AE 341: Flight mechanics of aircrafts and spacecrafts

Lateral-directional motion

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Recap of basics

- Recall the various angles, ϕ, ψ, β, χ
- Forces: X, Y, Z
- Moments: $\mathcal{L}, M, N$
- μ vs ϕ

Rate variables

- $q_b - q_w = \dot{\alpha}$
- $p_b - p_w = \dot{\beta} \sin \alpha$
- $r_b - r_w = -\dot{\beta} \cos \alpha$
- Note: $x_b - x_0$ is displacement from x_0 to x_b
- Note: β is positive from body to wind!
- This is why $p_b - p_w, r_b - r_w$ have the corresponding sign

$q1, p1$ and $q2, p2$ derivatives

- Subtle but important difference
- Key idea: wind axis quantities w.r.t. airplane motion
- $q1$ term when airplane body rotates w.r.t. wind axis
- OTOH $q2$ is when both wind and body are moving: curvature of airplane motion (phugoid, pull-up, roll etc.)

Aerodynamic derivative classifications

- Static effects: α, β, Mach number etc.
- Dynamic effects: Angular velocity of a/c w.r.t. that of the wind
- Flow curvature effects: Angular velocity of the wind axis due to motion of a/c
- Downwash lag effect: ϵ related effects

Basic lateral-directional equations

- Trim: $\gamma = 0, \theta = \alpha, \beta^* = 0 = \mu^* = \phi^*$

$$\begin{aligned}
\Delta\chi &= \Delta\beta + \Delta\psi \\
mV^*\Delta\dot{\chi} &= \Delta Y + mg\Delta\mu \\
I_{xx}\Delta\ddot{\phi} &= \Delta\mathcal{L} \\
I_{zz}\Delta\ddot{\psi} &= \Delta N \\
\dot{x_E} &= V^*\cos\chi \\
\dot{y_E} &= V^*\sin\chi
\end{aligned}$$

Timescales

- $Y = \bar{q}SC_Y$
- $T_s = V^*/g$ (10 secs)
- $T_f = \sqrt{I_{zz}/(\bar{q}Sb)}$ (1 s)
- $T_r = b/(2V^*)$ (0.1 s)

Aerodynamic derivatives

- $C_{Y\beta} = \frac{\partial C_Y}{\partial\beta}\Big|_*$
- $C_{Yp1} = \frac{\partial C_Y}{\partial(p_b - p_w)(b/2V)}\Big|_*$
- $C_{Yp2} = \frac{\partial C_Y}{\partial(p_w)(b/2V)}\Big|_*$
- $C_{Yr1} = \frac{\partial C_Y}{\partial(r_b - r_w)(b/2V)}\Big|_*$
- $C_{Yr2} = \frac{\partial C_Y}{\partial(r_w)(b/2V)}\Big|_*$
- Similar for others

Small perturbation equations

- $\Delta p_b - \Delta p_w = \Delta\dot{\beta}\sin\alpha^*$
- $\Delta r_b - \Delta r_w = -\Delta\dot{\beta}\cos\alpha^*$
- For small α^*
  - $\Delta p_b - \Delta p_w \approx 0$
  - $\Delta r_b - \Delta r_w = -\Delta\dot{\beta}$
- Can derive this (assume it here):
  - $\Delta p_w = \Delta\dot{\mu} - \Delta\dot{\chi}\sin\gamma^*$
  - $\Delta r_w = \Delta\dot{\chi}\cos\gamma^*\cos\Delta\mu$
  - Now see that $\gamma^* = 0$ and small $\Delta\mu$

Implications

- $\Delta p_w \approx \Delta\dot{\mu}$
- $\Delta r_w \approx \Delta\dot{\chi}$
- Since $\Delta p_b - \Delta p_w \approx 0$, the $p1$ derivatives are zero!
- We are left with $p2$ derivatives
- Recall that $p2$ derivatives have to do with the motion of the whole a/c

Small perturbation equations ...

- Can ignore $C_{Yp2}, C_{Yr1}, C_{Yr2}$ as they are small
- Writing $Y_\beta = \frac{\bar{q}S}{W}C_{Y\beta}$ and $N_\beta = \frac{\bar{q}S}{I_{zz}}C_{N\beta}$
- $\Delta\dot{\chi} = \frac{g}{V^*}\left[Y_\beta\Delta\beta + \Delta\mu\right]$
- $\Delta\ddot{\psi} = N_\beta\Delta\beta + N_{p2}\Delta\dot{\mu} + N_{r1}(-\Delta\dot{\beta}) + N_{r2}\Delta\dot{\chi}$
- Recall that, $\Delta\ddot{\psi} = \Delta\ddot{\chi} - \Delta\ddot{\beta}$
- So, $\Delta\ddot{\chi} = \frac{g}{V^*}\left[Y_\beta\Delta\dot{\beta} + \Delta\dot{\mu}\right]$

Small perturbation equations ...

- Simplifying, ignoring N_{p2}, for yawing and side force (T_f)

$$\begin{aligned}\Delta\ddot{\beta} + \left[-N_{r1} - \frac{g}{V^*}Y_\beta\right]\Delta\dot{\beta} + \left[N_\beta + \frac{g}{V^*}Y_\beta N_{r2}\right]\Delta\beta \\ -\frac{g}{V^*}\Delta\dot{\mu} + \frac{g}{V^*}N_{r2}\Delta\mu = 0\end{aligned}$$

- For rolling moment and side-force (T_r, T_s):

$$\Delta\ddot{\mu} = -\mathcal{L}_{r1}\Delta\dot{\beta} + \left[\mathcal{L}_\beta + \frac{g}{V^*}Y_\beta\mathcal{L}_{r2}\right]\Delta\beta + \mathcal{L}_{p2}\Delta\dot{\mu} + \frac{g}{V^*}\mathcal{L}_{r2}\Delta\mu$$

Approximations to the modes

- The key idea is to do this in order of fastest mode to slowest
- Roll rate mode (T_r)
- Dutch roll mode (T_f)
- Spiral mode (T_s)

Roll rate mode

- Split $\mu = \mu_r + \mu_f$
- μ_r is large and roll mode governed by:

$$\Delta\ddot{\mu}_r = \mathcal{L}_{p2}\Delta\dot{\mu}_r$$

$$-\mathcal{L}_{r1}\Delta\dot{\beta} + \left[\mathcal{L}_\beta + \frac{g}{V^*}Y_\beta\mathcal{L}_{r2}\right]\Delta\beta + \mathcal{L}_{p2}\Delta\dot{\mu}_f + \frac{g}{V^*}\mathcal{L}_{r2}\Delta\mu = 0$$

- Stability requires that $\mathcal{L}_{p2} < 0$

The residual

- Can solve for $\Delta\dot{\mu}_f$

$$\Delta\dot{\mu}_f = \frac{1}{\mathcal{L}_{p2}}\left\{\mathcal{L}_{r1}\Delta\dot{\beta} - \left[\mathcal{L}_\beta + \frac{g}{V^*}Y_\beta\mathcal{L}_{r2}\right]\Delta\beta - \frac{g}{V^*}\mathcal{L}_{r2}\Delta\mu\right\}$$

Dutch roll mode

- This is at the timescale T_f and is dominated by the $\Delta\beta$ variable, but uses $\Delta\dot{\mu}_f$

$$\Delta\ddot{\beta} + \left[-N_{r1} - \frac{g}{V^*}Y_\beta\right]\Delta\dot{\beta} + \left[N_\beta + \frac{g}{V^*}Y_\beta N_{r2}\right]\Delta\beta$$
$$-\frac{g}{V^*}\Delta\dot{\mu}_f + \frac{g}{V^*}N_{r2}\Delta\mu = 0$$

Dutch roll mode ...

- Plug in $\Delta\dot{\mu}_f$ from the earlier residual

$$\Delta\ddot{\beta} + \left[-N_{r1} - \frac{g}{V^*}Y_\beta\right]\Delta\dot{\beta} + \left[N_\beta + \frac{g}{V^*}Y_\beta N_{r2}\right]\Delta\beta$$
$$-\frac{g}{V^*}\frac{1}{\mathcal{L}_{p2}}\left\{\mathcal{L}_{r1}\Delta\dot{\beta} - \left[\mathcal{L}_\beta + \frac{g}{V^*}Y_\beta\mathcal{L}_{r2}\right]\Delta\beta - \frac{g}{V^*}\mathcal{L}_{r2}\Delta\mu\right\}$$
$$+\frac{g}{V^*}N_{r2}\Delta\mu = 0$$

- Simplify, drop terms of higher order in g/V^*: O(0.1)
- Again split $\Delta\beta = \Delta\beta_f + \Delta\beta_s$

Dutch roll mode ...

- Dutch roll:

$$\Delta\ddot{\beta}_f + \left[-N_{r1} - \frac{g}{V^*}(Y_\beta + \frac{\mathcal{L}_{r1}}{\mathcal{L}_{p2}})\right]\Delta\dot{\beta}_f +$$
$$\left[N_\beta + \frac{g}{V^*}(Y_\beta N_{r2} + \frac{\mathcal{L}_\beta}{\mathcal{L}_{p2}})\right]\Delta\beta_f = 0$$

- Residual

$$\left[N_\beta + \frac{g}{V^*}(Y_\beta N_{r2} + \frac{\mathcal{L}_\beta}{\mathcal{L}_{p2}})\right]\Delta\beta_s + \frac{g}{V^*}N_{r2}\Delta\mu = 0$$

Dutch roll mode ...

- From the first equation we have:

$$-\ \omega_{nDR}^2 = N_\beta + \tfrac{g}{V^*}\left(Y_\beta N_{r2} + \tfrac{\mathcal{L}_\beta}{\mathcal{L}_{p2}}\right)$$
$$-\ 2\zeta\omega_{nDR} = -N_{r1} - \tfrac{g}{V^*}\left(Y_\beta + \tfrac{\mathcal{L}_{r1}}{\mathcal{L}_{p2}}\right)$$

- The residual is now $\Delta\beta_s = \frac{\frac{-g}{V^*}N_{r2}}{\omega_{nDR}^2}\Delta\mu$

Dutch roll mode ...

- Typically Y_β is less significant
- Frequency depends on $N_\beta, \mathcal{L}_\beta$, i.e. $C_{n\beta}, C_{l\beta}$
- Damping on $N_{r1}, \mathcal{L}_{r1}$, i.e. C_{nr1}, C_{lr1}

Dutch roll mode ...

- One can show for stability:

$$C_{n\beta} + \frac{\frac{g}{V^*}\frac{2V^*}{b}}{\bar{q}Sb/I_{zz}} C_{l\beta}/C_{lp2} > 0$$
$$C_{n\beta} + \frac{T_f^2}{T_s T_r} C_{l\beta}/C_{lp2} > 0$$
$$C_{n\beta} + \epsilon C_{l\beta}/C_{lp2} > 0$$

- Also $C_{nr1} + \epsilon C_{lr1}/C_{lp2} < 0$

Spiral mode

- Consider residual of roll mode

$$\Delta\dot{\mu}_s = \frac{1}{\mathcal{L}_{p2}}\left\{\mathcal{L}_{r1}\Delta\dot{\beta}_s - \left[\mathcal{L}_\beta + \frac{g}{V^*}Y_\beta\mathcal{L}_{r2}\right]\Delta\beta_s - \frac{g}{V^*}\mathcal{L}_{r2}\Delta\mu\right\}$$

- Ignore $\Delta\dot{\beta}$; Plug $\Delta\beta_s$ from dutch role residual
- Dropping higher powers of (g/V^*) gives

$$\Delta\dot{\mu} = \frac{1}{\mathcal{L}_{p2}}\left[\frac{\mathcal{L}_\beta N_{r2} - N_\beta \mathcal{L}_{r2}}{\omega_{nDR}^2}\right]\frac{g}{V^*}\Delta\mu$$

Spiral mode ...

- Since $\mathcal{L}_{p2} < 0$
- Stability requires that $\left[\mathcal{L}_\beta N_{r2} - N_\beta \mathcal{L}_{r2}\right] > 0$

Roll damping derivative

- $C_{lp2} < 0$ for roll stability
- C_{lp2} is provided by the wing and vertical tail
- Easy to work out the contribution of each
- Both are negative
- Vertical tail also generates a Yaw

Roll control devices: ailerons

- Aileron deflection is positive in the sense of p, i.e. right down is +ve
- $C_{l\delta e}$ is the roll control derivative
- $\mathcal{L}_{\delta a} = (\bar{q}Sb/I_{xx})C_{l\delta e}$
- $\Delta\ddot{\mu} = \mathcal{L}_{p2}\Delta\dot{\mu} + \mathcal{L}_{\delta a}\Delta\delta a$
- Define $\tau = \frac{d\alpha}{d\delta a}$

Other roll control devices

- Ailerons not effective in supersonic flow
- Or in high angle of attack flow
- Or when $\bar{q}$ is small
- Spoilers can also be used
  - Left spoiler introduces drag and drops lift
  - So plane rolls to the left and also yaws left

Other roll control devices

- Differential tail deflection
- Or using rudder (positive is clockwise from above like r)

Yaw due to roll control

- Adverse yaw: generally preferred for pilot controls
- Proverse or pro yaw
- Ailerons, generate yaw due to drag; use different displacement to up and down
- Spoilers introduce a pro yaw
- Differential tail introduces pro-yaw because of the pressure difference across vertical tail
- Rudder introduces adverse yaw

Directional derivatives

- $C_{Y\beta}, C_{n\beta}$: Vertical tail is the main contributor
- $\Delta\beta \approx v/V^*$
- Angle of attack on the vertical tail and the moment arm l_v
- VTVR: $S_v l_v / Sb$
- Normally, $C_{Y\beta} < 0, \text{and } C_{n\beta} > 0$
- Wing body contribution to $C_{n\beta} < 0$

Lateral derivative

- $C_{l\beta}$
- Banking causes a sideslip
- Sideslip generates a roll: dihedral effect
- A dihedral angle, Γ makes for a negative $C_{l\beta}$
- Anhedral is when $\Gamma < 0$

Lateral derivative ...

- Positive sweep-back angle Λ contributes to negative $C_{l\beta}$
- Wing position on fuselage has an effect
  - Mid-wing: no effect
  - High-wing: $C_{l\beta} < 0$
  - Low-wing: $C_{l\beta} > 0$
- Vertical tail contributes $C_{l\beta} < 0$
- Ventral fin $C_{l\beta} > 0$

Damping derivatives

- C_{nr1}, C_{lr1}
- Recall that $C_{nr1} + \epsilon C_{lr1}/C_{lp2} < 0$
- Wing contributes: $C_{nr1} < 0$and $C_{lr1} > 0$
- Vertical tail: $C_{nr1} < 0$and $C_{lr1} > 0$

Spiral mode ...

- Since $\mathcal{L}_{p2} < 0$
- Stability requires that $\left[\mathcal{L}_\beta N_{r2} - N_\beta \mathcal{L}_{r2}\right] > 0$
- For a turning aircraft with positive $\Delta r_w = \Delta \dot{\chi}$
- $C_{lr2} > 0$ and $C_{nr2} < 0$

Some nice videos

- Dutch roll: https://www.youtube.com/watch?v=2tgfkGiHhxs
- Spiral and Phugoid https://www.youtube.com/watch?v=syR7KC9cODo