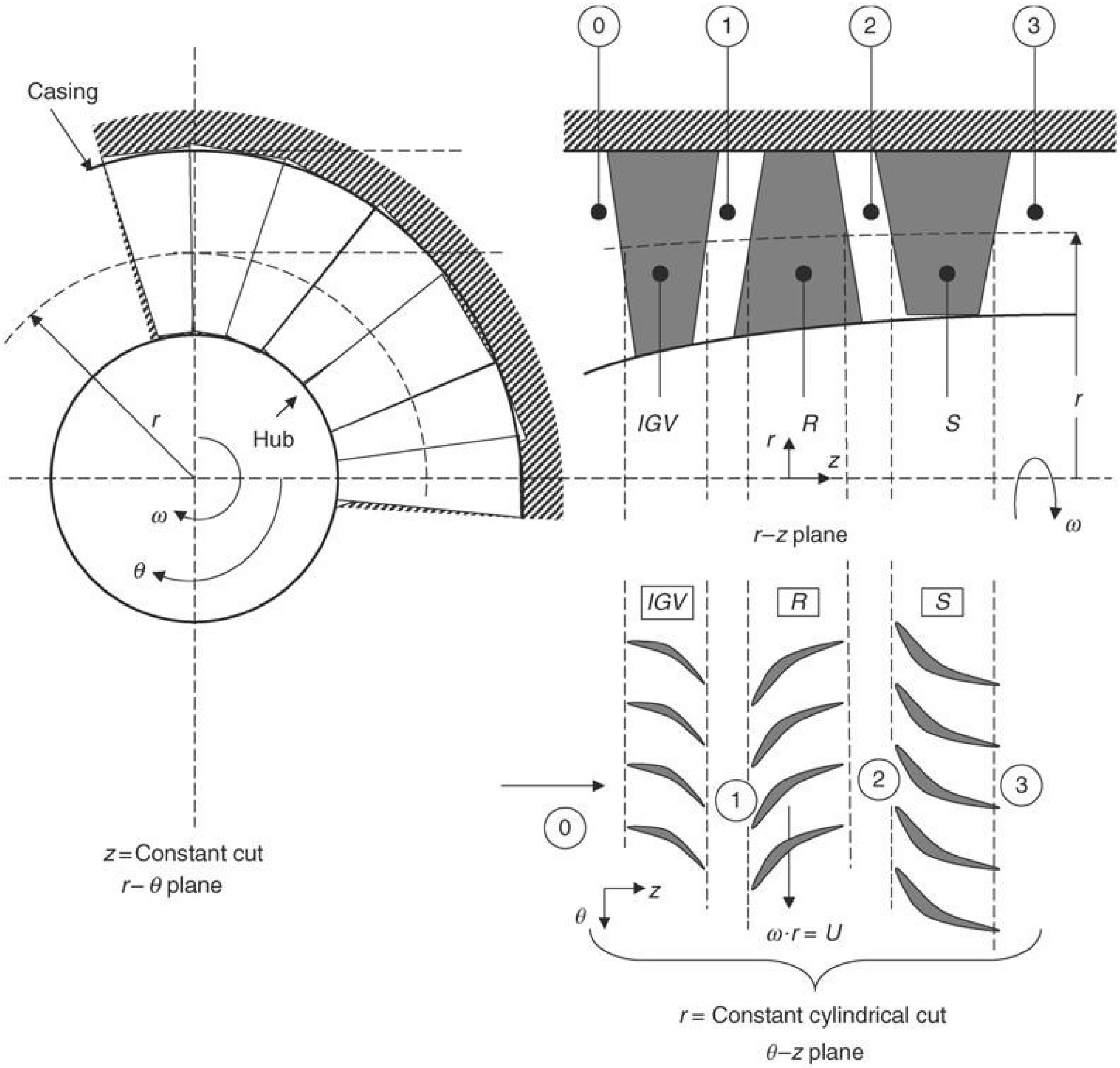
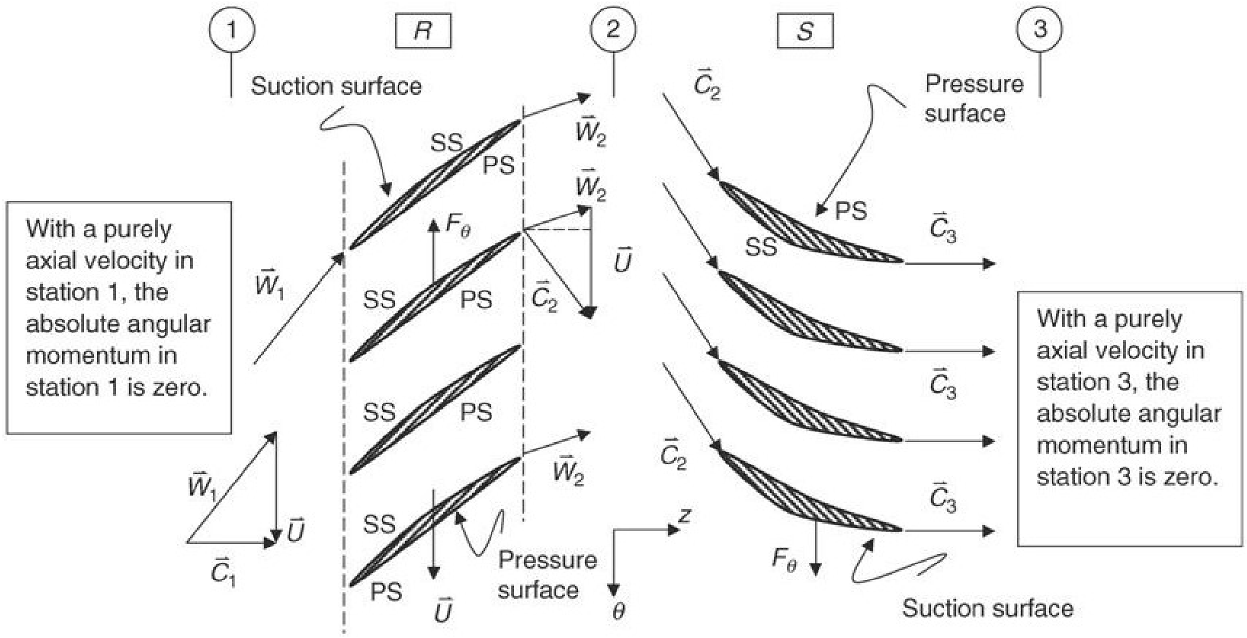
8.6 Axial-Flow Compressors and Fans

Axial-flow compressors and fans provide mechanical compression for the air stream that enters a gas turbine engine. Thermodynamically, their function is to increase the fluid pressure, efficiently. Hence, the boundary layers on the blades of compressors and fans, as well as their hub and casing, are exposed to an adverse, or rising, pressure gradient. Boundary layers exposed to an adverse pressure gradient, due to their inherently low momentum, cannot tolerate significant pressure rise. Consequently, to achieve a large pressure rise, axial-flow compressors and fans need to be staged. With this stipulation, a multistage machinery, or compression system, is born. In a stage, the rotor blade imparts angular momentum to the fluid, while the following stator blade row removes the angular momentum from the fluid. A definition sketch of a compressor stage with an inlet guide vane that imparts angular momentum to the incoming fluid is shown in Figure 8.7. A



■ **FIGURE 8.7 Definition sketch for station numbers and three different planes, r–θ, r–z, and θ–z, in a compressor stage with an inlet guide vane (IGV)**



■ **FIGURE 8.8** **Cylindrical (r = constant) cut of a compressor stage with velocity triangles showing the rotor imparting and the stator removing the swirl and angular momentum to the fluid**

compressor stage without an inlet guide vane is depicted in Figure 8.8. In either case, the principle of rotor increasing the fluid angular momentum and the stator blade removing the swirl (or angular momentum) is independent of any preswirl in the incoming flow to the stage. We may introduce an inlet guide vane upstream of the rotor blades that imparts a preswirl (in the direction of the rotor motion) to the incoming stream and yet the principle of rotor–stator angular momentum interactions with the fluid remains intact.

Based on the absolute velocity field in regions 1, 2, and 3, for a generic inlet condition that may include a preswirl $C_{\theta 1}$ created by an inlet guide vane, we may write the rotor and stator torques

$$\tau_{\text{rotor}} = -\tau_{\text{fluid}} = -\dot{m} \cdot r(C_{\theta 2} - C_{\theta 1}) \qquad \textbf{(8.14)}$$

$$\tau_{\text{stator}} = -\tau_{\text{fluid}} = -\dot{m} \cdot r(C_{\theta 3} - C_{\theta 2}) \qquad \textbf{(8.15)}$$

Assuming the absolute swirl and angular momentum across the stage remains the same, that is, $C_{\theta 1} = \mathrm{C}_{\theta 3}$ and $r_1 = r_3$ the rotor and stator torques become equal and opposite, namely,

$$\tau_{\text{rotor}} = -\tau_{\text{stator}} \qquad \textbf{(8.16)}$$

We observe the suction and pressure surfaces of the rotor and stator blades, as shown in Figure 8.8, and note that the blade aerodynamic forces in the θ-direction F_θ for the rotor and stator blades are in opposite directions. Since the moment of this tangential force, that is, $r{\cdot}F_\theta$ from the axis of rotation, is the blade torque, we conclude that the rotor and stator torques are opposite in direction and nearly equal in magnitude.

EXAMPLE 8.1

The absolute flow at the pitchline to a compressor rotor is swirl free. The exit flow from the rotor has a positive swirl, $C_{\theta 2} = 145$ m/s. The pitchline radius is $r_m = 0.5$ m, and the rotor angular speed is $\omega = 5600$ rpm. Calculate the specific work at the pitchline and the rotor torque per unit mass flow rate.

SOLUTION

Rotor tangential speed at pitchline is

$$\begin{aligned} U_m &= \omega r_m \\ &= (5600\ \text{rev/min})(2\pi\ \text{rad/rev})(\text{min}/60\ \text{s})(0.5\ \text{m}) \\ &= 293.2\ \text{m/s} \end{aligned}$$

$$\begin{aligned} w_c &\cong \omega r(\Delta C_\theta) = U(C_{\theta 2} - C_{\theta 1}) = 293.2\ \text{m/s}(145\ \text{m/s}) \\ &\approx 42.516\ \text{kJ/kg} \end{aligned}$$

$$\tau_{r,m}/\dot{m} = r_m(C_{\theta 2} - C_{\theta 1}) = 0.5\ \text{m}(145\ \text{m/s}) = 72.5\ \text{m}^2/\text{s}$$

8.6.1 Definition of Flow Angles

The flow angles are measured with respect to the axial direction, or axis of the machine, and are labeled as α and β, which correspond to the absolute and relative flow velocity vectors $\vec{C}$ and $\vec{W}$, respectively. Figure 8.9 is a definition sketch that shows the absolute and relative flow angles in a compressor stage.

We may use these absolute and relative flow angles to express the velocity components in the axial and the swirl direction as

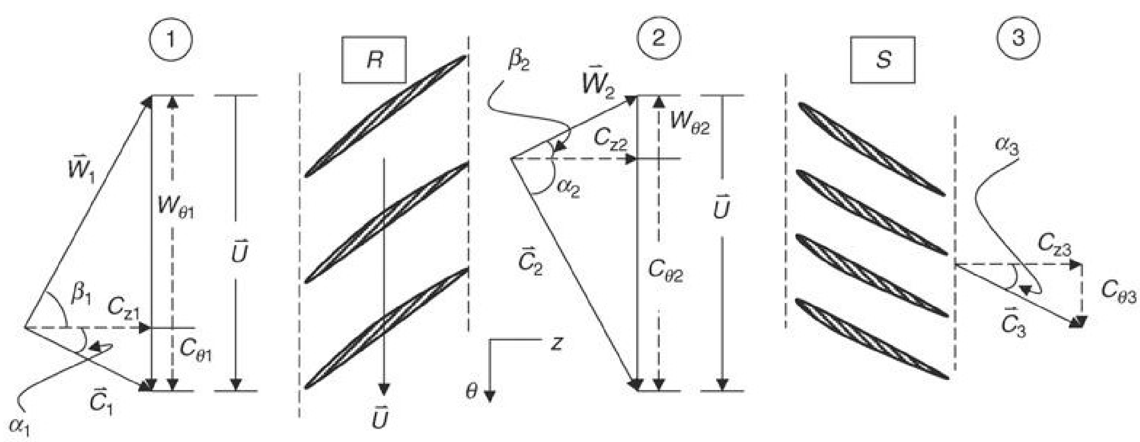
$$C_{\theta 1} = C_{z1} \cdot \tan(\alpha_1) \quad \textbf{(8.17)}$$

$$W_{\theta 2} = C_{z2} \cdot \tan(\beta_1) \quad \textbf{(8.18)}$$

$$C_{\theta 2} = C_{z2} \cdot \tan(\alpha_2) \quad \textbf{(8.19)}$$

$$W_{\theta 2} = C_{z2} \cdot \tan(\beta_2) \quad \textbf{(8.20)}$$

One method of accounting for positive and negative swirl velocities is through a convention for positive and negative flow angles. We observe that the absolute velocity vector upstream



■ **FIGURE 8.9 Definition sketch for the absolute and relative flow angles in a compressor stage (a cylindrical cut of the stage, r = constant)** of the rotor has a swirl component in the direction of the rotor rotation. Hence, the absolute flow angle α_1 is considered positive. The opposite is true for the relative velocity vector $\vec{W}_1$, which has a swirl component in the opposite direction to the rotor rotation. Both the relative flow angle β_1 and the swirl velocity facing it $W_{\theta 1}$ are thus negative.

A turbomachinery blade row is designed to maintain an attached boundary layer, under normal operating conditions. Hence, the flow angles at the exit of the blades are primarily fixed by the blade angle at the exit plane. For example, the relative velocity vector at the exit of the rotor, $\vec{W}_2$, should nearly be tangent to the rotor suction surface at the trailing edge. Hence, β_2 is fixed by the geometry of the rotor and remains nearly constant over a wide operating range of the compressor. The same statement may be made about α_1 or α_3. These angles remain constant over a wide range of the operation of the compressor as well. Again, remember that the exit flow angle argument is made for an attached (boundary layer) flow. The other flow angles, as in β_1 or α_2, change with rotor speed U (i.e., ωr). Consequently, use is made of the nearly constant exit flow angles α_1, β_2, and α_3 in expressing performance parameters of the compressor stage and the blade row.

The axial velocity components C_{z1}, C_{z2}, or C_{z3} contribute to the mass flow rate through the machine. A common (textbook) design approach in axial-flow compressors and fans maintains a constant axial velocity throughout the stages. We shall make use of this simplifying design approach repeatedly in this chapter. Figure 8.8 shows an example of a constant axial velocity in a compressor stage. Although a simplifying design assumption, the reader needs to be aware that the goal of constant axial velocity is nearly impossible to achieve in practice. This is due to a highly three-dimensional nature of the flowfield, set up by three-dimensional pressure gradients, in a turbomachinery stage. We will address this and other topics related to three-dimensional flow in Section 8.6.5.

A useful concept in turbomachinery design calls for a *repeated stage* (also referred to as *normal stage*). This implies that the velocity vectors at the exit and entrance to a stage are the same, that is,

$$\vec{C}_3 = \vec{C}_1 \tag{8.21a}$$

$$\alpha_3 = \alpha_1 \tag{8.21b}$$

The velocity triangles at the inlet and the exit of the stage shown in Figure 8.8 have made use of the repeated stage concept. Another concept in turbomachinery calls for a *repeated row* design, which leads to flow angle implications that are noteworthy. In a repeated row design, the exit relative flow angle has the same magnitude as the absolute inlet flow angle, and the inlet relative flow angle has the same magnitude as the exit absolute flow angle, that is,

$$|\beta_2| = |\alpha_1| \tag{8.22a}$$

$$|\alpha_2| = |\beta_1| \tag{8.22b}$$

The example of the compressor stage shown in Figure 8.9 has used the concept of repeated row. Note that a repeated row design leads to a repeated stage but the reverse is not necessarily correct. Namely, we may have a repeated stage design that does not use a repeated row concept. Figure 8.8 shows an example of a repeated stage that does not obey a repeated row design.

8.6.2 Stage Parameters

The Euler turbine equation that we derived earlier is the starting point for this section with the assumption of $r_1 \approx r_2 \approx r$,

$$\frac{T_{t2}}{T_{t1}} = 1 + \frac{U\left(C_{\theta 2} - C_{\theta 1}\right)}{c_p T_{t1}}$$

We may replace the swirl velocities by the flow angles and the axial velocity components, namely,

$$\frac{T_{t2}}{T_{t1}} = 1 + \frac{U\left(C_{z2}\tan\alpha_2 - C_{z1}\tan\alpha_1\right)}{c_p T_{t1}} = 1 + \left(\frac{U^2}{c_p T_{t1}}\right)\left(\frac{C_{z1}}{U}\right)\left(\frac{C_{z2}}{C_{z1}}\tan\alpha_2 - \tan\alpha_1\right) \tag{8.23}$$

Expression 8.23 for the total temperature rise across the rotor (or stage) involves nondimensional groups C_{z1}/U and $c_\mathrm{p}T_{t1}/U^2$. These groups appear throughout the turbomachinery literature and deserve a special attention. Also, the axial velocity ratio C_{z2}/C_{z1} appears that is often set equal to 1, as a first-order design assumption. Equation 8.23 is expressed in terms of the absolute flow angle at the exit of the rotor, which is not, however, a good choice, since it varies with the rotor speed. A better choice for the flow angle in plane 2, that is, downstream of the rotor, is the relative flow angle β_2. The relative exit flow angle β_2 remains nearly unchanged as long as the flow remains attached to the blades. To express the total temperature rise across the rotor to the flow angles α_1 and β_2, we replace the absolute swirl $C_{\theta 2}$ by the relative swirl speed, namely,

$$C_{\theta 2} = U + W_{\theta 2} \tag{8.24}$$

Therefore,

$$\frac{T_{t2}}{T_{t1}} = 1 + \left(\frac{U^2}{c_p T_{t1}}\right)\left[1 + \frac{C_{z2}}{U}\tan\beta_2 - \frac{C_{z1}}{U}\tan\alpha_1\right] \tag{8.25a}$$

If we assume a constant axial velocity design, that is, $C_{z1} = C_{z2}$, then we get

$$\frac{T_{t2}}{T_{t1}} = 1 + \left(\frac{U^2}{c_p T_{t1}}\right)\left[1 + \left(\frac{C_z}{U}\right)\left(\tan\beta_2 - \tan\alpha_1\right)\right] \tag{8.25b}$$

Here, we have expressed the stage total temperature ratio as a function of two nondimensional parameters. Note that β_2 is a negative angle and α_1 is a positive angle, according to our sign convention. Therefore, the contribution to the stage total temperature ratio falls with increasing (C_z/U) for a given wheel speed and inlet stagnation enthalpy. The ratio of axial-to-wheel speed is called the *flow coefficient* ϕ

$$\phi \equiv \frac{C_z}{U} \tag{8.26a}$$

We may divide both numerator and the denominator of Equation 8.26a by the speed of sound in plane 1, that is, a_1, to get the ratio of axial to blade (rotational) or tangential Mach number, namely,

$$\phi = \frac{C_z/a_1}{U/a_1} = \frac{M_z}{M_T} \tag{8.26b}$$

where M_z is the axial Mach number, and M_T is the blade tangential Mach number based on U and a_1. For example, a stream surface with an axial Mach number of 0.5 that approaches a section of a rotor that is spinning at Mach 1 has a flow coefficient of 0.5. Now, let us interpret the first nondimensional group $U^2/(c_pT_{t1})$. We may divide this expression by the square of the upstream speed of sound a_1^2 to get

$$\frac{U^2/a_1^2}{c_pT_{t1}/(\gamma RT_1)} = \frac{(\gamma-1)\,M_T^2}{1+\left(\frac{\gamma-1}{2}\right)M_1^2} \tag{8.27}$$

Noting that the absolute Mach number M_1 is expressible in terms of the axial Mach number and a constant preswirl angle α_1 as

$$M_1 = \frac{M_z}{\cos\alpha_1} \tag{8.28}$$

We conclude that

$$\frac{U^2}{c_pT_{t1}} = \frac{(\gamma-1)\,M_T^2}{1+\left(\frac{\gamma-1}{2}\right)\frac{M_z^2}{\cos^2\alpha_1}} \tag{8.29}$$

Based on Equations 8.26b and 8.29, we may recast the stage stagnation temperature ratio in terms of blade tangential and axial Mach numbers as

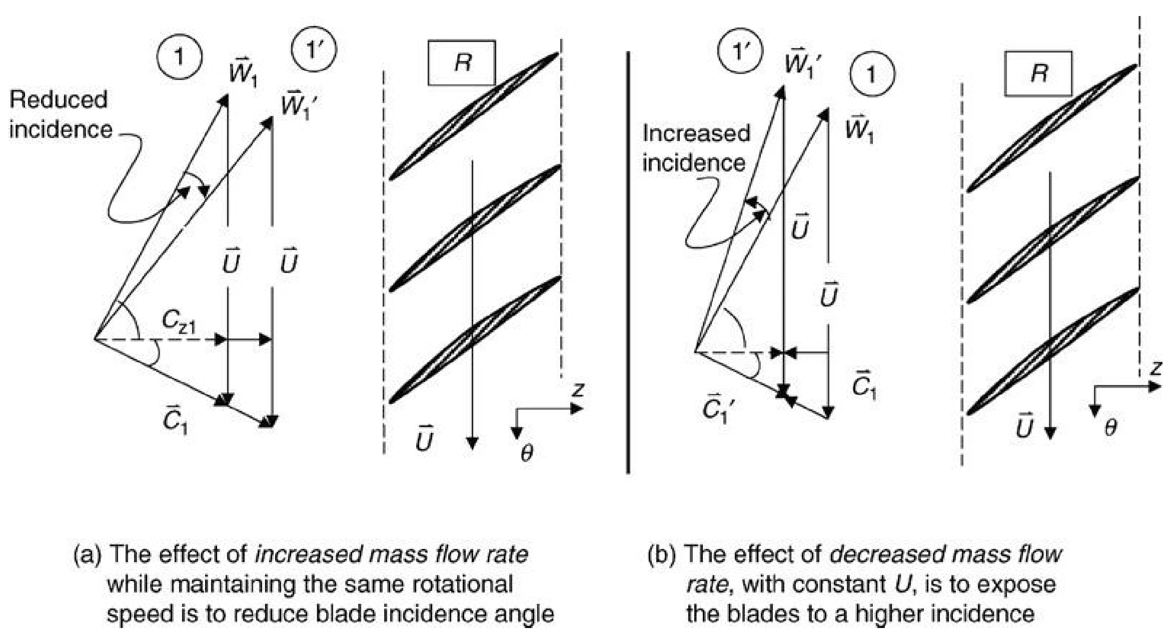
$$\frac{T_{t2}}{T_{t1}} = 1 + \left[\frac{(\gamma-1)\,M_T^2}{1+\left(\frac{\gamma-1}{2}\right)\frac{M_z^2}{\cos^2\alpha_1}}\right]\left[1+\left(\frac{M_z}{M_T}\right)\left(\tan\beta_2-\tan\alpha_1\right)\right] \tag{8.30}$$

Kerrebrock (1992) offers an insightful discussion of compressor aerodynamics based on this equation. Two Mach numbers, that is, the axial and the blade tangential Mach numbers, appear to influence the stage temperature ratio in three ways, namely, via M_z, M_T, and M_z/M_T influence in Equation 8.30. However, if we divide the first bracket on the right-hand side (RHS) of Equation 8.30 by the square of the blade tangential Mach number, we reduce this dependency to two parameters, namely,

$$\frac{T_{t2}}{T_{t1}} = 1 + \left[\frac{\gamma - 1}{\left(\frac{1}{M_T^2}\right) + \left(\frac{\gamma - 1}{2\cos^2 \alpha_1}\right)\left(\frac{M_z}{M_T}\right)^2}\right]\left[1 + \left(\frac{M_z}{M_T}\right)\left(\tan\beta_2 - \tan\alpha_1\right)\right] \tag{8.31}$$

Consequently, two parameters govern the stage temperature ratio (or pressure ratio) and these are the blade tangential Mach number M_T and the flow coefficient or the ratio of the axial-to-tangential Mach number M_z/M_T. For given flow angles α_1 and β_2, an increase in the flow coefficient reduces the temperature rise in the stage. We may interpret an increase in the flow coefficient as an increase in the axial Mach number or the mass flow rate through the machine. As the mass flow rate increases, keeping the blade rotational Mach number the same, the blade angle of attack, or in the language of turbomachinery the *incidence angle,* decreases, hence the total temperature ratio drops. The opposite effect is observed with a decreasing mass flow rate through the machine where the incidence angle increases and thus blade work on the fluid increases to produce higher temperature or pressure rise. To visualize the effect of throughflow on rotor work, temperature and pressure rise, a rotor blade at different axial Mach numbers (or flow rate) and the same blade tangential Mach number (or wheel speed) are shown in Figure 8.10.

A reduced flow rate leads to an increased compressor temperature (or pressure) ratio. However, there is a limitation on how low the flow rate, or axial Mach number, can sink before the blades stall, for a fixed shaft rotational speed. Consequently, at reduced flow rates we could enter a blade stall flow instability, which marks the lower limit of



■ **FIGURE 8.10** **Inlet velocity triangles for a compressor rotor with a changing flow rate** the axial flow speed on the compressor map for a given shaft speed. The increased flow rate that leads to a reduction of the stage total temperature rise has its own limitation. The phenomenon of negative stall could be entered as the inlet Mach number is increased significantly.

The second parameter is the blade tangential Mach number M_T. A higher blade tangential Mach number increases the stage total temperature rise as deduced from Equation 8.31. However, the limitations on the blade tangential Mach number are the appearance of strong shock waves at the tip as well as the structural limitations under centrifugal and vibratory stresses. The rotor shock losses increase (nonlinearly) with the relative tip Mach number; however, the advantage of higher work ($\propto M_T^2$) on the fluid outweigh the negatives of such a design at modest tip Mach numbers. The relative tip Mach number is defined as

$$M_{\text{tip,r}} = \sqrt{M_z^2 + \left(M_{\text{T,tip}}^2\right)} \tag{8.32}$$

where it represents the case of zero preswirl. The general case that includes a nonzero preswirl, may be written as

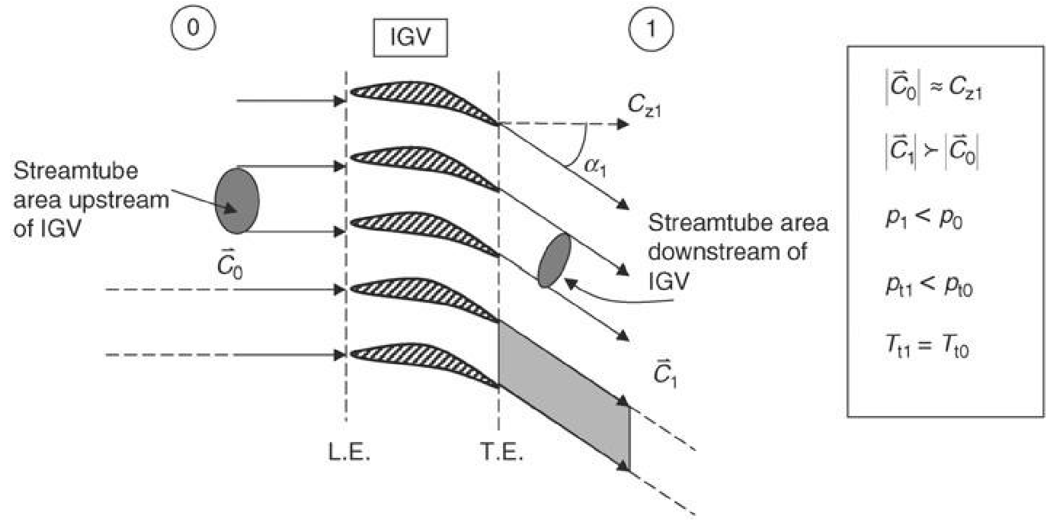
$$M_{\text{tip,r}} = \sqrt{M_z^2 + \left(M_{\text{T,tip}} - M_z \tan\alpha_1\right)^2} \tag{8.33}$$

The relative blade Mach numbers at the tip of operational fan blades have been supersonic for the past three decades. By necessity, these blade sections should be thin to avoid stronger bow shocks. A typical value for the relative tip Mach number is ~1.2 but may be designed as high as ~1.7. The thickness-to-chord ratio of supersonic blading may be as low as ~3%. High strength-to-weight ratio titanium alloys represented the enabling technology that allowed the production of supersonic (tip) fans in the early 1970s.

The inlet absolute flow angle to the rotor α_1 is a design choice that has the effect of reducing the rotor relative tip speed. The inlet preswirl is created by a set of inlet guide vanes, known as an IGV. To reduce the relative flow to the rotor tip, the IGV turns the flow in the direction of the rotor rotation, by α_1. The flowfield entering an IGV is swirl-free and thus the function of the IGV is to impart positive swirl (or positive angular momentum) to the incoming fluid. This in effect reduces the rotor blade loading whose purpose is to impart angular momentum to the incoming fluid. An inlet guide vane and its flowfield are shown in Figure 8.11.

We note that the blade passages in the IGV form a subsonic nozzle (i.e., contracting area) and cause flow acceleration across the blade row. The result of the flow acceleration is found in the static pressure drop due to flow acceleration, as well as a total pressure drop due to frictional losses of the blade passages. Hence, if the compressor design could avoid the use of an IGV, then certain advantages, including cost and weight savings, are gained. The advantage of operating at higher tip speeds at times outweighs the disadvantages of an IGV. The inlet guide vane may also be actuated rapidly if a quick response in thrust modulation is needed. For example, consider a lift fan, or a deflected engine exhaust flow, to support a VTOL aircraft in hover mode. The ability to modulate the jet lift (or vertical thrust) for stability and maneuver purposes may not be achieved through a spool up or spool down throttle sequence of the engine. Due to a large moment of inertia of the rotating parts in a turbomachinery, the rapid spooling is not fast enough for the control

■ **FIGURE 8.11** **An inlet guide vane is seen to impart swirl to the fluid (note the shrinking stream tube area implies flow acceleration and static pressure drop)**



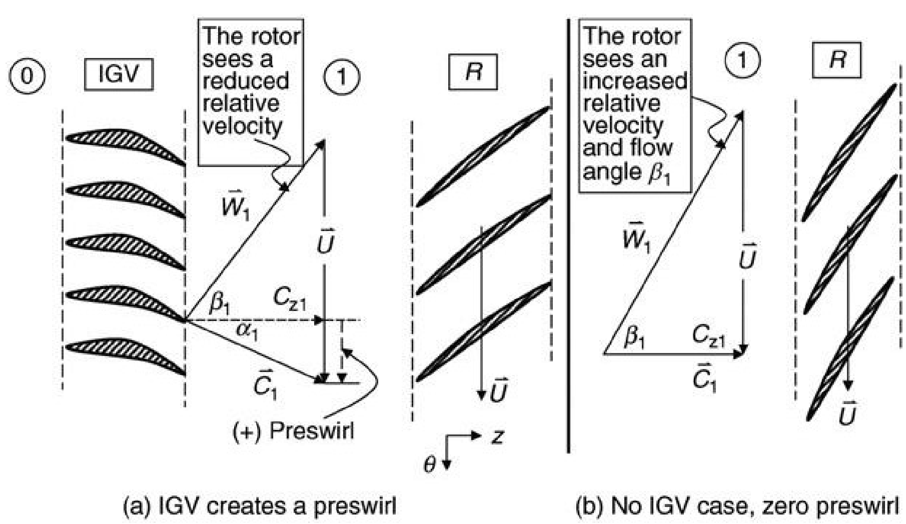
and stability purposes of a VTOL aircraft. In such applications, fast-acting IGV actuators could modulate the inlet flow to the rotor and hence the thrust/lift. An adjustable exit louver at the deflected nozzle end achieves the same goal. The inlet swirl angle α_1 may be zero or be adjustable in the range of ±30° or more using a variable geometry IGV to relieve the relative tip Mach number, improve efficiency at compressor off-design operation, and provide for rapid thrust/lift modulation in military aircraft.

An example of a velocity triangle upstream of the rotor blade with and without the inlet guide vane is shown in Figure 8.12. In both cases, we maintain the rotational speed and the mass flow rate, i.e., the axial velocity C_z constant.

The stage total pressure ratio is related to the stage total temperature ratio via a stage efficiency parameter η_s. Recalling from the chapter on cycle analysis,

$$\eta_s \equiv \frac{T_{t3s} - T_{t1}}{T_{t3} - T_{t1}} = \frac{\pi_s^{\frac{\gamma-1}{\gamma}} - 1}{\tau_s - 1} \tag{8.34}$$

■ **FIGURE 8.12** **Impact of IGV on the rotor relative flow for constant axial velocity and rotor speed**



Therefore, we may calculate the stage total pressure ratio from the velocity triangles that are used in the Euler turbine equation to establish τ_s and an efficiency parameter η_s to get

$$\pi_s = \left[1 + \eta_s\left(\tau_s - 1\right)\right]^{\frac{\gamma}{\gamma-1}} \tag{8.35}$$

Since the axial flow compressor pressure ratio per stage is small (i.e., near 1), the stage adiabatic efficiency and the polytropic efficiency are nearly equal. We recall that the polytropic efficiency was also called the "small-stage" efficiency, valid for an infinitesimal stage work. Although the approximation

$$\eta_s \cong e_c \tag{8.36}$$

is valid for low-pressure ratio axial flow compressor stages. The exact relationship between these parameters is derived in the cycle analysis chapter to be

$$\eta_s = \frac{\tau_s - 1}{\tau_s^{\frac{1}{e_c}} - 1} \tag{8.37}$$

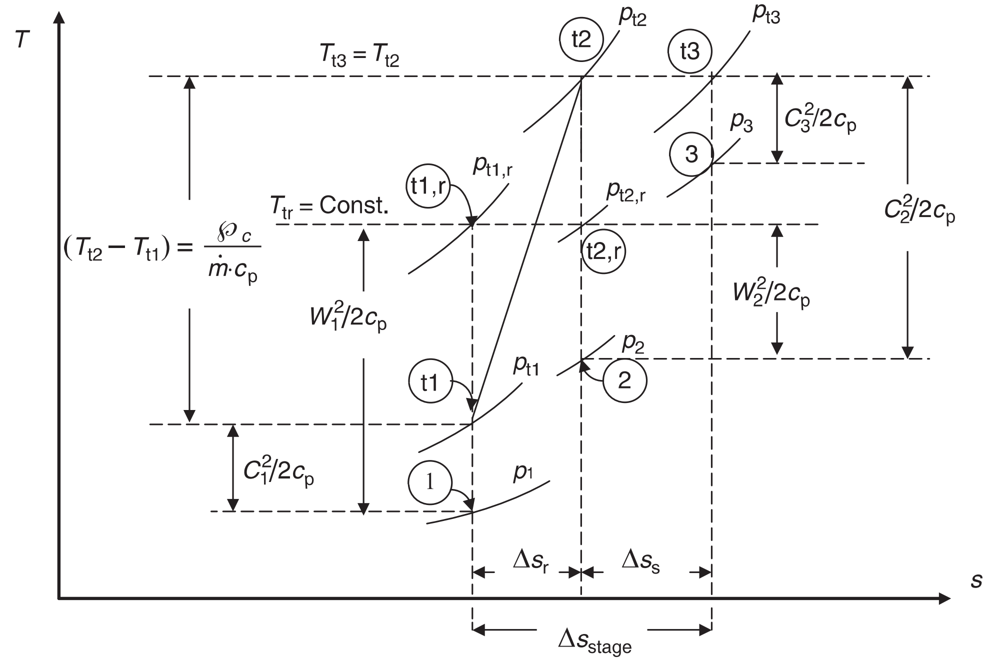
Therefore, by calculating the stage total temperature ratio from the Euler turbine equation and assuming polytropic efficiency, of say 0.90, we may calculate the stage adiabatic efficiency. Otherwise, we may calculate the stage total pressure ratio from the polytropic efficiency e_c and the stage total temperature ratio directly, via

$$\pi_s = \tau_s^{\frac{\gamma \cdot e_c}{\gamma-1}} \tag{8.38}$$

The frictional and shock losses are the main contributors to a total pressure loss in the relative frame of reference. We note that in a relative frame of reference, the blade passage is stationary and thus blade aerodynamic forces perform no work. Examining the fluid total pressure across the blade through the eyes of a relative observer, there are only losses to report. As noted, these losses stem from the viscous and turbulent dissipation of mechanical (kinetic) energy into heat as well as the flow losses associated with a shock. From the vantage point of an absolute observer, the blades are rotating and doing work on the fluid, thus increasing the fluid total temperature and pressure in the absolute frame of reference. These processes may be shown on a *T–s* diagram (Figure 8.13) that is instructive to review. For example, follow the static state of gas across the stage then follow the stagnation states in the absolute and relative frames. Explain the behavior of kinetic energy of gas as the fluid encounters the stationary and rotating blade rows, as seen by absolute and relative observers.

The *relative* total enthalpy is constant across the rotor along a stream surface (in steady flow without a radial shift in the stream surface),

$$h_{t,r} = h_1 + \frac{W_1^2}{2} = h_2 + \frac{W_2^2}{2} \tag{8.39}$$



■ **FIGURE 8.13**
Absolute and relative states of gas across a compressor rotor and stator

We may replace the total kinetic energy terms in Equation 8.39 by the sum of the kinetic energy of the component velocities, namely,

$$h_1 + W_{z1}^2/2 + W_{\theta 1}^2/2 = h_2 + W_{z2}^2/2 + W_{\theta 2}^2/2 \tag{8.40}$$

Let us substitute the absolute swirl minus the wheel speed ($C_\theta - U$) for the relative swirl in Equation 8.40, to get

$$h_1 + C_{z1}^2/2 + C_{\theta 1}^2/2 - C_{\theta 1} \cdot U = h_2 + C_{z2}^2/2 + C_{\theta 2}^2/2 - C_{\theta 2} U \tag{8.41}$$

The sum of the first three terms is the absolute total enthalpy on each side of the above equality, therefore,

$$h_{t1} - UC_{\theta 1} = h_{t2} - UC_{\theta 2} \tag{8.42}$$

This enthalpy constant in the rotor frame is known as "rothalpy," which may be rearranged to help us arrive at the Euler turbine equation, that is,

$$h_{t2} - h_{t1} = \frac{\wp}{\dot{m}} = w_c = U(C_{\theta 2} - C_{\theta 1}) \tag{8.43}$$

The rotor specific work on the fluid in nondimensional form is written as

$$\psi \equiv \frac{\Delta h_t}{U^2} = \frac{C_{\theta 2}}{U} - \frac{C_{\theta 1}}{U} = \frac{W_{\theta 2} + U}{U} - \frac{C_{\theta 1}}{U} = 1 + \left(\frac{C_z}{U}\right)\left(\tan\beta_2 - \tan\alpha_1\right) \tag{8.44}$$

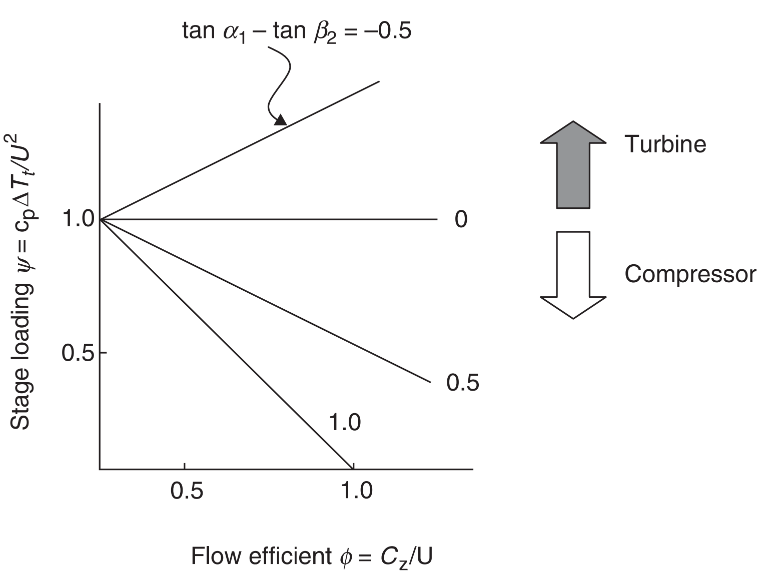
The function ψ is the nondimensional stage work parameter, which is called the *stage-loading factor or parameter* defined in Equation 8.44. The ratio of axial-to-wheel speed was called the flow coefficient ϕ, which forms another two-parameter family for the stage characteristics, namely,

$$\psi = 1 + \phi\left(\tan\beta_2 - \tan\alpha_1\right) \tag{8.45}$$

We note that the rotor exit relative flow angle is a negative quantity that leads to a stage-loading factor that is less than 1. In the case of zero preswirl, or no inlet guide vane, the stage loading factor increases with a decreasing rotor exit flow angle β_2. In the limit of zero relative swirl at the rotor exit and no inlet guide vane, the stage loading factor approaches unity. However, a rotor relative exit flow angle of zero implies significant turning in the rotor blade passage, which may lead to flow separation. The stage-loading factor is an alternate form of expressing the stage characteristics and, in essence, takes the place of the rotor tangential Mach number M_T, which was presented earlier. Horlock (1973) takes advantage of the linear dependence of the stage loading and the flow coefficient in Equation 8.45 to explore the off-design behavior of ideal turbomachinery stages. We shall discuss the off-design behavior of turbomachinery later in this chapter but for now show the linear dependence of the two parameters in Figure 8.14 (adapted from Horlock, 1973).

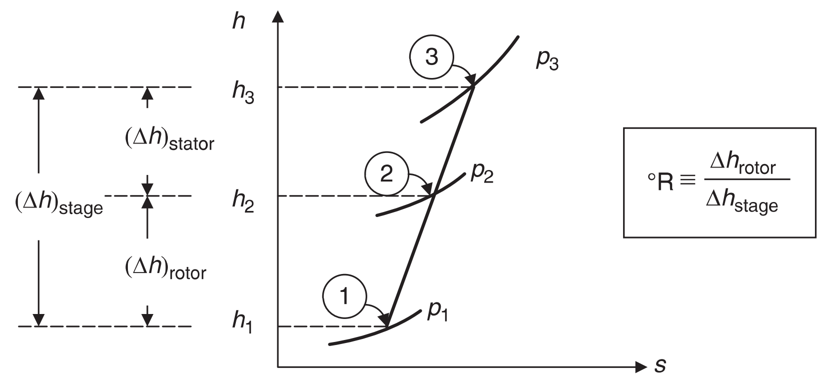
We define a stage *degree of reaction* °R as the fraction of static enthalpy rise across the stage that is accomplished by the rotor. Although the stator does no work on the fluid, it still acts as a diffuser that decelerates the fluid and thus causes an increase in fluid temperature, or static enthalpy. The stator takes out the swirl (kinetic energy) put in by the rotor and thus converts it to a static pressure rise. The degree of reaction measures the rotor share of the stage enthalpy rise as compared with the burden on the stator. This process is shown in an *h–s* diagram for the static states in a compressor stage. Remember that the static states are independent of the motion of the observer, hence they carry no subscript labels besides the station number (Figure 8.15).

$$°\mathrm{R} \equiv \frac{h_2 - h_1}{h_3 - h_1} = \frac{h_{t2} - h_{t1} - (C_2^2 - C_1^2)/2}{h_{t3} - h_{t1} - (C_3^2 - C_1^2)/2} \tag{8.46a}$$



■ FIGURE 8.14 Linear dependence of stage loading and flow coefficient parameters in ideal turbomachinery stages. Source: Adapted from Horlock 1973

■ **FIGURE 8.15**
Static states of gas in a compressor stage and a definition sketch for the stage degree of reaction °R



The stagnation enthalpy across the stator remains constant as the stator blades do no work on the fluid, hence $h_{t3} = h_{t2}$ and if we assume a repeated stage, with $C_1 = C_3$, we simplify the above expression to get

$$°\mathrm{R} \cong 1 - \frac{C_2^2 - C_1^2}{2(h_{t2} - h_{t1})} = 1 - \frac{C_{z2}^2 + C_{\theta 2}^2 - C_{z1}^2 + C_{\theta 1}^2}{2U(C_{\theta 2} - C_{\theta 1})} \tag{8.46b}$$

Now, for a constant axial velocity across the rotor, $C_{z2} = C_{z1}$, we get a simple expression for the stage degree of reaction, as

$$°\mathrm{R} \cong 1 - \frac{C_{\theta 2} + C_{\theta 1}}{2U} = 1 - \frac{C_{\theta,\mathrm{mean}}}{U} \tag{8.46c}$$

where $C_{\theta,\mathrm{mean}}$ is the average swirl across the rotor. Since the flow has to fight an uphill battle with an adverse pressure gradient throughout a compressor stage, it stands to reason to expect/design an equal burden of the static pressure rise in the rotor as that of a stator. Consequently, a 50% degree of reaction stage may be thought of as desirable. We may express the swirl velocity components across the rotor in terms of the absolute in flow and relative exit flow angles, α_1 and β_2, respectively,

$$°\mathrm{R} = 1 - \frac{W_{\theta 2} + U + C_{\theta 1}}{2U} = \frac{1}{2} - \frac{C_z \tan\beta_2 + C_z \tan\alpha_1}{2U} = \frac{1}{2} - \left(\frac{C_z}{U}\right)\left(\frac{\tan\beta_2 + \tan\alpha_1}{2}\right) \tag{8.47}$$

For a 50% degree of reaction stage (at some spanwise radius r), the rotor exit flow angle has to be equal in magnitude and opposite to the inlet absolute flow angle, which is dictated by an IGV, namely,

$$\beta_2 = -\alpha_1$$

This is the condition for a repeated row design, as noted in Equations 8.22a and 8.22b. Therefore a purely axial inflow (with no inlet guide vane) demands a purely axial relative outflow in order to produce a 50% degree of reaction. Again, we need to examine the net turning angle across the blade and assess the potential for flow separation. We shall introduce another parameter that will shed light on the state of the boundary layer at the blade exit and that is the diffusion factor.

On the degree of reaction, there is a body of experimental results that support the proposal that a boundary layer on a spinning blade, that is, the rotor, is more *stable* than a corresponding boundary layer on a stationary blade, that is, the stator, hence allocating a slightly higher burden of static pressure rise to the rotor. Here the word *stable* is used in the context of resistant to adverse pressure gradient or higher stalling pressure rise. Based on this, a degree of reaction of 60% may be a desirable split between the two blade rows in a compressor stage. As we shall see in the three-dimensional flow section in this chapter, the desirable degree of reaction is often compromised at different radii along a blade span to satisfy other requirements, namely, a healthy state of boundary layer flow.

EXAMPLE 8.2

An axial-flow compressor stage has a pitchline radius of $r_m = 0.5$ m. The rotational speed of the rotor at pitchline is $U_m = 212$ m/s. The absolute inlet flow to the rotor is described by $C_{zm} = 155$ m/s and $C_{\theta 1m} = 28$ m/s. The stage degree of reaction at pitchline is $°R_m = 0.60$, $\alpha_3 = \alpha_1$, and C_{zm} remains constant. Calculate

(a) rotor angular speed ω in rpm

(b) rotor exit swirl $C_{\theta 2m}$

(c) rotor specific work at pitchline w_{cm}

(d) relative velocity vector at the rotor exit

(e) rotor and stator torque per unit mass flow rate

(f) stage loading parameter at pitchline ψ_m

(g) flow coefficient ϕ_m.

SOLUTION

$\omega = U_m/r_m = (212 \text{ m/s})/0.5 \text{ m} = (424 \text{ rad/s}) (\text{rev}/2\pi \text{ rad}) (60 \text{ s/min}) \approx 4{,}049$ rpm

From Equation 8.46c, written at the pitchline

$$°R_m \cong 1 - \frac{C_{\theta 2m} + C_{\theta 1m}}{2U_m}$$

We isolate $C_{\theta 2m}$ to be

$$\begin{aligned} C_{\theta 2m} &= 2U_m(1 - °R_m) - C_{\theta 1m} \\ &= 2(212 \text{ m/s})(0.4) - 28 \text{ m/s} \approx 141.6 \text{ m/s} \end{aligned}$$

Euler turbine equation describes the rotor specific work,

$$w_c \equiv \frac{\wp_c}{\dot{m}_c} = \omega\Delta\left(rC_\theta\right)_c, \quad \text{therefore,}$$

$$\begin{aligned} w_{cm} &= U_m(C_{\theta 2m} - C_{\theta 1m}) \\ &= 212 \text{ m/s } (141.6 - 28)\text{m/s} \approx 24.08 \text{ kJ/kg} \end{aligned}$$

Rotor relative swirl at the exit is

$$W_{\theta 2m} = C_{\theta 2m} - U_m = (141.6 - 212)\text{m/s} = -71 \text{ m/s}$$

Since the axial component of velocity remains constant, we can write the vector

$$\vec{W}_{2m} = 155\hat{k} - 71\hat{e}_\theta$$

Since $\alpha_3 = \alpha_1$, the rotor and stator torques are equal and opposite to eachother, i.e.,

$$\begin{aligned} \tau_{rm}/\dot{m} &= r_m(C_{\theta 2m} - C_{\theta 1m}) = 0.5\text{m}(141.6 - 28) \text{ m/s} \\ &= 56.8\text{m}^2/\text{s} = -\tau_{sm}/\dot{m} \end{aligned}$$

The stage loading parameter and flow coefficients are

$$\begin{aligned} \psi_m &= \Delta C_\theta/U_m = (141.6 - 28)/212 \approx 0.5358 \\ \phi_m &= C_{zm}/U_m = 155/212 = 0.731 \end{aligned}$$

Another figure of merit for a compressor blade section that addresses the health of a boundary layer is, as noted earlier, the *Diffusion Factor*, or the *D*-factor. Its definitions for rotor and stator blades are, respectively

$$D_r \equiv 1 - \frac{W_2}{W_1} + \frac{|W_{\theta 2} - W_{\theta 1}|}{2\sigma_r W_1} \qquad \text{(rotor } D\text{-Factor)} \tag{8.48}$$

$$D_s \equiv 1 - \frac{C_3}{C_2} + \frac{|C_{\theta 3} - C_{\theta 2}|}{2\sigma_s C_2} \qquad \text{(stator } D\text{-Factor)} \tag{8.49}$$

where σ defines the blade solidity, that is, the ratio of blade chord c to spacing s

$$\sigma_r \equiv \frac{c_r}{s_r} \qquad \text{(rotor solidity)} \tag{8.50}$$

$$\sigma_s \equiv \frac{c_s}{s_s} \qquad \text{(stator solidity)} \tag{8.51}$$

Modern compressor design utilizes a high solidity ($\sigma_m \geq 1$) blading at the pitchline radius (i.e., r_m). Since the blade spacing increases linearly with radius, the solidity of a constant chord blade also decreases linearly with the blade span, that is,

$$s(r) = \frac{2\pi \cdot r}{N_b} \qquad \text{(blade spacing)} \tag{8.52}$$

where the N_b is the number of blades in the rotor or the stator row. We can see the variation of blade row solidity with blade span by substituting Equation 8.52 for the blade spacing, as

$$\sigma(r) = \frac{N_b \cdot c}{2\pi \cdot r} \qquad \text{(blade solidity)} \tag{8.53}$$

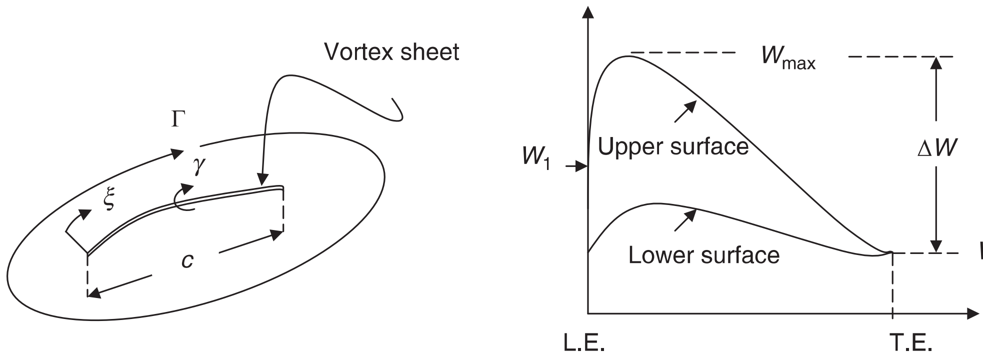
The hub section (r_h) has thus the highest solidity and the tip section (r_t) the lowest. The rationale for the definition of D-factor and its link to blade stall is made by Lieblein (1953, 1959, 1965). We review it here for its physical importance. First note that the definition of rotor diffusion factor is the same as the stator D-factor, except the parameters for the rotor are all in relative frame of reference. Next note that the change in swirl velocity across the rotor is the same regardless of the frame of reference, that is, the change in absolute swirl is the same in magnitude as the change in relative swirl, namely,

$$|W_{\theta 2} - W_{\theta 1}| = |C_{\theta 2} - C_{\theta 1}| \tag{8.54}$$

The blade circulation Γ is the integral of the vortex sheet strength γ over the chord, namely,

$$\Gamma \equiv \int_0^c \gamma(\xi) d\xi \approx \bar{\gamma} \cdot c \tag{8.55}$$

Figure 8.16 shows an element of a blade section represented by a vortex sheet of local strength $\gamma(\xi)$. The local strength is equal to the local tangential velocity jump across the



■ **FIGURE 8.16** **Blade airfoil section is represented by a vortex sheet and a velocity distribution**

blade. The average vortex sheet strength is thus the average velocity jump across the sheet, namely,

$$\bar{\gamma} \approx (\Delta W)_{avg} \approx \frac{W_{max} - W_1}{2} \tag{8.56}$$

Therefore, the average (positive) circulation around the blade section is

$$\Gamma_{avg} \approx \frac{c}{2}\left(W_{max} - W_1\right) \tag{8.57}$$

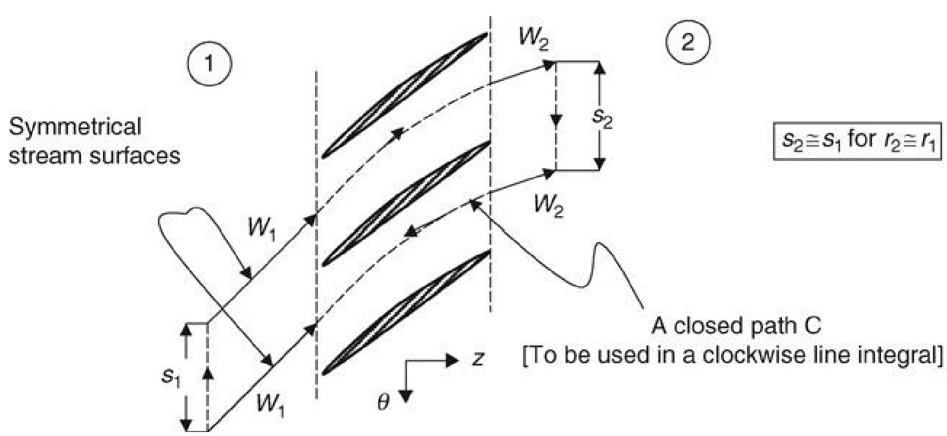
Now, let us consider a control volume symmetrically wrapped around a blade section, as shown in Figure 8.17.

By performing a closed line integral in the clockwise direction around the path C (Figure 8.18), we calculate the magnitude of blade circulation Γ as

$$\Gamma = s|W_{\theta 1} - W_{\theta 2}| \tag{8.58}$$

Therefore equating the two expressions for the (magnitude of) blade circulation, we get

$$W_{max} \approx W_1 + \frac{|W_{\theta 2} - W_{\theta 1}|}{2\sigma} \tag{8.59}$$



■ **FIGURE 8.17** **Control volume for determining the blade circulation Γ along a stream surface**

The maximum adverse pressure gradient on the blade suction surface leads to a maximum flow diffusion, which may be measured by the following parameter

$$\text{Maximum flow diffusion} \approx \frac{W_{max} - W_2}{W_2} \approx \frac{W_{max} - W_2}{W_1} \tag{8.60}$$

This parameter is called the *Diffusion Factor* D

$$D \equiv \frac{W_{max} - W_2}{W_1} = \frac{W_1 + (|\Delta C_\theta|/2\sigma) - W_2}{W_1} = 1 - \frac{W_2}{W_1} + \frac{|\Delta C_\theta|}{2\sigma W_1} \tag{8.61}$$

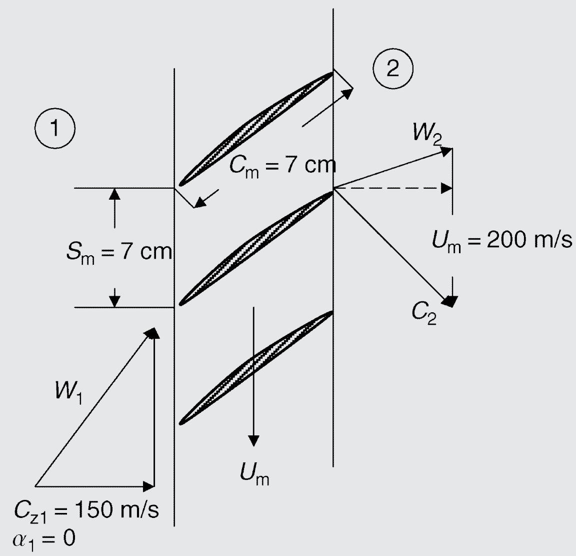
EXAMPLE 8.3

A rotor blade row is cut at pitchline, r_m. The velocity vectors at the inlet and exit of the rotor are shown.

Assuming that $U_{m1} = U_{m2} = 200$ m/s and $C_{z1} = C_{z2} = 150$ m/s

And $\beta_2 = -35°$, calculate

(a) W_{1m} and W_{2m}
(b) D-factor D_{rm}
(c) circulation Γ_m



SOLUTION

$$W_{1m} = [(150)^2 + (200)^2]^{1/2} = 250 \text{ m/s}$$

$$W_{\theta 2m} = C_{z2} \tan\beta_2 = (150 \text{ m/s}) \tan(-35°)$$

$$W_{\theta 2m} \approx -105 \text{ m/s}$$

$$W_{2m} = [(150)^2 + (105)^2]^{1/2} = 183.1 \text{ m/s}$$

The D-factor for the rotor at the pitchline is

$$D_{rm} \equiv 1 - \frac{W_{2m}}{W_{1m}} + \frac{|W_{\theta 2m} - W_{\theta 1m}|}{2\sigma_{rm} W_{1m}}$$

The rotor solidity at the pitchline is $\sigma_{rm} = c/s = 7/7 = 1.0$ The relative tangential speed at the inlet to the rotor is equal in magnitude to the rotor speed U_m, but it has a negative value, as it points in the opposite direction to the rotor rotation

$$W_{\theta 1m} = -200 \text{ m/s}$$

Therefore, we have

$$D_{rm} = 1 - (183.1/250) + |(-105 + 200)|/(2.1.250)$$
$$\approx 0.457$$

Since D-factor is less than ~0.5, the rotor boundary layer at the pitchline is expected to be attached.

Circulation around the rotor at pitchline is

$$\Gamma_m = s_m\, |W_{\theta 1m} - W_{\theta 2m}| = (0.07\text{m})|(-200 + 105)|$$
$$\approx 6.65 \text{ m}^2/\text{s}$$