efficiency of the engine, as the product of thermal and propulsive efficiencies, is tied to both the fuel economy as well as aircraft range. A unifying figure of merit for airbreathing engines and rockets is “specific impulse” with units of seconds.

Airbreathing engines incur *ram drag,* which is the product of air mass flow rate and the flight speed. The exhaust nozzle produces *gross thrust,* which is the sum of the momentum thrust and a pressure force contribution that occurs for a nozzle with imperfect expansion. The gross thrust is maximized for a *perfectly expanded nozzle* where $p_9 = p_0$. The sum of the gross thrust and the ram drag is the engine uninstalled thrust. The installation effects are primarily due to inlet and nacelle aft-end drag contributions.

Propulsive efficiency improves when the exhaust and flight speeds are closer to each other in magnitude, as in a turbofan engine. The parameter that brings the exhaust and flight speeds closer to each other in a turbofan engine is the *bypass ratio.* The trend in subsonic engine development/manufacturing is in developing ultra-high bypass (UHB) engines, with bypass ratio in 12–15 range. Pratt & Whitney has developed a geared UHB Turbofan engine, PW1000G, that represents the future of commercial aviation (shown in Section 3.6). Current conventional turbofan engines offer a bypass ratio of ~8. Since the turboprop affects a much larger airflow at an incrementally smaller speed increase, it offers the highest propulsive efficiency for a low-speed (i.e., subsonic) aircraft. Advanced turboprops may utilize counterrotating propellers and may include a *slimline nacelle*, as in a ducted fan configuration.

Thermal efficiency is a cycle-dependent parameter. The highest thermal efficiency of a heat engine operating between two temperature limits corresponds to the Carnot cycle. Consequently, a higher compression (Brayton) cycle, still bound by two temperature limits, offers a higher thermal efficiency. The trend in improving engine thermal efficiency is in developing high-pressure ratio compressors. The current maximum (compressor pressure ratio in aircraft gas turbine engines) is ~ 45–50.

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3. Lotter, K., “Aerodynamische Probleme der Integration von Triebwerk und Zelle beim Kampfflugzeugen,” *Proceedings of the 85th Wehrtechnischen Symposium,* Mannheim, Germany, 1977.
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Problems

3.1 The total pressures and temperatures of the gas in an afterburning turbojet engine are shown (J57 “B” from Pratt & Whitney, 1988). The mass flow rates for the air and fuel are also indicated at two engine settings, the Maximum Power and the Military Power. Use the numbers specified in this engine to calculate

(a) the fuel-to-air ratio f in the primary burner and the afterburner, at both power settings

(b) the low- and high-pressure spool compressor pressure ratios and the turbine pressure ratio (note that these remain constant with the two power settings)

(c) the exhaust velocity V_9 for both power settings by assuming the specified thrust is based on the nozzle gross thrust (because of sea level static) and *neglecting any pressure thrust* at the nozzle exit

(d) the thermal efficiency of this engine for both power settings (at the sea level static operation),

■ **FIGURE P3.1**
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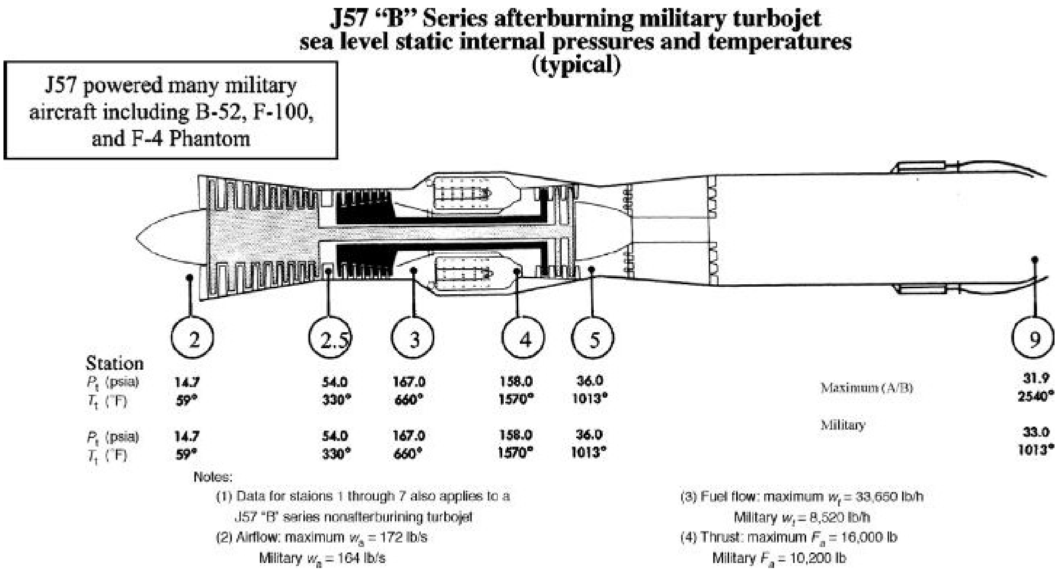

assuming the fuel heating value is 18,600 BTU/lbm and c_p = 0.24 BTU/lbm · °R. Explain the lower thermal efficiency of the Maximum power setting

(e) the thrust specific fuel consumption in lbm/h/lbf in both power settings

(f) the Carnot efficiency of a corresponding engine, i.e., operating at the same temperature limits, in both settings

(g) the comparison of percent thrust increase to percent fuel flow rate increase when we turn the afterburner on

(h) why don't we get proportional thrust increase with fuel flow increase (when it is introduced in the afterburner), i.e., doubling the fuel flow in the engine (through afterburner use) does not double the thrust

3.2 The total pressures and temperatures of the gas are specified for a turbofan engine with separate exhaust streams (JT3D-3B from Pratt & Whitney, 1974). The mass flow rates in the engine core (or primary) and the engine fan are also specified for the sea level static operation. Calculate

(a) the engine bypass ratio α defined as the ratio of fan-to-core flow rate

(b) from the total temperature rise across the burner, estimate the fuel-to-air ratio and the fuel flow rate in lbm/h, assuming the fuel heating value is Q_R ~ 18,600 BTU/ lbm and the specific heat at constant pressure is 0.24 and 0.26 BTU/lbm · °R at the entrance and exit of the burner, respectively

(c) the engine static thrust based on the exhaust velocities and the mass flow rates *assuming perfectly expanded nozzles* and compare your answer to the specified thrust of 18,000 lbs

(d) the engine thermal efficiency η_{th}

(e) the thermal efficiency of this engine compared to the afterburning turbojet of Problem 1. Explain the major contributors to the differences in η_{th} in these two engines

(f) the engine thrust specific fuel consumption in lbm/h/lbf

(g) the nondimensional engine specific thrust

(h) the Carnot efficiency corresponding to this engine

(i) the engine overall pressure ratio p_{t3} / p_{t2}

(j) fan nozzle exit Mach number [use $T_t = T + V^2/2c_p$ to calculate local static temperature at the nozzle exit, then local speed of sound $a = (\gamma RT)^{1/2}$]

3.3 A mixed exhaust turbofan engine (JT8D from Pratt and Whitney, 1974) is described by its internal pressures and temperature, as well as air mass flow rates and the mixed jet (exhaust) velocity. Let us examine a few parameters for this engine, for a ballpark approximation.

(a) Estimate the fuel flow rate from the total temperature rise across the burner assuming the fuel heating value is ~18,600 BTU/lbm and the specific heat at constant pressure is 0.24 and 0.26 BTU/lbm · °R at the entrance and exit of the burner, respectively

(b) Calculate the momentum thrust at the exhaust nozzle and compare it to the specified thrust of 14,000 lbs

(c) Estimate the thermal efficiency of this engine and compare it to Problems 3.1 and 3.2 as well as a Carnot cycle operating between the temperature extremes of this engine. Explain the differences

(d) Estimate the specific fuel consumption for this engine in lbm/h/lbf

(e) The overall pressure ratio (of the fan–compressor section) p_{t3}/ p_{t2}

■ **FIGURE P3.2**
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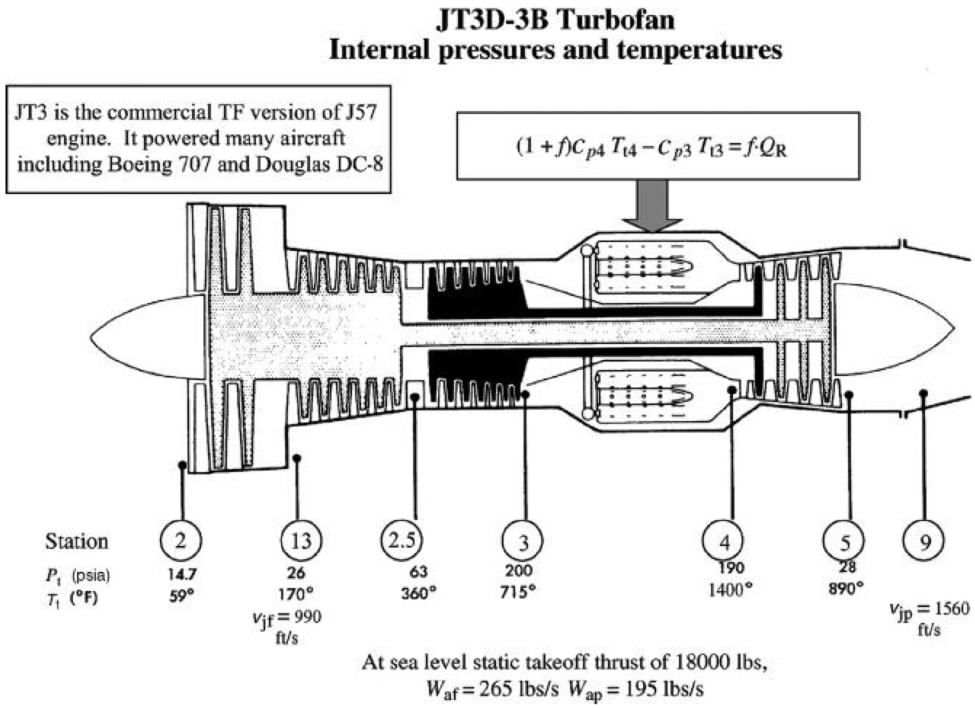

■ **FIGURE P3.3**
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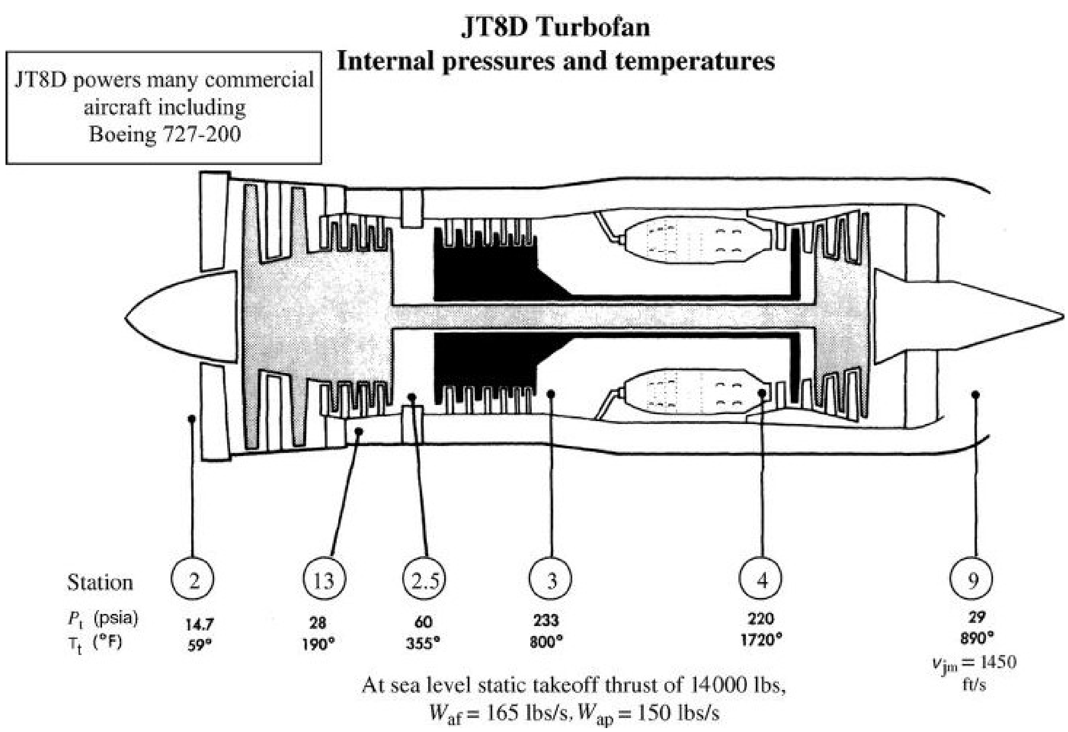


(f) What is the bypass ratio α for this engine at takeoff

(g) What is the Carnot efficiency corresponding to this engine

(h) Estimate nozzle exit Mach number [look at part (j) in Problem 3.2]

(i) What is the low-pressure compressor (LPC) pressure ratio $p_{t2.5}/\,p_{t2}$

(j) What is the high-pressure compressor (HPC) pressure ratio $p_{t3}\,/\,p_{t2.5}$

3.4 A large bypass ratio turbofan engine (JT9D engine from Pratt and Whitney, 1974) is described by its fan and core engine gas flow properties.

(a) What is the overall pressure ratio (OPR) of this engine

(b) Estimate the fan gross thrust $F_{g,fan}$ in lbf

(c) Estimate the fuel-to-air ratio based on the energy balance across the burner, assuming the fuel heating value is ~18,600 BTU/lbm and the specific heat at constant pressure is 0.24 and 0.26 BTU/lbm · °R at the entrance and exit of the burner, respectively

(d) Calculate the core gross thrust and compare the sum of the fan and the core thrusts to the specified engine thrust of 43,500 lbf

(e) Calculate the engine thermal efficiency and compare it to Problems 3.1–3.3. Explain the differences

(f) Estimate the thrust-specific fuel consumption (TSFC), in lbm/h/lbf

(g) What is the bypass ratio of this turbofan engine

(h) What is the Carnot efficiency η_{Carnot} corresponding to this engine

(i) What is the LPC pressure ratio $p_{t2.5}\,/\,p_{t2}$

(j) What is the HPC pressure ratio $p_{t3}\,/\,p_{t2.5}$

(k) Estimate the fan nozzle exit Mach number [see part (j) in Problem 3.2]

(l) Estimate the primary nozzle exit Mach number

3.5 An airbreathing engine flies at Mach $M_0 = 2.0$ at an altitude where the ambient temperature is $T_0 = -50$°C and ambient pressure is $p_0 = 10$ kPa. The airflow rate to the engine is 25 kg/s. The fuel flow rate is 3% of airflow rate and has a heating value of 42,800 kJ/kg. Assuming the exhaust speed is $V_9 = 1050$ m/s, and the nozzle is perfectly expanded, i.e., $p_9 = p_0$, calculate

(a) ram drag in kN

(b) gross thrust in kN

(c) net (uninstalled) thrust in kN

(d) thrust-specific fuel consumption in kg/h/N

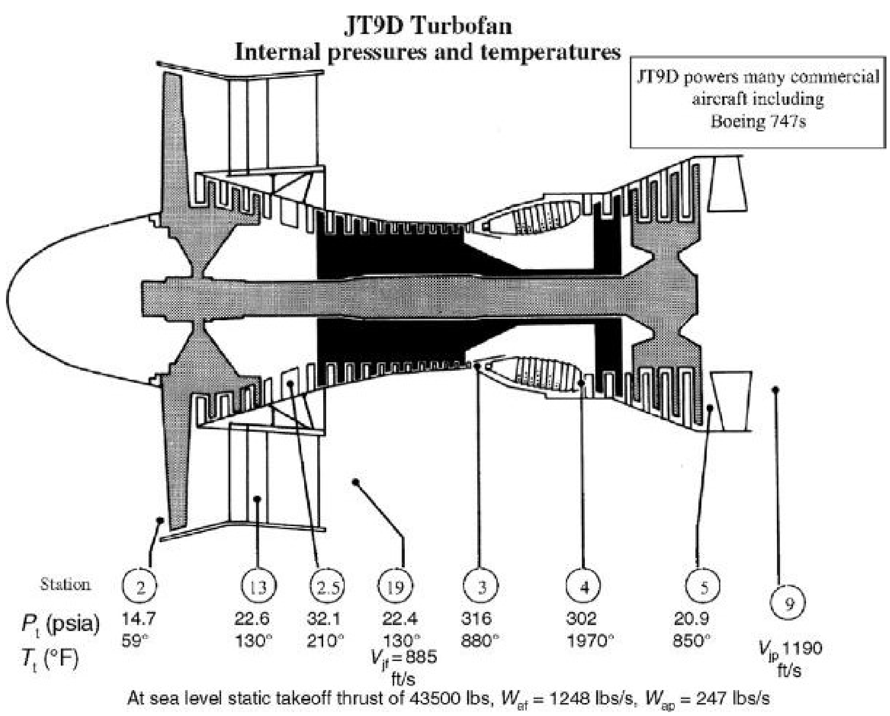
(e) engine thermal efficiency η_{th}

(f) propulsive efficiency η_p

(g) engine overall efficiency η_o

3.6 A turbo-propeller-driven aircraft is flying at $V_0 = 150$ m/s and has a propeller efficiency of $\eta_{pr} = 0.75$. The propeller thrust is $F_{prop} = 5000$ N and the airflow rate through the engine is 5 kg/s. The nozzle is perfectly expanded and produces 1000 N of gross thrust. Calculate

(a) the shaft power delivered to the propeller in kW



■ **FIGURE P3.4**
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(b) the nozzle exit velocity in m/s (neglect fuel flow rate in comparison to the air flow rate)

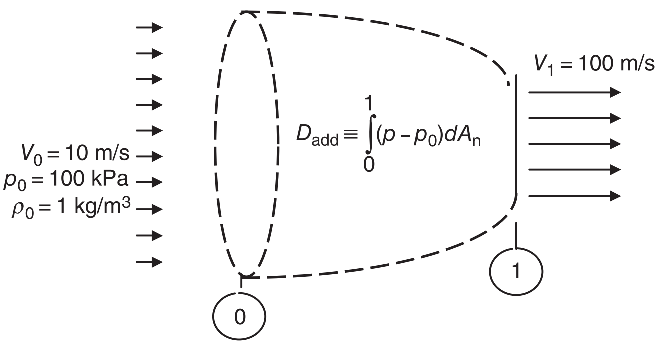
(c) in using Equation 3.56, $\eta_p \equiv \frac{F \cdot V_0}{\wp_s + \Delta \dot{KE}}$, first show that the contribution of the net kinetic power produced by the engine $\Delta K\dot{E}$ is small compared to the shaft power $\wp_s$ in denominator of Equation 3.56. Second, estimate the propulsive efficiency η_p for the turboprop engine from this equation.

3.7 Let us consider the control volume shown to represent the capture streamtube for an airbreathing engine at takeoff. The air speed is 10 m/s in area A_0 and 100 m/s in A_1.

(a) Use incompressible flow assumption to estimate capture area ratio A_0/A_1

(b) Use the Bernoulli equation to estimate p_1 / p_0

(c) Use momentum balance to estimate nondimensional additive drag $D_{add}/A_1\, p_0$



■ **FIGURE P3.7**

3.8 A rocket motor burns propellant at a rate of 50 kg/s. The exhaust speed is 3500 m/s and the nozzle is perfectly expanded. Calculate

(a) the rocket thrust in kN

(b) the rocket motor specific impulse I_s (s).

3.9 For the turbofan engine shown, calculate

(a) ram drag D_{ram} in kN

(b) primary nozzle gross thrust F_{g9} in kN

(c) fan nozzle gross thrust F_{g19} in kN

(d) the engine net thrust F_n in kN

(e) the propulsive efficiency η_p (–)

Hint: To calculate the pressure thrust for the primary and fan nozzles, you may calculate the flow areas at A_9 and A_{19} using the mass flow rate information as well as the density that you may calculate from pressure and temperature (via the speed of sound) using perfect gas law.

3.10 A ramjet is flying at Mach 2.0 at an altitude where $T_0 = -50$°C and the engine airflow rate is 10 kg/s. If the exhaust Mach number of the ramjet is equal to the flight Mach number, i.e., $M_9 = M_0$, with perfectly expanded nozzle and T_{t9} 2500 K, calculate

(a) the engine ram drag D_{ram} in kN

(b) the nozzle gross thrust F_g in kN

(c) the engine net thrust F_n in kN

(d) the engine propulsive efficiency η_p

Assume gas properties remain the same throughout the engine, i.e., assume $\gamma = 1.4$ and $c_p = 1004$ J/kg · K. Also, assume that the fuel flow rate is 4% of airflow rate.

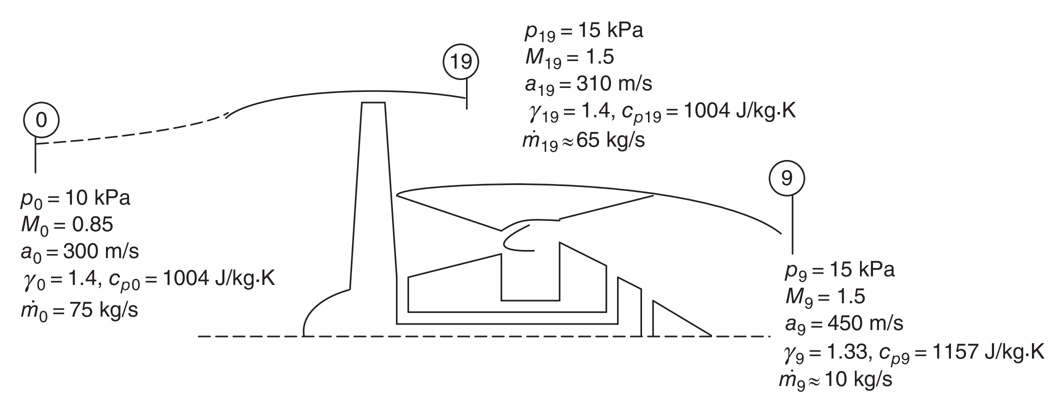
3.11 A turbojet-powered aircraft cruises at $V_0 = 300$ m/s while the engine produces an exhaust speed of 600 m/s. The air mass flow rate is 100 kg/s and the fuel mass flow rate is 2.5 kg/s. The fuel heating value is $Q_R = 42{,}000$ kJ/kg. Assuming that the nozzle is perfectly expanded, calculate

(a) engine ram drag in kN

(b) engine gross thrust in kN

(c) engine net thrust in kN

(d) engine thrust-specific fuel consumption (TSFC) in mg/s/kN



■ **FIGURE P3.9**

(e) engine thermal efficiency
(f) engine propulsive efficiency
(g) aircraft range R for L/D of 10 and the W_i / W_f of 1.25
(h) if this aircraft make it across the Atlantic Ocean?

3.12 We wish to investigate the range of a slender supersonic aircraft where its lift-to-drag ratio as a function of flight Mach number is described by

$$\frac{L}{D} \approx 3\frac{M_0 + 3}{M_0}$$

Using Equation 3.67, i.e.,

$$R = \left(M_0\frac{L}{D}\right)\frac{a_0/g_0}{\text{TSFC}}\ell n\frac{W_i}{W_f}$$

for range equation, vary the thrust specific fuel consumption TSFC between 1.0 and 2.0 lbm/h/lbf to graph R for flight Mach number ranging between 2.0 and 4.0. You may assume a_0 is 1000 ft/s and the aircraft initial-to-final weight ratio is $W_i/W_f = 2.0$.

3.13 A turboshaft engine consumes fuel with a heating value of 42,000 kJ/kg at the rate of 1 kg/s. Assuming the thermal efficiency is 0.333, calculate the shaft power that this engine produces.

3.14 A rocket engine consumes propellants at the rate of 1000 kg/s and achieves a specific impulse of $I_s = 400$ s. Assuming the nozzle is perfectly expanded, calculate

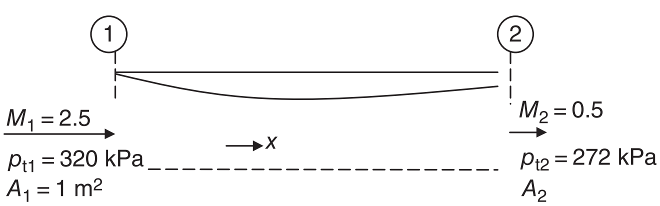
(a) the rocket exhaust speed V_9 in m/s
(b) the rocket thrust in MN

3.15 A rocket engine has a nozzle exit diameter of $D_9 = 2$ m. It is perfectly expanded at sea level. Calculate the rocket pressure thrust in vacuum.

3.16 A ramjet engine is in supersonic flight. Its inlet flow parameters are shown.

Assuming the flow is adiabatic and $\gamma = 1.4$, calculate

(a) the diffuser exit area A_2 in m^2
(b) impulse (in kN) in stations 1 and 2, I_1 and I_2
(c) internal force exerted (by the fluid) on the inlet in flight (or $-x$) direction

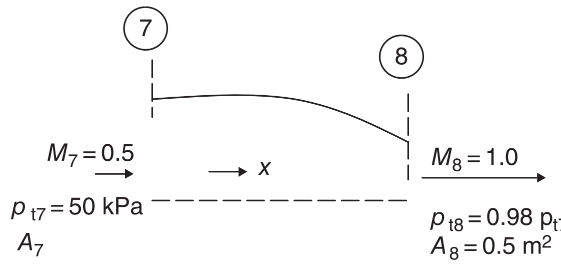


■ **FIGURE P3.16**

3.17 A convergent nozzle is perfectly expanded with exit Mach number $M_8 = 1$ 0. The exit total pressure is 98% of the inlet total pressure. The nozzle inlet Mach number is $M_7 = 0.5$ and the nozzle area at the exit is $A_8 = 0.5$ m^2.

Assuming the gas ratio of specific heats is $\gamma = 1.33$, and the flow is adiabatic, calculate

(a) nozzle inlet area A_7 in m^2
(b) nozzle inlet impulse I_7 in kN
(c) nozzle exit impulse I_8 in kN
(d) the axial force (i.e., in the x-direction) exerted by the fluid on the nozzle



■ **FIGURE P3.17**

3.18 A turboprop engine flies at $V_0 = 200$ m/s and produces a propeller thrust of $F_{prop} = 40$ kN and a core thrust of $F_{core} = 10$ kN. Engine propulsive efficiency η_p is 85%. Calculate

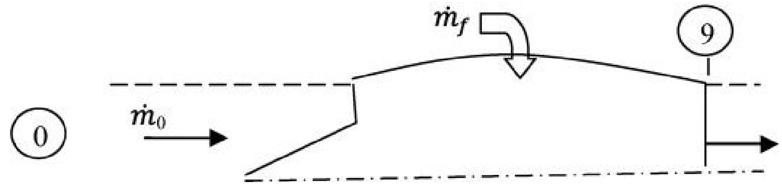
(a) total thrust produced by the turboprop in kN
(b) thrust power in MW
(c) shaft power produced by the engine $\wp_s$ in MW

3.19 A turbojet engine produces a net thrust of 40,000 N at the flight speed of V_0 of 300 m/s. For a propulsive efficiency of $\eta_p = 0.40$, estimate the turbojet exhaust speed V_9 in m/s.

3.20 Calculate the engine specific impulse in seconds for Problem 3.19. Also, assuming the fuel heating value is 42,000 kJ/kg and the thermal efficiency is 45%, estimate the fuel-to-air ratio consumed in the burner.

3.21 A turbojet engine is shown in cruise condition. The flight condition is known to be: $M_0 = 2.0$, $p_0 = 20$ kPa, $T_0 = -35°C$ with $\gamma = 1.4$ and $R = 287$ J/kg.K. The air mass flow rate into the engine is known to be $\dot{m}_0 = 110$ kg/s. The fuel flow rate to the combustor is $\dot{m}_f = 4.2$ kg/s with heating value $Q_R = 42{,}800$ kJ/kg. If the nozzle is perfectly expanded and the exhaust velocity is $V_9 = 1{,}200$ m/s, calculate:

(a) Ram drag, D_r, in kN and lbf
(b) Gross thrust, F_g, in kN and lbf
(c) Net un-installed thrust in kN
(d) TSFC in mg/s/N and lbm/hr/lbf
(e) Engine thermal efficiency, η_{th}
(f) Propulsive efficiency, η_p



■ FIGURE P3.21

3.22 An aircraft gas turbine engine is shown in flight. Calculate:

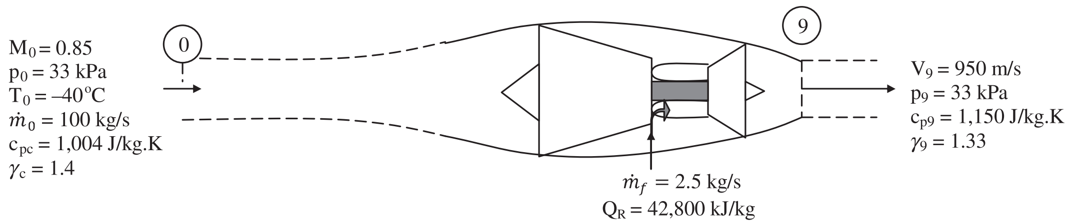
(a) ram drag in kN and lbf

(b) gross thrust in kN and lbf

(c) specific fuel consumption in mg/s/N and lbm/hr/lbf

(d) thermal efficiency, η_{th}

(e) propulsive efficiency, η_p



■ FIGURE P3.22