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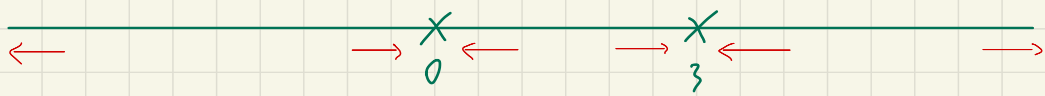
①

$$f(x) = e^{\frac{3x+3}{x^2-3x}}$$

$$D(f) = \{ x \in \mathbb{R} \mid x^2 - 3x \neq 0 \} =$$

$$= \mathbb{R} \setminus \{0, 3\}$$



Per calcolare $\lim e^{\frac{3x+3}{x^2-3x}}$ è necessario calcolare : $\lim \frac{3x+3}{x^2-3x}$

$$\lim_{x \to \pm\infty} \frac{3x+3}{x^2-3x} = \lim_{x \to \pm\infty} \frac{1}{x} \cdot \frac{3 + \frac{3}{x^2}}{1 - \frac{3}{x}} = 0$$

$$\lim_{x \to 0^-} \frac{3x+3}{x^2-3x} = \lim_{x \to 0^-} \underbrace{\frac{3x+3}{x-3}}_{-1} \cdot \frac{1}{x} = +\infty$$

$x \to 0+$ $\qquad (-\infty)$

$$\lim_{\substack{x \to 3^- \\ x \to 3^+}} \frac{3x+3}{x^2-3x} = \lim_{x \to 3^-} \underbrace{\frac{3x+3}{x}}_{2} \cdot \frac{1}{x-3} = -\infty \quad (+\infty)$$

$\Longrightarrow$

$$\lim_{x \to \pm\infty} e^{\frac{3x+3}{x^2-3x}} = e^0 = 1$$

$$\lim_{\substack{x \to 0^- \\ x \to 3^+}} e^{\frac{3x+3}{x^2-3x}} = +\infty$$

$$\lim_{\substack{x \to 0^+ \\ x \to 3^-}} e^{\frac{3x+3}{x^2-3x}} = 0$$

$$f(x) = e^{\frac{3x+3}{x^2-3x}}$$

$$f'(x) = e^{\frac{3x+3}{x^2-3x}} \cdot \frac{3(x^2-3x) - (2x-3)\cdot(3x+3)}{(x^2-3x)^2}$$

$$= e^{\frac{3x+3}{x^2-3x}} \cdot \frac{3x^2 - \cancel{9x} - 6x^2 - 6x + \cancel{9x} + 9}{(x^2-3x)^2} =$$

$$= e^{\frac{3x+3}{x^2-3x}} \cdot \frac{-3x^2 - 6x + 9}{(x^2-3x)^2} =$$

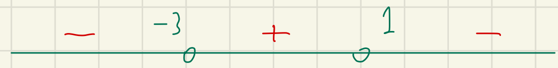
$$= \frac{3 \cdot e^{\frac{3x+3}{x^2-3x}}}{(x^2-3x)^2} \cdot \boxed{\left(-x^2 - 2x + 3\right)}$$

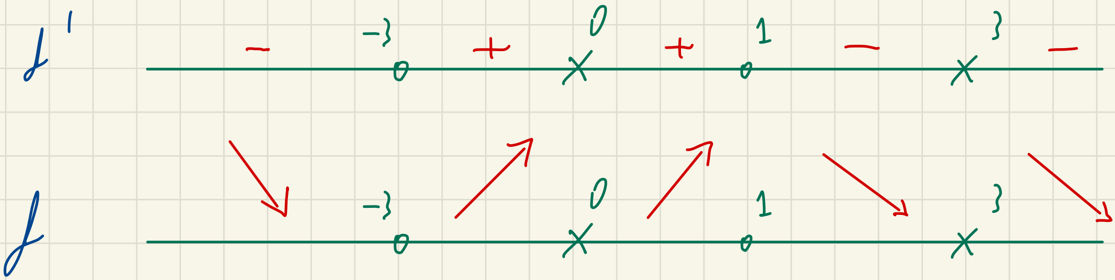
$$\Delta = 4 + 12 = 16 \qquad x_{1,2} = \frac{2 \pm 4}{-2} \begin{cases} 1 \\ -3 \end{cases}$$

È sufficiente studiare:

$$-x^2 - 2x + 3$$

$-x^2-2x+3$





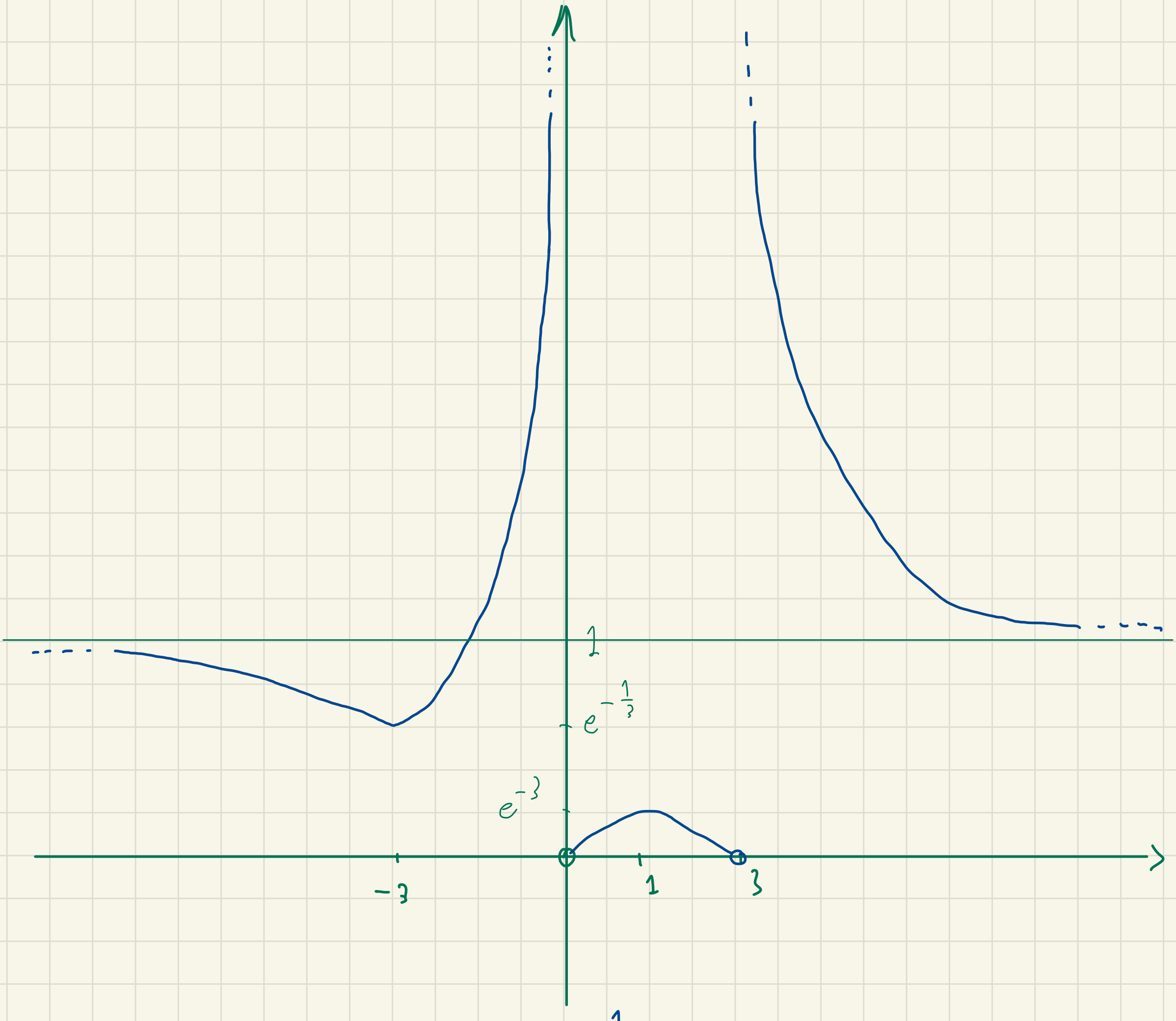
$$f(-3) = e^{\frac{-6}{18}} = e^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e}}$$

$\left(-3, \frac{1}{\sqrt[3]{e}}\right)$ p. di MIN.

$$f(1) = e^{\frac{6}{-2}} = e^{-3} = \frac{1}{e^3}$$

$\left(1, e^{-3}\right)$ p. di MAX

Grafico "qualitativo"



$$\operatorname{Im} f = \left]0, e^{-3}\right[\cup \left]e^{-\frac{1}{3}}, +\infty\right[$$

$f(x) = K$ 1 sol $\iff K = \{e^{-3}, e^{-\frac{1}{3}}, 1\}$

Grafico vero (a puro scopo di esempio!)

2

$$\lim_{x\to 0} \frac{\ln(1+\arctan(x)) + \frac{1}{2}\ln(1+x^2) - x}{x^4}$$

$$\ln(1+\arctan(x)) = ?$$

$$t = \arctan x \implies t \approx x$$

$$o(t^n) = o(x^n) \stackrel{?}{=} o(x^4) \implies n = 4$$

$$\ln(1+\arctan(x)) = \arctan x - \frac{1}{2}(\arctan x)^2 +$$

$$+ \frac{1}{3}(\arctan x)^3 - \frac{1}{4}(\arctan x)^4 + o(x^4) =$$

$$= \left(x - \frac{x^3}{3} + o(x^4)\right) - \frac{1}{2}\left(x - \frac{x^3}{3} + o(x^4)\right)^2 +$$

$$+ \frac{1}{3}\left(x + o(x^2)\right)^3 - \frac{1}{4}\left(x + o(x^4)\right)^4 + o(x^4)$$

$$= \left(x - \frac{x^3}{3} + o(x^4)\right) - \frac{1}{2}\left(x - \frac{x^3}{3} + o(x^4)\right)^2 +$$

$$+ \frac{1}{3}\left(x + o(x^2)\right)^3 - \frac{1}{4}\left(x + o(x^4)\right)^4 + o(x^4)$$

$$= x - \frac{x^3}{3} - \frac{1}{2}\left(x^2 - \frac{2}{3}x^4 + o(x^4)\right) +$$

$$+ \frac{1}{3}\left(x^3 + o(x^4)\right) - \frac{1}{4}\left(x^4 + o(x^4)\right) + o(x^4)$$

$$= x - \cancel{\frac{x^3}{3}} - \frac{x^2}{2} + \frac{1}{3}x^4 + \cancel{\frac{1}{3}x^3} - \frac{1}{4}x^4 + o(x^4)$$

$$= x - \frac{x^2}{2} + \frac{x^4}{12} + o(x^4)$$

$$\ln(1 + x^2) = x^2 - \frac{x^4}{2} + o(x^4)$$

$$\frac{\ln\left(1+\mathrm{arctg}(x)\right)+\frac{1}{2}\ln\left(1+x^2\right)-x}{x^4}=$$

$$=\frac{\cancel{x}-\cancel{\frac{x^2}{2}}+\frac{x^4}{12}+\cancel{\frac{x^2}{2}}-\frac{x^4}{4}-\cancel{x}+o(x^4)}{x^4}$$

$$=\frac{\frac{x^4}{12}-\frac{x^4}{4}+o(x^4)}{x^4}=\frac{-\frac{x^4}{6}+o(x^4)}{x^4}$$

$$=-\frac{1}{6}+\frac{o(x^4)}{x^4}\xrightarrow[x\to 0]{}-\frac{1}{6}$$