Lateral – Directional Dynamics

$$\Delta\dot{v} = Y_v\Delta v + Y_p\Delta p - \left(u_0 - Y_r\right)\Delta r + \left(g\cos\theta_0\right)\Delta\phi + Y_{\delta r}\delta_r$$

$$\Delta\dot{p} = L_v\Delta v + L_p\Delta p + L_r\Delta r + \left({I_{zx}}\middle/{I_{xx}}\right)\Delta\dot{r} + L_{\delta a}\Delta\delta_a + L_{\delta r}\Delta\delta_r$$

$$\Delta\dot{r} = N_v\Delta v + N_p\Delta p + N_r\Delta r + \left({I_{zx}}\middle/{I_{zz}}\right)\Delta\dot{p} + N_{\delta a}\Delta\delta_a + N_{\delta r}\Delta\delta_r$$

$$\Delta\dot{\phi} = \Delta p$$

$$\Delta\dot{\psi} = \Delta r$$

$$\Delta\dot{y}_I = \Delta v$$

4^{th} Order Simplified Model

$$\begin{Bmatrix}\Delta\dot{v}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\phi}\end{Bmatrix}=\begin{bmatrix}Y_v & Y_p & -(u_0-Y_r) & g\cos\theta_0\\ L_v & L_p & L_r & 0\\ N_v & N_p & N_r & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta v\\ \Delta p\\ \Delta r\\ \Delta\phi\end{Bmatrix}+\begin{bmatrix}0 & Y_{\delta r}\\ L_{\delta a} & L_{\delta r}\\ N_{\delta a} & N_{\delta r}\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta\delta_a\\ \Delta\delta_r\end{Bmatrix};\quad\left(\frac{I_{xz}}{I_{xx}},\frac{I_{xz}}{I_{zz}}\approx 0\right)$$

$$\begin{Bmatrix}\Delta\dot{\beta}\\ \Delta\dot{p}\\ \Delta\dot{r}\\ \Delta\dot{\phi}\end{Bmatrix}=\begin{bmatrix}\frac{Y_v}{u_0} & \frac{Y_p}{u_0} & -\left(1-\frac{Y_r}{u_0}\right) & \frac{g\cos\theta_0}{u_0}\\ L_v & L_p & L_r & 0\\ N_v & N_p & N_r & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta\beta\\ \Delta p\\ \Delta r\\ \Delta\phi\end{Bmatrix}+\begin{bmatrix}0 & \frac{Y_{\delta r}}{u_0}\\ L_{\delta a} & L_{\delta r}\\ N_{\delta a} & N_{\delta r}\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\Delta\delta_a\\ \Delta\delta_r\end{Bmatrix}$$

'Δϕ' and 'Δβ' coupling is weak as u_0 is a large value and, it is **also** more convenient to replace 'Δ**v**' with 'Δβ', as shown above.

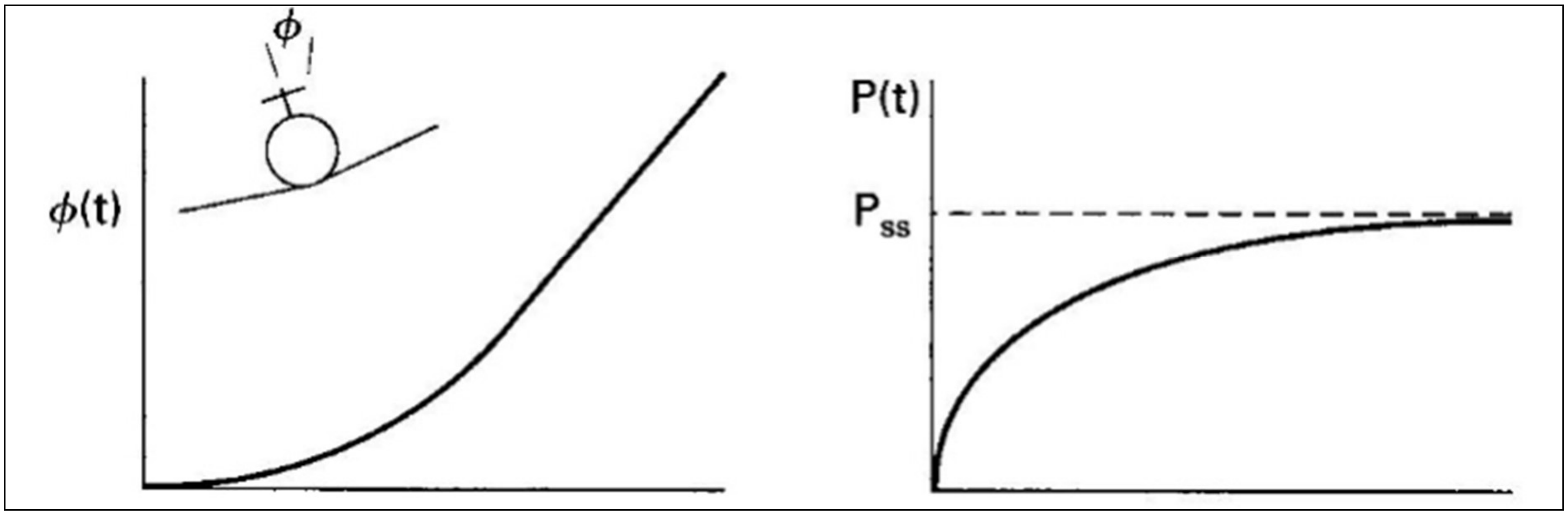
Lateral-Directional Dynamics Example

$$
\begin{aligned}
&Y_u = -0.254 / s, \quad Y_\beta = -45.72\, ft / s^2, \quad Y_p = 0 \\
&Y_r = 0, \quad L_v = -0.091 (ft / s)^{-1}, \quad L_\beta = -16.02 / s^2 \\
&L_p = -8.4 / s, \quad L_r = 2.19 / s, \quad N_v = 0.025 (ft / s)^{-1} \\
&N_\beta = 4.49 / s^2; \quad N_p = -0.35 / s, \quad N_r = -0.76 / s \\
&u_0 = 171\, ft / s; \quad \theta_0 = 0; \quad g = 32.2\, ft / s^2
\end{aligned}
$$

$$
[A] = \begin{bmatrix} -0.254 & 0.0000 & -1.0000 & 0.1820 \\ -16.02 & -8.40 & 2.1900 & 0.0000 \\ 4.4880 & -0.350 & -0.7600 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \end{bmatrix}
$$

$$
\begin{aligned}
&-0.00877, \ -8.435, \\
&-0.487 \pm j2.335
\end{aligned}
$$

Roll Subsidence Mode

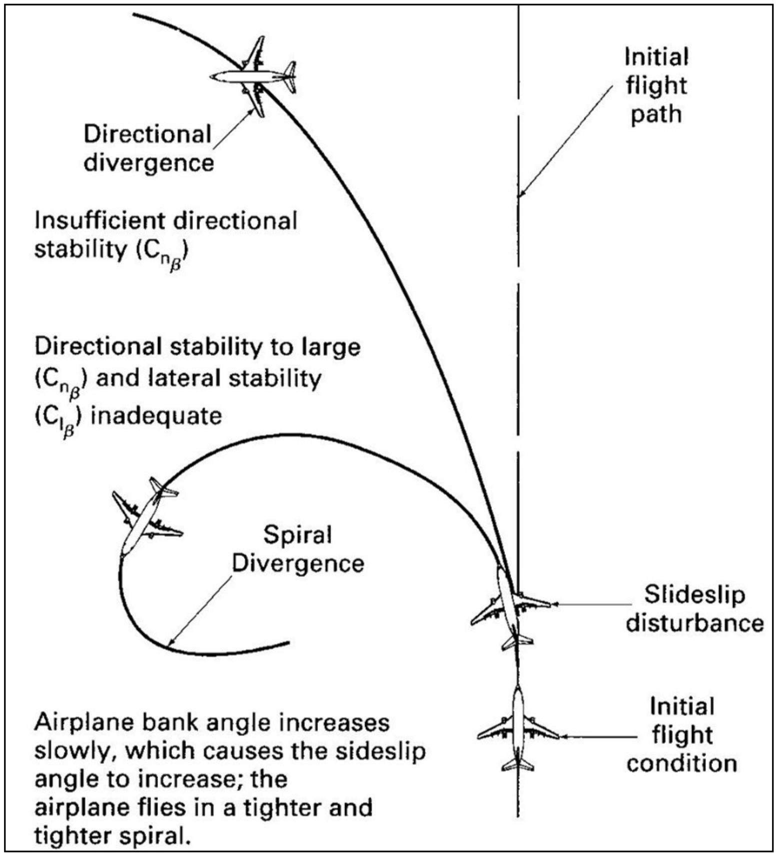


Directional Divergence and Spiral Motion

Figure alongside shows two instances of impact of **weak** and strong directional stability, coupled with weak **roll** stability.

Directional divergence is a result of insufficient $C_{N\beta}$, **that** results in large 'β'.

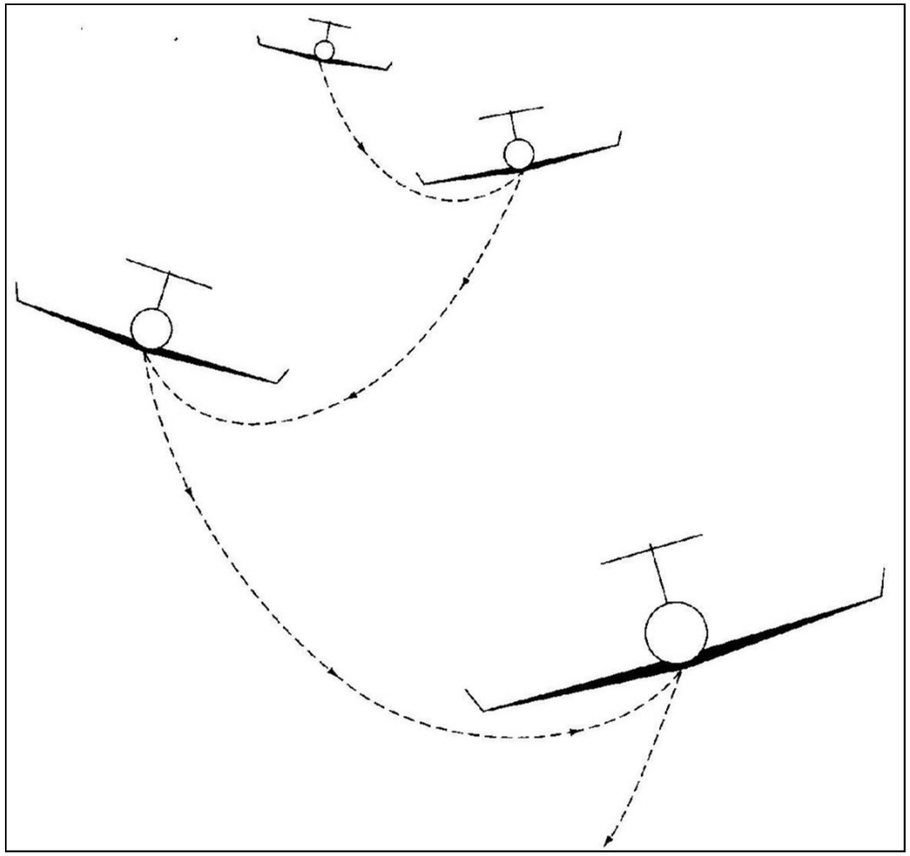
On the other hand, spiral mode is a result of **large** positive $C_{N\beta}$ and insufficient $C_{l\beta}$.



Dutch Roll Motion of Aircraft

In Dutch roll motion, there is a significant yaw **along** with side-slipping, and some small amount of **roll** and name comes from the fact that motion is **somewhat** similar to the movements of an ice-skater**.**

In aircraft, dutch-roll mode appears as shown **alongside**.



Dutch Roll Mode Description

Dutch-roll occurs when there is a weak directional **stability** combined with strong **lateral** stability.

Generally, the motion starts with a roll **disturbance** that creates side-slipping to one **side**, which the aircraft tries to **restore**, due to directional stability.

However, as roll stability is higher, aircraft becomes **wings-**level faster, but the corrective **side-slip** action lags behind and **continues** to side-slip in the other direction.

Lateral-Directional Approximations – Roll Mode

Roll subsidence can be approximated by **assuming** that '$\Delta\beta$' & 'Δr' motions are **negligible** so that roll **acceleration** is only due to rolling moment caused by 'Δp'.

$$\tau\Delta\dot{p} + \Delta p = 0, \quad \tau = -\frac{1}{\lambda_{roll}} = -\frac{1}{L_p}$$

It is seen that a large value of 'L_p' results in small **value** of 'τ', which is a desirable **feature** for combat aircraft.

Lateral-Directional Approximations – Spiral

Spiral approximation is based on the premise that **motion** predominantly involves roll angle '$\Delta\phi$', along with **large** changes in yaw angle '$\Delta\psi$'.

Further, it is found that though '$\Delta\beta$' is small, it **cannot** be neglected as all aerodynamic moments **depend** on it.

Thus, while we ignore 'ΔY', '$\Delta\phi$', equations, **moments** due to '$\Delta\beta$' and 'Δr' are included as shown **alongside**.

$$L_\beta \Delta\beta + L_r \Delta r = 0$$

$$\Delta\dot{r} = N_\beta \Delta\beta + N_r \Delta r$$

$$\Delta\dot{r} + \frac{L_r N_\beta - L_\beta N_r}{L_\beta} \Delta r = 0$$

$$\lambda_{Spiral} = -\frac{L_r N_\beta - L_\beta N_r}{L_\beta}$$

Spiral Approximation Motion Features

It is to be noted that for most aircraft, L_β and N_r are **both** negative, while N_β and L_r are both positive**.**

In such a case, we can find the condition for a stable **spiral** mode as shown alongside.

We can see that aircraft with large dihedral and **small** directional stability will have a stable spiral **mode**.

$$-\frac{L_r N_\beta - L_\beta N_r}{L_\beta} < 0; \quad (L_\beta < 0)$$

$$L_r N_\beta - L_\beta N_r < 0 \rightarrow L_\beta N_r > L_r N_\beta$$

Dutch Roll Approximation

As Dutch roll consists mainly of side-**slipping** and yawing motion, we can neglect the **roll** dynamics completely and obtain the a **2nd** order system, as shown alongside.

$$\begin{Bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{Bmatrix} = \begin{bmatrix} \dfrac{Y_\beta}{u_0} & -\left(1 - \dfrac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{Bmatrix} \Delta\beta \\ \Delta r \end{Bmatrix}; \quad D(\lambda) = \det(\lambda I - A) = 0$$

$$\lambda^2 - \left(\frac{Y_\beta + u_0 N_r}{u_0}\right)\lambda + \frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0} = 0$$

$$\omega_{nDR} = \sqrt{\frac{Y_\beta N_r - N_\beta Y_r + u_0 N_\beta}{u_0}}, \quad \zeta_{DR} = -\frac{1}{2\omega_{nDR}}\left(\frac{Y_\beta + u_0 N_r}{u_0}\right)$$