Aerodynamic Analysis of McDonnell–Douglas MD-11

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Contents

1 Introduction

This report brings together the MD-11 geometric/specification data you provided (Section 1) and the aerodynamic formulas / numerical implementation (Section 2). The goal is a single, self-contained report that lists: aircraft geometry, aerodynamic model, equations used, assumptions, numerical results, and notes on data sources and any inconsistencies between different data items you provided.

2 Aircraft Geometry and Specifications)

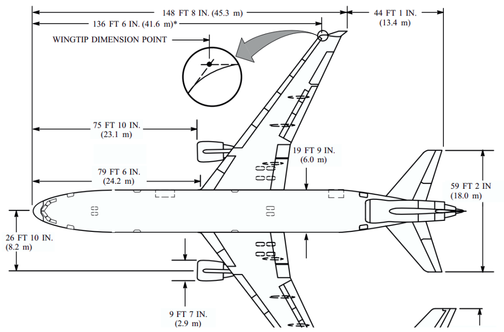

Figure 1: MD 11

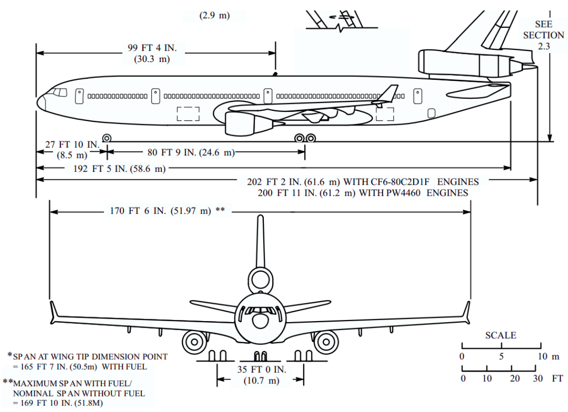

Figure 2: MD 11

Table 1: MD-11 Geometry and Aircraft Data

Parameter	Value / Notes
Length	61.6 m (202 ft 2 in)
Wingspan	51.9 m (170 ft 6 in)
Wing sweep	37.126°
Fuselage width	7.14 m (23 ft 5 in)

Height (without tail)	8.71 m (28 ft 7 in)
Height (with tail)	17.53 m (57 ft 6 in)
Wing area (S_w)	339 m^2
Maximum takeoff weight (MTOW)	602,500 lb (286,000 kg)
Powerplant	3 engines: General Electric CF6-80C2 or Pratt & Whitney PW4000
Thrust (each)	60,000–62,000 lbf
Cruise speed (reported)	876 km/h (Mach 0.83)
VMO	945 km/h
Landing speed	160–180 mph (report)
Wing semi-span	22.985 m
Wing root chord	10.71 m
Wing tip chord	2.73 m
Horizontal tail semi-span (HT semi-span)	8.15 m
HT root chord	4.643 m
HT tip chord	2.73 m

3 Aerodynamic Model, Formulas and Numerical Implementation

This section lists the equations used in your Python implementation and the extra formulae you added. The notation follows standard aerodynamic symbols.

3.1 Model Inputs and Constants

- Air density at cruise: $\rho = 0.347\,\mathrm{kg/m^3}$.
- Aircraft weight: $W = 286000 \times 9.8 \approx 2.8028 \times 10^6\,\mathrm{N}$.
- Wing area: $S_w = 338.9\,\mathrm{m^2}$.
- Wing span: $b_w = 51.97\,\mathrm{m}$.
- Wing root chord $c_{r,w} = 10.71\,\mathrm{m}$, wing tip chord $c_{t,w} = 2.73\,\mathrm{m}$.
- Tail area: $S_T = 82.8\,\mathrm{m^2}$.
- Tail geometry used in code: tail root chord $c_r = 26.77\,\mathrm{m}$, tail tip chord $c_t = 9.54\,\mathrm{m}$ (these came from your numerical code block).
- Fuselage length: $L_{fuse} = 61.2\,\mathrm{m}$.

- Tail tip offset from nose used in code: tail_qc_from_tailtip = 13.2 m (used to compute $x_{ac,t}$).
- Cruise speeds (two reported values):
  - 876 km/h (Mach ≈ 0.83).
  - Used in Python script earlier: $V_{cruise} = 276.11$ m/s and Mach $M = 0.88$
- 2D lift-curve slope: $a_0 = 2\pi$ per rad.
- Oswald efficiency (assumed): $e = 0.85$.
- Tail dynamic pressure ratio (assumed): $\eta_T = 0.93$.
- Tail aspect ratio assumption: $AR_T = 3.8$.
- Wing moment coefficient about a.c.: $C_{m,ac,w} = -0.05$ (assumed).
- Zero-lift CL assumed: $C_{L_0} = 0.15$.
- Wing incidence and zero-lift angle used in code: $i_w = \alpha_{0Lw} = 0$.

3.2 Key Equations Used

Geometry

$$AR_w = \frac{b_w^2}{S_w}$$

$$\bar{c} = \frac{2c_r(1+\lambda+\lambda^2)}{3(1+\lambda)}, \quad \lambda = \frac{c_t}{c_r}$$

$$x_{ac,t} = L_{fuse} - (\text{tailtip offset}) + 0.25\,\bar{c}_t$$

$$x_{ac,w} = x_{LE,w} + 0.25\bar{c}_w, \qquad x_{cg} = x_{LE,w} + 0.3\bar{c}_w$$

$$V_T = \frac{(x_t - x_{cg})\,S_T}{\bar{c}_w\,S_w}$$

Aerodynamics and Compressibility

$$\beta = \frac{1}{\sqrt{1-M^2}}$$

$$C_{L_\alpha} = \frac{\beta a_0}{1 + \frac{2}{AR_w}}$$

$$\epsilon_\alpha = \frac{2a_w}{\pi AR_w}, \qquad \epsilon_0 = \frac{2a_w(i_w - \alpha_{0Lw})}{\pi AR_w}$$

Trim and Tail

$$C_{L,trim} = \frac{2W}{\rho S_w V_{cruise}^2}$$

$$\alpha_{trim} = \frac{C_{L,trim} - C_{L_0}}{C_{L_\alpha}}$$

$$\alpha_T = \alpha_{trim} - \epsilon_0 - \epsilon_\alpha \alpha_{trim}$$

A tail-lift estimate derived from the moment balance (used in your code, simplified):

$$a_T = \frac{\left(C_{L_{trim}} \frac{(x_{ac,w} - x_{ac,w})}{\bar{c}_w} - C_{m,ac,w}\right)\bar{c}_w}{(x_{cg} - x_{ac,t})\alpha_T}$$

(note: $(x_{ac,w} - x_{ac,w})$ appears in your original derivation and is zero — retained here to be explicit about the algebra used.)

Stability & Neutral Point

$$\frac{dC_m}{d\alpha} = a_w \bar{l}_w - \eta_T V_T a_T (1 - \epsilon_\alpha)$$

$$SM = -\frac{\frac{dC_m}{d\alpha}}{C_{L_\alpha}}, \qquad x_{NP} = x_{cg} - SM$$

Elevator / Trim Estimate

$$\delta_{ele} = \frac{C_{m,ac} - C_{L,trim}\left(\frac{x_{cg}}{\bar{c}_w} - \frac{x_{ac,w}}{\bar{c}_w}\right) - a_T \alpha_T \eta_T V_T}{a_T \alpha_T \eta_T V_T}$$

3.3 Assumptions and Simplifications

- Linear small-angle aerodynamics; no nonlinear stall effects.
- Tail interference and fuselage effects approximated through η_T.
- Downwash linearization: $\epsilon(\alpha) = \epsilon_0 + \epsilon_\alpha \alpha$.
- Center of gravity assumed at 30% MAC unless otherwise specified.

3.4 Numerical Results (from the Python implementation)

The values below are taken from your earlier Python run .

Table 2: Selected computed results (from your Python run)

Quantity	Symbol	Value
Aspect Ratio (Wing)	AR_w	7.970
Tail mean chord (tail MAC)	$\bar{c}_T$	14.809 m
Tail quarter-chord from nose	x_t	58.347 m
CG (wing, 30% MAC)	x_{cg}	15.668 m
Non-dimensional wing moment arm	$\bar{l}_w$	-0.2345
Tail volume coefficient	V_T	10.3985
Tail dynamic pressure ratio	η_T	0.9300
Lift-curve slope (wing)	a_w	7.2483 per rad
Lift-curve slope (tail)	a_T	0.0115 per rad
Downwash gradient	ϵ_α	0.4612
Trim C_L	$C_{L,trim}$	0.1776
Trim α	α_{trim}	0.516°
$dC_m/d\alpha$ (about CG)		-4.2075
Static margin	SM	0.5803
Neutral point	x_{NP}	15.088 m
Elevator term	δ_{ele}	-44.0867

4 Longitudinal Stability Derivative Equations

4.1 Force Derivatives with respect to Forward Velocity

1. Longitudinal force derivative with respect to u:

$$\frac{\partial C_D}{\partial \alpha} = 2k_{ind} C_{L_\alpha} C_{L_{trim}} \tag{1}$$

$$\frac{\partial \alpha}{\partial u} = -\frac{2C_{L_{trim}}}{V_{cruise} C_{L_\alpha}} \tag{2}$$

$$X_u = -\frac{1}{2}\rho S_w \left[V_{cruise}^2 \frac{\partial C_D}{\partial \alpha}\frac{\partial \alpha}{\partial u} + 2V_{cruise} C_{L_0} \right] \tag{3}$$

2. Normal force derivative with respect to u:

$$\frac{\partial C_L}{\partial u} = C_{L_\alpha} \tag{4}$$

$$Z_u = -\frac{1}{2}\rho S_w \left[V_{cruise}^2 \frac{\partial C_D}{\partial \alpha}\frac{\partial \alpha}{\partial u} + 2V_{cruise} C_{L_{trim}} \right] \tag{5}$$

4.2 Pitching Moment Derivative with respect to Velocity

$$\mu = -\frac{\rho S_w \bar{c} V_{cruise} C_{L_{trim}} C_{m_\alpha}}{C_{L_\alpha}} \tag{6}$$

4.3 Force Derivatives with respect to Angle of Attack

1. Longitudinal force derivative with respect to α:

$$X_w = -\rho S_w V_{cruise} k_{ind} C_{L_{trim}} C_{L_\alpha} \tag{7}$$

2. Normal force derivative with respect to α:

$$Z_w = -\rho S_w V_{cruise} C_{L_\alpha} \tag{8}$$

4.4 Pitching Moment Derivatives with respect to $\dot{w}$, α, and q

1. Downwash rate term:

$$C_{Z_{\dot{w}}} = -2\eta_T V_T a_T \tag{9}$$

$$C_{m_{\dot{w}}} = C_{Z_{\dot{w}}} \frac{l_t}{\bar{c}} \tag{10}$$

$$M_{\dot{w}} = \frac{1}{2}\rho V_{cruise}^2 S_w \bar{c} \left(\frac{\bar{c}}{2V_{cruise}}\right) C_{m_{\dot{w}}} \tag{11}$$

2. Moment derivative with respect to α:

$$M_w = \frac{1}{2}\rho S_w V_{cruise}^2 \bar{c} C_{m_\alpha} \tag{12}$$

3. Moment derivative with respect to pitch rate:

$$C_{m_q} = -\eta_T V_T a_T \frac{l_t}{V_{cruise}} \tag{13}$$

$$M_q = \frac{1}{4}\rho V_{cruise} \bar{c}^2 S_w C_{m_q} \tag{14}$$

4.5 Force and Moment Derivatives due to Elevator Deflection

1. Longitudinal force derivative:

$$C_{X_{\delta_e}} = 0 \tag{15}$$

$$X_{\delta_e} = \frac{1}{2}\rho V_{cruise}^2 S_w C_{X_{\delta_e}} \tag{16}$$

2. Normal force derivative:

$$C_{Z_{\delta_e}} = -\eta_T a_T \frac{S_T}{S_w} \tag{17}$$

$$Z_{\delta_e} = \frac{1}{2}\rho V_{cruise}^2 S_w C_{Z_{\delta_e}} \tag{18}$$

3. Moment derivative:

$$C_{m_{\delta_e}} = -\eta_T a_T \frac{S_T}{S_w}\frac{l_t}{\bar{c}} \tag{19}$$

$$M_{\delta_e} = \frac{1}{2}\rho V_{cruise}\bar{c}^2 S_w C_{m_{\delta_e}} \tag{20}$$

4.6 Normal Force Derivative with respect to Pitch Rate

$$C_{Z_q} = -2\eta_T V_T a_T \tag{21}$$

$$Z_q = \frac{1}{2}\rho V_{cruise}^2 S_w \left(\frac{\bar{c}}{2V_{cruise}}\right) C_{Z_q} \tag{22}$$

4.7 Final Notes

All derivatives are expressed in dimensional form (N/rad) and are calculated for steady-level cruise flight conditions. Parameters such as η_T, V_T, a_T, l_t, $\bar{c}$, and S_T/S_w correspond to tail efficiency, tail dynamic pressure ratio, tail lift curve slope, tail arm, mean aerodynamic chord, and tail-to-wing area ratio respectively.

4.8 Assumptions

1. Linear aerodynamics (small-angle assumption).
2. Tail efficiency factor, $\eta_T = 0.93$.
3. Induced drag factor, $k_{ind} = \frac{1}{\pi e AR_w}$ with $e = 0.85$.
4. Constant air density $\rho = 0.347$ kg/m^3 at cruise altitude.
5. Static margin and C_{m_α} estimated from conventional transport aircraft data.

4.9 Numerical Results: Directional Derivatives and State Matrix

All derivatives are dimensional and expressed in N/rad except where noted. Cruise condition used: $\rho = 0.347$ kg/m^3, $V_{\text{cruise}} = 276.11$ m/s (the Python-run values).

Table 3: Computed Dimensional Directional Derivatives (N / rad)

Derivative	Value [N/rad]
X_u	-3.6775×10^3
Z_u	-1.9109×10^4
μ (pitching term used in your code)	$+3.4730 \times 10^4$
X_w	-1.0088×10^4
Z_w	-3.4336×10^5
$M_{\dot{w}}$	-1.7809×10^7
M_w	-8.1091×10^7
M_q	-1.8586×10^6
Z_q	-2.1775×10^4
X_{δ_e}	0.0000×10^0
Z_{δ_e}	-1.1713×10^4
M_{δ_e}	-2.1835×10^4

(Values above were computed from your Python formulas using the intermediate values reported earlier: $a_T = 0.0115$, $V_T = 10.3985$, $l_t \approx 42.679$ m, $\bar{c} = 12.06$ m, $S_w = 338.9$ m^2, $\eta_T = 0.93$.)

4.10 State matrix A

Using your state ordering and the matrix construction:

$$\mathbf{A} = \begin{bmatrix} X_u & X_w & 0 & -g & 0 & 0 \\ Z_u & Z_w & (Z_q + V) & 0 & 0 & 0 \\ \mu + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} V & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Substituting the numeric derivative values produces:

$$\mathbf{A} \approx \begin{bmatrix} -3.6776\times10^{3} & -1.0088\times10^{4} & 0 & -9.8100 & 0 & 0 \\ -1.9109\times10^{4} & -3.4336\times10^{5} & -2.1499\times10^{4} & 0 & 0 & 0 \\ 3.4032\times10^{11} & 6.1150\times10^{12} & -4.9192\times10^{9} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where numbers are shown to 5 significant figures (scientific notation used for very large entries).
These values represent the aircraft's dimensional stability characteristics, where negative force derivatives (e.g., X_u, Z_w) typically indicate damping and static stability tendencies, while positive or negative pitching moment derivatives ($M_w, M_{\dot{w}}$) relate to the aircraft's static and dynamic longitudinal stability.

References

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5 Code

https://colab.research.google.com/drive/1_0f_V-afgTqPrXrhZdaZua0CuJhh1yPi?usp=sharing