that the gas speed starts at almost zero, since the captured flow area A_0 is fairly large, and continues to grow to the engine face speed of V_2.

In summary, we have learned that

- the inlet flow may be considered to be adiabatic, that is, $h_{t2} = h_{t0}$
- the inlet flow is always irreversible, that is, $p_{t2} < p_{t0}$, with viscous dissipation in the boundary layer and in a shock as the sources of irreversibility ($s_2 > s_0$)
- there are two figures of merit that describe the extent of *losses* in the inlet and these are η_d and π_d
- the two figures of merit are related (via Equation 4.6b)
- in cruise, $A_0 < A_2$ and hence a diffusing passage and at low speed or takeoff, $A_0 > A_2$, that is, a nozzle
- the outer nacelle geometry of an inlet dictates the drag divergence and high angle of attack characteristics of the inlet and is crucial for the installed performance.

EXAMPLE 4.1

An aircraft is flying at an altitude where the ambient static pressure is $p_0 = 10$ kPa and the flight Mach number is $M_0 = 0.85$. The total pressure at the engine face is measured to be $p_{t2} = 15.88$ kPa. Assuming the inlet is adiabatic and $\gamma = 1.4$, calculate

(a) the inlet total pressure recovery π_d
(b) the inlet adiabatic efficiency η_d
(c) the nondimensional entropy rise caused by the inlet $\Delta s_d/R$

SOLUTION

We first calculate the flight total pressure p_{t0}, and from definition of π_d (i.e., Equation 4.6), the inlet total pressure recovery.

$$p_{t0} = p_0[1 + (\gamma - 1)M_0^2/2]^{\frac{\gamma}{\gamma-1}} = 10\,\text{kPa}[1 + 0.2(0.85)^2]^{3.5}$$
$$= 16.04\,\text{kPa}$$
$$\pi_d \equiv p_{t2}/p_{t0} = 15.88/16.04 = 0.990$$

Inlet adiabatic efficiency η_d is calculated from Equation 4.4

$$\eta_d = \left[\left(\frac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \Bigg/ \left(\frac{\gamma-1}{2}M_0^2\right)$$
$$= [(15.88/10)^{0.2857} - 1]/[0.2(0.85)^2] \cong 0.9775$$

The entropy rise is linked to the inlet total pressure loss parameter π_d via Equation 4.2,

$$\Delta s_d/R = -\ln(\pi_d) = -\ln(0.99) \cong 0.010$$

4.3.1.2 *The Compressor.*

The thermodynamic process in a gas generator begins with the mechanical compression of air in the compressor. As the compressor discharge contains higher energy gas, that is, the compressed air, it requires *external power* to operate. The power comes from the turbine via a shaft, as shown in Figure 4.1 in case of an operating gas turbine engine. Other sources of external power may be used to *start* the engine, which are in the form of electric motor, air turbine, and hydraulic starters. The flow of air in a compressor is considered to be an essentially adiabatic process, which suggests that only a *negligible* amount of heat transfer takes place between the air inside and the ambient air outside the engine. Therefore, even in a real compressor analysis, we will still treat the flow as adiabatic. Perhaps a more physical argument in favor of neglecting the heat transfer in a compressor can be made by examining the order of magnitude of the energy transfer sources in a compressor. The power delivered to the medium in a compressor is achieved by one or more rows of rotating blades (called rotors) attached to one or more spinning shafts (typically referred to as *spools*). Each rotor blade, which changes the *spin* (or swirl) of a medium, will experience a countertorque as a reaction to its own action on the fluid. If we denote the rotor torque as τ, then the power delivered to the medium by the rotor spinning at the angular speed ω follows Newtonian mechanics, that is,

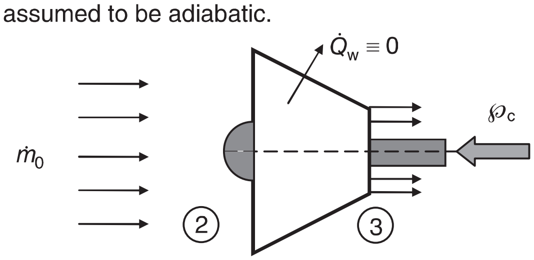
$$\wp = \tau \cdot \omega \tag{4.7}$$

A typical axial-flow compressor contains hundreds of rotor blades (distributed over several stages), which interact with the medium according to the above equation. Therefore, the rate of mechanical energy transfer in a typical modern compressor is usually measured in mega-Watt (MW) and is several orders of magnitude larger than heat transfer through the compressor wall. Symbolically, we may present this as

$$\wp_c \ggg \dot{Q}_w \tag{4.8}$$

Therefore, a real compressor flow is assumed to be adiabatic.

$$\wp_c = \dot{m}_0(h_{t3} - h_{t2})$$
$$\pi_c \equiv (p_{t3}/p_{t2})$$
$$\tau_c \equiv T_{t3}/T_{t2}$$

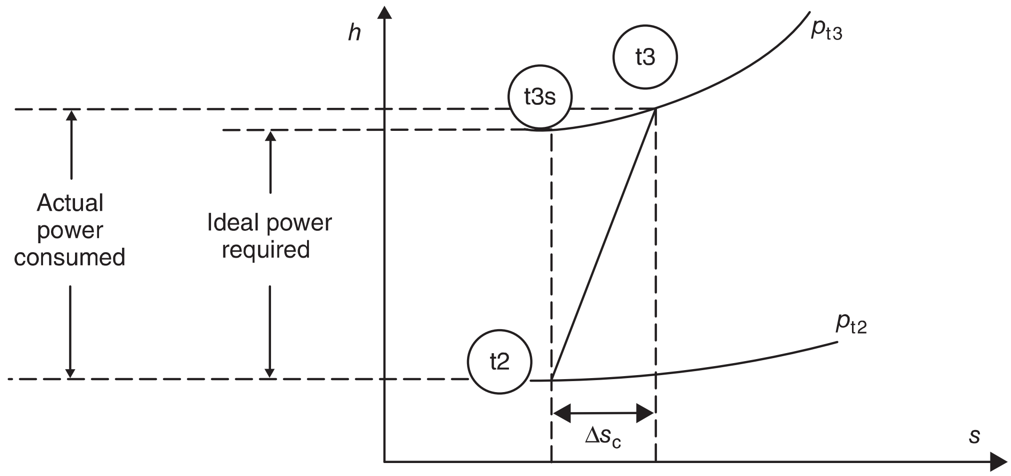


As a real process, however, the presence of wall friction acting on the medium through the boundary layer and shock waves caused by the relative supersonic flow through compressor blades will render the process irreversible. Therefore, the sources of irreversibility in a compressor are due to the viscosity of the medium and its consequences (boundary layer formation, wakes, vortex shedding) and the shock formation in relative supersonic passages.

The measure of irreversibility in a compressor may be thermodynamically defined through some form of compressor efficiency. There are two methods of compressor efficiency definitions. These are

- compressor adiabatic efficiency η_c
- compressor polytropic efficiency e_c.

To define the compressor adiabatic efficiency η_c, we depict a "real" compression process on an *h–s* diagram, as shown in Figure 4.6 and compare it to an ideal, that is, isentropic process. The state "t2" represents the *total* (or stagnation) state of the gas entering the compressor, typically designated by p_{t2} and T_{t2}. An actual flow in a



■ **FIGURE 4.6** **Enthalpy–entropy (*h*–*s*) diagram of an actual and ideal compression process (note Δs_c)**

compressor will follow the solid line from "t2" to "t3", thereby experiencing an entropy rise in the process, Δs_c. The actual total state of the gas is designated by "t3" in Figure 4.6. The ratio p_{t3}/p_{t2} is known as the compressor pressure ratio, with a shorthand notation π_c. The compressor total temperature ratio is depicted by, the shorthand notation, $\tau_c \equiv T_{t3}/T_{t2}$. Since the state "t3" is the actual state of the gas at the exit of the compressor and is not achieved via an isentropic process, we cannot expect the isentropic relation between τ_c and π_c, to hold, that is,

$$\tau_c \neq \pi_c^{\frac{\gamma-1}{\gamma}} \tag{4.9}$$

It can be seen from Figure 4.6 that the actual τ_c is larger than the ideal, that is, isentropic τ_c, which is denoted by the end state T_{t3s}. This fact actually helps with the exponent memorization in a real compression process. The compressor adiabatic efficiency is the ratio of the ideal power required to the power consumed by the compressor, that is,

$$\eta_c \equiv \frac{h_{t3s} - h_{t2}}{h_{t3} - h_{t2}} = \frac{\Delta h_{t,\text{isentropic}}}{\Delta h_{t,\text{actual}}} \tag{4.10}$$

The numerator in Equation 4.10 is the power-per-unit mass flow rate in an *ideal compressor* and the denominator is the power-per-unit mass flow rate in the actual compressor. If we divide the numerator and denominator of Equation 4.10 by h_{t2}, we get

$$\eta_c = \frac{T_{t3s}/T_{t2} - 1}{T_{t3}/T_{t2} - 1} \tag{4.11}$$

Since the thermodynamic states "t3s" and "t2" are on the same isentrope, the temperature and pressure ratios are then related via the isentropic formula, that is,

$$\frac{T_{t3s}}{T_{t2}} = \left(\frac{p_{t3s}}{p_{t2}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{t3}}{p_{t2}}\right)^{\frac{\gamma-1}{\gamma}} = \pi_c^{\frac{\gamma-1}{\gamma}} \tag{4.12}$$

Therefore, compressor adiabatic efficiency may be expressed in terms of compressor pressure and temperature ratios as

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} \tag{4.13}$$

Equation 4.13 involves three parameters, η_c, π_c, and τ_c. It can be used to calculate τ_c for a given compressor pressure ratio and adiabatic efficiency. As compressor pressure ratio and efficiency are typically known and assumed quantities in a gas turbine cycle analysis, the only unknown in Equation 4.13 is τ_c.

A second efficiency parameter in a compressor is *polytropic efficiency* e_c. As might be expected, compressor adiabatic and polytropic efficiencies are related. The definition of compressor polytropic efficiency is

$$e_c \equiv \frac{dh_{ts}}{dh_t} \tag{4.14}$$

It is interesting to compare the definition of compressor adiabatic efficiency, involving finite jumps (Δh_t), and the polytropic efficiency, which takes infinitesimal steps (dh_t). The conclusion can be reached that the polytropic efficiency is actually the adiabatic efficiency of a compressor with *small* pressure ratio. Consequently, compressor polytropic efficiency is also called *small stage efficiency*. From the combined first and second law of thermodynamics, we have

$$T_t ds = dh_t - \frac{dp_t}{\rho_t} \tag{4.15}$$

We deduce that for an isentropic process, that is, $ds = 0$, $dh_t = dh_{ts}$ and, therefore,

$$dh_{ts} = \frac{dp_t}{\rho_t} \tag{4.16}$$

If we substitute Equation 4.16 in 4.14 and replace density with pressure and temperature from the perfect gas law, we get

$$e_c = \frac{\dfrac{dp_t}{p_t}}{\dfrac{dh_t}{RT_t}} = \frac{\dfrac{dp_t}{p_t}}{\dfrac{C_p dT_t}{RT_t}} = \frac{\dfrac{dp_t}{p_t}}{\dfrac{\gamma}{\gamma - 1}\dfrac{dT_t}{T_t}} \tag{4.17}$$

$$\frac{dp_t}{p_t} = \frac{\gamma e_c}{\gamma - 1}\frac{dT_t}{T_t} \tag{4.18}$$

which can now be integrated between the inlet and exit of the compressor to yield

$$\frac{p_{t3}}{p_{t2}} = \pi_c = \left(\frac{T_{t3}}{T_{t2}}\right)^{\frac{\gamma e_c}{\gamma-1}} = \left(\tau_c\right)^{\frac{\gamma e_c}{\gamma-1}} \tag{4.19}$$

To express the compressor total temperature ratio in terms of compressor pressure ratio and polytropic efficiency, Equation 4.19 can be rewritten as

$$\tau_c = \pi_c^{\frac{\gamma-1}{\gamma e_c}} \tag{4.20}$$

The presence of e_c in the denominator of the above exponent (Equation 4.20) causes the exponent of π_c to be greater than its isentropic exponent (which is $(\gamma - 1)/\gamma$), therefore,

$$\tau_{c,real} \succ \tau_{c,isentropic} \quad \text{or} \quad T_{t3} > T_{t3s} \tag{4.21}$$

as noted earlier (Figure 4.6). The physical argument for higher actual T_t than the isentropic T_t (to achieve the same compressor pressure ratio) can be made on the grounds that *lost work* to overcome the irreversibility in the real process (friction, shock) is converted into heat, a higher exit T_t is reached in a real machine due to dissipation. On the contrary, for a given compressor pressure ratio π_c, an ideal compressor consumes less power than an actual compressor (the factor being η_c, as defined earlier). Again, the absence of dissipative mechanisms, leading to lost work, is cited as the reason for a reversible flow machine to require less power to run.

Now, to relate the two types of compressor efficiency description, e_c and η_c, we may substitute Equation 4.20 into Equation 4.13, to get

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\pi_c^{\frac{\gamma-1}{\gamma . e_c}} - 1} \tag{4.22}$$

Equation 4.22 is plotted in Figure 4.7.

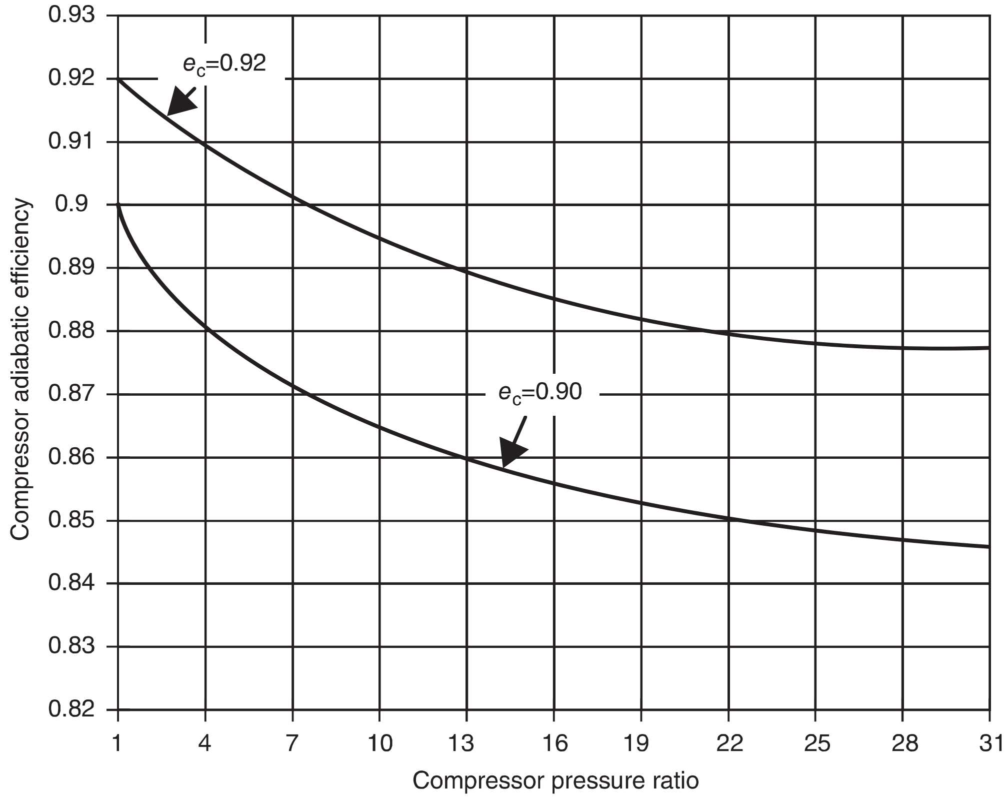
The compressor adiabatic efficiency η_c is a function of compressor pressure ratio, while the polytropic efficiency is independent of it. Consequently, in a cycle analysis, we usually assume the polytropic efficiency e_c as the figure of merit for a compressor (and turbine) and then we can maintain e_c as constant in our engine off-design analysis. Typical values for the polytropic efficiency in modern compressors are in the range 88–92%.

So far, we have studied the thermodynamics of the mechanical compression through the stagnation states of the working medium. It is also instructive to examine the static states of the gas in a compressor as well. The mental connection between these states will help us with the fluid dynamics of the compressors. Figure 4.8 is basically an elaborate version of the earlier Figure 4.6, to which we have added the static states of the gas at the inlet and exit of the compressor. These states, that is, the local stagnation and static states, when plotted on the *h–s* diagram, are distanced from each other only by the amount of local (specific) kinetic energy $V^2/2$ and on the same isentrope.

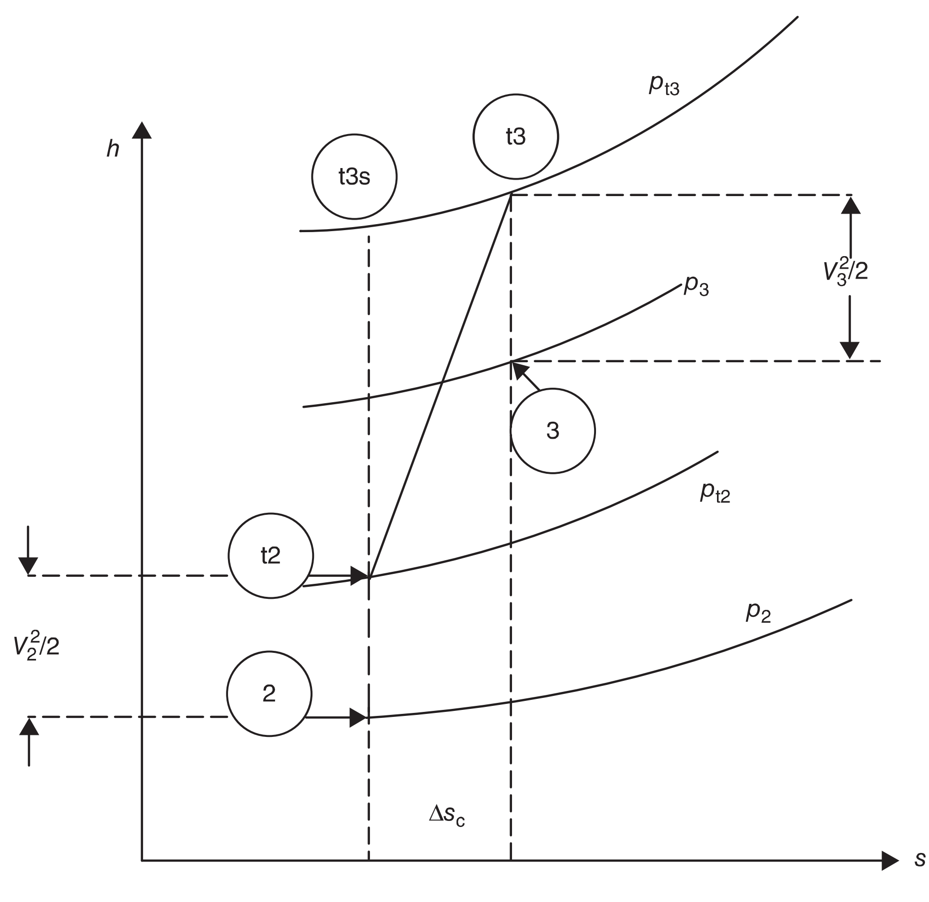
The flow velocity at the compressor inlet, V_2, is nearly the same as the fluid velocity at the compressor exit, V_3, by design. This explains why the vertical gap shown in Figure 4.8, between the static and stagnation states at 2 and 3 is nearly the same, that is,

$$V_2^2/2 \approx V_3^2/2 \tag{4.23}$$

This design philosophy, that is, $V_2 = V_3$, is a part of the so-called constant through flow assumption often used in compressor aerodynamics. We will return to this topic, in detail, in Chapter 8.



■ **FIGURE 4.7** **Variation of compressor adiabatic efficiency with the pressure ratio and polytropic efficiency**



■ **FIGURE 4.8** **Static and stagnation states at the compressor inlet and exit on an *h–s* diagram**

In summary, we have learned that

- a real compressor flow may be considered adiabatic, that is, $Q_{\text{compressor}} \approx 0$
- the energy transfer to the fluid due to the shaft in a compressor is several orders of magnitude higher than any heat transfer that takes place through the casing, thus heat transfer is neglected
- viscous dissipation in the wall boundary layer and shocks account for the sources of irreversibility in a compressor
- there are two figures of merit that describe the compressor efficiency, one is the adiabatic compressor efficiency η_c (sometimes referred to as "isentropic" efficiency) and the second is the polytropic or small-stage efficiency e_c
- the two compressor efficiencies are interrelated, that is, $\eta_c = \eta_c\,(\pi_c, e_c)$
- the compressor polytropic efficiency is independent of compressor pressure ratio π_c
- the compressor adiabatic efficiency is a function of π_c and decreases with increasing pressure ratio
- to achieve a high-pressure ratio in a compressor, multistage and multispool configurations are needed
- in a gas turbine engine, the compressor power is derived from a shaft that is connected to a turbine.

EXAMPLE 4.2

A multistage axial-flow compressor has a mass flow rate of 50 kg/s and a total pressure ratio of 35. The compressor polytropic efficiency is $e_c = 0.90$. The inlet flow condition to the compressor is described by $T_{t2} = 288$ K and $p_{t2} = 100$ kPa. Assuming the flow in the compressor is adiabatic, and constant gas properties throughout the compressor are assumed, i.e., $\gamma = 1.4$ and $c_p = 1004$ J/kg · K, calculate

(a) compressor exit total temperature T_{t3} in K
(b) compressor adiabatic efficiency η_c
(c) compressor shaft power $\wp_c$ in MW

SOLUTION

Following Equation 4.20, we relate compressor total temperature and pressure ratio via polytropic efficiency,

$$\tau_c = \pi_c^{\frac{\gamma-1}{\gamma . e_c}} = (35)^{0.31746} \cong 3.0916$$

Therefore, the exit total temperature is $T_{t3} = 3.0916\, T_{t2} = 3.0916(288\text{ K}) \cong 890.4$ K.

Compressor adiabatic efficiency is related to the polytropic efficiency and compressor pressure ratio, via Equation 4.13

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} = \frac{35^{0.2857} - 1}{3.0916 - 1} \approx 0.8422$$

Therefore, compressor adiabatic efficiency is $\eta_c \cong 84.22\%$.

Compressor shaft power is proportional to the mass flow rate (i.e., the size of the compressor) as well as the total enthalpy rise across the compressor, according to

$$\begin{aligned}\wp_c &= \dot{m}(h_{t3} - h_{t2}) = \dot{m}c_p(T_{t3} - T_{t2}) \\ &= (50\,\text{kg/s})(1004\,\text{J/kg}\cdot\text{K})(890.4 - 288)\,\text{K} \\ &\approx 30.24\,\text{MW}\end{aligned}$$

Therefore the shaft power delivered to the compressor is $\wp_c \approx 30.24$ MW

4.3.1.3 The Burner. In the combustor, the air is mixed with the fuel and a chemical reaction ensues which is *exothermic,* that is, it results in a heat release. The ideal burner is considered to behave like a reversible heater, which in the combustion context, means very slow burning, $M_b \approx 0$, and with no friction acting on its walls. Under such circumstances, the total pressure remains conserved.

In a real combustor, due to wall friction, turbulent mixing and chemical reaction at finite Mach number, the total pressure drops, that is,

$$\pi_b = \frac{p_{t4}}{p_{t3}} < 1 \quad \text{“\textit{real} combustion chamber”} \tag{4.24}$$

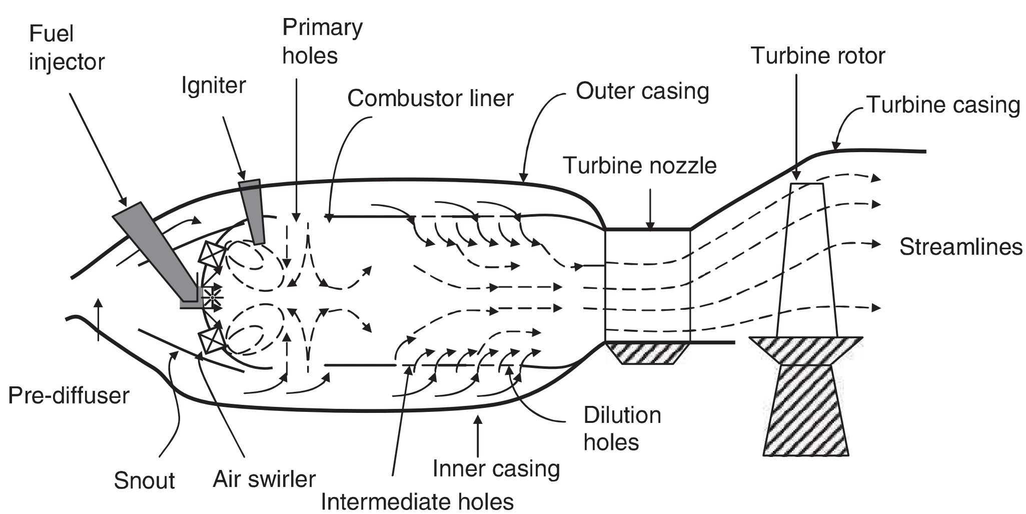
$$\pi_b = 1 \quad \text{“\textit{ideal} combustion chamber”} \tag{4.25}$$

Kerrebrock gives an approximate expression for π_b in terms of the average Mach number of the gas in the burner, M_b, as

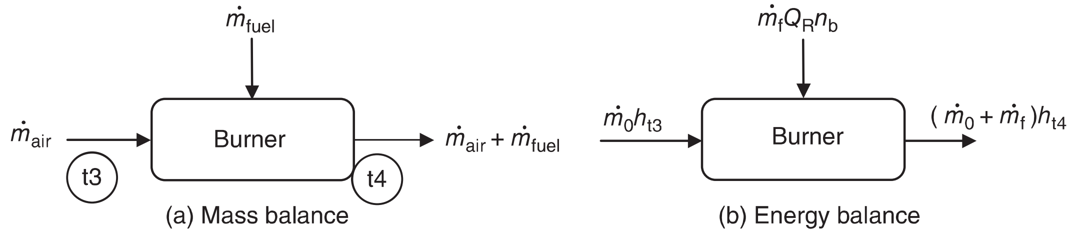
$$\pi_b \approx 1 - \varepsilon \frac{\gamma}{2} M_b^2 \quad \text{where} \quad 1 < \varepsilon < 2 \tag{4.26}$$

The total pressure loss in a burner is then proportional to the average dynamic pressure of the gases inside the burner, that is, $\propto \gamma/2 M_b^2$, where the proportionality coefficient is ε. Assuming an average Mach number of gases of 0.2 and $\varepsilon = 2$, we get $\pi_b \approx 0.95$ (for a $\gamma \approx 1.33$). This formula is obviously not valid for supersonic combustion throughflows, rather it points out the merits of *slow* combustion in a conventional burner. A schematic diagram of a combustion chamber, and its essential components, is shown in Figure 4.9.

A preliminary discussion of the components of the combustion chamber is useful at this time. The inlet diffuser decelerates the compressor discharge flow to a Mach number of about 0.2–0.3. The low-speed flow will provide an efficient burning environment in the combustor. Mixing improvement with the fuel in the combustor primary zone is achieved via the air swirler. A recirculation zone is created that provides the necessary stability in the primary combustion zone. To create a fuel-rich environment in the primary zone to sustain combustion, a large percentage of air is diverted around a dome-like structure. The



■ **FIGURE 4.9** **Components of a conventional combustion chamber and the first turbine stage**



■ **FIGURE 4.10** **Block diagram of a burner with mass and energy balance**

airflow that has bypassed the burner primary zone will enter the combustor as the cooling flow through a series of cooling holes (as shown). The fuel-air mixture is ignited in the combustor primary zone via an igniter properly positioned in the dome area.

For the purposes of *cycle analysis*, a combustor flow is analyzed only at its inlet and outlet. Thus, we will not consider the details of combustion processes such as atomization, vaporization, mixing, chemical reaction, and dilution in the cycle analysis phase. Also, we do not consider pollutant formation and the means of reducing it in this chapter. We will address these issues in a later chapter dealing with combustor flow and design considerations. Obviously, with the details of flow and reaction omitted, we need to make assumptions regarding the loss of total pressure and burner efficiency in the cycle analysis only. A block diagram representation of a combustor, useful in cycle analysis, is shown in Figure 4.10. Figures 4.10a and 4.10b are the steady-state mass and the energy balance applied to the combustion chamber, that is,

$$\dot{m}_4 = \dot{m}_0 + \dot{m}_f = \dot{m}_0(1 + f) \tag{4.27}$$

where, f is the fuel-to-air ratio and $f \equiv \frac{\dot{m}_f}{\dot{m}_0}$. Using the energy balance in Figure 4.10b gives

$$\dot{m}_0 h_{t3} + \dot{m}_f Q_R \eta_b = \left(\dot{m}_0 + \dot{m}_f\right) h_{t4} = \dot{m}_0(1 + f) h_{t4} \tag{4.28}$$

The fuel is characterized by its *energy content per unit mass*, that is, the amount of thermal energy inherent in the fuel, capable of being released in a chemical reaction. This parameter is heat of reaction and is given the symbol Q_R. The unit for this parameter is energy/mass, which in the metric system is kJ/kg and in the British system of units is BTU/lbm. In an actual combustion chamber, primarily due to volume limitations, the entirety of the Q_R cannot be realized. The fraction that can be realized is called *burner efficiency* and is given the symbol η_b. Therefore,

$$\eta_b \equiv \frac{Q_{R,Actual}}{Q_{R,Ideal}} \tag{4.29}$$

The ideal heat of reaction, or heating value of typical hydrocarbon fuels, to be used in our cycle analysis is

$$Q_R = 42{,}000 \text{ kJ/kg} \tag{4.30a}$$

or

$$Q_R = 18{,}000\ \mathrm{BTU/lbm} \tag{4.30b}$$

However, the most energetic fuel is the hydrogen, which is capable of releasing roughly three times the energy of typical hydrocarbon fuels per unit mass, that is,

$$Q_R = 127{,}500\ \mathrm{kJ/kg} \tag{4.30c}$$

or

$$Q_R = 55{,}400\ \mathrm{BTU/lbm} \tag{4.30d}$$

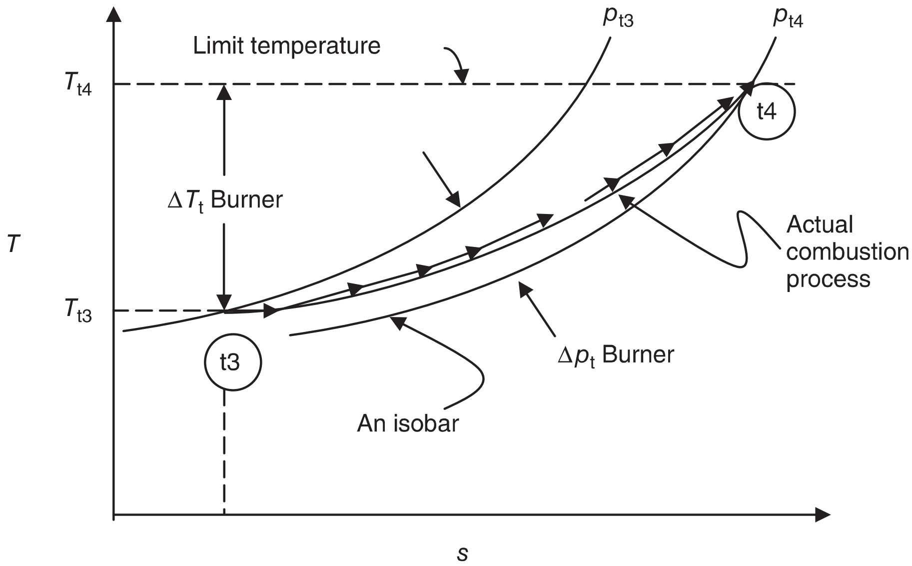
Consistent with the theory of *no free lunch*, we need to note the drawbacks of a fuel such as hydrogen. The low-molecular weight of hydrogen makes it the lightest fuel (with a density ratio of about 1/10 of typical hydrocarbon fuels, such as octane). This implies a comparatively very large volume requirement for the hydrogen fuel. We may want to think of this as *volumetric efficiency* of the hydrogen is the lowest of all fuels. Secondly, hydrogen in liquid form is cryogenic, which means a very low boiling point temperature, that is, −423°F or 20 K at ambient pressure. The cryogenic aspect of hydrogen requires thermally insulated fuel tanks, fuel lines, valves, and the associated weight penalty. Therefore, due to space limitations and system requirement weight penalties on board conventional atmospheric-flight aircraft, conventional hydrocarbon fuels are preferred over hydrogen. An exception to this is found in rocketry and hypersonic airbreathing propulsion where the regenerative cooling of the engine components requires a cryogenic fuel such as hydrogen to withstand the thermal loads of aerodynamic heating.

Typically in modern gas turbine engines, the burner efficiency can be as high as 98–99%. In a cycle analysis, we need to make assumptions about the loss parameters in every component, which in a combustion chamber are $\pi_b < 1$ and $\eta_b < 1$. The real and ideal combustion process can be depicted on a T–s diagram, which later will be used to perform cycle analysis. Figure 4.11 shows the burner thermodynamic process on a T–s diagram.

The isobars p_{t3} and p_{t4} drawn at the entrance and exit of the combustor in Figure 4.11, clearly show a total pressure drop in the burner, $\Delta p_{t,\ \mathrm{burner}}$. The maximum temperature limit T_{t4} is governed by the level of cooling technology, material selection, and the protective thermal coating used in the turbine. Typical current values for the maximum T_{t4} are about 3200–3,600 °R or 1775–2,000 K. Another burner parameter is the temperature rise ΔT_t across the combustion chamber, as shown in Figure 4.11. The thermal power invested in the engine (by the fuel) is proportional to the temperature rise across the combustor, that is, it is nearly equal to $\dot{m}_0 c_p (\Delta T_t)_{\mathrm{burner}}$.

The application of energy balance across the burner, that is, Equation 4.28, will yield the fuel-to-air ratio f as the only unknown parameter. To derive an expression for the fuel-to-air ratio, we will divide Equation 4.28 by $\dot{m}_0$, the air mass flow rate, to get

$$h_{t3} + f Q_R \eta_b = (1 + f) h_{t4} \tag{4.31}$$



■ **FIGURE 4.11**
Actual flow process in a burner (note total pressure loss Δp_t across the burner)

The unknown parameter f can be isolated and expressed as

$$f = \frac{h_{t4} - h_{t3}}{Q_R \eta_b - h_{t4}} \tag{4.32}$$

Knowing the fuel property Q_R, assuming burner efficiency η_b, and having specified a turbine inlet temperature T_{t4}, the denominator of Equation 4.32 is fully known. The compressor discharge temperature T_{t3} is established via compressor pressure ratio, efficiency, and inlet condition, as described in the compressor section and in Equation 4.19, which renders the numerator of Equation 4.32 fully known as well. Therefore, application of energy balance to a burner usually results in the establishment of fuel-to-air ratio parameter f. It is customary to express Equation 4.32 in terms of nondimensional parameters, by dividing each term in the numerator and denominator by the flight static enthalpy h_0 to get

$$f = \frac{\dfrac{h_{t4}}{h_0} - \dfrac{h_{t3}}{h_0}}{\dfrac{Q_R \eta_b}{h_0} - \dfrac{h_{t4}}{h_0}} = \frac{\tau_\lambda - \tau_r \tau_c}{\dfrac{Q_R \eta_b}{h_0} - \tau_\lambda} \tag{4.33}$$

where we recognize the product $\tau_r \tau_c$ as h_{t3}/h_0, and τ_λ as the cycle thermal limit parameter h_{t4}/h_0.

In summary, we learned that

- the fuel is characterized by its heating value Q_R (maximum releasable thermal energy per unit mass)
- the burner is characterized by its efficiency η_b, and its total pressure ratio π_b
- burning at finite Mach number, frictional losses on the walls and turbulent mixing are identified as the sources of irreversibility, that is, losses, in a burner

- the fuel-to-air ratio f and the burner exit temperature T_{t4} are the thrust control/engine design parameters
- the application of the energy balance across the burner yields either f or T_{t4}.

EXAMPLE 4.3

A gas turbine combustor has inlet condition $T_{t3} = 800$ K, $p_{t3} = 2$ Mpa, air mass flow rate of 50 kg/s, $\gamma_3 = 1.4$, $c_{p3} = 1004$ J/kg · K.

A hydrocarbon fuel with ideal heating value $Q_R = 42{,}000$ kJ/kg is injected in the combustor at a rate of 1 kg/s. The burner efficiency is $\eta_b = 0.995$ and the total pressure at the combustor exit is 96% of the inlet total pressure, i.e., combustion causes a 4% loss in total pressure. The gas properties at the combustor exit are $\gamma_4 = 1.33$ and $c_{p4} = 1156$ J/kg · K. Calculate

(a) fuel-to-air ratio f

(b) combustor exit temperature T_{t4} in K and p_{t4} in MPa

SOLUTION

The air and fuel flow rates are specified at 50 and 1 kg/s, respectively, in the problem, therefore, $f = 1/50 = 0.02$ or 2%.

We calculate combustor exit temperature by energy balance,

$$\dot{m}_0 h_{t3} + \dot{m}_f Q_R \eta_b = (\dot{m}_0 + \dot{m}_f) h_{t4} = \dot{m}_0 (1 + f) h_{t4}$$

Therefore

$$h_{t3} + f Q_R \eta_b = (1 + f) h_{t4}$$

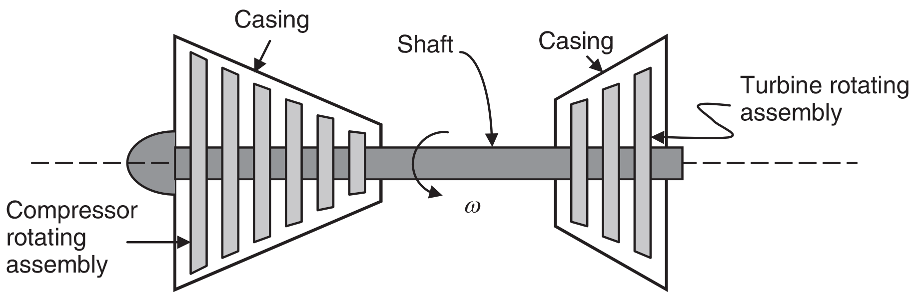
$$T_{t4} = \frac{(c_{p3}/c_{p4}) T_{t3} + f Q_R \eta_b / c_{p4}}{1 + f}$$

$$= \frac{(1004/1156)\,800\,\text{K} + 0.02\,(42000\,\text{kJ/kg})\,(0.995)\,/1.156\,\text{kJ/kg}\cdot\text{K}}{1.02}$$

Therefore, combustor exit temperature is $T_{t4} \cong 1390$ K and $p_{t4} = 0.96(2\ \text{MPa}) = 1.92$ MPa

4.3.1.4 The Turbine. The high pressure and temperature gas that leaves the combustor is directed into a turbine. The turbine may be thought of as a *valve* because on one side it has a high-pressure gas and on the other side it has a low-pressure gas of the exhaust nozzle or the tailpipe. Therefore, the first *valve*, that is, the throttle station, in a gas turbine engine is at the turbine. The throat of an exhaust nozzle in a supersonic aircraft is the second and final throttle station in an engine. Thus, the flow process in a turbine (and exhaust nozzle) involves significant (static) pressure drop and, in harmony with it, the (static) temperature drop, which is called *flow expansion*. The flow expansion produces the necessary power for the compressor and the propulsive power for the aircraft. The turbine is connected to the compressor via a common shaft, which provides the shaft power to the compressor (Figure 4.12). In drawing an analogy, we can think of the expansion process in a gas turbine engine as the counterpart of *power stroke* in an intermittent combustion engine. However, in a turbine, the power transmittal is continuous.

Due to high temperatures of the combustor exit flow, the first few stages of the turbine, that is, the high-pressure turbine (HPT), need to be cooled. The coolant is the air bleed from the compressor, which may be bled from different compression stages, for example, between the low- and high-pressure compressor and at the compressor exit. It has been customary, however, to analyze an *uncooled* turbine in the preliminary cycle analysis, followed by an analysis of the cycle with cooling effects in the turbine and

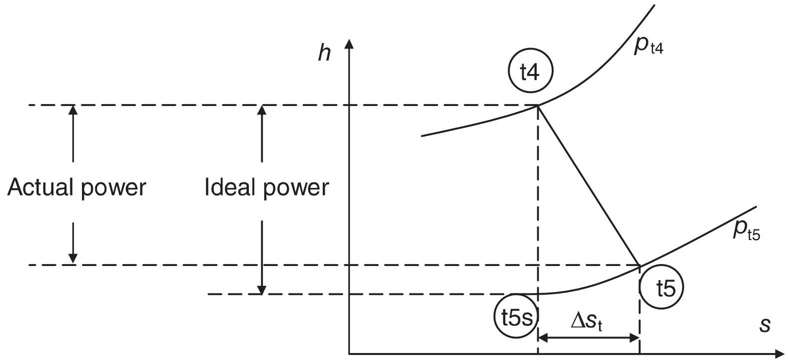


■ **FIGURE 4.12** **Common shaft in a gas generator connects the compressor and turbine (*ω* is the shaft rotational speed)**

the exhaust nozzle. Other cooling media, such as water, have been used in stationary gas turbine power plants. However, in the design-to-weight environment of an aircraft carrying extra water to cool the turbine blades is not feasible. A cooling solution, which uses the engine cryogenic fuel, such as hydrogen or methane, as the coolant to cool the engine and aircraft components is called *regenerative cooling* and has proven its effectiveness in liquid propellant chemical rocket engines for decades.

A real flow process in an uncooled turbine involves irreversibilities such as frictional losses in the boundary layer, tip clearance flows, and shock losses in transonic turbine stages. The viscous dominated losses, that is, boundary layer separation, reattachment, and tip vortex flows, are concentrated near the end walls, thus a special attention in turbine flow optimization is made on the *end-wall* regions. We will address the end wall losses and treatments in the turbomachinery chapters. Another source of irreversibility in a real turbine flow is related to the cooling losses. Coolant is typically injected from the blade attachment (to the hub or casing) into the blade, which provides internal convective cooling and usually external film cooling on the blades. The turbulent mixing associated with the coolant stream and the hot gases is the primary mechanism for (the turbine stage) cooling losses. We will examine the question of turbine cooling and cycle thermal efficiency later in this chapter.

The thermodynamic process for an uncooled turbine flow may be shown in an *h–s* diagram (Figure 4.13). The actual expansion process in the turbine is depicted by the solid line connecting the total (or stagnation) states t4 and t5 (Figure 4.13). The isentropically reached exit state t5s represents the ideal, loss-free flow expansion in the turbine to the same backpressure p_{t5}. The relative height, on the enthalpy scale in Figure 4.13, between the inlet total and outlet total condition of the turbine represents the power production



■ **FIGURE 4.13** **Expansion process in an uncooled turbine (note the entropy rise across the turbine, Δs_t)**

potential (i.e., ideal) and the actual power produced in a turbine. The ratio of these two heights is called the turbine adiabatic efficiency η_t, that is,

$$\wp_{t,actual} = \dot{m}_t(h_{t4} - h_{t5}) = \dot{m}_t \Delta h_{t,actual} \tag{4.34}$$

$$\wp_{t,ideal} = \dot{m}_t(h_{t4} - h_{t5s}) = \dot{m}_t \Delta h_{t,isentropic} \tag{4.35}$$

$$\eta_t \equiv \frac{h_{t4} - h_{t5}}{h_{t4} - h_{t5s}} = \frac{\Delta h_{t,actual}}{\Delta h_{t,isentropic}} \tag{4.36}$$

In Equations 4.34 and 4.35, the turbine mass flow rate is identified as $\dot{m}_t$, which accounts for the air and fuel mass flow rate that emerge from the combustor and expand through the turbine, that is,

$$\dot{m}_t = \dot{m}_0 + \dot{m}_f = (1 + f)\dot{m}_0 \tag{4.37}$$

The numerator in Equation 4.36 is the actual power produced in a *real uncooled turbine*, and the denominator is the *ideal* power that in a reversible and adiabatic turbine could be produced. If we divide the numerator and denominator of Equation 4.36 by h_{t4}, we get

$$\eta_t = \frac{1 - T_{t5}/T_{t4}}{1 - T_{t5s}/T_{t4}} \tag{4.38}$$

Since the thermodynamic states "t5s" and "t4" are on the same isentrope, the temperature and pressure ratios are then related via the isentropic formula, that is,

$$\frac{T_{t5s}}{T_{t4}} = \left(\frac{p_{t5s}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{t5}}{p_{t4}}\right)^{\frac{\gamma-1}{\gamma}} = \pi_t^{\frac{\gamma-1}{\gamma}} \tag{4.39}$$

Therefore, turbine adiabatic efficiency may be expressed in terms of the turbine total pressure and temperature ratios as

$$\eta_t = \frac{1 - \tau_t}{1 - \pi_t^{\frac{\gamma-1}{\gamma}}} \tag{4.40}$$

We may also define a *small-stage* efficiency for a turbine, as we did in a compressor, and call it the turbine polytropic efficiency e_t. For a small expansion, representing a small stage, we can replace the finite jumps, that is, Δs, with incremental step d in Equation 4.36 and write

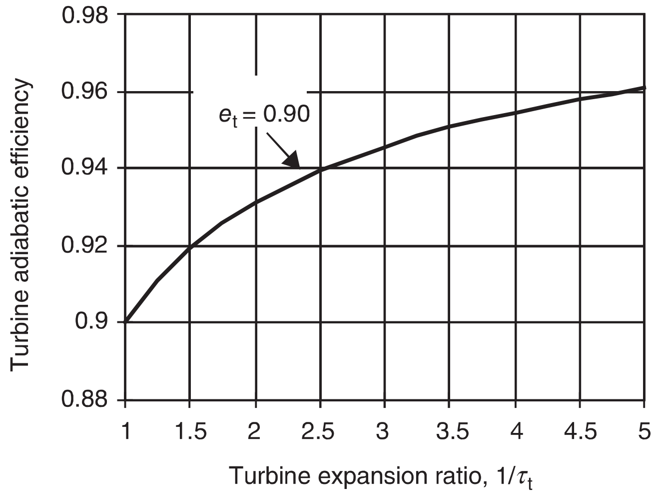
$$e_t \equiv \frac{dh_t}{dh_{ts}} = \frac{dh_t}{\frac{dp_t}{\rho_t}} \tag{4.41}$$

Expressing enthalpy in terms of temperature and specific heat at constant pressure as $dh_t = c_p dT_t$, which is suitable for a perfect gas, and simplifying Equation 4.41 similar to the compressor section, we get

$$\tau_t = \pi_t^{\frac{(\gamma-1)e_t}{\gamma}} \quad \text{or} \quad \pi_t = \tau_t^{\frac{\gamma}{(\gamma-1)e_t}} \tag{4.42}$$

$$\tau_t \equiv T_{t5}/T_{t4} \tag{4.43}$$

$$\pi_t \equiv p_{t5}/p_{t4} \tag{4.44}$$



■ **FIGURE 4.14** **Variation of turbine adiabatic efficiency η_t with the inverse of turbine expansion ratio $1/\tau_t$**

We note that in Equation 4.42, in the limit of e_t approaching 1, that is, isentropic expansion, we will recover the isentropic relationship between the temperature and pressure ratio, as expected. Replacing π_t in Equation 4.39 by its equivalent expression (from Equation 4.42), we derive a relation between the two types of turbine efficiencies, η_t and e_t, as

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}} \tag{4.45}$$

Equation 4.45 is plotted in Figure 4.14. For a small-stage efficiency of 90%, that is, $e_t = 0.90$, the turbine adiabatic efficiency η_t grows with the inverse of turbine expansion parameter $1/\tau_t$. This may initially defy logic that how is it possible to become more efficient if we *add* more stages to the turbine? Wouldn't it add to losses? The opposite of this trend occurred in the compressor, that is, compressor adiabatic efficiency for a finite-size compressor *was* lower than the efficiency of a small-stage compressor (Figure 4.7). The explanation is that the energy transfer occurs from the fluid to the rotor in the turbine; therefore, more stages offer more opportunities to convert fluid energy into shaft power. The opposite happens in a compressor, that is, the compressor stages *consume* power, which means that the additional stages add losses to the process. In simple terms, the turbine stages may be thought of as *opportunities* and compressor stages as the *burden*.

The temperature ratio parameter τ_t across the turbine is established via a power balance between the turbine, compressor, and other shaft power extraction (e.g., electric generator) on the gas generator. Let us first consider the power balance between the turbine and compressor in its simplest form and then try to build on the added parameters. Ideally, the compressor absorbs all the turbine shaft power, that is,

$$\wp_t = \wp_c \tag{4.46a}$$

$$\dot{m}_0(1+f)(h_{t4} - h_{t5}) = \dot{m}_0(h_{t3} - h_{t2}) \tag{4.46b}$$

which simplifies to the following nondimensional form

$$(1+f)\tau_\lambda(1 - \tau_t) = \tau_r(\tau_c - 1) \tag{4.46c}$$

The only unknown in the above equation is τ_t, as all other parameters either flow from upstream components, for example, the combustor will provide f, the compressor produces τ_c, and so on, or are design parameters such as τ_λ or τ_r. Hence, the turbine expansion parameter τ_t can be written as

$$\tau_t = 1 - \frac{\tau_r(\tau_c - 1)}{(1+f)\tau_\lambda} \tag{4.46d}$$

Next, we consider the practical issue of hydrodynamic (frictional) losses in (radial) bearings holding the shaft in place and provide dynamic stability under operating conditions to the rotating assemblies of turbine and compressor. Therefore, a small fraction of the turbine power output is dissipated through viscous losses in the bearings, that is,

$$\wp_t = \wp_c + \Delta\wp_{\text{bearings}} \tag{4.47}$$

where $\Delta\wp_{\text{bearings}}$ is the power loss due to bearings. In addition, an aircraft has electrical power needs for its flight control system and other aircraft subsystems, which requires tapping into the turbine shaft power. Consequently, the power balance between the compressor and turbine should account for the electrical power extraction, which usually accompanies the gas generator, that is,

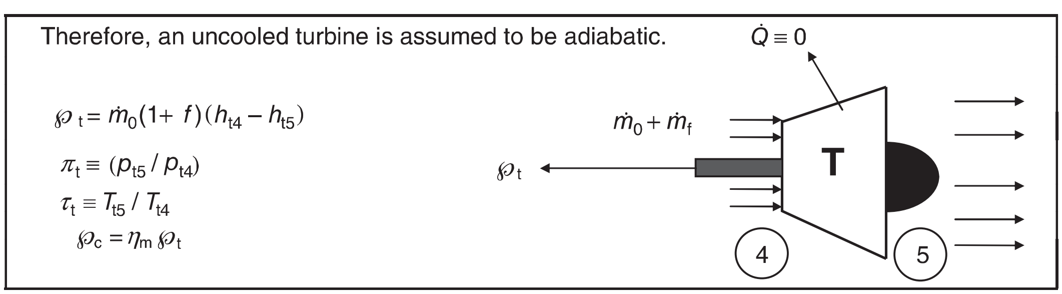
$$\wp_t = \wp_c + \Delta\wp_{\text{bearing}} + \Delta\wp_{\text{electric generator}} \tag{4.48a}$$

$$\wp_c = \wp_t - \Delta\wp_{\text{bearing}} - \Delta\wp_{\text{electric generator}} \tag{4.48b}$$

In a simple cycle analysis, it is customary to lump all power dissipation and power extraction terms into a single *mechanical efficiency* parameter η_m that is multiplied by the turbine shaft power, to derive the compressor shaft power, that is,

$$\wp_c = \eta_m \wp_t \tag{4.49}$$

where η_m is the mechanical efficiency parameter that needs to be specified a'priori, for example, $\eta_m = 0.95$.



To cool the high-pressure turbine stages, a small fraction of compressor air can be diverted from various stages of the compression. Engine cooling is essentially a *pressure-driven* process, which calls for a pressure scheduling of the coolant to achieve the highest cooling efficiency. For example, to cool the first nozzle and the first rotor in a turbine,