 \rho\ \eta^2 c^3/2 \tag{23}$$

It is interesting to find that this is independent of the modulus and wall thickness, provided they satisfy (19) and is proportional to the tube cross section. As the capture width (15) is also proportional to the cross section we have a good match at all tube diameters.

For a 10 s wave with a pressurised tube and maximum strain $\eta = 50\%$, then $P_b = d^2 \times 0.33\ MW$ where d is the diameter of the tube. For tube diameter 7 m the bulge power capability would be 16 MW !! This is much larger than our requirement.

Cyclic strains up to 50% in rubber produce negligible fatigue. So it appears that this is an ideal material for the Anaconda. The grade of rubber, filling and wall structure to furnish the correct combination of E and h with small hysteresis loss at optimum cost are being investigated.