(2)

$(1+\sin(x))^x$ $\quad x=\pi$

$\hookrightarrow f'(\pi) = -\pi$

$$f(x) = (1+\cos x)^x =$$

$$= e^{\ln(1+\cos x)^x} =$$

$$= e^{x\ln(1+\cos x)}$$

$$f'(x) = e^{x\ln(1+\cos x)} \cdot \left(\ln(1+\cos x) + \right.$$

$$\left. + x \cdot \frac{1}{1+\cos x} \cdot (-\sin x)\right) =$$

$$= (1+\cos x)^x \cdot \left(\ln(1+\cos x) - \frac{x \cdot \sin x}{1+\cos x}\right)$$

$$f\left(\frac{\pi}{2}\right) = \left(1+\cos\frac{\pi}{2}\right)^{\frac{\pi}{2}} = 1^{\frac{\pi}{2}} = 1$$

$$f'\left(\frac{\pi}{2}\right) = \left(\ln\left(1+\cos\frac{\pi}{2}\right) - \frac{\frac{\pi}{2}\cdot\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}\right) =$$

$$= -\frac{\pi}{2}$$

$$y = f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right)$$

$$y = -\frac{\pi}{2}\cdot\left(x - \frac{\pi}{2}\right) + 1$$

$$\boxed{y = -\frac{\pi}{2}x + \frac{\pi^2}{4} + 1}$$

2'

$$(1+\sin(x))^x \quad x=\pi$$

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$$f(x) = (1+\sin x)^x =$$

$$= e^{\ln(1+\sin x)^x} =$$

$$= e^{x\ln(1+\sin x)}$$

$$f'(x) = e^{x\ln(1+\sin x)} \cdot \left(\ln(1+\sin x) + \right.$$

$$\left. + x \cdot \frac{1}{1+\sin x} \cdot (\cos x)\right) =$$

$$= (1+\sin x)^x \cdot \left(\ln(1+\sin x) + \frac{x \cdot \cos x}{1+\sin x}\right)$$

$$f(\pi) = (1+\sin\pi)^\pi = 1^\pi = 1$$

$$f'(\pi) = \left(\ln(1+\sin\pi) + \frac{\pi \cdot \cos\pi}{1+\sin\pi}\right) =$$

$$= -\pi$$

$$y = f'(\pi)(x - \pi) + f(\pi)$$

$$y = -\pi \cdot (x - \pi) + 1$$

$$\boxed{y = -\pi \cdot x + \pi^2 + 1}$$

③ $$\lim_{x\to 0} \frac{e^{\frac{\sin x}{2}} - \sqrt{1+\ln(1+x)} - \frac{x^2}{2}}{\sin(x^3)}$$

$\sin(x^3) = x^3 + o(x^3)$

$$\sqrt{1+\boxed{\ln(1+x)}} = \,?$$

$\ln(1+x) = t$

$t \approx x \implies o(t^n) = o(x^n) = o(x^3)$

$\implies n = 3$

$$\sqrt{1+t} = (1+t)^{\frac{1}{2}} =$$

$$= 1 + \frac{1}{2}t + \boxed{\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}}t^2 +$$

$\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!} = -\frac{1}{8}$

$$+ \boxed{\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}}t^3 + o(t^3)$$

$$\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6} = \frac{3}{48} = \frac{1}{16}$$

$$\sqrt{1+\ln(1+x)} = 1 + \frac{1}{2}\ln(1+x) - \frac{1}{8}(\ln(1+x))^2 + \frac{1}{16}(\ln(1+x))^3 + o(x^3)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$(\ln(1+x))^2 = \left(x - \frac{x^2}{2} + o(x^2)\right)^2 =$$

$$= x^2 + 2\cdot x\cdot\left(-\frac{x^2}{2}\right) + o(x^3)$$

$$= x^2 - x^3 + o(x^3)$$

$$(\ln(1+x))^3 = (x+o(x))^3 = x^3 + o(x^3)$$

$$\sqrt{1+\ln(1+x)} = 1 + \frac{1}{2}\left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right) -$$

$$- \frac{1}{8}\left(x^2 - x^3 + o(x^3)\right) + \frac{1}{16}\left(x^3 + o(x^3)\right) =$$

$$= 1 + \frac{x}{2} - \frac{3}{8}x^2 + \frac{17}{48}x^3 + o(x^3)$$

$$e^{\boxed{\frac{1}{2}\sin x}^{=t}}$$

$t = x + o(x) \implies t \simeq x$

$o(t^n) = o(x^n) = o(x^3) \implies n = 3$

$$e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + o(t^3)$$

$$e^{\frac{1}{2}\sin x} = 1 + \frac{\sin x}{2} + \frac{1}{2}\left(\frac{\sin x}{2}\right)^2 +$$

$$+ \frac{1}{6}\left(\frac{\sin x}{2}\right)^3 + o(x^3)$$

$$\frac{1}{2}\sin x = \frac{1}{2}\left(x - \frac{x^3}{6} + o(x^3)\right) = \frac{x}{2} - \frac{x^3}{12} + o(x^3)$$

$$\left(\frac{1}{2}\sin x\right)^2 = \frac{1}{4}\left(x + o(x^2)\right)^2 = \frac{x^2}{4} + o(x^3)$$

$$\left(\frac{1}{2}\sin x\right)^3 = \frac{1}{8}\left(x + o(x)\right)^3 = \frac{x^3}{8} + o(x^3)$$

$$e^{\frac{1}{2}\sin x} = 1 + \frac{x}{2} - \frac{x^3}{12} + \frac{x^2}{8} + \frac{x^3}{48} + o(x^3)$$

$$= 1 + \frac{x}{2} + \frac{x^2}{8} - \frac{3}{48}x^3 + o(x^3)$$

$$\frac{e^{\frac{\sin x}{2}} - \sqrt{1 + \ln(1+x)} - \frac{x^2}{2}}{\sin(x^3)} =$$

$$= \frac{\cancel{1} + \cancel{\frac{x}{2}} + \frac{x^2}{8} - \frac{3}{48}x^3 - \left(\cancel{1} + \cancel{\frac{x}{2}} - \frac{3}{8}x^2 + \frac{17}{48}x^3\right) - \frac{x^2}{2} + o(x^3)}{x^3 + o(x^3)}$$

$$= \frac{\cancel{\frac{x^2}{8}} - \frac{3}{48}x^3 + \cancel{\frac{3}{8}x^2} - \frac{17}{48}x^3 - \cancel{\frac{x^2}{2}} + o(x^3)}{x^3 + o(x^3)} =$$

$$= \frac{-\frac{20}{48}x^3 + o(x^3)}{x^3 + o(x^3)} = \frac{-\frac{20}{48} + \frac{o(x^3)}{x^3}}{1 + \frac{o(x^3)}{x^3}} \xrightarrow{x \to 0} -\frac{20}{48} = -\frac{5}{12}$$

3'

Usando gli sviluppi calcolati nell'esercizio precedente:

$$\frac{\sqrt{1+\ln(1+x)} - e^{\frac{\sin x}{2}} + \frac{x^2}{2}}{\ln(1+x^3)} =$$

$$= \frac{\cancel{1} + \cancel{\frac{x}{2}} - \cancel{\frac{3}{8}x^2} + \frac{17}{48}x^3 - \cancel{1} - \cancel{\frac{x}{2}} - \cancel{\frac{x^2}{8}} + \frac{3}{48}x^3 + \cancel{\frac{x^2}{2}}}{x^3 + o(x^3)}$$

$$= \frac{\frac{20}{48}x^3 + o(x^3)}{x^3 + o(x^3)} \longrightarrow \frac{20}{48} = \frac{5}{12}$$

(4) $f(x) = \left(x^2 - 3x + 2 + \frac{1}{e}\right) \ln\left(x^2 - 3x + 2 + \frac{1}{e}\right)$

$D(f) = ?$

$x^2 - 3x + 2 + \frac{1}{e}$

$\Delta = 9 - 4\left(2 + \frac{1}{e}\right) = 1 - \frac{4}{e} < 0$

infatti: $1 - \frac{4}{e} < 0 \iff 1 < \frac{4}{e}$

$\iff e < 4$

(in quanto $e < 3$)

Quindi: $x^2 - 3x + 2 + \frac{1}{e} > 0 \quad \forall x \in \mathbb{R}$

$\implies D(f) = \mathbb{R}$

OSS.: f non è né pari né dispari

$$\lim_{\substack{x\to+\infty \\ (-\infty)}} \left(x^2-3x+2+\frac{1}{e}\right)\ln\left(x^2-3x+2+\frac{1}{e}\right) = +\infty$$

$$f'(x) = (2x-3)\ln\left(x^2-3x+2+\frac{1}{e}\right) +$$

$$+\cancel{\left(x^2-3x+2+\frac{1}{e}\right)} \cdot \frac{1}{\cancel{x^2-3x+2+\frac{1}{e}}} \cdot (2x-3) =$$

$$= (2x-3)\left(\ln\left(x^2-3x+2+\frac{1}{e}\right)+1\right)$$

$$2x-3>0 \longrightarrow x > \frac{3}{2}$$

$$- \qquad \frac{3}{2} \qquad +$$

$$\ln\left(x^2-3x+2+\frac{1}{e}\right)+1>0$$

$$\Longrightarrow \ln\left(x^2-3x+2+\frac{1}{e}\right) > -1$$

$$x^2-3x+2+\cancel{\frac{1}{e}} > \cancel{\frac{1}{e}}$$

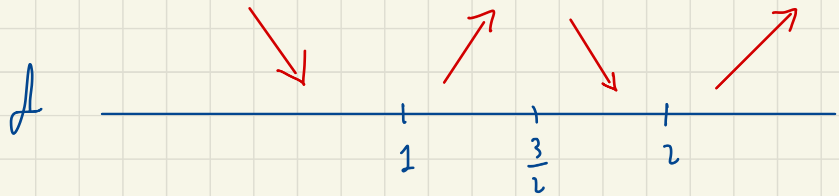
$$x^2 - 3x + 2$$

$$\Delta = 9 - 8 = 1 > 0$$

$$x_{1,2} = \frac{3 \pm 1}{2} = \begin{cases} 1 \\ 2 \end{cases}$$

x		1		2	
	$+$		$-$		$+$

x		1		$\frac{3}{2}$		2	
$2x - 3$	$-$		$-$		$+$		$+$
$\ln\left(x^2 - 3x + 2 + \frac{1}{e}\right) + 1$	$+$		$-$		$-$		$+$
f'	$-$		$+$		$-$		$+$
f	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$



$x = 1$ p. di minimo relativo

$$f(1) = \frac{1}{e} \ln\left(\frac{1}{e}\right) = -\frac{1}{e}$$

$x = \frac{3}{2}$ p. di max relativo

$$f\left(\frac{3}{2}\right) = \left(\frac{9}{4} - \frac{9}{2} + 2 + \frac{1}{e}\right) \ln\left(\frac{9}{4} - \frac{9}{2} + 2 + \frac{1}{e}\right)$$

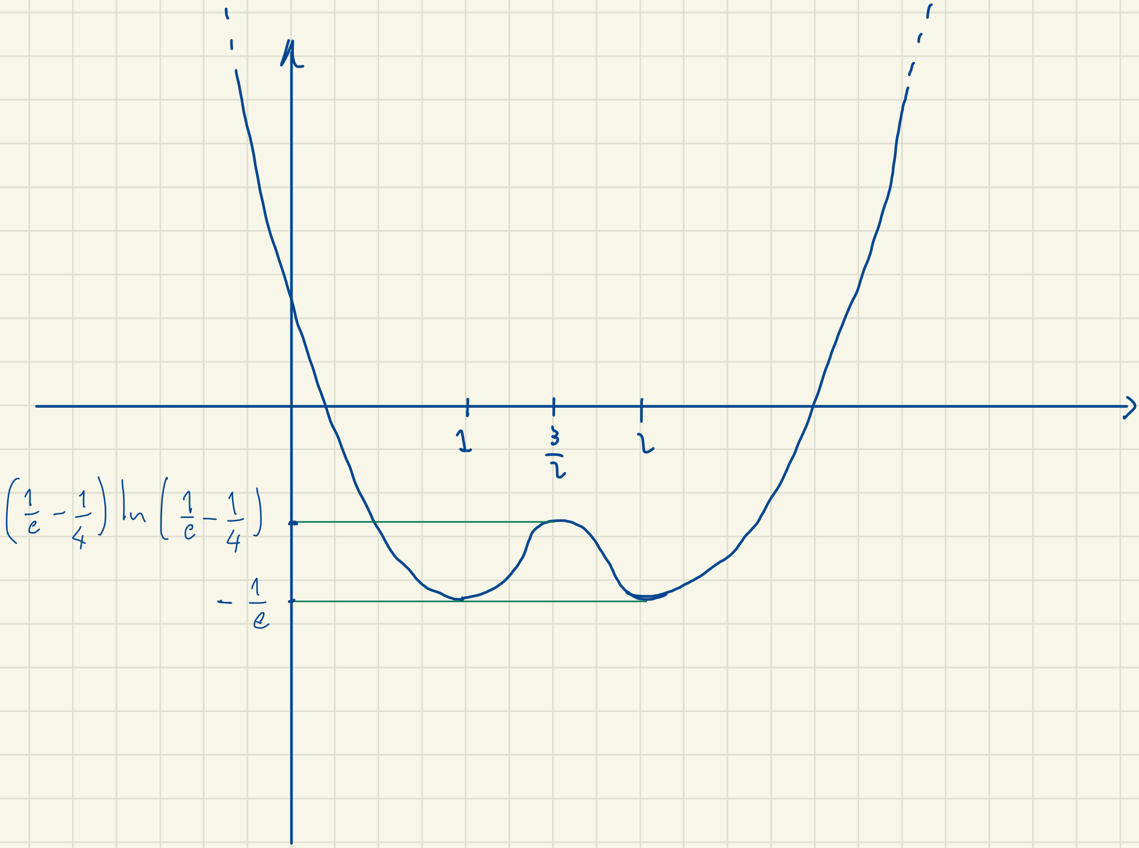
$$= \left(-\frac{1}{4} + \frac{1}{e}\right) \ln\left(-\frac{1}{4} + \frac{1}{e}\right)$$

$$= \underbrace{\left(\frac{1}{e} - \frac{1}{4}\right)}_{>0} \underbrace{\ln\left(\frac{1}{e} - \frac{1}{4}\right)}_{<0}$$

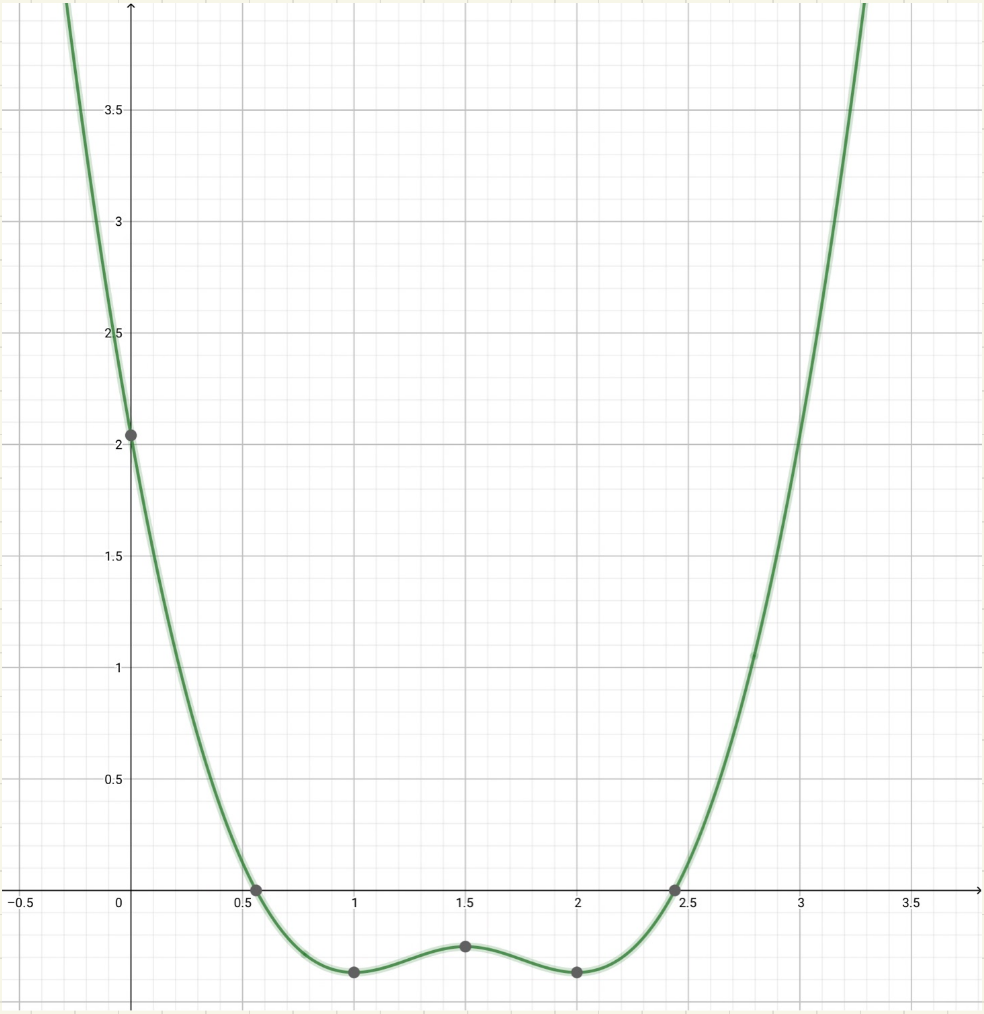
$$0 < \frac{1}{e} - \frac{1}{4} < \frac{1}{3}$$

$x = 2$ p. di min relativo

$$f(2) = \frac{1}{e} \ln \frac{1}{e} = -\frac{1}{e}$$



$$\operatorname{Im} f = \left[-\frac{1}{e} , +\infty \right[$$



$f(x) = \lambda$ ha 2 radici distinte

se $\lambda = -\frac{1}{e}$ o $\lambda > \left(\frac{1}{e} - \frac{1}{4}\right) \ln\left(\frac{1}{e} - \frac{1}{4}\right)$