Directional Motion and Sideslip

Aircraft flight, in the normal course, is in x – z **plane**, which also is its primary **motion** configuration**.**

However, there are situations where motion **direction** has non-zero angle with **x-axis** in horizontal plane (i.e. x-y), **leading** to forces that are not restricted to x-z plane**.**

Motion, so generated in y-direction, is termed **directional** which is characterized by a **side** velocity 'v', resulting in **sideslip** angle, denoted by 'β'.

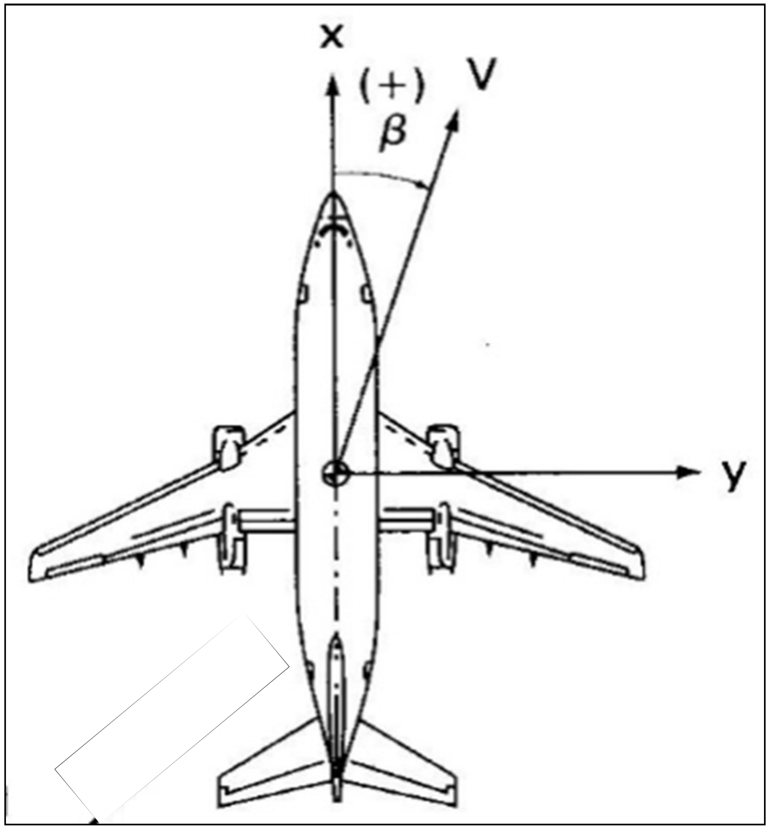
Directional Motion Features

Though, these effects are small, there is still a need to **examine** the stability of aircraft trim motion (in x-z plane) **under** such disturbances**.**

Consider an aircraft in asymmetric flight, as shown **alongside**.

Side velocity, ‘v’, gives rise to sideslip angle, ‘β’, **which** is given by the following expression.

$$\boxed{\beta = \sin^{-1}\left(\frac{v}{V}\right)}$$



Impact of Sideslip

Sideslip generates a side force (Y) on the aircraft, **along** with a yawing moment (N), **so** that it starts to rotate about **z-axis**, raising the issue of changes to trim condition.

Important questions that need to be **answered** are; (1) What are the changes to the **trim** and (2) Whether or not the **resulting** motion is stable.

As stability requires that, under the action of **disturbance**, aircraft should tend to **nullify** it, we conclude that for a **stable** aircraft, 'N' should reduce the value of 'β'.

Directional Stability Definition

This is possible only if x-axis has the tendency to **align** itself with vector ‘V’, which **indicates** that yawing moment ‘N’ > 0 for ‘β’ > 0, or that (dN/dβ) > 0.

The implication of the above stability requirement is **that** aircraft, if directionally **stable**, would turn into the wind, **until** ‘β’ becomes zero.

However, this behaviour results in aircraft **deviating** from the intended flight **path**, and moving along a trajectory **decided** by the resultant velocity vector**.**

Contributions to Yawing Moment - Wings

Contributions to directional stability **derivative** (i.e. $dN/d\beta$) arise from all **components** of aircraft and there is a **need** to understand these effects.

Wings lie in x-y plane, so that their in-plane motion is **not** expected to produce any **significant** side force, or N_β, **though** there is some effect of sweep, as shown below.

$$\boxed{C_{N\beta} = \frac{N_\beta}{qS_w b} = 0.00006 \times \left(\Lambda^o\right)^{1/2}}$$

Axi-symmetric Fuselage Contribution

Fuselage, being an axi-symmetric slender body, **produces** yawing moments which **are** destabilizing, i.e. aircraft **rotates** in such a manner so as to increase ‘β’.

Negative fuselage contribution to yawing moment **for** a positive ‘β’, for axi-symmetric **bodies**, can be adequately **modelled**, through the fuselage volume**,** as follows.

$$\boxed{C_{N\beta f} = -2\frac{V_f}{S_w b_w}; \quad V_f \rightarrow \text{Fuselage Volume}}$$

Propeller Contribution

In case of aircraft with propellers, a running propeller has **significant** effect on overall $C_{N\beta}$.

Consider a propeller aircraft, as shown alongside, whose **contribution** to $C_{N\text{-}\beta}$ is as given below.

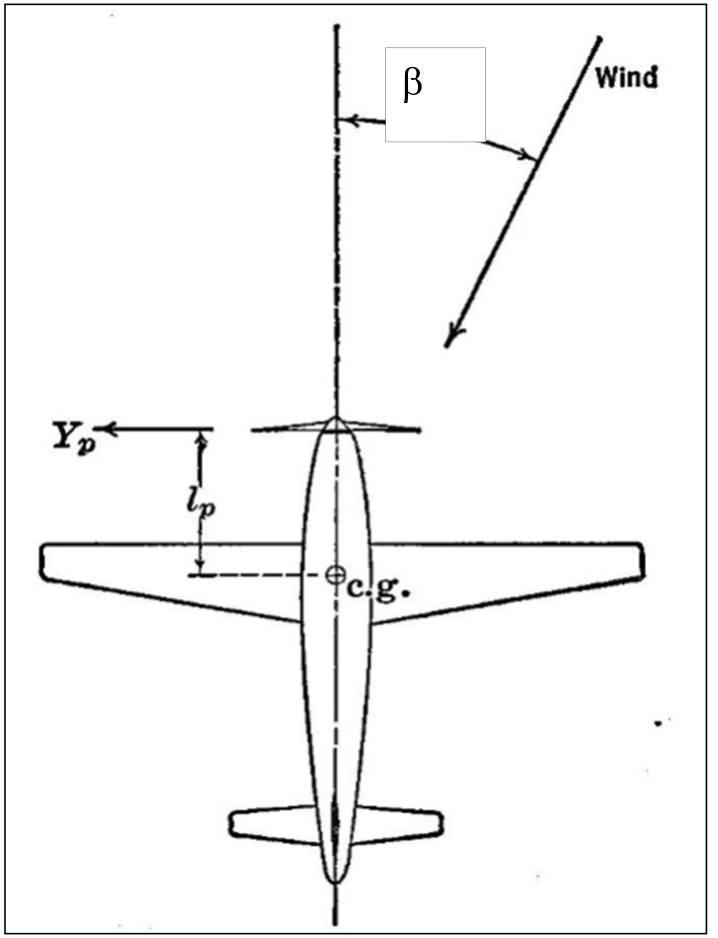
$$C_N = -\frac{Y_p \times l_p}{qS_w b} = -\frac{C_{Yp} \times \left(\pi D^2\right) \times l_p \times n}{4S_w b}; \quad D \rightarrow \text{Diameter}$$

$$C_{N\beta} = -\frac{C_{Yp\beta} \times \left(\pi D^2\right) \times l_p \times n}{4S_w b}; \quad n \rightarrow \text{No. of Propellers}$$

$C_{Yp\beta}$ (Wind-milling)

2-bladed: 0.00165; 3-bladed: 0.00235; 4-bladed: 0.00296

6-bladed: 0.00510 (Counter-rotating)



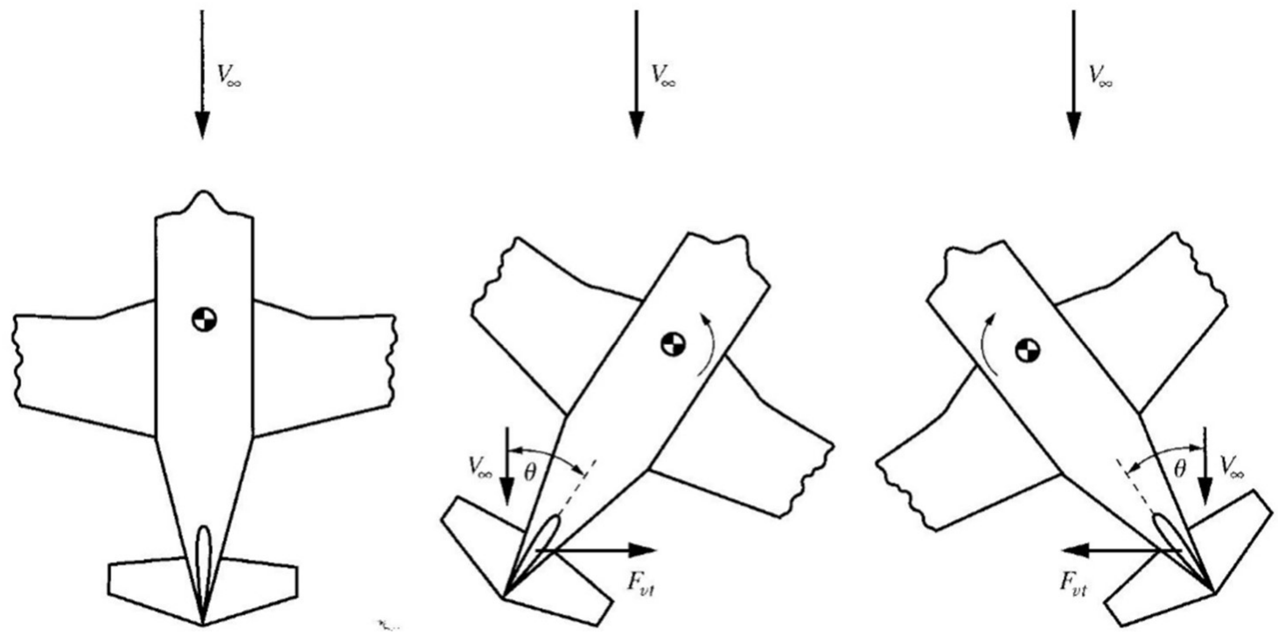
Aircraft Directional Stability

It is seen that the combination of wing, fuselage and **HT** is directionally unstable so **that** we use vertical tail (VT) **at** the back to provide basic directional stability.

VT (also called fin), generates a positive yawing **moment** for a positive sideslip **angle**, so that the aircraft turns into the **wind**, until ‘β’ becomes zero**.**

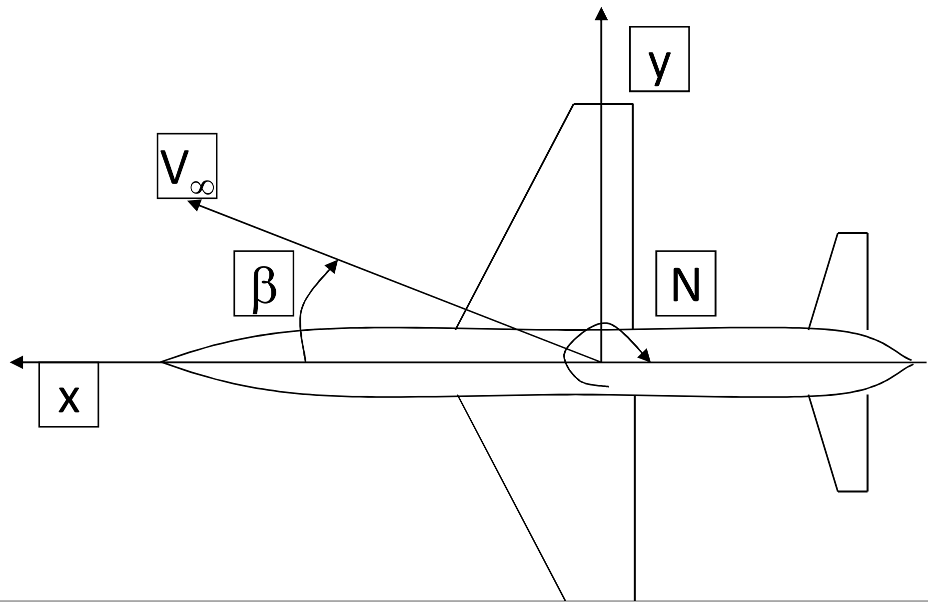
This is called ‘weathercock’ stability, drawing **analogy** from a device of **similar** name, which is used for **predicting** wind direction in the horizontal plane.

Directional Stability Scenarios from VT



(*a*) No yaw (*b*) Yaw to the right (*c*) Yaw to the left

Fuselage + VT Forces and Moments



$\mathbf{z_w}$: y-distance between wing root c/4 point and fuselage centre line

'**d**': maximum fuselage depth

$\boldsymbol{\Lambda_{c/4w}}$: sweep of wing quarter chord.

$$Y_V = -q_V S_V a_V(\beta+\sigma); \quad N_V = -Y_V\left(\bar{x}_{cg} - \bar{x}_{acV}\right)$$

$$N_V = q_V S_V a_V(\beta+\sigma) l_V; \quad C_{N\beta V} = \frac{N}{q_\infty S_w b_w}$$

$$C_{N\beta V} = \frac{Q_V S_V a_V(\beta+\sigma) l_V}{q_\infty S_w b_w} = \eta_V V_V a_V\left(1+\sigma_\beta\right)$$

$$\eta_V = \frac{q_V}{q_\infty} \to \text{Efficiency;} \quad V_V = \frac{S_V l_V}{S_w b_w} \to \text{Fin Volume Ratio}$$

$$a_V = \left(\frac{dC_Y}{d\beta}\right)_V, \quad \sigma_\beta = \frac{d\sigma}{d\beta} \to \text{Sidewash Derivative}$$

$$\text{For directional stability: } S_v l_v > \frac{2V_f}{\eta_v a_v(1+\sigma_\beta)}$$

$$\eta_v(1+\sigma_\beta) \approx 0.724 + \frac{3.06\times\left(S_V/S_w\right)}{1+\cos\Lambda_{c/4w}} + 0.4\frac{z_w}{d} + 0.009 AR_w$$

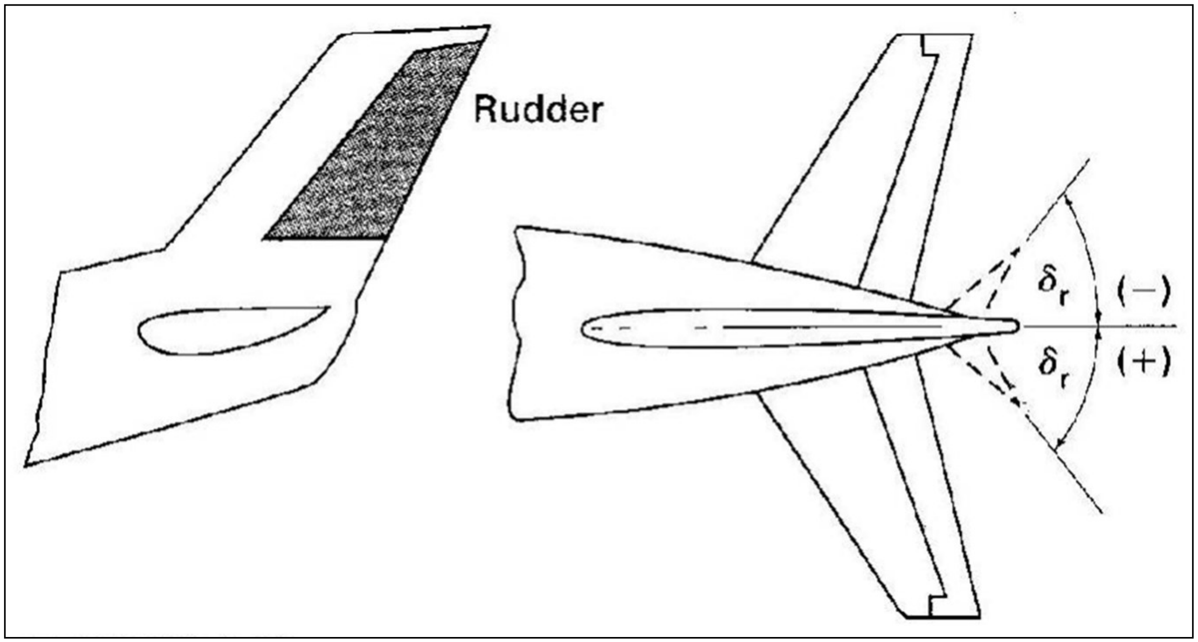
Directional Control

Similar to the longitudinal control using elevator to **fly** at different trim, there is also a **need** for a directional control**,** **under** various conditions, as listed below**.**

Flight under crosswind, yaw due to propeller slip-**stream**, spin manoeuvre, adverse yaw **due** to roll, one-engine off etc. are **some** of the situations that need directional control.

Therefore, a small part of fin, called 'rudder', is **made** moveable with respect to **fin** and is operated using **'**pedals' at the **pilot** station, to generate the yawing moment.

Rudder Concept



$$N = -L_V \times l_V \rightarrow C_N = -\frac{C_{LV} S_V q_V}{q_\infty S_w b}$$

$$\frac{dC_N}{d\delta_r} = -a_V \frac{d\alpha_V}{d\delta_r} V_V \eta_V ; \quad \frac{d\alpha_V}{d\delta_r} = \tau$$