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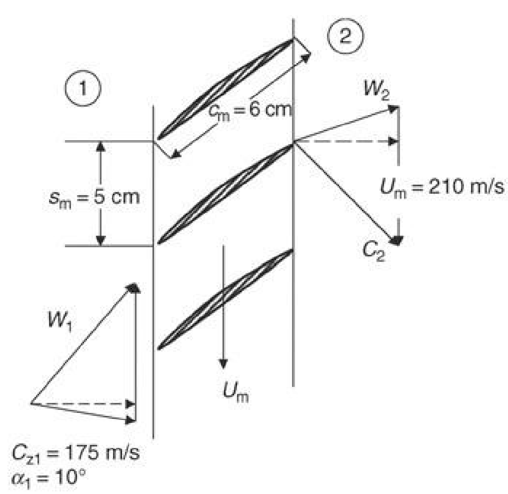
Problems

8.1 The absolute flow at the pitchline to a compressor rotor has a coswirl with $C_{\theta 1} = 78$ m/s. The exit flow from the rotor has a positive swirl, $C_{\theta 2} = 172$ m/s. The pitchline radius is at $r_m = 0.6$ m and the rotor angular speed is $\omega = 5220$ rpm. Calculate the specific work at the pitchline and the rotor torque per unit mass flow rate.

8.2 An axial-flow compressor stage has a pitchline radius of $r_m = 0.6$ m. The rotational speed of the rotor at pitchline is $U_m = 256$ m/s. The absolute inlet flow to the rotor is described by $C_{zm} = 155$ m/s and $C_{\theta 1m} = 28$ m/s. Assuming that the stage degree of reaction at pitchline is $°R_m = 0.50$, $\alpha_3 = \alpha_1$, and C_{zm} remains constant, calculate

- **(a)** rotor angular speed ω in rpm
- **(b)** rotor exit swirl $C_{\theta 2m}$
- **(c)** rotor specific work at pitchline, w_{cm}
- **(d)** relative velocity vector at the rotor exit
- **(e)** rotor and stator torques per unit mass flow rate
- **(f)** stage loading parameter at pitchline, ψ_m
- **(g)** flow coefficient φ_m

8.3 A rotor blade row is cut at pitchline, r_m. The velocity vectors at the inlet and exit of the rotor are shown.



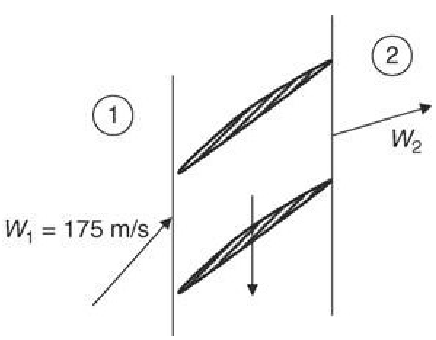
■ FIGURE P8.3

Assuming that $U_{m1} = U_{m2} = 210$ m/s and $C_{z1} = C_{z2} = 175$ m/s, $\rho_1 = 1$ g/m^3 $\beta_2 = -25°$, and $\varpi_r = 0.03$, calculate

- **(a)** $W_{\theta 1}$ and $W_{\theta 2}$
- **(b)** W_{1m} and W_{2m}
- **(c)** D-factor D_{rm}
- **(d)** Circulation Γ_m
- **(e)** rotor lift at pitchline per unit span
- **(f)** lift coefficient at pitchline
- **(g)** rotor-specific work at r_m
- **(h)** loading coefficient ψ_m
- **(i)** degree of reaction $°R_m$

8.4 A rotor blade row at the hub radius is shown. The rotor total pressure loss coefficient at this radius is $\varpi_{rm} = 0.04$.

- **(a)** How much deceleration is allowed in the rotor under de Haller criterion? i.e., What is the minimum W_2?
- **(b)** What is the static pressure rise coefficient, assuming incompressible flow and W_{2min} from de Haller criterion?
- **(c)** Compare the C_p in part (b) to the Arbitrary $C_{p\max}$ shown in Figure 8.30 at the hub.



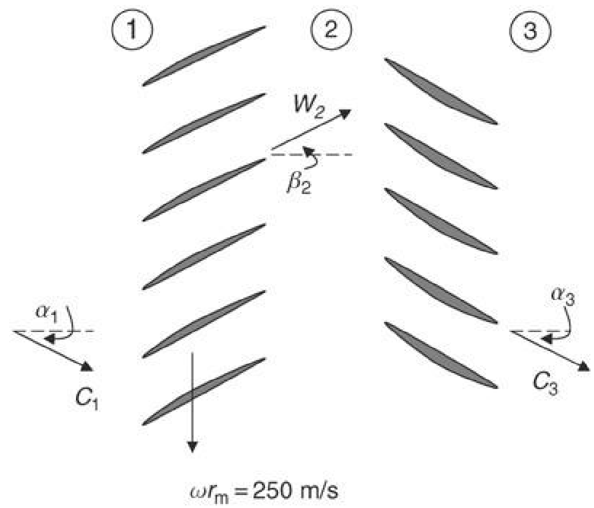
■ **FIGURE P8.4**

8.5 A compressor stage develops a pressure ratio of $\pi_s = 1.6$. Its polytropic efficiency is $e_c = 0.90$. Calculate the stage total temperature ratio τ_s and compressor stage adiabatic efficiency η_s. Assume $\gamma = 1.4$.

8.6 An axial-flow compressor stage is shown at the pitchline. Assuming

$\alpha_1 = \alpha_3$	
$\beta_2 = -30°$	$p_{t1} = 10^5$ Pa
$C_{z1} \cong C_{z2} \cong C_{z3}$	$T_{t1} = 290$ K
$r_1 \cong r_2 \cong r_3$	$M_1 = 0.5$
$\varpi_r = 0.03$	$\alpha_1 = 30°$
$\varpi_s = 0.02$	$\gamma = 1.4$
Calculate	$c_p = 1{,}004$ J/kg · K

- **(a)** w_c (in kJ/kg)
- **(b)** T_{t3}/T_{t1}
- **(c)** M_{2r}
- **(d)** p_{t2}/p_{t1}
- **(e)** p_{t3}/p_{t2}
- **(f)** η_s
- **(g)** $°R_m$



■ **FIGURE P8.6**

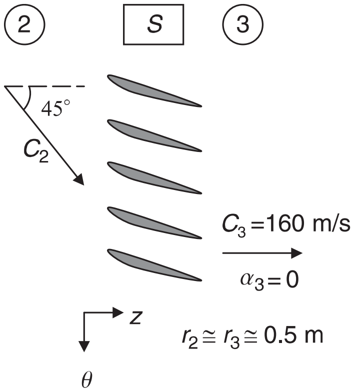
8.7 The flow at the entrance to an axial-flow compressor rotor has zero preswirl and an axial velocity of 175 m/s. The shaft angular speed is 5000 rpm. If at a radius of 0.5 m, the rotor exit flow has zero relative swirl, calculate at this radius

- **(a)** rotor specific work w_c in kJ/kg
- **(b)** degree of reaction $°R$

8.8 The absolute flow angle at the inlet of a stator blade in a compressor is $\alpha_2 = 45°$, as shown. The absolute total pressure and temperature in station 2 are $p_{t2} = 150$ kPa and $T_{t2} = 300$ K, respectively. The total pressure loss coefficient for this section of the the stator blade is $\varpi_s = 0.02$. Assuming the axial velocity remains constant and gas properties are $\gamma = 1.4$ and $c_p = 1.004$ kJ/kg · K, calculate

- **(a)** entrance Mach number M_2
- **(b)** exit total pressure p_{t3}
- **(c)** exit Mach number M_3
- **(d)** stator torque for a mass flow rate of $\dot{m} = 100$ kg/s
- **(e)** static pressure rise, $\Delta p = p_3 - p_2$
- **(f)** static temperature rise $\Delta T = T_3 - T_2$
- **(g)** entropy rise $(s_3 - s_2)/R$

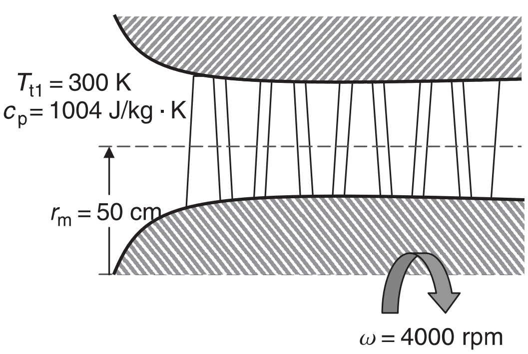
Note that the radius of this cut (section) is at $r \cong 0.5$ m from the axis of rotation, as shown in the diagram.



■ **FIGURE P8.8**

8.9 An axial-flow compressor with four stages is shown. Assuming a repeated stage design, with constant throughflow speed, C_z = 150 m/s, and 50% degree of reaction at the pitch-line with zero preswirl, calculate

(a) rotor specific work at the pitchline (kJ/kg)
(b) stage pressure ratio for an $\eta_s = 0.90$
(c) compressor pressure ratio π_c
(d) shaft power (in MW) for a mass flow rate of $\dot{m}_0$ = 100 kg/s
(e) D-factor for the first rotor at the pitchline for σ_r = 2.0



■ **FIGURE P8.9**

8.10 An axial-flow compressor stage is designed on the principle of constant through flow speed. The flow at the entrance to the rotor has 100 m/s of positive swirl and 180 m/s of axial velocity. Assuming we are at the pitchline radius r_m = 0.5 m, where the rotor rotational speed is U_m = 230 m/s, the degree of reaction $°R_m$ = 0.5, the radial shift in the stream-tube is negligible, i.e., $r_{1m} \approx r_{2m} \approx r_{3m}$, and also assuming a repeated stage design principle is implemented, calculate

(a) α_{1m} and β_{1m}
(b) α_{2m} and β_{2m}
(c) rotor specific work at the pitchline w_{cm} in kJ/kg
(d) stator torque at the pitchline per unit mass flow rate, $\tau_s/\dot{m}$

8.11 In an axial-flow compressor test rig with no inlet guide vanes, a 1-m diameter fan rotor blade spins with a sonic tip speed, i.e., U_{tip}/a_1 = 1.0. If the speed of sound in the laboratory is a_0 = 300 m/s, and the axial velocity to the fan is C_{z1} = 150 m/s, calculate the fan rotational speed ω in rpm.

$$R = 287\ \mathrm{J/kg \cdot K} \text{ and } \gamma = 1.4$$

8.12 An axial-flow compressor rotor has an angular velocity of ω = 5000 rpm. The flow entering the compressor rotor has zero preswirl and an axial velocity of C_{z1} = 150 m/s. Assuming the axial velocity is constant throughout the stage, and the rotor specific work at the radius r = 0.5 m is w_c = 62 kJ/kg (γ = 1.4 and R = 287 J/kg · K) calculate

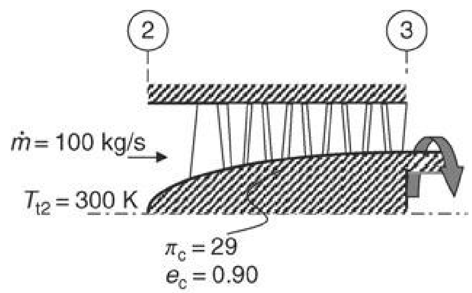
(a) stage degree of reaction, $°R$, at this radius
(b) total pressure ratio across the rotor, p_{t2}/p_{t1}, at this radius, assuming a polytropic efficiency of 90% and T_1 = 20 °C.

8.13 An axial-flow compressor rotor at the pitchline has a radius of r_m = 0.35 m. The shaft rotational speed is ω = 5000 rpm. The inlet flow to the rotor has zero preswirl and the axial velocity is $C_{z1} = C_{z2} = C_{z3}$ = 175 m/s. The rotor has a 50% degree of reaction at the pitchline. The stage adiabatic efficiency is nearly equal to the polytropic efficiency $\eta_s \cong e_c$ = 0.92. Assuming the inlet total temperature is T_{t1} = 288 K and c_p = 1.004 kJ/kg · K, calculate

(a) rotor specific work at $r = r_m$
(b) stage loading ψ at $r = r_m$
(c) flow coefficient at $r = r_m$
(d) rotor relative Mach number at the pitchline, $M_{1r,m}$
(e) stage total pressure ratio at the pitchline

8.14 For the multistage compressor, as shown, calculate

(a) compressor adiabatic efficiency η_c
(b) shaft power $\dot{W}_e$
(c) exit total temperature T_{t3}
(d) average π_s for ten stages, i.e., N = 10
Assume γ = 1.4 and c_p = 1004 J/kg · K.



■ **FIGURE P8.14**

8.15 A compressor test rig operates in a laboratory where θ_2 = 0.95 and δ_2 = 0.95. The mass flow rate is measured at the compressor face to be $\dot{m}$ = 100 kg/s and the shaft power is measured to be $\wp_c$ = 50 MW. Assuming compressor poly-tropic efficiency is e_c = 0.90, calculate

(a) compressor (total) pressure ratio π_c
(b) compressor corrected mass flow rate $\dot{m}_{c2}$
(c) compressor adiabatic efficiency η_c

8.16 A compressor adiabatic efficiency is measured to be $\eta_c = 0.85$ for a compressor total pressure ratio of $\pi_c = 20$. What is the "small-stage" efficiency for this compressor?

8.17 A compressor has a polytropic efficiency of e_c = 0.92 and a pressure ratio, $\pi_c = 25$ for an inlet condition of T_{t1} = 520 °R and c_p = 0.24 BTU/lbm · °R, calculate

(a) exit total temperature T_{t2}
(b) compressor adiabatic efficiency η_c
(c) compressor specific work w_c
(d) shaft power $\wp_s$ for a 100 lbm/s flow rate

8.18 A multistage compressor develops a total pressure ratio π_c = 25, and is designed with eight identical (i.e., "repeated") stages. The compressor polytropic efficiency is e_c = 0.92. Calculate

(a) *average* stage total pressure ratio π_s
(b) stage adiabatic efficiency η_s
(c) compressor total temperature ratio τ_c

8.19 Plot compressor adiabatic efficiency η_c versus π_c ranging from 1.0 to 50, for the following polytropic efficiencies e_c = 0.95, 0.90, and 0.85 as the running parameter.

8.20 A rotor blade row has a solidity of 1.0 at its pitchline. The absolute flow enters the rotor with no preswirl at C_{z1m} = 500 fps, and the rotor rotational speed is U_m = 1200 fps (at the pitchline). If the rotor exit flow angle at r_m is $\beta_{2m} = -30°$, calculate

(a) exit swirl velocities $W_{\theta 2m}$ and $C_{\theta 2m}$
(b) blade torque (per unit mass flow rate) at the pitchline assuming r_m = 1.0 ft
(c) the nondimensional total temperature rise across the rotor $\Delta T_t/T_{t1}$
(d) exit speed of sound a_2 assuming inlet speed of sound is a_1 = 1100 fps
(e) exit absolute Mach number M_{2m}
(f) inlet static pressure p_1 for an inlet total pressure of p_{t1} = 14.7 psia
(g) exit total pressure, assuming rotor adiabatic efficiency is 0.9
(h) rotor static pressure rise, $C_{Pm} = \Delta p_m/(1/2\ \rho W_{1m}^2)$
(i) shaft rotational speed ω (rpm)
(j) rotor torque at r_m per unit mass flow rate
(k) the axial force on the blades at r_m, assuming the mean chord c_m = 4 in.
(l) tangential force on the blade at r_m
(m) sectional lift-to-drag ratio, L'/D'
(n) rotor specific work at r_m

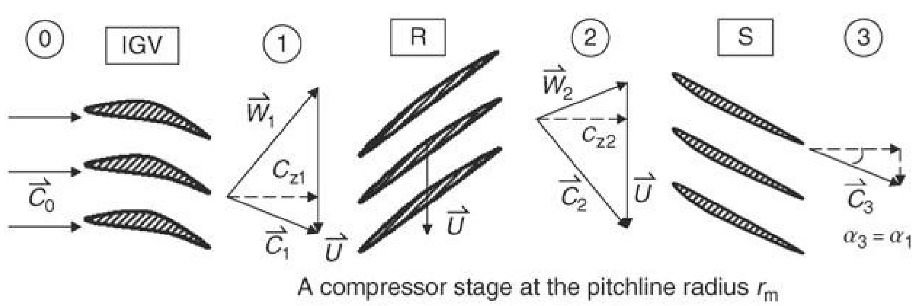
8.21 Calculate the Reynolds number based on chord at the pitchline for the rotor blade described in Problem 8.20, assuming fluid coefficient of viscosity is $\mu = 1.8 \times 10^{-5}$ kg/m · s. Compare the Reynolds number that you calculate to the upper critical Reynolds number in a compressor.

8.22 Calculate the circulation at the pitchline for the rotor blade row described in Problem 8.20. Also calculate the fraction of "ideal" lift that was destroyed by the total pressure losses in the blade row. Is there any indication of shock losses at the pitchline?

8.23 A rotor blade row has a hub-to-tip radius ratio of 0.5, solidity at the pitchline of 1.0, the axial velocity is 160 m/s, and zero preswirl. The mean section has a design diffusion factor of D_m = 0.5. Calculate and plot where appropriate

(a) exit swirl at the pitchline assuming the shaft rpm of 6000 and r_m = 1.0 ft (0.3 m)
(b) downstream swirl distribution $C_{\theta 2}$ (r) assuming a freevortex design rotor
(c) the radial distribution of degree of reaction °R along the blade span
(d) radial distribution of diffusion factor $D_r(r)$.

8.24 An axial-flow compressor stage is downstream of an IGV that turns the flow 15° in the direction of the rotor rotation, as shown. The axial velocity component remains constant throughout the stage at C_z = 150 m/s. The rotor rotational speed is ω = 3000 rpm and the pitchline radius is r_m = 0.5 m. The rotor relative exit flow angle is $\beta_2 = -15°$.



■ **FIGURE P8.24**

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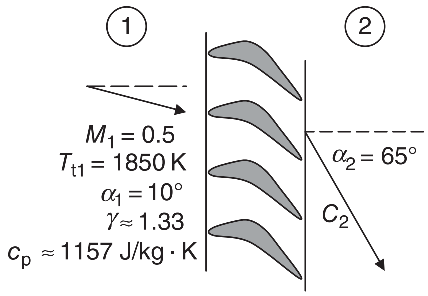
Problems

10.1 The combustor discharge into a turbine nozzle has a total temperature of 1850 K and inlet Mach number of 0.50, as shown.



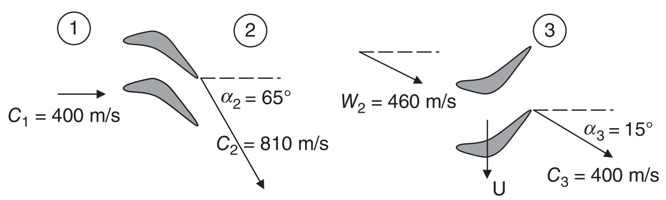
■ **FIGURE P10.1**

Assuming that the nozzle is uncooled, the axial velocity remains constant across the nozzle and the absolute flow angle at the nozzle exit is $\alpha_2 = 65°$, calculate

(a) inlet velocity C_1 in m/s
(b) the exit absolute Mach number M_2 and
(c) nozzle torque per unit mass flow rate for $r_1 \approx r_2 = 0.40$ m

10.2 Calculate the nozzle exit flow angle α_2 in Problem 10.1, if we wish the exit Mach number to be $M_2 = 1.0$, i.e., choked.

10.3 A turbine stage at pitchline has the following velocity vectors, as shown.



■ **FIGURE P10.3**

Calculate:

(a) the axial velocities up- and downstream of the rotor
(b) relative flow angle β_2 in degrees
(c) the rotor velocity U_m
(d) the degree of reaction at this radius
(e) rotor specific work, w_m in kJ/kg

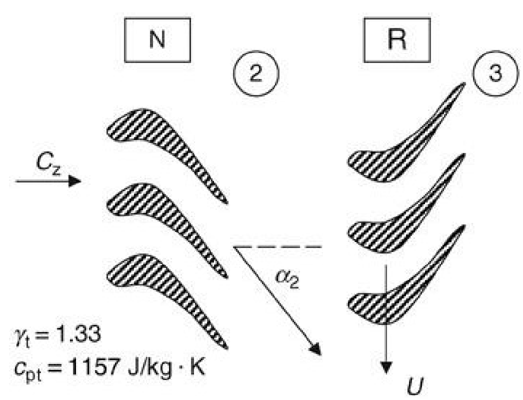
10.4 A turbine stage is designed with a constant axial velocity of 250 m/s and zero exit swirl. For a rotor rotational speed U_m at the pitchline of 600 m/s.
Calculate

(a) the nozzle exit flow angle, α_2 in degrees for $°R_m = 0.50$
(b) the nozzle exit flow angle, α_2 in degrees for $°R_m = 0.0$
(c) the rotor specific work at the pitchline radius, for $°R_m = 0.50$ and $°R_m = 0.0$

10.5 An axial-flow turbine nozzle turns the flow from an axial direction in the inlet to an exit flow angle of $\alpha_2 = 70°$. The rotor wheel speed is $U = 400$ m/s at the pitchline. The rotor is of impulse design and the exit flow from the rotor has zero swirl, i.e., $\alpha_3 = 0$. Calculate

(a) the rotor-specific work
(b) the stage loading at the pitchline

10.6 An axial-flow turbine stage at the pitchline is shown.
The flow entering and exiting the turbine stage is axial, i.e., $\alpha_1 = \alpha_3 = 0$



■ **FIGURE P10.6**

The nozzle exit flow is $\alpha_2 = 65°$. The shaft speed is $\omega = 5500$ rpm and the pitchline radius is $r_m = 50$ cm. Assuming $C_z = 250$ m/s = constant.
Calculate

(a) turbine-specific work w_t(kJ/kg)
(b) β_3 (degrees)
(c) $°R_m$

10.7 The combustor discharge total temperature and pressure are $T_{t1} = 2000$ K and $p_{t1} = 2$ MPa, respectively, with $\gamma_t = 1.30$ and $c_{pt} = 1244\,\mathrm{J/kg\cdot K}$. The flow speed is 400 m/s and is in the axial direction. Calculate

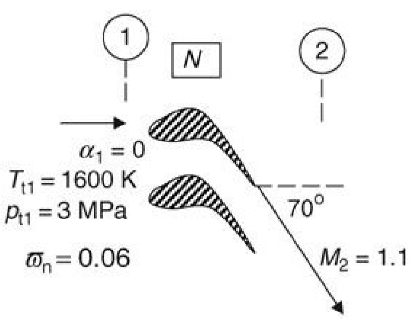
(a) the combustor exit static temperature T_1 in K
(b) the Mach number M_1
(c) the combustor exit static pressure p_1 (in kPa)

10.8 The inlet flow condition to a turbine nozzle is characterized by $T_{t1} = 1800$ K and $p_{t1} = 2.4$ MPa, $M_1 = 0.5$, $\alpha = 5°$, with $\gamma_t = 1.30$ and $c_{pt} = 1244\,\mathrm{J/kg\cdot K}$. Assuming the nozzle is designed for constant axial velocity, i.e., C_z = constant, calculate the nozzle exit flow angle α_2 that will produce the exit Mach number of $M_2 = 1.1$.

10.9 For the flow condition across a nozzle as shown, calculate

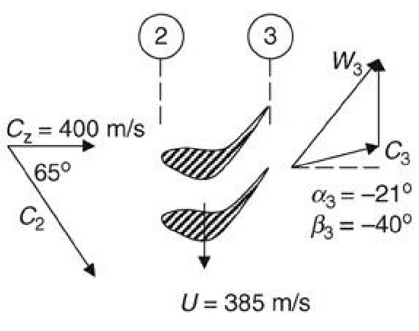
(a) T_2 in K
(b) C_{z2} in m/s
(c) $C_{\theta 2}$ in m/s
(d) M_1
(e) p_{t2} in MPa
(f) p_2 in kPa

Assume C_z = constant, $\gamma_t = 1.30$, and $c_{pt} = 1244\,\mathrm{J/kg\cdot K}$.



■ **FIGURE P10.9**

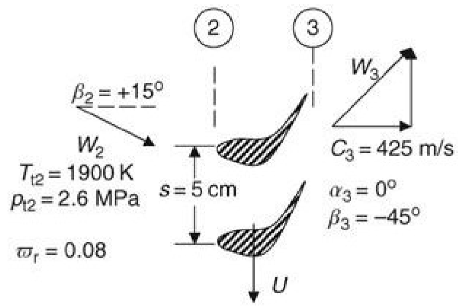
10.10 The velocity triangles across a turbine rotor are shown. The axial velocity remains constant across the rotor. If the total temperature at the rotor inlet is 1850 K with $\gamma_t = 1.30$ and $c_{pt} = 1244\,\mathrm{J/kg\cdot K}$, calculate



■ **FIGURE P10.10**

(a) inlet absolute Mach number M_2
(b) inlet relative Mach number M_{2r}
(c) the degree of reaction °R
(d) rotor specific work w_t in kJ/kg
(e) exit static temperature T_3(K)
(f) exit relative Mach number M_{3r}.

10.11 A rotor is designed for constant axial velocity. The velocity triangles are as shown. The rotor total pressure loss coefficient is known to be 0.08 with $\gamma_t = 1.30$ and $R_t = 287\,\mathrm{J/kg\cdot K}$.



■ **FIGURE P10.11**

Calculate

(a) rotor speed U in m/s
(b) rotor specific work w_t in kJ/kg
(c) stage degree of reaction °R
(d) rotor circulation Γ in m^2/s
(e) inlet absolute Mach number M_2
(f) inlet gas static density ρ_2 in kg/m^3
(g) exit relative Mach number M_{3r}
(h) exit total pressure p_{t3} in MPa
(i) exit static density ρ_3 in kg/m^3.

10.12 Coolant air is bled from a compressor exit at $T_{tc} = 800$ K with $c_{pc} = 1004\,\mathrm{J/kg\cdot K}$. The coolant is given a (positive) preswirl before it enters the rotor blade root in the direction of the rotor rotation. Assuming the port of coolant entry into the rotor is at $r_c = 42$ cm with rotor angular speed $\omega = 12000$ rpm, and the coolant enters the rotor blade root axially, as shown in Figure 10.64, calculate the coolant relative total temperature as it enters the rotor blade.

10.13 A multistage turbine is to be designed with constant axial velocity. The total temperature ratio across the entire turbine is $\tau_t = 0.72$ and the turbine polytropic efficiency is $e_t = 0.85$ with $\gamma_t = 1.30$ and $c_{pt} = 1244\,\mathrm{J/kg\cdot K}$. Calculate

(a) The turbine total pressure ratio π_t
(b) the turbine adiabatic efficiency η_t
(c) turbine area ratio, A_5/A_4, for choked inlet, i.e., $M_4 = 1.0$, $T_{t4} = 1675$ K and $C_z = 445$ m/s and swirl-free exit flow

Assume γ and R remain constant across the turbine.