

AE 339 : High speed aerodynamics

(VI) External aerodynamics: Transonic flow
past bodies

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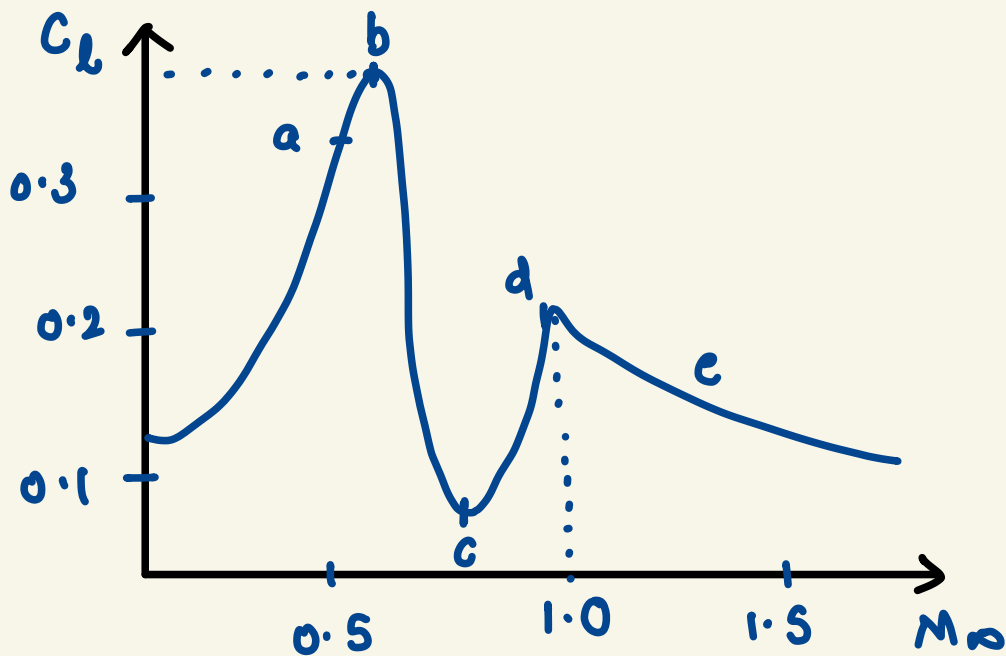
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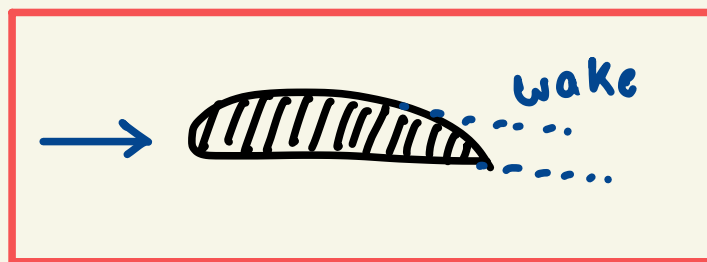
Transonic flow past airfoils

There are complex changes in the flow-field through the transonic speed range.

Lift coefficient

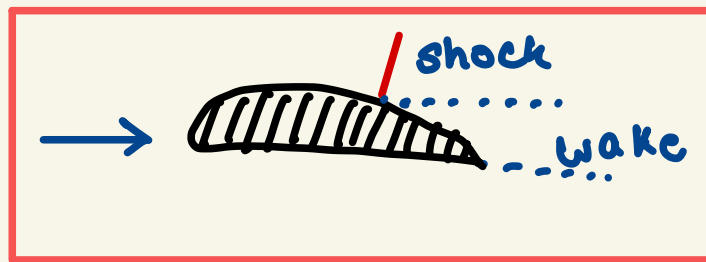


(a) $M_\infty = M_a$



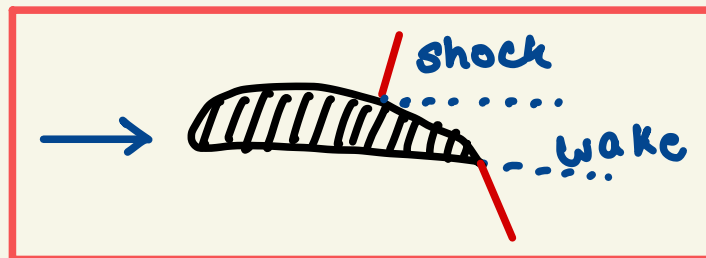
The flow past the upper surface decelerates from local flow velocities which are supersonic without a shock wave. Section lift coefficient is approximately 60% greater than the low speed values at same α .

(b) $M_\infty = M_b$



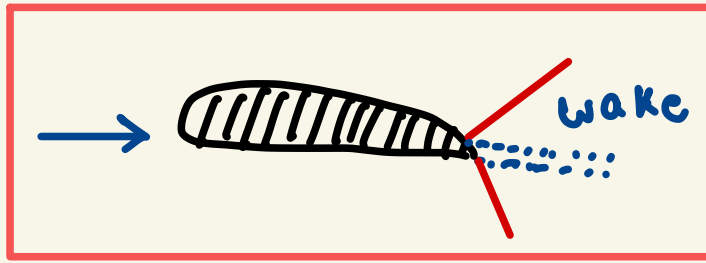
C_L reaches maximum value, approximately twice the low speed value. Flow is supersonic over 70% of surface, terminating in a shock wave. The flow on the lower surface is subsonic everywhere. As viscous flow separates at the shock, wake is wider.

(c) $M_\infty = M_c$



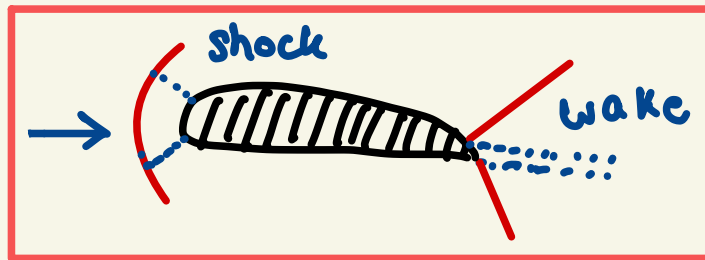
The flow is supersonic over nearly the entire lower surface and deceleration to subsonic speed occurs through a shock wave at the trailing edge \Rightarrow lower surface pressures less than at $M_\infty = M_b$. Pressure on upper surface greater than lower near trailing edge. Lift is drastically reduced

(d) $M_\infty = M_d$



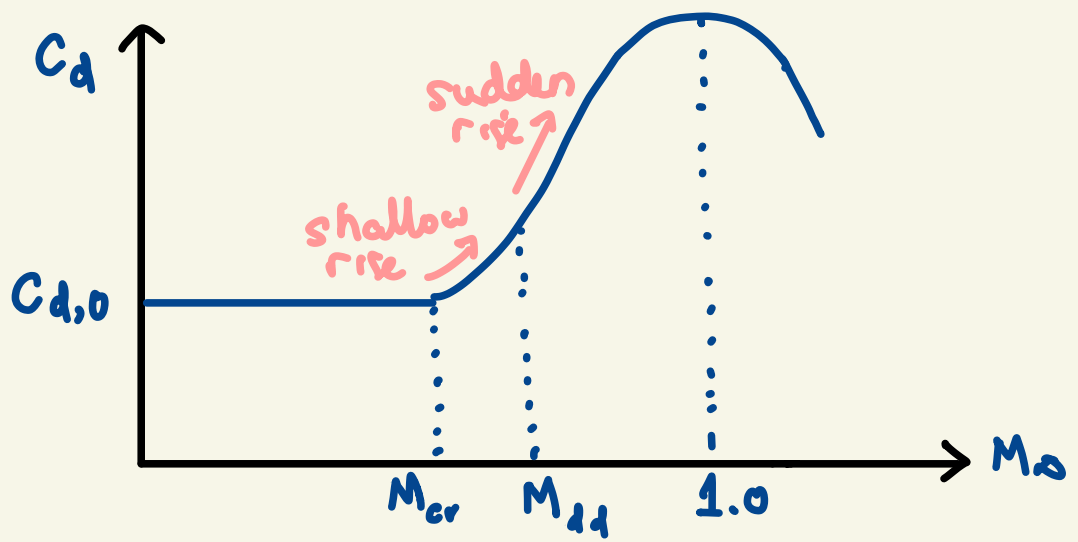
Both shock waves have reached the trailing edge. The local Mach number is supersonic for most of the airfoil.

(e) $M_\infty = M_c$



A bow shock wave is generated. The flow around the airfoil is supersonic everywhere except very near the rounded nose. Trailing edge shocks become weaker.

Drag coefficient



Let $C_{d,0}$ be the drag coefficient at subsonic speed. We have $C_d \approx C_{d,0}$ for $M \leq M_{cr}$.

M_{cr} (Critical Mach number) is the lowest freestream Mach number for which the maximum value of the local velocity first becomes sonic.

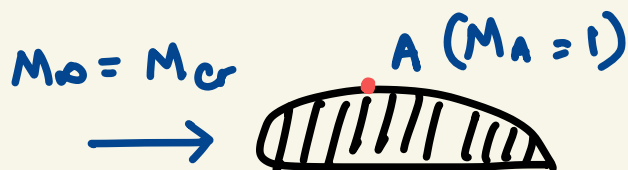
For $M > M_{cr}$, we have transonic flows with mixed subsonic / supersonic flow-fields.

The value of M_∞ at which a sudden rise in drag is observed is defined as the drag divergence

Mach number (M_{dd})

The large increase in drag is associated with shock waves that cause severe flow separation.

Estimation of M_{cr}



$$\frac{p_A}{p_\infty} = \frac{p_A / p_0}{p_\infty / p_0} = \left[\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M_A^2} \right]^{\frac{\gamma}{\gamma-1}}$$

$$C_{p,A} = \frac{2}{\gamma M_\infty^2} \left[\frac{p_A}{p_\infty} - 1 \right]$$

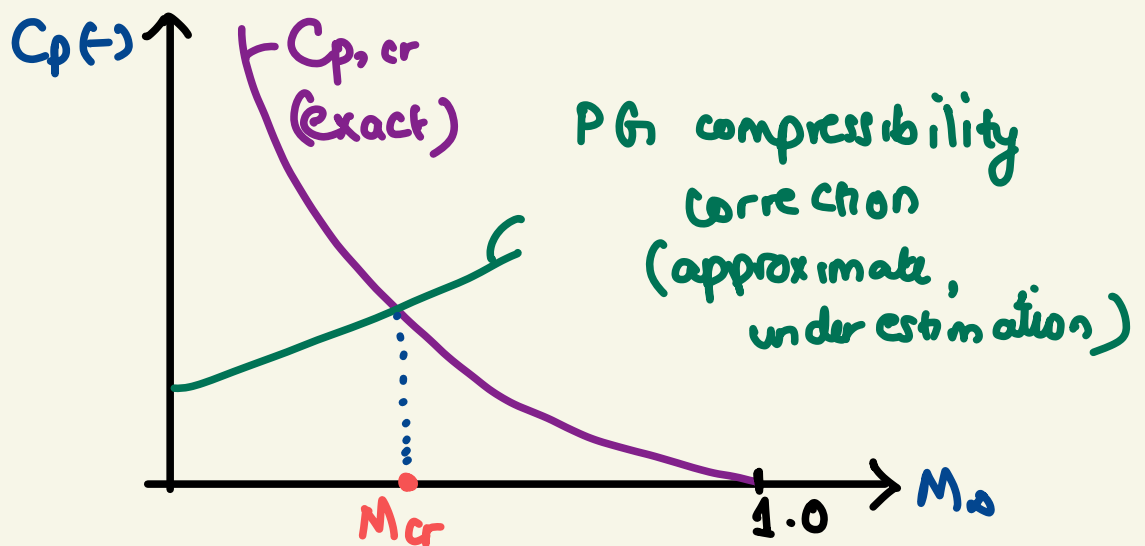
$$= \frac{2}{\gamma M_\infty^2} \left[\left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M_A^2} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right]$$

Local value of C_p depends only on local Mach number for given fluid and M_∞

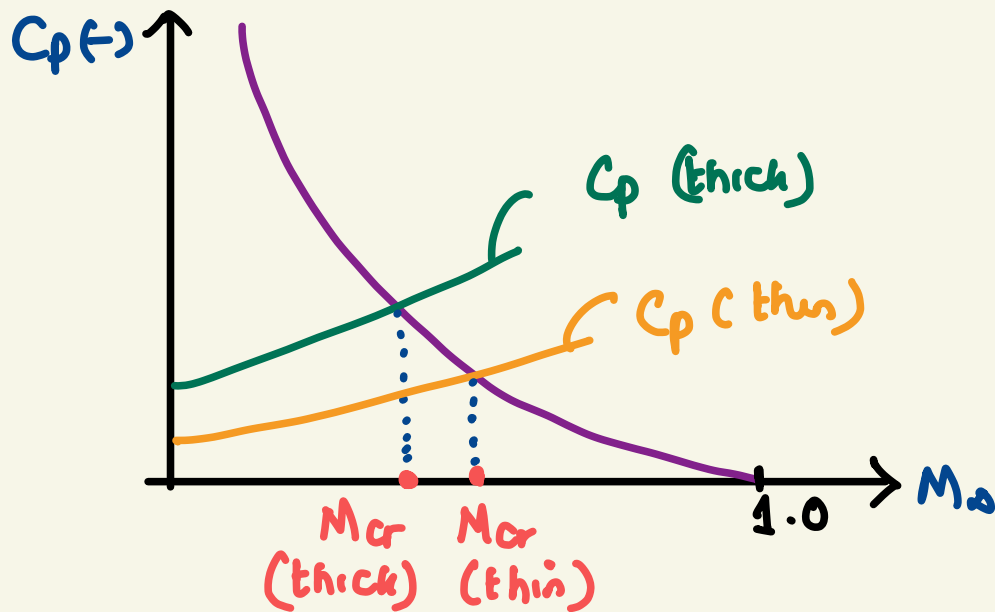
If $M_\infty = M_{cr}$, $M_A = 1$ and $C_p = C_{p,cr}$ (critical pressure coefficient)

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2} \left[\left(\frac{1 + \frac{\gamma-1}{2} M_\infty^2}{\frac{\gamma+1}{2}} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right] \quad - (*)$$

This is simply an aerodynamic relation for an isentropic flow and has no connection with the shape of the airfoil.



Thin vs thick airfoils



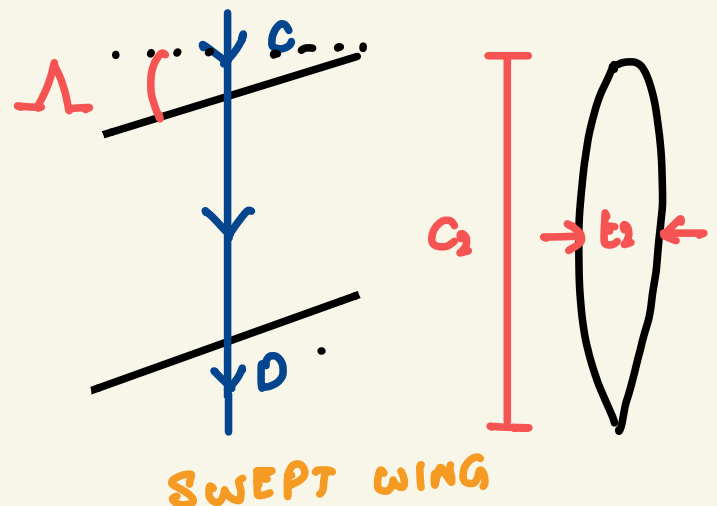
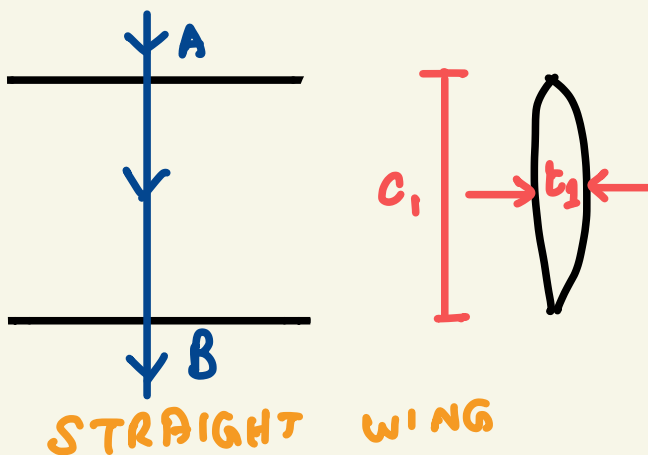
Thin airfoil \Rightarrow smaller $\hat{u} \Rightarrow$ smaller C_p

Thick airfoil has a lower Ma_{cr} than the thin airfoil. Hence, modern high-speed subsonic airplanes have relatively thin airfoils.

However, airfoil requires a certain thickness for

1. structural strength
2. storage of fuel.

Swept wings (Busemann 1935)



$$C_2 = \frac{C_1}{\cos \Lambda} > C_1 \quad \text{and} \quad t_2 = t_1$$

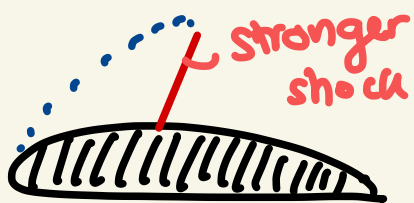
$$\Rightarrow \frac{t_2}{C_2} < \frac{t_1}{C_1} \quad \text{effectively a thinner airfoil}$$

By sweeping the wing, the flow behaves as if the airfoil section is thinner, with a consequent increase in M_{cr} .

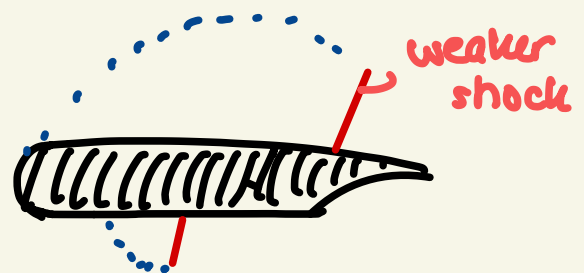
↳ delays the formation of shock waves thus delaying transonic drag rise.

Supercritical airfoil sections (Whitcomb 1965)

These airfoil sections increase the Mach number increment between M_{cr} and M_{dd} .



NACA 64-SERIES
($M_{cr} = 0.69$)



SUPERCritical AIRFOIL
($M_{cr} = 0.79$)

The supercritical airfoil has a relatively flat top, resulting in a weaker terminating shock
 \Rightarrow higher M_{dd}

Forward 60% of supercritical airfoils typically have -ve camber which lowers the lift. To compensate, lift is increased by having extreme positive camber on the rearward 30% of the airfoil.

The area rule (Whitcomb)

To reduce peak drag near Mach 1, the variation of cross-sectional area for the airplane should be smooth with no discontinuities

