Compressible Flow Calculator

High Speed Aerodynamics

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October 14, 2024

1 Isentropic Flow Calculator

These are the formula used to calculating the parameters for Isentropic flow:

$$\frac{p}{p_t} = \left(\frac{\rho}{\rho_t}\right)^{\gamma} = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}} \tag{1}$$

$$\frac{p}{p_t} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{2}$$

$$\frac{T}{T_t} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1} \tag{3}$$

$$\frac{\rho}{\rho_t} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{1}{\gamma-1}} \tag{4}$$

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5}$$

$$\frac{p}{p^*} = 0.528 \tag{6}$$

$$\frac{\rho}{\rho^*} = 0.634 \tag{7}$$

$$\frac{T}{T^*} = 0.833 \tag{8}$$

Where, M is the local Mach number of the gas.
γ is the ratio of specific heats of the gas.
p is the pressure of the gas (Pa).
ρ is the density of the gas (kg/m^3).
T is the temperature of the gas (K).
A is the cross sectional area of the nozzle at the point of interest (m^2).
A* is the cross sectional area of the nozzle at the sonic point, or the point where gas velocity is Mach 1 (m^2). Ideally this will occur at the nozzle throat.
p*, $\rho*$ and T* are the critical pressure, density and temperature at sonic point (M=1).

Important Points:

- $\frac{p}{p_t} < 1$: The static pressure is always less than or equal to the stagnation pressure.
- $\frac{\rho}{\rho_t} < 1$: The static density is less than or equal to the stagnation density.
- $\frac{T}{T_t} < 1$: The static temperature is less than or equal to the stagnation temperature.
- $\frac{A}{A^*}$:
  - > 1 for $M < 1$ (subsonic flow).
  - $= 1$ for $M = 1$ (sonic flow).
  - < 1 for $M > 1$ (supersonic flow).

2 Normal Shock Flow Calculator

. These are the formula used to calculating the parameters for Normal Shock:

$$M_2 = \sqrt{\frac{M_1^2(\gamma-1)+2}{2\gamma M_1^2-(\gamma-1)}} \tag{9}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2-(\gamma-1)}{\gamma+1} \tag{10}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \tag{11}$$

$$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)}{M_1^2\left(\frac{2\gamma}{\gamma-1}+\frac{\gamma-1}{2}\right)} \tag{12}$$

$$\frac{p_{02}}{p_{01}} = \left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{1}{\frac{2\gamma M_1^2}{\gamma+1}-\frac{\gamma-1}{\gamma+1}}\right)^{\frac{1}{\gamma-1}} \tag{13}$$

Where, M_1 is the upstream Mach number.
M_2 is the post normal shock Mach number.
γ is ratio of specific heats.
p_2 is the post normal shock pressure.
p_1 is the upstream ambient pressure.
ρ_2 is the post normal shock density.
ρ_1 is the upstream ambient density.
T_2 is post normal shock temperature.
T_1 is the upstream ambient temperature.
p_{01} is the upstream stagnation pressure.
p_{02} is the post normal stagnation pressure.

Important Points:

- $M_1 > 1$: The upstream Mach number must be greater than 1 for a normal shock to occur.
- $M_2 < 1$: The post-shock Mach number must be less than 1.
- $\frac{p_2}{p_1} > 1$: The pressure increases after the shock.
- $\frac{\rho_2}{\rho_1} > 1$: The density increases after the shock.
- $\frac{T_2}{T_1} > 1$: The temperature increases after the shock.
- $\frac{p_{02}}{p_{01}} < 1$: The stagnation pressure decreases across the shock.

3 Oblique Shock Flow Calculator

These are the formula used to calculating the parameters for Oblique Shock:

$$M_{n1} = M\sin(s) \tag{14}$$

$$\frac{T_2}{T_1} = \frac{(2\gamma M_{n1}^2 - (\gamma-1))((\gamma-1)M_{n1}^2 + 2)}{((\gamma+1)M_{n1})^2} \tag{15}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_{n1}^2 - (\gamma-1)}{\gamma+1} \tag{16}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2 + 2} \tag{17}$$

$$\frac{P_{t2}}{P_{t1}} = \left(\frac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2 + 2}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{\gamma+1}{2\gamma M_{n1}^2 - (\gamma-1)}\right)^{\frac{1}{\gamma-1}} \tag{18}$$

$$a = \arcsin\left(\frac{M_{n1}}{M}\right) \tag{19}$$

Where, M is upstream Mach number.
M_{n1} is the Normal Mach number before the shock.
s is the shock angle in degrees.
a is the deflection angle in degrees.
γ is ratio of specific heats.
p_2 is the post oblique shock pressure.
p_1 is the upstream ambient pressure.
ρ_2 is the post oblique shock density.
ρ_1 is the upstream ambient density.
T_2 is post oblique shock temperature.
T_1 is the upstream ambient temperature.
p_{01} is the upstream stagnation pressure.
p_{02} is the post oblique shock stagnation pressure.

Important Points:

- $M > 1$: The upstream Mach number must be greater than 1 for an oblique shock.
- $M_{n1} \leq M$: The normal Mach number before the shock is always less than or equal to the upstream Mach number.
- $\frac{p_2}{p_1} > 1$: The pressure increases after the oblique shock.
- $\frac{\rho_2}{\rho_1} > 1$: The density increases after the oblique shock.
- $\frac{T_2}{T_1} > 1$: The temperature increases after the oblique shock.
- $\frac{p_{02}}{p_{01}} < 1$: The stagnation pressure decreases across the oblique shock.