From Figure 4.3, we also note that the states (t0) and (0) are separated *isentropically* by the amount of $V_0^2/2$, as expected from the definition of stagnation state and its relation to the static state. The states (t2s) and (0) are separated isentropically by a kinetic energy amount, which produces p_{t2} as its stagnation state and obviously the rest of the kinetic energy, that is, the gap between the states (t0) and (t2s), represents the amount of dissipated kinetic energy into heat. Therefore, the smaller the gap between the states (t2s) and (t0), the more efficient will the diffuser flow process be. Consequently, we use the fictitious state (t2s) in a definition of inlet efficiency (or a figure of merit) known as the *inlet adiabatic efficiency*. Symbolically, the inlet adiabatic efficiency is defined as

$$\eta_d \equiv \frac{h_{t2s} - h_0}{h_{t2} - h_0} = \frac{(V^2/2)_{ideal}}{V_0^2/2} \tag{4.3}$$

The practical form of the above definition is derived when we divide the numerator and denominator by h_0 to get

$$\eta_d = \frac{\dfrac{h_{t2s}}{h_0} - 1}{\dfrac{h_{t2}}{h_0} - 1} = \frac{\dfrac{T_{t2s}}{T_0} - 1}{\dfrac{h_{t0}}{h_0} - 1} = \frac{\left(\dfrac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma-1}{2}M_0^2} = \frac{\left(\dfrac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\tau_r - 1} \tag{4.4}$$

where we have used the isentropic relation between the states (t2s) and (0). Note that the only unknown in Equation 4.4 is p_{t2} for a given flight altitude p_0, flight Mach number M_0, and an inlet adiabatic efficiency η_d. We can separate the unknown term p_{t2} and write the following expression:

$$\frac{p_{t2}}{p_0} = \left\{ 1 + \eta_d \frac{\gamma-1}{2} M_0^2 \right\}^{\frac{\gamma}{\gamma-1}} \tag{4.5}$$

It is interesting to note that Equation 4.5 recovers the isentropic relation for a 100% efficient inlet or $\eta_d = 1.0$. Another parameter, or a *figure of merit*, that describes the inlet performance is the total pressure ratio between the compressor face and the (total) flight condition. This is given a symbol π_d and is often referred to as the *inlet total pressure recovery*:

$$\pi_d \equiv \frac{p_{t2}}{p_{t0}} \tag{4.6}$$

As expected, the two figures of merit for an inlet, that is, η_d or π_d, are not independent from each other and we can derive a relationship between η_d and π_d working the left-hand side of Equation 4.5, as follows:

$$\frac{p_{t2}}{p_0} = \frac{p_{t2}}{p_{t0}} \frac{p_{t0}}{p_0} = \left\{ 1 + \eta_d \frac{\gamma-1}{2} M_0^2 \right\}^{\frac{\gamma}{\gamma-1}} \tag{4.6a}$$

$$\pi_d = \frac{\left\{ 1 + \eta_d \dfrac{\gamma-1}{2} M_0^2 \right\}^{\frac{\gamma}{\gamma-1}}}{\dfrac{p_{t0}}{p_0}} = \left\{ \frac{1 + \eta_d \dfrac{\gamma-1}{2} M_0^2}{1 + \dfrac{\gamma-1}{2} M_0^2} \right\}^{\frac{\gamma}{\gamma-1}} = \frac{\left[1 + \eta_d \dfrac{\gamma-1}{2} M_0 \right]^{\frac{\gamma}{\gamma-1}}}{\pi_r} \tag{4.6b}$$

Brayton cycle is the net output of the cycle. To put it differently, in an ideal cycle, the difference between the power production and consumption can be entirely converted into useful cycle power output. Now, let us manipulate Equation 4.68 in order to cast it in terms of useful nondimensional quantities, that is,

$$
\begin{aligned}
w_c &\cong h_{t4}\left(1 - \frac{T_9}{T_{t4}}\right) - h_{t3}\left(1 - \frac{T_0}{T_{t3}}\right) \\
&= h_{t4}\left\{1 - \left(\frac{p_9}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}}\right\} - h_{t3}\left\{1 - \left(\frac{p_0}{p_{t3}}\right)^{\frac{\gamma-1}{\gamma}}\right\}
\end{aligned}
\tag{4.69}
$$

But since $p_9 = p_0$ and $p_{t4} = p_{t3}$ in an ideal cycle with a perfectly expanded nozzle, we can rewrite Equation 4.69 in the following form;

$$
\begin{aligned}
w_c &\cong (h_{t4} - h_{t3})\left(1 - \frac{T_0}{T_{t3}}\right) = h_0\left(\tau_\lambda - \tau_c\tau_r\right)\left(1 - \frac{1}{\tau_c\tau_r}\right) \\
&= h_0\left(\tau_\lambda - \frac{\tau_\lambda}{\tau_r\tau_c} - \tau_r\tau_c + 1\right)
\end{aligned}
\tag{4.70}
$$

where $\tau_\lambda \equiv h_{t4}/h_0$ is the engine thermal limit parameter, $\tau_r \equiv T_{t0}/T_0 = 1 + \gamma - 1/2\,M_0^2$ is the ram temperature ratio, and $\tau_c \equiv T_{t3}/T_{t2}$ is the compressor temperature ratio. To find a maximum value for the specific work of the cycle, for a constant turbine inlet temperature, that is, τ_λ, we may treat the product $\tau_r\tau_c$ as one and only variable in Equation 4.70. Therefore, we can differentiate w_c/h_0 with respect to $\tau_r\tau_c$ and set it equal to zero,

$$
\frac{d\left(\dfrac{w_c}{h_0}\right)}{d(\tau_r\tau_c)} = \frac{\tau_\lambda}{(\tau_r\tau_c)^2} - 1 = 0
\tag{4.71}
$$

which implies that

$$
\tau_r\tau_c = \sqrt{\tau_\lambda}
\tag{4.72}
$$

Therefore, the product $\tau_r\tau_c$, which satisfies Equation 4.72, yields the *largest area under the curve* in the *T–s* diagram of the Brayton cycle. The area within the cycle walls, on a *T–s* diagram, we learned in thermodynamics that represents the net cycle specific work. We can isolate the compressor contribution to the temperature ratio as

$$
\tau_c = \frac{\sqrt{\tau_\lambda}}{\tau_r} = \frac{\sqrt{\tau_\lambda}}{1 + \dfrac{\gamma - 1}{2}M_0^2}
\tag{4.73}
$$

Equation 4.73 is useful in telling us that for a given burner temperature, the compressor pressure ratio requirement for the maximum cycle output (i.e., the *optimum* compressor