

References

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Problems

4.1 An aircraft is flying at an altitude where the ambient static pressure is $p_0 = 25 \text{ kPa}$ and the flight Mach number is $M_0 = 2.5$. The total pressure at the engine face is measured to be $p_{12} = 341.7 \text{ kPa}$. Assuming the inlet flow is adiabatic and $\gamma = 1.4$, calculate

- (a) the inlet total pressure recovery π_d
- (b) the inlet adiabatic efficiency η_d
- (c) the nondimensional entropy rise caused by the inlet $\Delta s_d/R$

4.2 A multistage axial-flow compressor has a mass flow rate of 100 kg/s and a total pressure ratio of 25. The compressor polytropic efficiency is $e_c = 0.90$. The inlet flow condition to the compressor is described by $T_{12} = -35^\circ\text{C}$ and $p_{12} = 30 \text{ kPa}$. Assuming the flow in the compressor is adiabatic, and constant gas properties throughout the compressor are assumed, i.e., $\gamma = 1.4$ and $c_p = 1004 \text{ J/kg} \cdot \text{K}$, calculate

- (a) compressor exit total temperature T_{13} , in K
- (b) compressor adiabatic efficiency η_c
- (c) compressor shaft power φ_c in MW

4.3 A gas turbine combustor has inlet condition $T_{13} = 900 \text{ K}$, $p_{13} = 3.2 \text{ MPa}$, air mass flow rate of 100 kg/s, $\gamma_3 = 1.4$, $c_{p3} = 1004 \text{ J/kg} \cdot \text{K}$.

A hydrocarbon fuel with ideal heating value $Q_R = 42,800 \text{ kJ/Kg}$ is injected in the combustor at a rate of 2 kg/s. The burner efficiency is $\eta_b = 0.99$ and the total pressure at the combustor exit is 97% of the inlet total pressure, i.e., combustion causes a 3% loss in total pressure. The gas properties at the combustor exit are $\gamma_4 = 1.33$ and $c_{p4} = 1,156 \text{ J/kg} \cdot \text{K}$. Calculate

- (a) fuel-to-air ratio f
- (b) combustor exit temperature T_{14} in K and p_{14} in MPa

4.4 An uncooled gas turbine has its inlet condition the same as the exit condition of the combustor described in Problem 4.3. The turbine adiabatic efficiency is 85%. The turbine produces a shaft power to drive the compressor and other accessories at $\dot{\varphi}_t = 60 \text{ MW}$. Assuming that the gas properties in the turbine are the same as the burner exit in Problem 4.3, calculate

- (a) turbine exit total temperature T_{t5} in K
- (b) turbine polytropic efficiency e_t
- (c) turbine exit total pressure p_{t5} in kPa
- (d) turbine shaft power $\dot{\varphi}_t$ based on turbine expansion ΔT_t

4.5 In a turbine nozzle blade row, hot gas mass flow rate is 100 kg/s and $h_{tg} = 1900 \text{ kJ/kg}$. The nozzle blades are internally cooled with a coolant mass flow rate of 1.2 kg/s and $h_{tc} = 904 \text{ kJ/kg}$ as the coolant is ejected through nozzle blades trailing edge. The coolant mixes with the hot gas and causes a reduction in the mixed-out enthalpy of the gas. Calculate the mixed-out total enthalpy after the nozzle. Also for the $c_{p,\text{mixed-out}} = 1594 \text{ J/kg}\cdot\text{K}$, calculate the mixed out total temperature.

4.6 Consider the internally cooled turbine nozzle blade row of Problem 4.5. The hot gas total pressure at the entrance of the nozzle blade is $p_{t4} = 2.2 \text{ MPa}$, $c_{pg} = 1156 \text{ J/kg}\cdot\text{K}$, and $\gamma_g = 1.33$. The mixed-out total pressure at the exit of the

4.7 A convergent-divergent nozzle with a pressure ratio, NPR = 12. The gas properties are $\gamma = 1.33$ and $c_p = 1156 \text{ J/kg}\cdot\text{K}$ and remain constant in the nozzle. The nozzle adiabatic efficiency is $\eta_n = 0.94$. Calculate

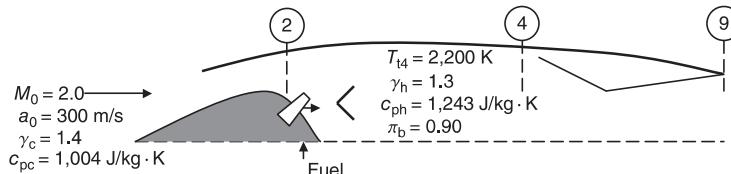
- (a) nozzle total pressure ratio π_n
- (b) nozzle area ratio A_9/A_8 for a perfectly expanded nozzle
- (c) nozzle exit Mach number M_9 (perfectly expanded)

4.8 Calculate the propulsive efficiency of a turbojet engine under the following two flight conditions that represent takeoff and cruise, namely

- (a) $V_0 = 100 \text{ m/s}$ and $V_9 = 2000 \text{ m/s}$
- (b) $V_0 = 750 \text{ m/s}$ and $V_9 = 2000 \text{ m/s}$

4.9 A ramjet is in supersonic flight, as shown. The inlet pressure recovery is $\pi_d = 0.90$. The combustor burns hydrogen with $Q_R = 117,400 \text{ kJ/kg}$ at a combustion efficiency of $\eta_b = 0.95$. The nozzle expands the gas perfectly, but suffers from a total pressure loss of $\pi_n = 0.92$. Calculate

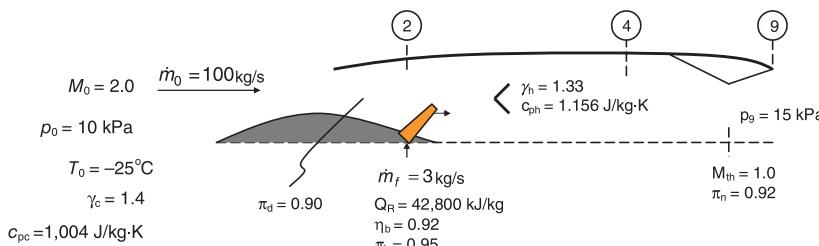
- (a) fuel-to-air ratio f
- (b) nozzle exit Mach number M_9
- (c) specific (net) thrust F_n/\dot{m}_0 (in N/kg/s)
- (d) η_{th} , engine thermal efficiency
- (e) η_p , engine propulsive efficiency



■ FIGURE P4.9

nozzle has suffered 5% loss due to both mixing and frictional losses in the blade row boundary layers. Calculate the entropy change $\Delta s/R$ across the turbine nozzle blade row.

4.10 A ramjet takes in 100 kg/s of air at a Mach 2 flight condition at an altitude where $p_0 = 10 \text{ kPa}$ and $T_0 = -25^\circ\text{C}$. The engine throttle setting allows 3 kg/s of fuel flow rate



■ FIGURE P4.10

in the combustor where a hydrocarbon fuel of 42,800 kJ/kg heating value is burned. The ramjet component efficiencies are all listed on the engine cross section. Note that the exhaust nozzle is not perfectly expanded. We intend to establish some performance parameters for this engine as well as some flow areas (i.e., physical sizes) of this engine.

Assuming the gas properties are split into a *cold* section and a hot section (perfect gas properties), namely, $\gamma_c = 1.4$ and $c_{pc} = 1,004 \text{ kJ/kg} \cdot \text{K}$ and $\gamma_h = 1.33$ and $c_{ph} = 1,156 \text{ kJ/kg} \cdot \text{K}$ (subscript "c" stands for "cold" and "h" for "hot"), calculate

- (a) ram drag in kN
- (b) the inlet capture area A_0 in m^2
- (c) p_{t4} in kPa
- (d) T_{t4} in K
- (e) p_{t9} in kPa
- (f) exit Mach number M_9
- (g) exhaust velocity V_9
- (h) nozzle exit area A_9 in m^2
- (i) gross thrust in kN
- (j) thermal efficiency $^*\eta_{th}$
- (k) propulsive efficiency $^*\eta_p$

Note:

For exhaust nozzles that are not perfectly expanded the thermal and propulsive efficiencies are defined as

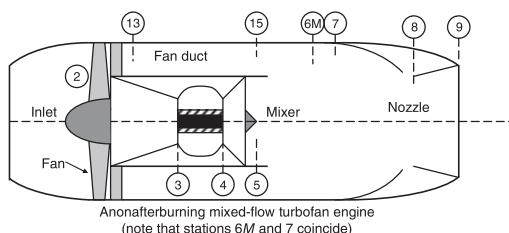
$$\eta_{th} \equiv \frac{\Delta K\dot{E}}{\dot{m}_1 Q_R \eta_b} = \frac{\dot{m}_9 V_{9\text{eff}}^2 - \dot{m}_0 V_0^2}{2\dot{m}_1 Q_R \eta_b}$$

$$\eta_p \equiv \frac{2F_n V_0}{\dot{m}_9 V_{9\text{eff}}^2 - \dot{m}_0 V_0^2}$$

where the effective exhaust velocity $V_{9\text{eff}}$ is defined as

$$V_{9\text{eff}} \equiv \frac{F_g}{\dot{m}_9}$$

4.11 A mixed-exhaust turbofan engine is described by the following design and limit parameters:



■ FIGURE P 4.11

Flight:

$$M_0 = 2.2, p_0 = 10 \text{ kPa},$$

$$T_0 = -50^\circ\text{C},$$

$$R = 287 \text{ J/kg} \cdot \text{K}, \gamma = 1.4$$

$$(1 + \alpha) \dot{m}_0 = 25 \text{ kg/s},$$

$$\pi_d = 0.85$$

$$\pi_c = 15, e_c = 0.90,$$

$$\pi_f = 1.5, e_f = 0.90$$

$$\pi_b = 0.95, \eta_b = 0.98,$$

$$Q_R = 42,800 \text{ kJ/kg},$$

$$T_{t4} = 1400^\circ\text{C}$$

$$\epsilon_t = 0.92, \eta_m = 0.95, M_5 = 0.5$$

$$\pi_{M,f} = 0.98$$

None

$$\pi_n = 0.90, p_9/p_0 = 1.0$$

Calculate

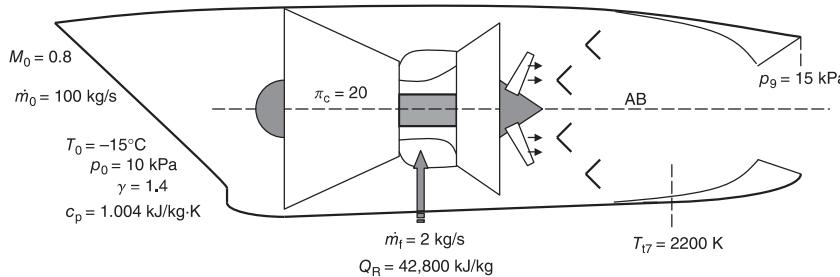
- (a) ram drag D_R in kN
- (b) p_{t12} in kPa, T_{t12} in K
- (c) p_{t13} in kPa, T_{t13} in K
- (d) p_{t113} in kPa, T_{t113} in K
- (e) p_{t14} in kPa, T_{t15} in K
- (f) fuel-to-air ratio f
- (g) bypass ratio α , \dot{m}_{core} in kg/s, and \dot{m}_{fan} in kg/s
- (h) T_{t6M} in K
- (i) p_{t9} in kPa, T_{t9} in K
- (j) M_9, V_9 in m/s
- (k) gross thrust F_g in kN
- (l) thrust-specific fuel consumption in mg/s/N

You may assume constant gas properties γ and R throughout the engine.

We may also assume that the flow in the fan duct, i.e., between stations 13 and 15, is frictionless and adiabatic.

4.12 In the afterburning turbojet engine shown, assume constant gas properties and ideal components to calculate

- (a) ram drag
- (b) compressor shaft power \wp_c
- (c) fuel-to-air ratio in the primary burner
- (d) τ_λ , the limit enthalpy parameter in the gas generator
- (e) turbine expansion parameter τ_t
- (f) turbine shaft power \wp_t
- (g) $\tau_{\lambda AB}$, the afterburner limit enthalpy parameter
- (h) fuel-to-air ratio in the afterburner
- (i) nozzle gross thrust
- (j) engine thrust specific fuel consumption
- (k) engine net uninstalled thrust
- (l) engine thermal efficiency
- (m) engine propulsive efficiency



■ FIGURE P4.12

- 4.13** An ideal separate-exhaust turbofan engine has the following design parameters:

$$\begin{aligned} T_0 &= -20^\circ\text{C}, p_0 = 15 \text{ kPa}, M_0 = 0.85 \\ \pi_c &= 30 \\ \pi_f &= 1.60, \alpha = 6 \\ \tau_\lambda &= 6.5, Q_R = 42,800 \text{ kJ/kg} \\ \text{Fan and core nozzles are convergent.} \end{aligned}$$

Assuming the gas is calorically perfect with $\gamma = 1.4$ and $c_p = 1.004 \text{ kJ/kg}\cdot\text{K}$, calculate

- (a) compressor exit pressure p_{13} in kPa
- (b) fan exit temperature T_{113} in K
- (c) fuel-to-air ratio f
- (d) turbine exit temperature T_{15} in K
- (e) fan nozzle exit Mach number M_{19}
- (f) core nozzle exit Mach number M_9
- (g) core nozzle exit velocity V_9 in m/s
- (h) The ratio of fan-to-core thrust $F_{\text{fan}}/F_{\text{core}}$

- 4.14** In a mixed-exhaust turbofan engine, we have calculated the parameters shown on the engine diagram.

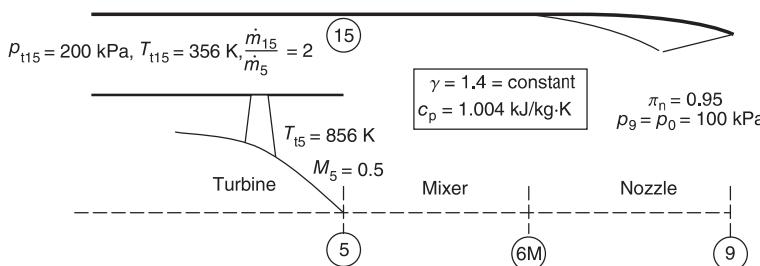
Assuming constant gas properties between the two streams and constant total pressure between the hot and cold gas streams, calculate

- (a) the mixer exit total temperature T_{16M}
- (b) M_9
- (c) V_9
- (d) $\frac{F_g}{m_5}$

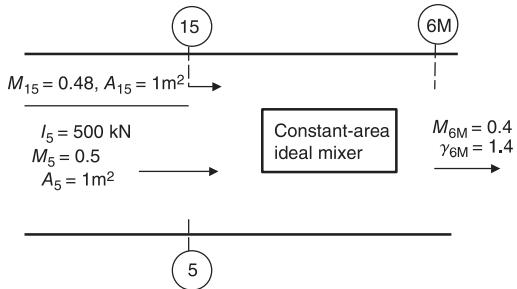
- 4.15** In an afterburning gas turbine engine, the exhaust nozzle is equipped with a variable area throat. Calculate percentage increase in nozzle throat area needed to accommodate the engine flow in the afterburning mode. With the afterburner on, the nozzle mass flow rate increases by 3%, the nozzle total temperature doubles, i.e. $(T_{18})_{\text{AB-ON}}/(T_{18})_{\text{AB-OFF}} = 2$, and the total pressure at the nozzle entrance is reduced by 20%. You may assume the gas properties γ and R remain constant and the nozzle throat remains choked.

- 4.16** For the constant-area ideal mixer shown, assuming *constant gas properties*, calculate

- (a) p_5 (kPa)
- (b) p_{15} (kPa)
- (c) p_{6M} (kPa)



■ FIGURE P4.14

**■ FIGURE P4.16**

4.17 A turbojet engine has the following parameters at the on-design operating point:

$M_0 = 0, p_0 = p_{\text{STD}}, T_0 = T_{\text{STD}}, \gamma = 1.4, c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$
 $\pi_d = 0.99$
 $\pi_c = 30, e_c = 0.9$
 $\tau_x = 6.0, \eta_b = 0.98, \pi_b = 0.95$, and $Q_R = 42,800 \text{ kJ/kg}$
 $e_t = 0.90, \eta_m = 0.95$
 $\pi_n = 0.98$ and $p_9/p_0 = 1.0$

Calculate

- (a) P_{13} and τ_c
- (b) fuel-to-air ratio f and p_{t4}
- (c) τ_t

Now for the following off-design condition:

$M_0 = 0.85$ at 20 km U.S. standard altitude and throttle ratio, $T_{t4} = 0.9 T_{t4\text{-design}}$, assuming that τ_t remains constant between on-design and off-design, calculate

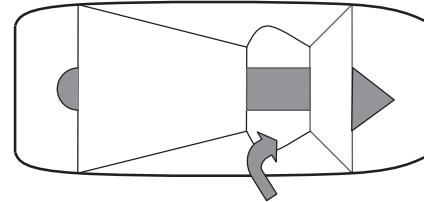
- (d) compressor pressure ratio at the off-design operation

4.18 A turbojet engine has the following design point parameters:

$M_0 = 0, p_0 = 100 \text{ kPa}, T_0 = 15^\circ\text{C}$
 $\pi_d = 0.98$
 $\pi_c = 25, e_c = 0.90$
 $Q_R = 42,800 \text{ kJ/kg}, \pi_b = 0.95, \eta_b = 0.98, \tau_\lambda = 6.0$
 $e_t = 0.85, \eta_m = 0.98$
 $\pi_n = 0.97, p_9 = p_0$

Calculate

- (a) fuel-to-air ratio f
- (b) turbine total temperature ratio τ_t

**■ FIGURE P4.18**

For the following off-design operation:

$$M_0 = 0.85, p_0 = 10 \text{ kPa}, T_0 = -15^\circ\text{C}$$

$$\tau_\lambda = 6.5$$

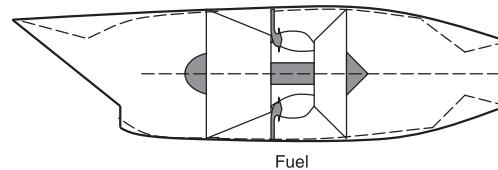
Assume $\gamma = 1.4, c_p = 1004 \text{ kJ/kg} \cdot \text{K}$ and calculate

- (c) $\pi_{c\text{-Off-Design}}$ if $\tau_{t\text{-Design}} = \tau_{t\text{-Off-Design}}$

4.19 An ideal turbojet engine has the following design and limit parameters, namely,

$M_0 = 2.0$, altitude 37 kft
compressor pressure ratio π_c
maximum enthalpy ratio $\tau_\lambda = 7.0$
fuel type is hydrocarbon with $Q_R = 42,800 \text{ kJ/kg}$
assume constant gas properties $c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$ and
 $\gamma = 1.4$.

For a range of compressor pressure ratios, namely, $1 \leq \pi_c \leq 40$, calculate and graph (using MATLAB or a spreadsheet)

**■ FIGURE P4.19**

- (a) engine (nondimensional) specific thrust $F_n/(m \cdot a_0)$
- (b) thrust specific fuel consumption in mg/s/N
- (c) in order to assess the effect of gas property variations with temperature on the engine performance parameters, repeat parts (a) and (b) for the following gas properties:

Engine cold section: $\gamma_c = 1.40, c_{pc} = 1.004 \text{ kJ/kg} \cdot \text{K}$
Engine hot section: $\gamma_t = 1.33, c_{pt} = 1.156 \text{ kJ/kg} \cdot \text{K}$

- (d) To assess the effect of inlet total pressure recovery on the engine performance, calculate and graph the engine specific thrust and fuel consumption for a single compressor pressure ratio of $\pi_c = 20$, but vary π_d from 0.50 to 1.0. Use gas properties of part (c)

- (e) Now, for the following component efficiencies

$$\begin{aligned}\pi_d &= 0.90, e_c = 0.90, \pi_b = 0.98, \eta_b = 0.98, e_t = 0.91, \\ \eta_m &= 0.99, \pi_n = 0.95, \pi_c = 20, \tau_\lambda = 7, \text{ and } p_9 = p_0,\end{aligned}$$

calculate the engine performance parameters

1. specific thrust (nondimensional)
2. specific fuel consumption
3. thermal efficiency
4. propulsive efficiency

4.20 For an ideal ramjet, derive an expression for the flight Mach number in terms of the cycle limit enthalpy, τ_λ that will lead to an engine thrust of zero.

4.21 Derive an expression for an optimum Mach number that maximizes the engine-specific thrust in an ideal ramjet.

4.22 For an ideal ramjet with a perfectly expanded nozzle, show that the nozzle exit Mach number M_9 is equal to the flight Mach number M_0 .

4.23 Assuming the component efficiencies in a real ramjet are

$$\begin{aligned}\pi_d &= 0.90, \pi_n = 0.95, \eta_b = 0.98, \pi_c = 0.90 \text{ and} \\ p_9/p_0 &= 1.0 \text{ flying at 37 kft altitude.}\end{aligned}$$

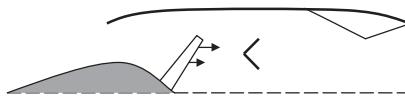
For the maximum enthalpy ratio $\tau_\lambda = 8.0$, the fuel heating value of 42,000 kJ/kg and a cold and hot section gas properties

$$\begin{aligned}\text{Engine cold section: } \gamma_c &= 1.40, c_{pc} = 1.004 \text{ kJ/kg} \cdot \text{K} \\ \text{Engine hot section: } \gamma_t &= 1.33, c_{pt} = 1.156 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

calculate the optimum flight Mach number corresponding to the maximum specific thrust.

4.24 A ramjet uses a hydrocarbon fuel with $Q_R = 42,800$ kJ/kg flying at Mach 2 (i.e., $M_0 = 2$) in an atmosphere where $a_0 = 300$ m/s. Its exhaust is perfectly expanded and the exhaust velocity is $V_9 = 1200$ m/s. Assuming the inlet total pressure recovery is $\pi_d = 0.90$, the burner losses are $\pi_b = 0.98$ and $\eta_b = 0.96$, the nozzle total pressure ratio is $\pi_n = 0.98$ and $\lambda = 1.4$ and $c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$ are constant throughout the engine, calculate

- (a) τ_λ
- (b) fuel-to-air ratio f
- (c) nondimensional-specific thrust $F_n/(m_0 u_0)$
- (d) propulsive efficiency
- (e) thermal efficiency



■ FIGURE P4.24

4.25 A large bypass ratio turbofan engine has the following design and limit parameters:

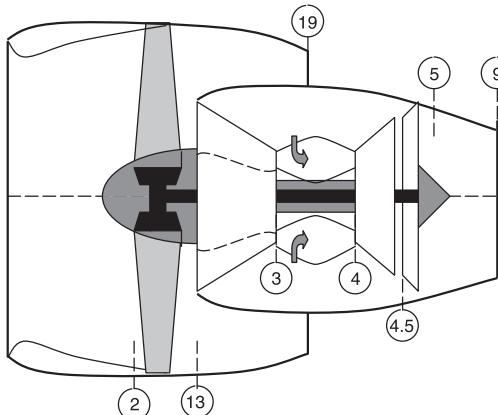
$$\begin{aligned}M_0 &= 0.8, \text{ altitude} = 37 \text{ kft U.S. standard atmosphere} \\ \pi_d &= 0.995 \\ \pi_c &= 40, e_c = 0.90 \\ \alpha &= 6, \pi_f = 1.6, e_f = 0.90, \pi_{fn} = 0.98, \text{ fan nozzle is convergent} \\ \tau_\lambda &= 7.0, Q_R = 42,800 \text{ kJ/kg}, \pi_b = 0.95, \eta_b = 0.98 \\ e_t &= 0.90, \eta_m = 0.975 \\ \pi_n &= 0.98, \text{ core nozzle is convergent}\end{aligned}$$

Assuming the gas properties may be described by two sets of parameters, namely, cold and hot stream values, i.e.,

$$\begin{aligned}\text{Engine cold section: } \gamma_c &= 1.40, c_{pc} = 1.004 \text{ kJ/kg} \cdot \text{K} \\ \text{Engine hot section: } \gamma_t &= 1.33, c_{pt} = 1.156 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Calculate

- (a) the ratio of compressor to fan shaft power φ_c/φ_f
- (b) fuel-to-air ratio f
- (c) the ratio of fan nozzle exit velocity to core nozzle exit velocity V_{19}/V_9
- (d) the ratio of two gross thrusts, $F_{g,fan}/F_{g,core}$
- (e) the engine thermal efficiency and compare it to an ideal Carnot cycle operating between the same temperature limits

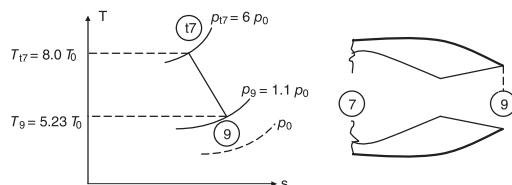


■ FIGURE P4.25

- (f) the engine propulsive efficiency and compare it to the turbojet propulsive efficiency of Problem 4.19
 (g) engine thrust specific fuel consumption
 (h) engine (fuel)-specific impulse I_s in seconds

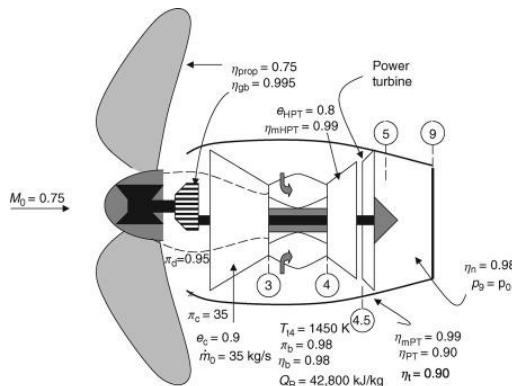
4.26 The flow expansion in an exhaust nozzle is shown on a $T-s$ diagram. Assuming $\gamma = 1.4$, $c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$, calculate

- (a) nozzle adiabatic efficiency η_n
 (b) V_9/a_0
 (c) nondimensional pressure thrust, i.e., $\frac{(p_9 - p_0)A_9}{\dot{m}_9 \cdot a_0}$



■ FIGURE P4.26

4.27 An advanced turboprop engine is flying at $M_0 = 0.75$ at 37 kft standard altitude. The component parameters are designated on the following engine drawing.



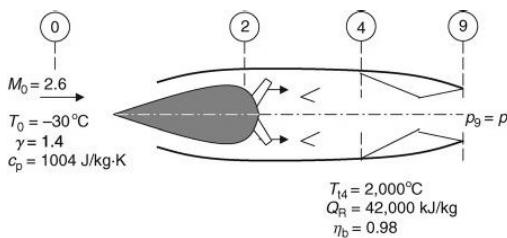
■ FIGURE P4.27

Assuming an optimum power split α_{opt} that leads to a maximum engine thrust, calculate

- (a) core thrust
 (b) propeller thrust
 (c) thrust-specific fuel consumption
 (d) engine overall efficiency

4.28 Consider a ramjet, as shown. The diffuser total pressure ratio is $\pi_d = 0.80$, the burner total pressure ratio is $\pi_b = 0.96$, and the nozzle total pressure ratio is $\pi_n = 0.90$. Calculate

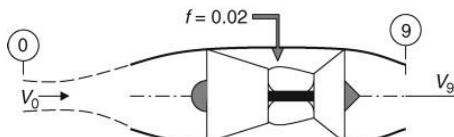
- (a) fuel-to-air ratio f
 (b) exit Mach number M_9
 (c) nondimensional-specific thrust, i.e., $F_n/\dot{m}_0 a_0$



■ FIGURE P4.28

4.29 A turbojet engine flies at $V_0 = 250 \text{ m/s}$ with an exhaust velocity of $V_9 = 750 \text{ m/s}$. The fuel-to-air ratio is 2% and the actual fuel heating value is $Q_{R,\text{actual}} = 40,800 \text{ kJ/kg}$. Estimate the engine propulsive and thermal efficiencies, η_p and η_{th} .

Assume the nozzle is perfectly expanded.



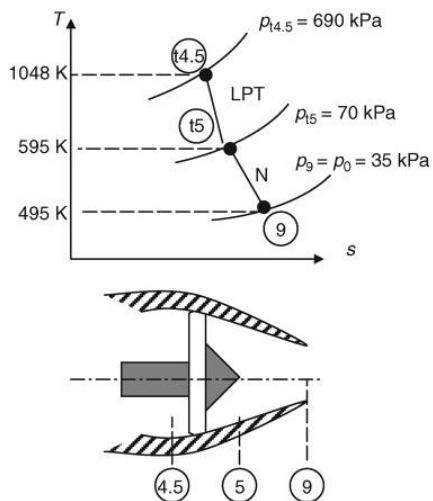
■ FIGURE P4.29

4.30 Consider a turboprop engine with the following parameters (note that the nozzle is convergent):

$$\begin{aligned} \dot{m}_9 &= 20 \text{ kg/s} \\ V_0 &= 220 \text{ m/s} \\ \eta_{\text{prop}} &= 0.80 \\ \eta_{\text{gb}} &= 0.995 \\ \eta_{\text{ml,LP}} &= 0.99 \end{aligned}$$

Calculate

- (a) propeller thrust F_{prop} in kN
 (b) nozzle gross thrust $F_{\text{g,core}}$ in kN
 (c) M_9

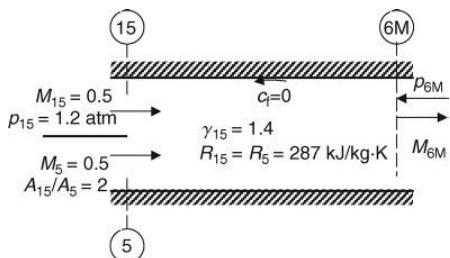


■ FIGURE P4.30

- (d) core net thrust $F_{n,\text{core}}$ in kN, assume $f = 0.02$
- (e) nozzle adiabatic efficiency η_n
- (f) power turbine adiabatic efficiency η_{PT}

Assume $\gamma = 1.4$, $c_p = 1,004 \text{ J/kg} \cdot \text{K}$.

4.31 A constant-area mixer operates with the inlet conditions as shown.



■ FIGURE P4.31

Assuming the hot stream has a total temperature of

$$T_5 = 4 T_{t15}$$

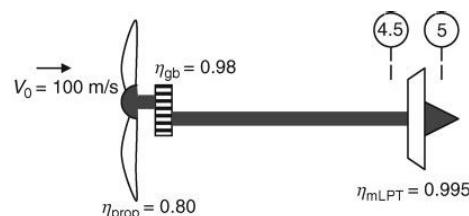
Calculate

- (a) the ratio of mass flow rates \dot{m}_{15}/\dot{m}_5
- (b) A_{6M}/A_5

- (c) $p_{6M}(1 + \gamma M_{6M}^2)$ from impulse
- (d) T_{t6M}/T_{t5}

4.32 Consider a turboprop engine with the power turbine driving a propeller, as shown. The power turbine inlet and exit total temperatures are $T_{t4.5} = 783 \text{ K}$ and $T_{t5} = 523 \text{ K}$. The mass flow rate through the turbine is 25 kg/s. Assuming $c_p = 1,100 \text{ J/kg} \cdot \text{K}$. Calculate

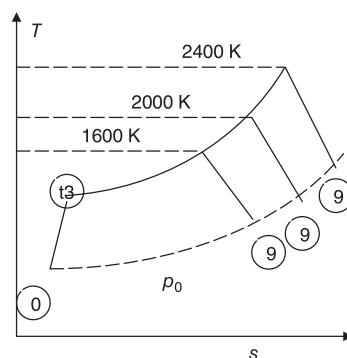
- (a) the power produced by the turbine
- (b) the power delivered to the propeller
- (c) the propeller thrust F_{prop}



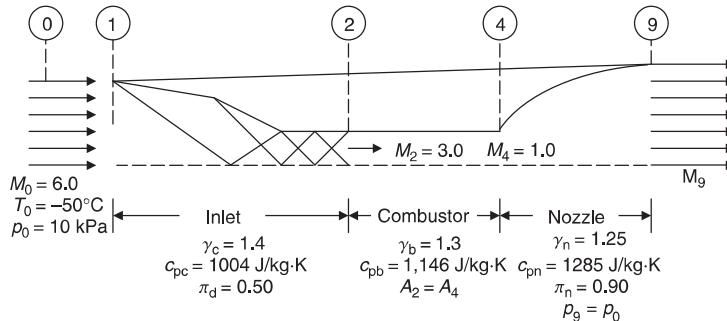
■ FIGURE P4.32

4.33 Let us study a family of turbojet engines, all with the same component parameters except the burner, as shown on the $T-s$ diagram. The family of turbojets are at the standard sea-level condition and stationary, i.e., $M_0 = 0$. The fixed engine parameters are

$$\begin{aligned} \pi_d &= 0.995 \\ \pi_c &= 20, e_c = 0.90, \gamma_c = 1.4, c_{pc} = 1,004 \text{ J/kg} \cdot \text{K} \\ \pi_b &= 0.95, \eta_b = 0.98, Q_R = 42,800 \text{ kJ/kg} \\ \eta_m &= 0.99, e_t = 0.85, \gamma_t = 1.33, c_{pt} = 1,146 \text{ J/kg} \cdot \text{K} \\ \pi_n &= 0.98 \\ p_9/p_0 &= 1 \end{aligned}$$



■ FIGURE P4.33



■ FIGURE P4.35

The burner exit temperature ranges from $T_{14} = 1600 \text{ K}$ to 2400 K. Calculate and graph

- (a) nondimensional-specific gross thrust $F_g/m_0 a_0$ versus T_{14}
- (b) specific impulse I_s (based on fuel flow rate) in seconds versus T_{14}
- (c) T_9/T_0 versus T_{14}
- (d) thermal efficiency, η_{th} versus T_{14}

4.34 A ramjet is in flight at an altitude where $T_0 = -23^\circ\text{C}$, $p_0 = 10 \text{ kPa}$, and the flight Mach number is M_0 . Assuming $T_{14} = 2500 \text{ K}$ and the nozzle is perfectly expanded, calculate the “optimum” flight Mach number such that ramjet specific thrust is maximized. Assume that all components are ideal, with constant γ and c_p throughout the engine and $Q_R = 42,600 \text{ kJ/kg}$. Would the fuel heating value affect the “optimum” flight Mach number?

4.35 Consider a scramjet in a Mach-6 flight. The fuel of choice for this engine is hydrogen with $Q_R = 120,000 \text{ kJ/kg}$. The inlet uses multiple oblique shocks with a total pressure recovery of $\pi_d = 0.5$. The combustor entrance Mach number is $M_2 = 3.0$. Use Rayleigh flow approximations in the supersonic combustor to estimate the fuel-to-air ratio f for a choking exit

condition, as shown, after you calculate the combustor exit total temperature T_{14} .

Also calculate

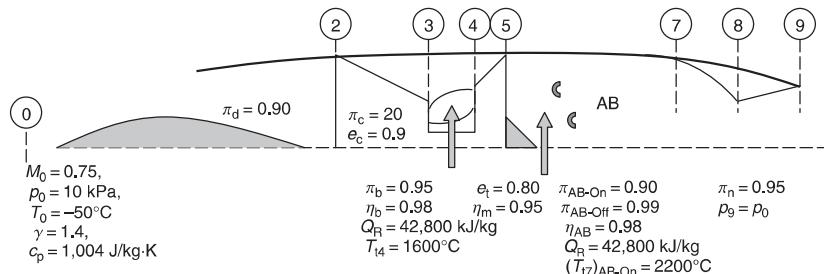
- (a) nozzle exit Mach number M_9
- (b) nondimensional ram drag $D_{ram}/p_0 A_1$ (note that $A_0 = A_1$)
- (c) nondimensional gross thrust $F_g/p_0 A_1$
- (d) fuel specific impulse I_s in seconds

4.36 We are interested in calculating the thrust boost of an afterburning turbojet engine when the afterburner is turned on. The flight condition and engine parameters are shown.

For simplicity of calculations, we assume that gas properties γ , c_p remain constant throughout the engine.

Calculate

- (a) percentage thrust gained when afterburner is turned on
- (b) percentage increase in exhaust speed with afterburner is turned on
- (c) static temperature rise at the nozzle exit when the afterburner is turned on



■ FIGURE P4.36

- (d) percentage fuel-air ratio increase when the afterburner is turned on
- (e) percentage increase in thrust specific fuel consumption with afterburner on
- (f) percentage drop in thermal efficiency when the afterburner is turned on

4.37 A separate-flow turbofan engine is designed with an aft-fan configuration, as shown. The fan and core engine nozzles are of convergent design.

For simplicity, you may assume constant gas properties in the engine, i.e., let γ be 1.4 and $c_p = 1,004 \text{ J/kg} \cdot \text{K}$.

Calculate

- (a) ram drag (in kN)
- (b) compressor adiabatic efficiency η_c
- (c) shaft power produced by the high-pressure turbine in MW
- (d) shaft power consumed by the fan in MW
- (e) core net thrust in kN
- (f) fan net thrust in kN
- (g) thrust-specific fuel consumption in mg/g/N
- (h) engine thermal efficiency η_{th}
- (i) engine propulsive efficiency η_p

Is there an obvious advantage to an aft-fan configuration?

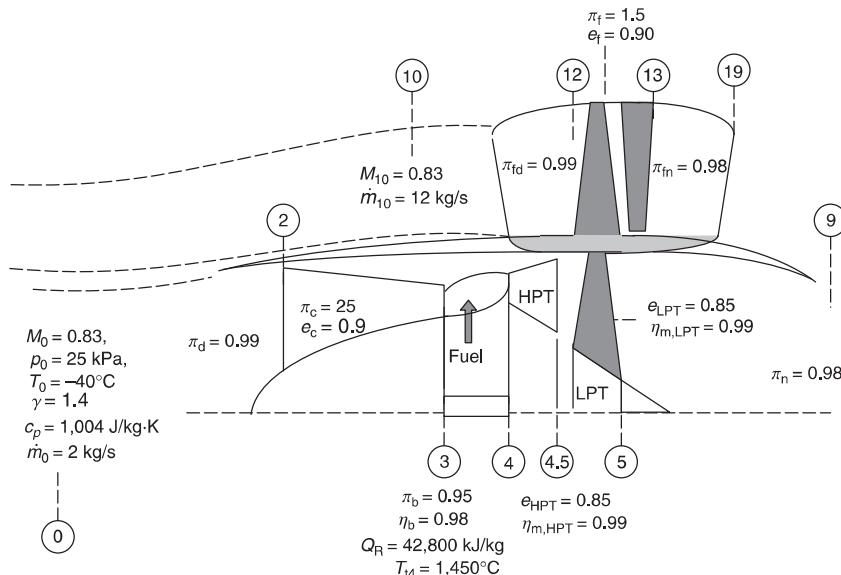
Is there an obvious disadvantage to this design?

4.38 An advanced turboprop flies at $M_0 = 0.82$ at an altitude where $p_0 = 30 \text{ kPa}$ and $T_0 = -15^\circ\text{C}$. The propeller efficiency is $\eta_{prop} = 0.85$. The inlet captures airflow rate at 50 kg/s and has a total pressure recovery of $\pi_d = 0.99$. The compressor pressure ratio is $\pi_c = 35$ and its polytropic efficiency is $e_c = 0.92$. The combustor has an exit temperature $T_{14} = 1650 \text{ K}$ and the fuel heating value is $Q_R = 42,000 \text{ kJ/kg}$, with a burner efficiency of $\eta_b = 0.99$ and the total pressure loss in the burner is $\pi_b = 0.96$. The HPT has a polytropic efficiency of $e_{HPT} = 0.80$, and a mechanical efficiency of $\eta_{m,HPT} = 0.99$. The power split between the LPT and the engine nozzle is at $\alpha = 0.75$ and the mechanical efficiency of the LPT is $\eta_{m,LPT} = 0.99$, the LPT adiabatic efficiency is $\eta_{LPT} = 0.88$. A reduction gearbox is used with an efficiency of $\eta_{gb} = 0.995$. The exhaust nozzle is convergent with an adiabatic efficiency of $\eta_n = 0.95$. We will describe gas properties in the engine based only on two temperature zones (cold and hot):

Inlet and compressor sections (cold): $\gamma_c = 1.4$,
 $c_{pc} = 1004 \text{ J/kg} \cdot \text{K}$
Turbines and nozzle sections (hot): $\gamma_t = 1.33$,
 $c_{pt} = 1,152 \text{ J/kg} \cdot \text{K}$

Calculate

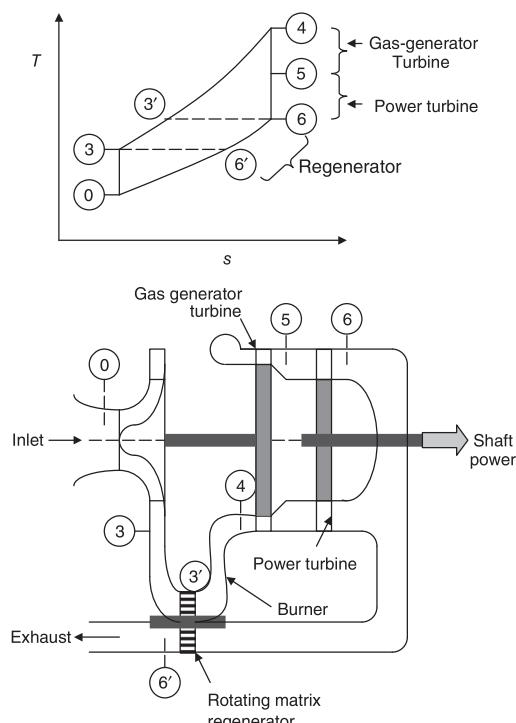
- (a) total pressure and temperature throughout the engine (include fuel-to-air ratio in mass/energy balance)
- (b) engine core thrust in kN



■ FIGURE P4.37

- (c) propeller thrust in kN
- (d) power-specific fuel consumption in mg/s/kW
- (e) thrust-specific fuel consumption in mg/s/N
- (f) thermal and propulsive efficiencies η_{th} and η_p
- (g) engine overall efficiency η_o

4.39 An ideal regenerative (Brayton) cycle is shown. The cycle compression is between states 0 and 3. The compressor discharge is preheated between states 3 and 3'. The source of this thermal energy is the hot exhaust gas from the engine. The burner is responsible for the temperature rise between states 3' and 4. The expansion in the turbine is partly between states 4 and 5 that supplies the shaft power to the compressor and partly between states 5 and 6 that produces shaft power for an external load (e.g., propeller, helicopter rotor, or electric generator). The total power production as shown in the expansion process is unaffected by the heat exchanger between states 6 and 6'. Note that the turbine exit temperature T_6 has to be higher than the compressor discharge temperature T_3 for the regenerative cycle to work. Therefore low-pressure ratio cycles can benefit from this (regenerative) concept. Also note that $T_{6'} = T_3$ and $T_{3'} = T_6$.



■ FIGURE P4.39

Show that the thermal efficiency of this cycle is

$$\eta_{th} = 1 - \frac{T_3}{T_4}$$

Calculate the thermal efficiency of a Brayton cycle with cycle pressure ratio of 10, i.e., $p_3 / p_0 = 10$ and the maximum cycle temperature ratio of $T_4 / T_0 = 6.5$ with and without regeneration.

4.40 A mixed-exhaust turbofan engine with afterburner is flying at $M_0 = 2.5$, $p_0 = 25$ kPa, and $T_0 = -35^\circ\text{C}$. The engine inlet total pressure loss is characterized by $\pi_d = 0.85$. The fan pressure ratio is $\pi_f = 1.5$ and polytropic efficiency of the fan is $e_f = 0.90$.

The flow in the fan duct suffers 1% total pressure loss, i.e., $\pi_{fd} = 0.99$. The compressor pressure ratio and polytropic efficiency are $\pi_c = 12$ and $e_c = 0.90$, respectively. The combustor exit temperature is $T_{14} = 1800$ K, fuel heating value is $Q_R = 42,800$ kJ/kg, total pressure ratio $\pi_b = 0.94$, and the burner efficiency is $\eta_b = 0.98$. The turbine polytropic efficiency is $e_t = 0.80$, its mechanical efficiency is $\eta_m = 0.95$, and the turbine exit Mach number is $M_5 = 0.5$. The constant-area mixer suffers a total pressure loss due to friction, which is characterized by $\pi_{M,f} = 0.95$. The afterburner is on with $T_{17} = 2200$ K, $Q_{R,AB} = 42,800$ kJ/kg, $\pi_{AB,On} = 0.92$, and afterburner efficiency $\eta_{AB} = 0.98$. The nozzle has a total pressure ratio of $\pi_n = 0.95$ and $p_9 / p_0 = 2.6$.

The gas behavior in the engine is dominated by temperature (in a thermally perfect gas), thus we consider four distinct temperature zones:

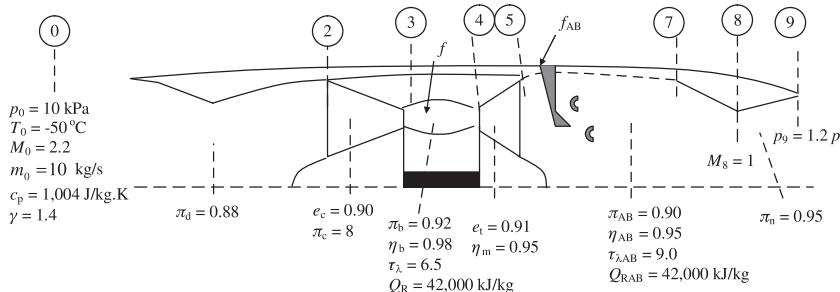
- Inlet, fan, and compressor section: $\gamma_c = 1.4$, $c_{pc} = 1,004 \text{ J/kg} \cdot \text{K}$
 Turbine section: $\gamma_t = 1.33$, $c_{pt} = 1,152 \text{ J/kg} \cdot \text{K}$
 Mixer exit: γ_{6M} , c_{p6M} (to be calculated based on mixture of gases)
 Afterburner and nozzle section: $\gamma_{AB} = 1.30$, $c_{p,AB} = 1,241 \text{ J/kg} \cdot \text{K}$

Calculate

- (a) total pressure and temperature throughout the engine, the fan bypass ratio α , and include the contributions of fuel-to-air ratio in the primary and afterburner, f and f_{AB} and
- (b) engine performance parameters, i.e., TSFC in mg/s/N, specific thrust and cycle efficiencies

4.41 An afterburning turbojet engine is in supersonic flight, as shown. The flight condition and cycle parameters are specified. Assuming constant gas properties throughout the engine, calculate

- (a) ram drag, D_r in kN
- (b) compressor shaft power, ϕ_c in MW
- (c) fuel-to-air ratio in the primary burner, f

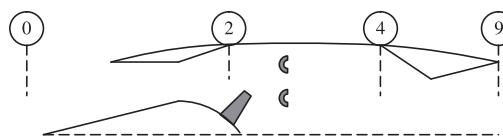
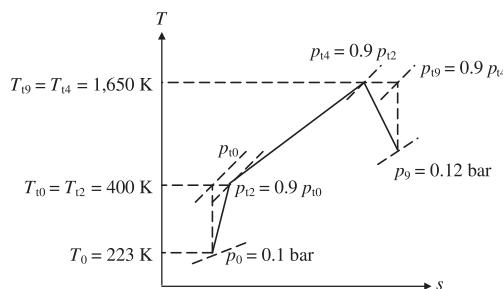


- (d) fuel-to-air ratio in the afterburner, f_{AB}
- (e) gas speed at the nozzle throat, V_8 , in m/s
- (f) exit Mach number, M_9
- (g) exit flow area, A_9 , in m^2

For Intermediate Steps Calculate These Parameters:

$\tau_r =$	$\tau_c =$	$\tau_t =$
$\pi_r =$	$\pi_c =$	$\pi_t =$

- 4.42 The thermodynamic state of gas in a ramjet is shown on a $T-s$ diagram.



Assuming constant gas properties, $\gamma = 1.4$ and $c_p = 1,004 \text{ J/kg} \cdot \text{K}$, calculate

- (a) the flight Mach number, M_0
- (b) the exhaust Mach number, M_9
- (c) the exhaust velocity, V_9 , in m/s

- 4.43 The air mass flow rate in a turbojet engine at takeoff is 100 kg/s at standard sea-level conditions ($p_0 = 100 \text{ kPa}$, $T_0 = 15^\circ\text{C}$). The fuel-to-air ratio is 0.035 and the nozzle exhaust speed is 1000 m/s. The nozzle is underexpanded with $p_9 = 150 \text{ kPa}$. Assuming the nozzle exit temperature is $T_9 = 1,176 \text{ K}$ with $\gamma_9 = 1.33$ and $c_{p9} = 1,156 \text{ J/kg} \cdot \text{K}$, calculate

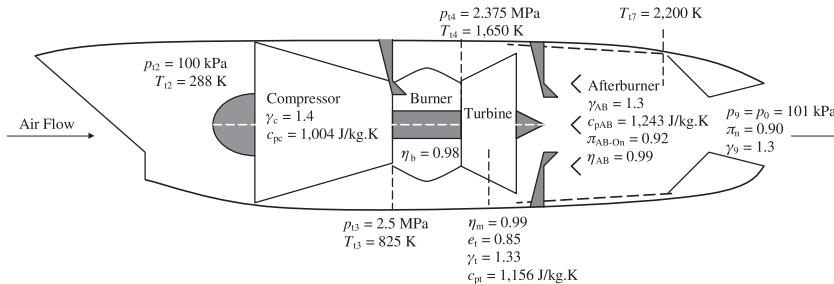
- (a) Nozzle exit area, A_9 , in m^2
- (b) Effective exhaust speed, V_{eff} , in m/s
- (c) Takeoff thrust, $F_{\text{T.O.}}$, in kN
- (d) Fuel-specific impulse, I_s , at takeoff in seconds

- 4.44 An afterburning turbojet engine is shown in "wet mode". Calculate

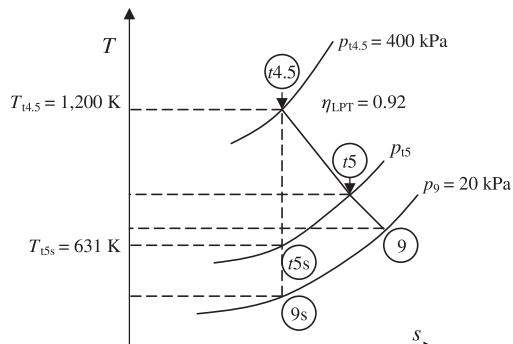
- (a) fuel-to-air ratio in the primary burner, f , for $Q_R = 42,000 \text{ kJ/kg}$
- (b) turbine exit total temperature, T_{15} (K)
- (c) turbine exit total pressure, p_{15} , in kPa
- (d) fuel-to-air ratio in the afterburner, f_{AB} , for $Q_{R,AB} = 42,000 \text{ kJ/kg}$
- (e) nozzle exit Mach number, M_9

- 4.45 The $T-s$ diagram shows the power split between the propeller and the nozzle. Assuming the mass flow rate is $\dot{m} = 37 \text{ kg/s}$ with $\gamma = 1.33$ and $c_p = 1,152 \text{ J/kg} \cdot \text{K}$, calculate

- (a) ideal power available in station 4.5, \wp_i in MW
- (b) LPT exit pressure, p_{15} , in kPa
- (c) LPT exit temperature, T_{15} , in K
- (d) LPT power (actual) in MW
- (e) nozzle exit velocity, V_9 , in m/s, for $\eta_n = 0.95$



■ FIGURE P4.44



■ FIGURE P4.45

4.46 A turboprop aircraft flies at $V_0 = 150 \text{ m/s}$ and its engine produces 20 kN of propeller thrust and 2 kN of core thrust. For a propeller efficiency of $\eta_{\text{prop}} = 0.85$, estimate the engine propulsive efficiency, η_p . [Hint: neglect ΔKE]

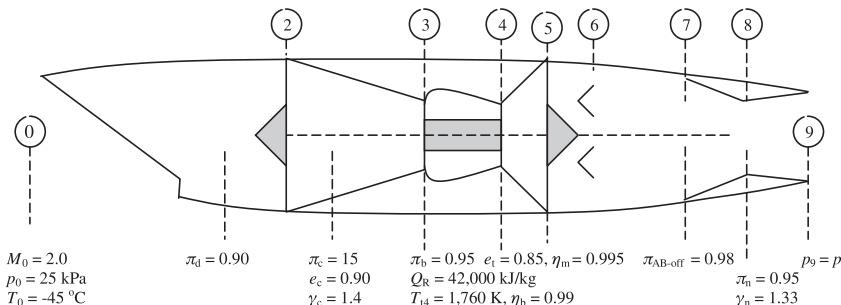
4.47 Consider an afterburning turbojet engine, with afterburner off, as shown.

The gas is thermally perfect with two zones of "cold" and "hot" described by the gas properties in the compressor and turbine sections as $\gamma_c = 1.4$, $c_{pc} = 1,004 \text{ J/kg} \cdot \text{K}$ and $\gamma_t = 1.33$ and $c_{pt} = 1,156 \text{ J/kg} \cdot \text{K}$, respectively. Assuming the air flow rate is $\dot{m}_0 = 100 \text{ kg/s}$, calculate

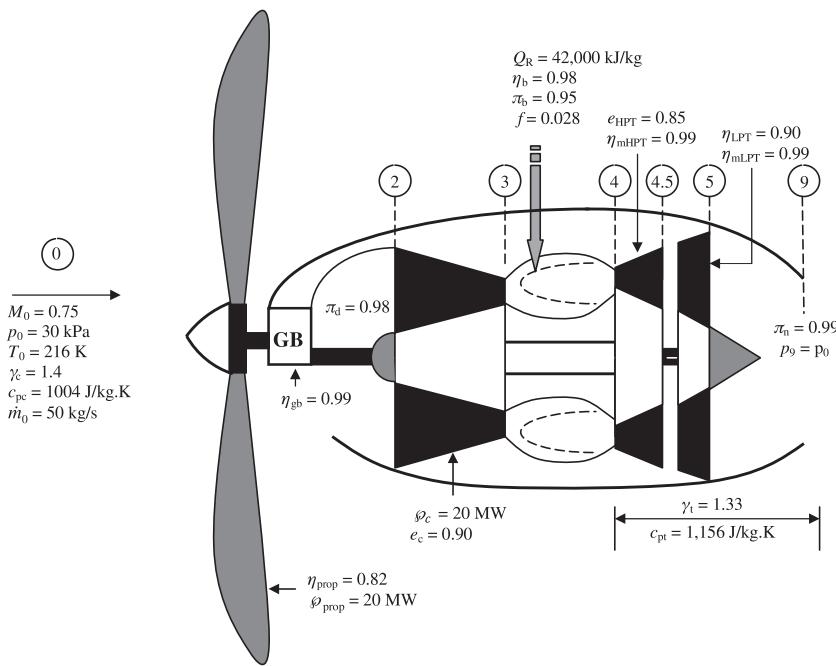
- D_r , ram drag, in kN and lbf
- f , fuel-to-air ratio
- T_{15} in K and °R
- M_9
- F_n , net uninstalled thrust in kN and lbf

4.48 An exhaust nozzle has an inlet total pressure and temperature, $p_{17} = 75 \text{ kPa}$ and $T_{17} = 900 \text{ K}$, respectively. The nozzle exit static pressure is $p_9 = 30 \text{ kPa}$ where the ambient pressure is $p_0 = 20 \text{ kPa}$. Assuming nozzle total pressure ratio is $\pi_n = 0.95$, $\gamma_n = 1.33$ and $c_{pn} = 1,156 \text{ J/kg} \cdot \text{K}$, calculate

- nozzle exhaust speed, V_9 , in m/s and fps
- the nozzle exhaust speed (in m/s and fps) if the nozzle were perfectly expanded, i.e., $p_9 = 20 \text{ kPa}$



■ FIGURE P4.47



■ FIGURE P4.49

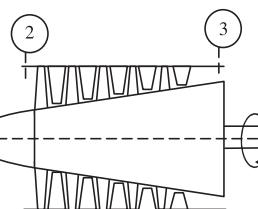
4.49 An advanced turboprop engine cruises at Mach 0.75 at an altitude where $p_0 = 30 \text{ kPa}$ and $T_0 = 216 \text{ K}$, as shown. The parameters related to each component is identified on the graph, e.g., $\pi_d = 0.98$. Also the gas constants representative of the cold and hot section are also specified. The air mass flow rate in the engine is noted to be 50 kg/s . Calculate

- total pressure and temperature at the engine face, p_{12} and T_{12} , in kPa and K, respectively
- total pressure and temperature, p_{13} and T_{13} , in kPa and K, respectively
- combustor exit temperature and pressure, T_{14} and p_{14} , in K and kPa, respectively
- total temperature and pressure at the exit of HPT, $T_{14.5}$ and $p_{14.5}$, in K and kPa respectively
- total temperature and pressure at the exit of LPT, T_{15} and p_{15} , in K and kPa, respectively
- ram drag, D_r , in kN and lbf
- propeller thrust, F_{prop} , in kN and lbf
- core nozzle exit Mach number, M_9
- core thrust, F_{core} , in kN and lbf
- thrust-specific fuel consumption, TSFC, in mg/s/N (and lbm/hr/lbf)

4.50 A compressor in a turbojet engine consumes 40 MW of shaft power to handle 100 kg/s of air flow rate and create a compressor total pressure ratio of 15.4. Assuming the inlet condition to the compressor is $p_{12} = 100 \text{ kPa}$, $T_{12} = 288 \text{ K}$, calculate

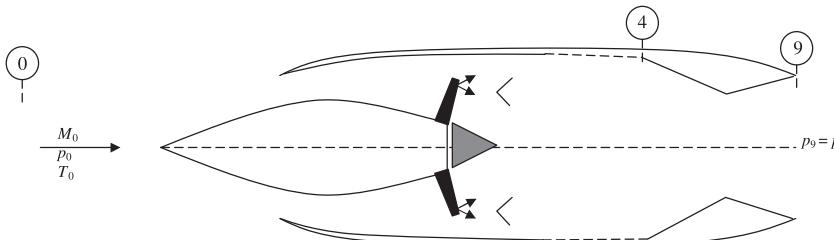
- the exit total temperature, T_{13} , in K
- compressor polytropic efficiency, e_c

Assume $\gamma = 1.4$ and $c_{pc} = 1004 \text{ J/kg} \cdot \text{K}$

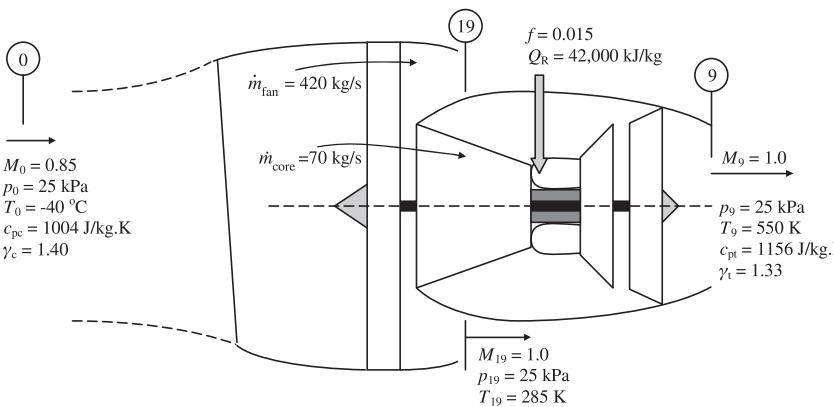


■ FIGURE P4.50

4.51 A ramjet engine flies at Mach 2 at an altitude where $p_0 = 20 \text{ kPa}$ and $T_0 = 245 \text{ K}$.



■ FIGURE P4.51



■ FIGURE P4.52

The inlet total pressure recovery is $\pi_{\text{d}} = 0.90$ and the combustor exit temperature is $T_{\text{t4}} = 1800 \text{ K}$.

The fuel heating value is $Q_R = 42,000 \text{ kJ/kg}$ the burner efficiency is $\eta_b = 0.98$ and the burner total pressure ratio is $\pi_b = 0.95$. The nozzle is perfectly expanded with $\pi_n = 0.92$.

Assume constant γ of 1.4 and constant c_p of $1004 \text{ J/kg} \cdot \text{K}$. Calculate

- flight speed, V_0 , in m/s and fps
- fuel-to-air ratio in the combustor, f
- exhaust velocity, V_9 , in m/s and fps
- ratio of gross thrust to ram drag, F_g/D_r

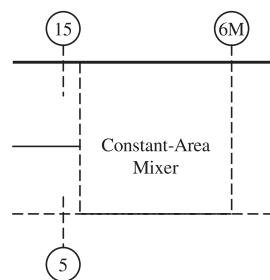
4.52 For the separate exhaust turbofan engine shown, calculate: (a) ram drag in kN, (b) fan nozzle gross thrust in kN, (c) net uninstalled thrust in kN, (d) thermal efficiency, (e) propulsive efficiency, (f) thrust specific fuel consumption in mg/s/N.

4.53 Consider a constant-area mixer, as shown. The mass flow ratio between the cold and hot streams is 3, i.e., $\frac{\dot{m}_{15}}{\dot{m}_5} = 3$.

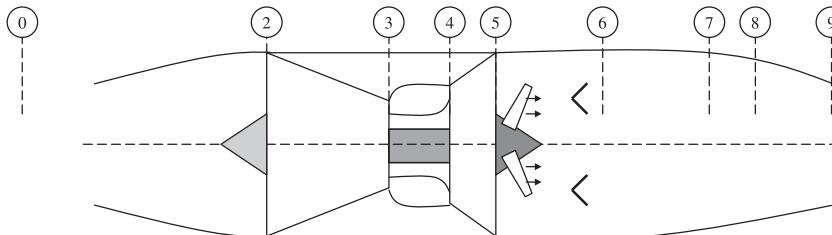
The gas properties are: $c_{p15} = 1,004 \text{ J/kg} \cdot \text{K}$, $\gamma_{15} = 1.4$ $c_{p5} =$

$1,156 \text{ J/kg} \cdot \text{K}$ and $\gamma_5 = 1.33$. The flow conditions in the inlet to the mixer are:

$$p_{15} = p_{t15} = 150 \text{ kPa}, T_{t15} = 500 \text{ K} \text{ and } T_{15} = 880 \text{ K}.$$



■ FIGURE P4.53



■ FIGURE P4.54

Assuming the hot gas Mach number is $M_5 = 0.4$, calculate

- (a) gas properties at the mixed exit, c_{p6M} and γ_{6M}
- (b) Mach number of the cold stream, M_{15}
- (c) area ratio, A_{15}/A_5
- (d) total temperature at the mixed exit, T_{t6M} in K

4.54 An afterburning turbojet engine operates at an altitude where the ambient pressure and temperatures are: $p_0 = 15 \text{ kPa}$ and $T_0 = 223 \text{ K}$, respectively. The flight Mach number is $M_0 = 2.5$ and the ambient air is characterized by $c_p = 1,004 \text{ J/kg} \cdot \text{K}$ and $\gamma_c = 1.4$. The engine has the following operating parameters and efficiencies: $\pi_d = 0.85$, $\pi_e = 6$, $e_c = 0.92$, $\tau_a = 7.7$, $\pi_b = 0.95$, $\eta_b = 0.98$, $Q_p = 42,600 \text{ kJ/kg}$, $e_t = 0.85$, $\eta_m = 0.99$, $\tau_{AB} = 10.5$, $Q_{RAB} = 42,600 \text{ kJ/kg}$, $\pi_{AB} = 0.94$, $\eta_{AB} = 0.97$, $\pi_n = 0.95$ and $p_0 = 15 \text{ kPa}$. Assuming the air flow rate is 50 kg/s in the engine, and gas properties in the turbine and afterburner/nozzle are described by $c_{p1} = 1,156 \text{ J/kg} \cdot \text{K}$, $\gamma_1 = 1.33$, $c_{pAB} = 1,243 \text{ J/kg} \cdot \text{K}$ and $\gamma_{AB} = 1.30$, respectively, calculate

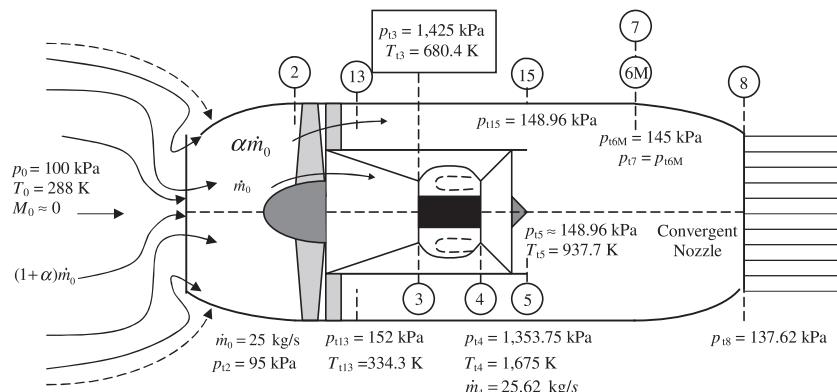
- (a) ram drag, D_r , in kN and lbf
- (b) compressor shaft power in MW and hp

- (c) fuel-to-air ratio, f , in the main burner
- (d) turbine discharge p_{t5} and T_{t5} in kPa and K, respectively
- (e) fuel-to-air-ratio in the afterburner, f_{ab}
- (f) nozzle gross thrust in kN and lbf
- (g) thrust specific fuel consumption, TSFC, in mg/s/N and lbm/hr/lbf
- (h) thermal efficiency, η_{th}
- (i) propulsive efficiency, η_p

4.55 The total pressures, temperatures and mass flow rates at some stations inside a nonafterburning, mixed-flow turbofan engine, at takeoff, are shown.

For simplicity of analysis, assume the gas is calorically perfect with constant properties ($\gamma = 1.4$ and $c_p = 1004 \text{ J/kg} \cdot \text{K}$) throughout the engine. Calculate

- (a) bypass ratio, α
- (b) fuel-to-air ratio, f
- (c) mixer exit total temperature, T_{t6M} , in K
- (d) exhaust Mach number, M_8 (note that the exhaust nozzle is of convergent type)
- (e) (un-installed) takeoff thrust, $F_{T.O.}$, in kN and lbf



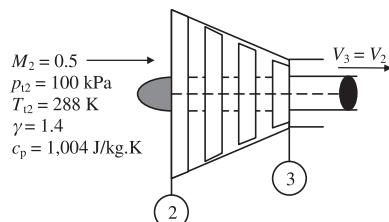
■ FIGURE P4.55

4.56 The power turbine in a turboprop engine produces a shaft power of 4.53 MW, working on a gas flow rate of 10.25 kg/s with $c_{pt} = 1,152 \text{ J/kg} \cdot \text{K}$ and $\gamma = 1.33$. Assuming $\eta_{\text{mPT}} = 0.99$, $\eta_{\text{gb}} = 0.99$ and $\eta_{\text{prop}} = 0.85$ at flight speed of 200 m/s, calculate

- the total temperature drop across the power turbine in K
- the shaft power delivered to propeller, \wp_{prop} (MW)
- the propeller thrust, F_{prop} , in kN

4.57 A multistage compressor develops a total pressure ratio of $\pi_c = 35$ with a polytropic efficiency of $e_c = 0.90$. The air mass flow rate through the compressor is $\dot{m} = 200 \text{ kg/s}$. Assuming γ and c_p remain constant, calculate

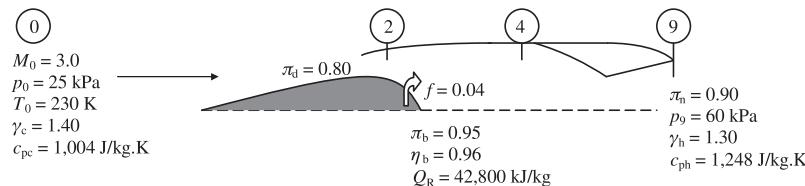
- Compressor shaft power, \wp_c , in MW
- Flow area in 2, i.e., A_2 , in m^2
- Density of air in station 3, ρ_3 , in kg/m^3 [note: the axial velocity at the compressor exit, $V_3 = V_2$]
- The nondimensional entropy rise across the compressor, $\Delta s/R$



■ FIGURE P 4.57

4.58 A ramjet in flight is shown. The inlet total pressure recovery is $\pi_d = 0.80$ and the nozzle is underexpanded. Calculate

- ram temperature and pressure ratios, τ_r and π_r
- the total temperature at the combustor exit, T_{14} , in K and the corresponding τ_λ
- exhaust velocity, V_9 , in m/s



■ FIGURE P 4.58

(d) pressure thrust as a fraction of nozzle momentum thrust, i.e., $\frac{(p_9 - p_0)A_9}{\dot{m}_9 V_9}$

(e) the propulsive efficiency of the ramjet, η_p (note that the nozzle is not perfectly expanded, so you need to calculate $V_{9,\text{eff}}$)

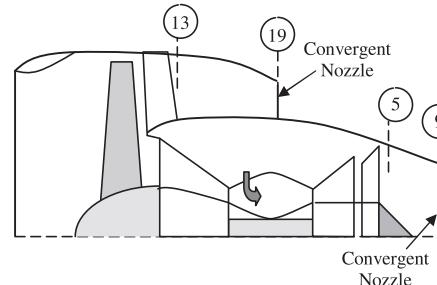
(f) thrust-specific fuel consumption, TSFC, in mg/s/N and lbm/hr/lbf

4.59 A separate exhaust turbofan engine has **convergent nozzles** with the following parameters:

$$\begin{aligned} M_0 &= 0.85 \\ p_0 &= 30 \text{ kPa}, T_0 = 240 \text{ K}, \gamma_c = 1.4, c_{pc} = 1004 \text{ J/kg} \cdot \text{K} \\ \pi_d &= 0.98 \\ \pi_i &= 1.55, e_f = 0.90 \\ \pi_{fn} &= 0.97 \end{aligned}$$

Calculate

- fan exit total pressure, p_{13} , in kPa
- fan exit total temperature, T_{13} , in K
- exit Mach number, M_{19}
- nozzle exit static pressure, p_{19} (or p_{18}), in kPa
- exhaust speed, V_{19} (or V_{18}), in m/s
- effective exhaust velocity, $V_{19,\text{eff}}$, in m/s

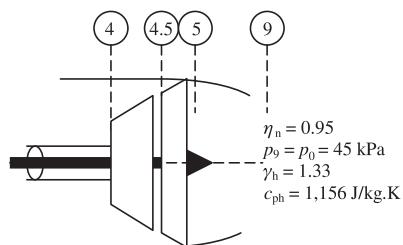


■ FIGURE P 4.59

4.60 A turboprop uses a power split of 0.96 between the power turbine and the exhaust nozzle, i.e. $\alpha = 0.96$. The

adiabatic efficiency of the LPT (or power turbine) is $\eta_{\text{LPT}} = 0.88$. The flow at the inlet of the LPT has $p_{14,5} = 99 \text{ kPa}$ and $T_{14,5} = 845 \text{ K}$. The nozzle adiabatic efficiency is $\eta_n = 0.95$. Calculate

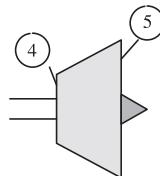
- the shaft power per unit mass flow rate of the LPT in kJ/kg
- the exhaust velocity, V_9 , in m/s



■ FIGURE P4.60

4.61 An un-cooled turbine has entrance and exit flow conditions: $p_{t4} = 2.5 \text{ MPa}$, $T_{t4} = 1760 \text{ K}$, $T_{t5} = 1000 \text{ K}$. The gas mass flow rate in the turbine is $\dot{m}_4 = 103 \text{ kg/s}$ and the turbine polytropic efficiency is $e_t = 0.85$. Assuming $\gamma_t = 1.33$ and $c_{pt} = 1156 \text{ J/kg}\cdot\text{K}$, calculate

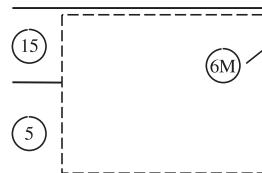
- turbine exit total pressure, p_{t5} , in kPa
- turbine shaft power in MW
- turbine adiabatic efficiency, η_t



■ FIGURE P4.61

4.62 The flow condition at the entrance to a constant-area mixer is that the mass flow rate from the fan side, $\dot{m}_{15} = 250 \text{ kg/s}$ and the mass flow rate from the core is $\dot{m}_5 = 51.5 \text{ kg/s}$. The entrance total temperatures are $T_{15} = 360 \text{ K}$ and $T_5 = 895 \text{ K}$. The cold and hot streams have $\gamma_c = 1.4$, $c_{pc} = 1004 \text{ J/kg}\cdot\text{K}$, $\gamma_t = 1.33$ and $c_{pt} = 1156 \text{ J/kg}\cdot\text{K}$. Calculate

- the gas properties at the mixer exit, γ_{6M} and c_{p6M}
- the mixer exit total temperature, T_{6M} , in K

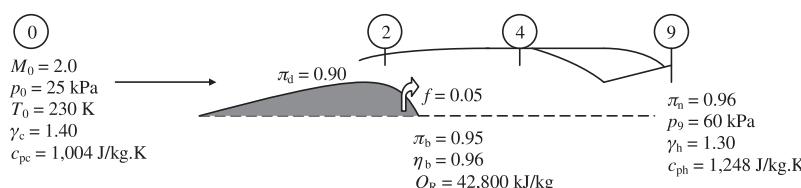


■ FIGURE P4.62

4.63 A ramjet is shown in supersonic flight. The inlet total pressure recovery is $\pi_d = 0.90$ and the nozzle is under-expanded (note that $p_9 \neq p_0$). Calculate

- ram temperature and pressure ratios, τ_r and π_r
- the total temperature at the combustor exit, T_{t4} , in K and the corresponding τ_λ
- Exhaust velocity, V_9 , in m/s
- pressure thrust as a fraction of nozzle momentum thrust, i.e., $\frac{(p_9 - p_0)A_9}{\dot{m}_9 V_9}$
- the propulsive efficiency of the ramjet, η_p (note that the nozzle is not perfectly expanded, so you need to calculate $V_{9\text{eff}}$)
- thrust-specific fuel consumption, TSFC, in mg/s/N and lbf/hr/lbf

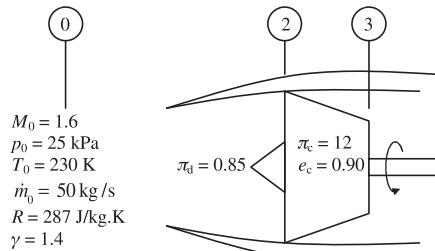
4.64 A turbojet engine is flying at Mach 1.6, with ambient pressure, $p_0 = 25 \text{ kPa}$ and temperature, $T_0 = 230 \text{ K}$. The inlet total pressure recovery is $\pi_d = 0.85$ and the compressor pressure ratio is $\pi_c = 12$ with $e_c = 0.90$.



■ FIGURE P4.63

Assuming the air flow rate is 50 kg/s, calculate

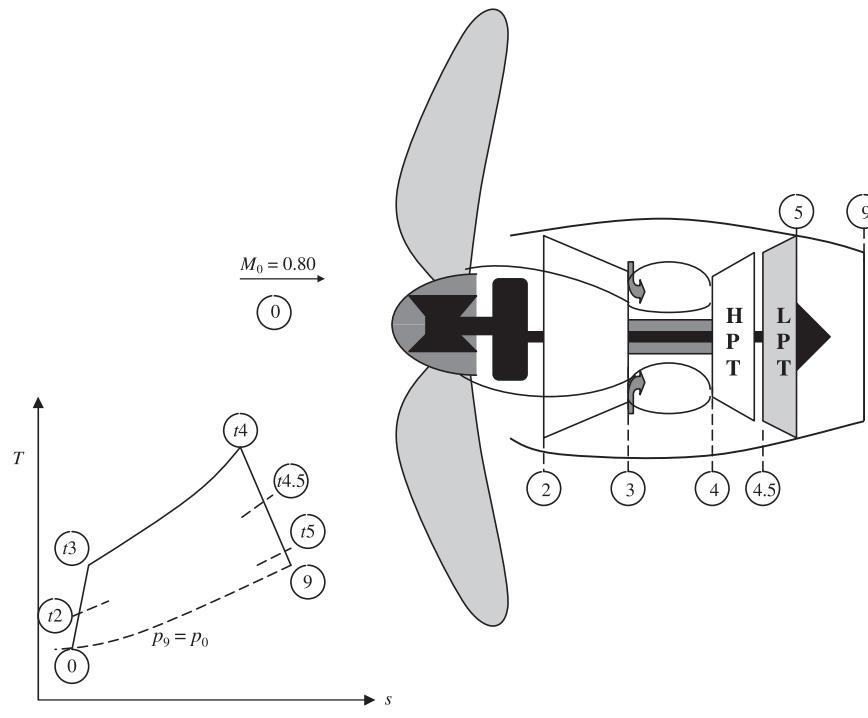
- ram drag in kN
- compressor exit total pressure, p_{13} , in kPa
- compressor shaft power, φ_c , in MW
- inlet adiabatic efficiency, η_d



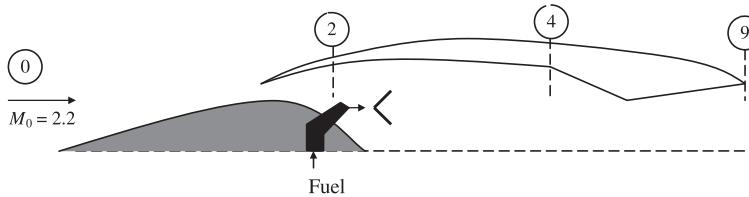
■ FIGURE P4.64

4.65 A turboprop is in a Mach 0.80 flight at an altitude where $p_0 = 22 \text{ kPa}$ and temperature is $T_0 = 245 \text{ K}$ with $\gamma_c = 1.4$ and $c_{pc} = 1004 \text{ J/kg} \cdot \text{K}$. The inlet total pressure loss is 3% of flight dynamic pressure, i.e., $p_{01}p_{12} = 0.03 q_0$. The compressor pressure ratio is $\pi_c = 35$ and its polytropic efficiency is $e_c = 0.90$. The combustor achieves an exit total temperature of $T_{14} = 1,650 \text{ K}$ while burning a hydrocarbon fuel with an ideal heating value of 42,000 kJ/kg, at a burner efficiency of $\eta_b = 0.99$ and a total pressure ratio $\pi_b = 0.95$. The gas in the hot section is characterized by $\gamma_t = 1.33$ and $c_{pt} = 1,152 \text{ J/kg} \cdot \text{K}$. The high-pressure turbine has $\eta_{mHPT} = 0.995$ and a polytropic efficiency of $e_{HPT} = 0.85$. The power split parameter between the low-pressure turbine and the nozzle is $\alpha = 0.85$. The low-pressure turbine has adiabatic and mechanical efficiencies of $\eta_{LPT} = 0.90$ and $\eta_{m,LPT} = 0.995$ respectively. The propeller is gearbox-driven with $\eta_{gb} = 0.995$ and the propeller efficiency is $\eta_{prop} = 0.80$. Assuming the nozzle is perfectly expanded, $p_9 = p_0$, and $\eta_n = 0.96$, calculate:

- flight dynamic pressure, q_0 , in kPa
- compressor discharge (total) temperature, T_{13} in K
- the fuel-to-air ratio, f , in the burner



■ FIGURE P4.65



■ FIGURE P4.66

- (d) the total pressure and temperature at the exit of HPT, i.e., $p_{4,5}$ (in kPa) and $T_{4,5}$ in K
- (e) the total pressure and temperature at the exit of the LPT, $p_{5,6}$ in kPa and $T_{5,6}$ in K
- (f) the nozzle exit Mach number, M_0
- (g) the nozzle exit velocity, V_0
- (h) the propeller thrust, F_{prop} in kN

4.66 A ramjet flies at Mach 2.2 at an altitude where the speed of sound is $a_0 = 294$ m/s and the pressure is $p_0 = 20$ kPa. The air is characterized as perfect gas with $\gamma = 1.4$ and $R = 287 \text{ J/kg} \cdot \text{K}$. The inlet total pressure recovery is $\pi_d = 0.90$, combustor losses are characterized by $\pi_b = 0.92$, $\eta_b = 0.99$ and the nozzle total pressure loss parameter is $\pi_n = 0.94$. Assuming nozzle is perfectly expanded, combustor exit temperature is $T_{t4} = 2,000$ K, fuel heating value is $Q_R = 42,600$ kJ/kg, and gas properties (γ and R) remain constant in the ramjet, calculate

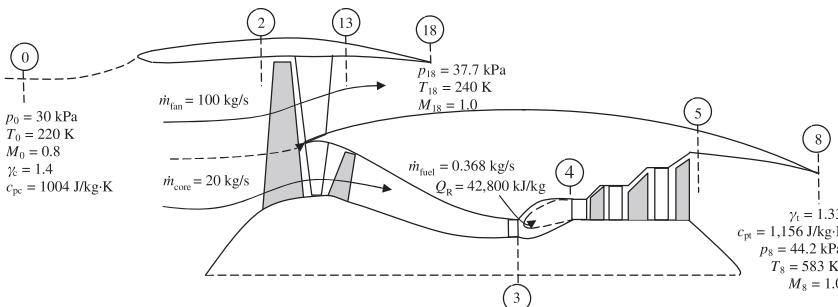
- (a) enthalpy ratio, τ_λ
- (b) fuel-to-air ratio, f
- (c) exhaust speed, V_0 , in m/s
- (d) non-dimensional specific thrust, $F_n/\dot{m}_0 a_0$
- (e) propulsive efficiency, η_p
- (f) thermal efficiency, η_{th}

4.67 A separate-flow turbofan engine is shown at cruise condition. The flight condition, air and fuel flow rates, nozzle exit conditions and fuel properties are all labeled in the schematic drawing (Figure P4.67). The primary and fan nozzles are of *convergent* type and both are choked, as shown.

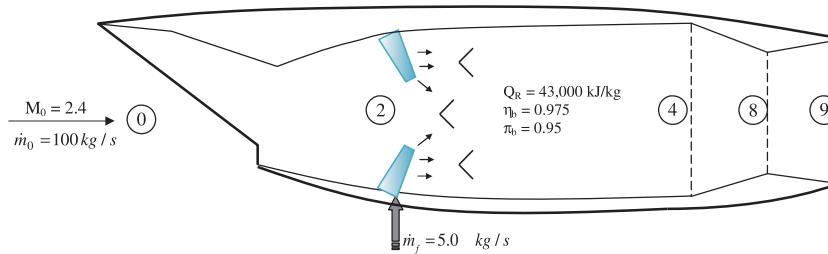
Calculate

- (a) ram drag in kN and lbf
- (b) fan nozzle exit area, A_{18} , in m^2 and ft^2
- (c) fan nozzle gross thrust in kN and lbf
- (d) core nozzle exit area, A_8 , in m^2 and ft^2
- (e) core nozzle gross thrust in kN and lbf
- (f) TSFC in mg/s/N and lbm/hr/lbf
- (g) fan total temperature ratio, τ_f
- (h) fan total pressure ratio, π_f , if the fan polytropic efficiency is $e_f = 0.90$
- (i) fan nozzle *effective exhaust speed*, $V_{18,\text{eff}}$, in m/s and fps
- (j) core nozzle *effective exhaust speed*, $V_{8,\text{eff}}$, in m/s and fps
- (k) engine propulsive efficiency, η_p (%)
- (l) engine thermal efficiency, η_{th} (%)

4.68 A ramjet is shown at Mach 2.4 flight at an altitude where $p_0 = 25$ kPa and $T_0 = 240$ K. The inlet total pressure



■ FIGURE P4.67

**FIGURE P4.68**

recovery is 90%. The air and fuel mass flow rates are 100 and 5.0 kg/s respectively, as shown.

The nozzle total pressure ratio is 95% and it is perfectly expanded. The gas properties for the cold and hot sections of the engine are: $\gamma_c = 1.4$, $c_p = 1,004 \text{ J/kgK}$ and $\gamma_t = 1.3$, $c_{pt} = 1,243 \text{ J/kg \cdot K}$, respectively. Calculate

- (a) ram drag, D_r , in kN and lbf
- (b) combustor exit total pressure, p_{14} , in kPa

- (c) combustor exit total temperature, T_{14} , in K
- (d) nozzle exit Mach number and velocity, M_9 and V_9 , in m/s
- (e) nozzle gross thrust, F_g , in kN and lbf
- (f) thrust-specific fuel consumption, TSFC, in mg/s/N and lbm/hr/lbf
- (g) engine thermal efficiency, η_{th}
- (h) engine propulsive efficiency, η_p