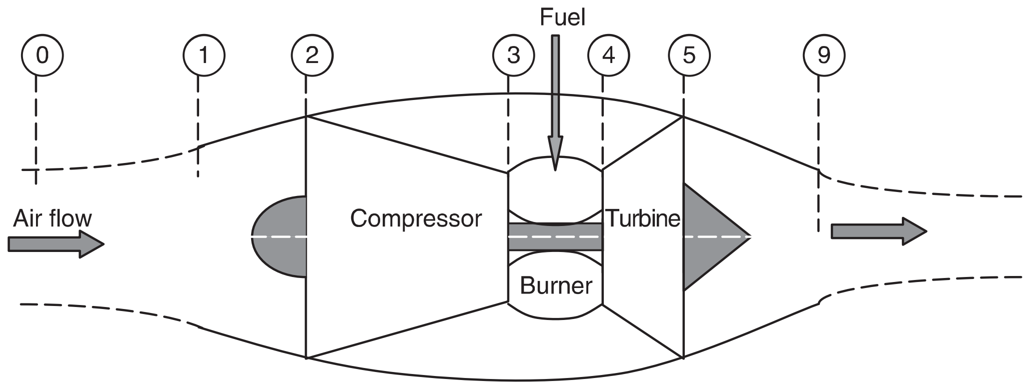


■ **FIGURE 4.2**
Schematic drawing of a turbojet engine

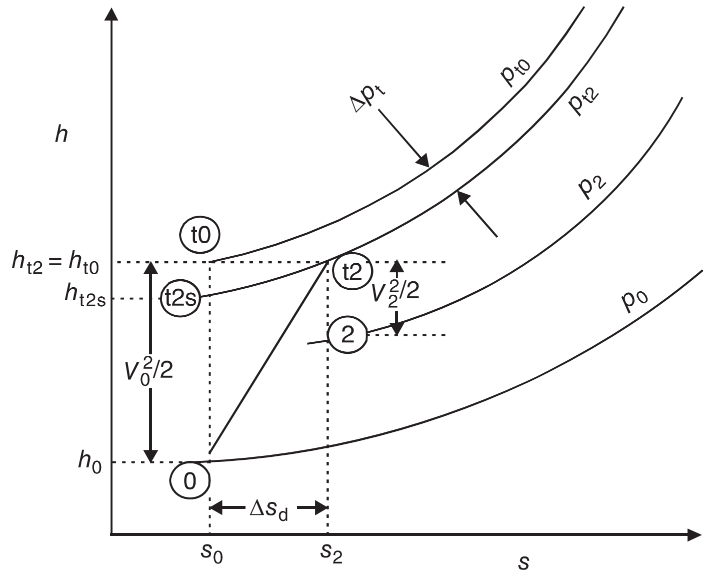
The additional parameters that are needed to calculate the performance of a turbojet engine are the inlet and exhaust component efficiencies. These parameters will be defined in the next section. The station numbers in a turbojet engine are defined at the unperturbed flight condition, 0, inlet lip, 1, compressor face, 2, compressor exit, 3, burner exit, 4, turbine exit, 5, and the nozzle exit plane, 9.

4.3.1.1 The Inlet. The *basic* function of the inlet is to deliver the air to the compressor at the *right* Mach number M_2 and the *right* quality, that is, low distortion. The subsonic compressors are designed for an axial Mach number of $M_2 = 0.5$–0.6. Therefore, if the flight Mach number is higher than 0.5 or 0.6, which includes all commercial (fixed wing) transports and military (fixed wing) aircraft, then the inlet is required to *decelerate* the air efficiently. Therefore, the main function of an inlet is to *diffuse or decelerate* the flow, and hence it is also called a diffuser. Flow deceleration is accompanied by the static pressure rise or what is known as the *adverse pressure gradient* in fluid dynamics. As one of the first principles of fluid mechanics, we learned that the boundary layers, being of a low-energy and momentum-deficit zone, facing an adverse pressure gradient environment tend to separate. Therefore, one of the challenges facing an inlet designer is to prevent inlet boundary layer separation. One can achieve this by tailoring the geometry of the inlet to avoid rapid diffusion or possibly through variable geometry inlet design. Now, it becomes obvious why an aircraft inlet designer faces a bigger challenge if the inlet has to decelerate a Mach 2 or 3 stream to the compressor face Mach number of 0.5 than an aircraft that flies at Mach 0.8 or 0.9. In the present section, we will examine the *thermodynamics* of an aircraft inlet. This exciting area of propulsion, that is, inlet aerodynamics, will be treated in more detail in Chapter 6.

An ideal inlet is considered to provide a *reversible and adiabatic*, that is, isentropic, compression of the captured flow to the engine. The adiabatic aspect is actually met in real inlets as well. This requirement says that there is no heat exchange between the captured stream and the ambient air, through the diffuser walls. We remember (from the Fourier law of heat conduction) that for heat transfer to take place through the inlet wall, we need to set up a *temperature gradient* across the wall, such that

$$q_{\mathrm{n}} \equiv \frac{\dot{Q}_{\mathrm{n}}}{A} = -k\frac{\partial T}{\partial n} \tag{4.1}$$

where n denotes the direction of heat conduction, k the thermal conductivity of the wall, and q_{n} the heat flux, which is defined as the heat transfer rate $\dot{Q}_{\mathrm{n}}$ per unit area A.



■ **FIGURE 4.3** ***h–s* diagram of an aircraft engine inlet flow under real and ideal conditions**

Equation 4.1 signifies the importance of a *gradient* to set up the heat transfer. The temperature gradient across the inlet wall (i.e., nacelle) is negligibly small, and consequently the inlet aerodynamics is considered to be *adiabatic,* even in a real flow. Therefore, it is only the *reversible* aspect of our ideal inlet flow assumption that negates the realities of wall friction and any shocks, which are invariably present in *real* supersonic flows. The process of compression in a real inlet can be shown on the *h-s* diagram of Figure 4.3.

Now, let us describe the information we observe in Figure 4.3. First, four isobars are shown on the *h-s* diagram. From the lowest to the highest value, these are p_0, the static pressure at the flight altitude, p_2, the static pressure at the engine face, p_{t2}, the total pressure at the inlet discharge (or compressor face), and p_{t0}, the flight total pressure. We also note five thermodynamic states identified, as the flight static (0), the flight total (t0), the compressor face or inlet discharge static (2), and total (t2) and a stagnation state (t2s) that does not actually exist! Note, that the first four states, that is, 0, t0, 2, and t2, do all in reality exist and are measured. The states 0 and t0 are measured by a pitot-static tube on an aircraft, and the static and total pressures at the engine face are measured by a pressure rake or inlet pitot tubes. We note that the state (t2s) is at the intersection of s_0 and p_{t2}, which is arrived at isentropically from state (0) to a state that shares the same total pressure as that of a real inlet (t2). We note an entropy rise Δs_d across the inlet as well. The total enthalpy of the real inlet h_{t2} is identified to be the same as the total enthalpy of flight h_{t0}. This is based on the earlier assertion that a *real* inlet flow may be considered to be *adiabatic*. It is to be noted that although the total energy of the fluid in a real inlet is not changing, there is an *energy conversion* that takes place in the inlet. The inlet kinetic energy is partially converted to the static pressure (rise) and partially it is dissipated into heat. It is the latter portion, that is, the dissipation, which renders the process irreversible and cause the entropy to rise. The entropy rise in an adiabatic process leads to a total pressure loss Δp_t following the combined first and second law of thermodynamics, that is, Gibbs equation, according to

$$\frac{p_{t2}}{p_{t0}} = e^{-\frac{s_2 - s_0}{R}} = e^{-\frac{\Delta s}{R}} \tag{4.2}$$

From Figure 4.3, we also note that the states (t0) and (0) are separated *isentropically* by the amount of $V_0^2/2$, as expected from the definition of stagnation state and its relation to the static state. The states (t2s) and (0) are separated isentropically by a kinetic energy amount, which produces p_{t2} as its stagnation state and obviously the rest of the kinetic energy, that is, the gap between the states (t0) and (t2s), represents the amount of dissipated kinetic energy into heat. Therefore, the smaller the gap between the states (t2s) and (t0), the more efficient will the diffuser flow process be. Consequently, we use the fictitious state (t2s) in a definition of inlet efficiency (or a figure of merit) known as the *inlet adiabatic efficiency*. Symbolically, the inlet adiabatic efficiency is defined as

$$\eta_d \equiv \frac{h_{t2s} - h_0}{h_{t2} - h_0} = \frac{(V^2/2)_{\text{ideal}}}{V_0^2/2} \tag{4.3}$$

The practical form of the above definition is derived when we divide the numerator and denominator by h_0 to get

$$\eta_d = \frac{\dfrac{h_{t2s}}{h_0} - 1}{\dfrac{h_{t2}}{h_0} - 1} = \frac{\dfrac{T_{t2s}}{T_0} - 1}{\dfrac{h_{t0}}{h_0} - 1} = \frac{\left(\dfrac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma - 1}{2} M_0^2} = \frac{\left(\dfrac{p_{t2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\tau_r - 1} \tag{4.4}$$

where we have used the isentropic relation between the states (t2s) and (0). Note that the only unknown in Equation 4.4 is p_{t2} for a given flight altitude p_0, flight Mach number M_0, and an inlet adiabatic efficiency η_d. We can separate the unknown term p_{t2} and write the following expression:

$$\frac{p_{t2}}{p_0} = \left\{1 + \eta_d \frac{\gamma - 1}{2} M_0^2\right\}^{\frac{\gamma}{\gamma-1}} \tag{4.5}$$

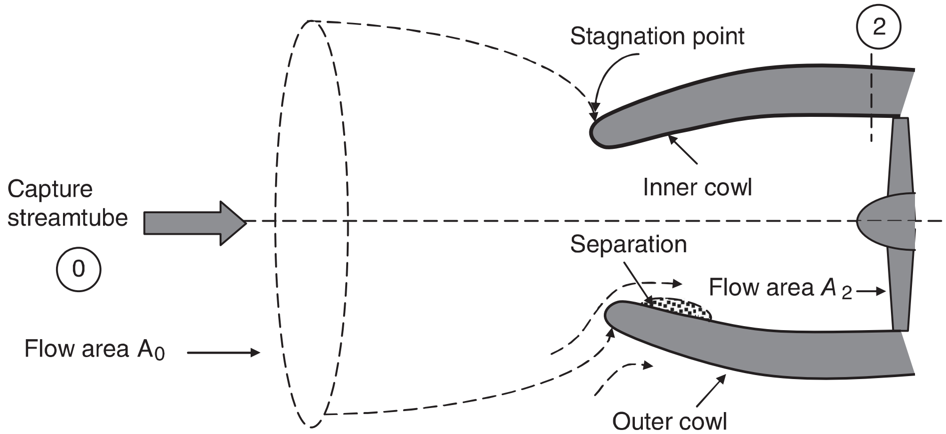
It is interesting to note that Equation 4.5 recovers the isentropic relation for a 100% efficient inlet or $\eta_d = 1.0$. Another parameter, or a *figure of merit*, that describes the inlet performance is the total pressure ratio between the compressor face and the (total) flight condition. This is given a symbol π_d and is often referred to as the *inlet total pressure recovery*:

$$\pi_d \equiv \frac{p_{t2}}{p_{t0}} \tag{4.6}$$

As expected, the two figures of merit for an inlet, that is, η_d or π_d, are not independent from each other and we can derive a relationship between η_d and π_d working the left-hand side of Equation 4.5, as follows:

$$\frac{p_{t2}}{p_0} = \frac{p_{t2}}{p_{t0}} \frac{p_{t0}}{p_0} = \left\{1 + \eta_d \frac{\gamma - 1}{2} M_0^2\right\}^{\frac{\gamma}{\gamma-1}} \tag{4.6a}$$

$$\pi_d = \frac{\left\{1 + \eta_d \dfrac{\gamma - 1}{2} M_0^2\right\}^{\frac{\gamma}{\gamma-1}}}{\dfrac{p_{t0}}{p_0}} = \left\{\frac{1 + \eta_d \dfrac{\gamma - 1}{2} M_0^2}{1 + \dfrac{\gamma - 1}{2} M_0^2}\right\}^{\frac{\gamma}{\gamma-1}} = \frac{\left[1 + \eta_d \dfrac{\gamma - 1}{2} M_0\right]^{\frac{\gamma}{\gamma-1}}}{\pi_r} \tag{4.6b}$$

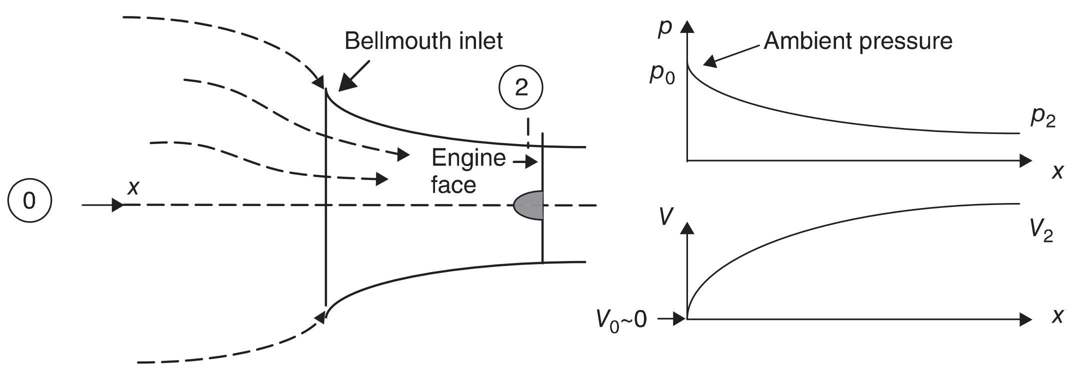


■ **FIGURE 4.4** **Schematic drawing of an inlet flow at low-speed and/or takeoff flight condition (note: $A_0 > A_2$)**

Therefore, Equation 4.6b relates the inlet total pressure recovery π_d to the inlet adiabatic efficiency η_d at any flight Mach number M_0. We note that Equation 4.6b as $\eta_d \to 1$, then $\pi_d \to 1$ as well, as expected. Figure 4.3 also shows the static state 2, which shares the same entropy as the total state t2 and lies below it by the amount of kinetic energy at 2, namely, $V_2^2/2$. Compare the kinetic energy at 2 and at 0, shown in Figure 4.3, and then justify the static pressure rise achieved in the inlet (diffuser), $p_2 - p_0$.

So far, we have considered the cruise condition or the high-speed end of the flight envelope and have thought of inlets as diffusers. Under low-speed or takeoff conditions, the captured stream tube will instead undergo acceleration to the engine face and as such the inlet acts like a nozzle! In Figure 4.4, the schematics of a captured stream tube, under a low-speed flight condition, are shown. Note that the stagnation point is on the outer cowl and flow accelerates to the engine face ($A_0 > A_2$). The aerodynamics of the outer nacelle geometry affects the drag divergence of the inlet and plays a major role in the propulsion system integration studies of engine installation. However, the cycle analysis phase usually disregards the external performance and concentrates on the internal evaluation of the propulsion system.

The nature of the flow path into the inlet at low forward speed or under static engine testing conditions requires the inlet to act as a nozzle; therefore, it explains the use of a bellmouth in static test rigs. Figure 4.5 shows the schematic drawing of an inlet configuration in a static test rig. In Figure 4.5, we note that the static pressure along the stream lines drop from the ambient p_0 to the engine face pressure of p_2. Also, we note



■ **FIGURE 4.5** **Bellmouth inlet guides the flow smoothly into the engine on a static test rig**

- the turbine nozzle is choked (i.e., the throat Mach number is 1), over a wide operating range of the engine, and as such is the first *throttle station* of the engine.

EXAMPLE 4.4

Consider an uncooled gas turbine with its inlet condition the same as the exit condition of the combustor described in Example 4.3. The turbine adiabatic efficiency is 88%. The turbine produces a shaft power to drive the compressor and other accessories at $\wp_t = 45$ MW. Assuming that the gas properties in the turbine are the same as the burner exit in Example 4.3, calculate

(a) turbine exit total temperature T_{t5} in K
(b) turbine polytropic efficiency, e_t
(c) turbine exit total pressure p_{t5} in kPa
(d) turbine shaft power $\wp_t$ based on turbine expansion ΔT_t

SOLUTION

The turbine shaft power is proportional to the mass flow rate through the turbine, which from Example 4.3 is 51 kg/s (50 for air and 1 for fuel flow rate), as well the total enthalpy drop, i.e.,

$$\wp_t = \dot{m}_t(h_{t4} - h_{t5})$$

Therefore, we isolate h_{t5} from above equation to get

$$\begin{aligned} h_{t5} &= h_{t4} - \wp_t/\dot{m}_t = c_{p4}T_{t4} - 35 \times 10^6/51 \\ &= 1.156\,\mathrm{kJ/kg\cdot K}\,(1390\,\mathrm{K}) \\ &\quad - 45{,}000\,\mathrm{kW}/51\,\mathrm{kg/s} \approx 724.5\,\mathrm{kJ/kg} \end{aligned}$$

$$\begin{aligned} T_{t5} &= h_{t5}/c_{p5} \\ &= (724.5\,\mathrm{kJ/kg})/(1.156\,\mathrm{kJ/kg\cdot K}) \approx 626.7\,\mathrm{K} \end{aligned}$$

Turbine polytropic efficiency e_t may be related to its adiabatic efficiency and τ_t via equation

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}}$$

The turbine expansion parameter $\tau_t = 626.7/1390 = 0.4509$; and if we isolate e_t from above equation we get

$$\begin{aligned} e_t &= \ln(\tau_t)/\ln[1 - (1 - \tau_t)/\eta_t] \\ &= \ln(0.4509)/\ln[1 - 0.5491/0.88] \cong 0.8144 \end{aligned}$$

We know that turbine pressure and temperature ratios are related by the polytropic efficiency via

$$\pi_t = \tau_t^{\frac{\gamma}{(\gamma-1)e_t}} = (0.4509)^{\frac{1.33}{0.33(0.8144)}} \cong 0.01941$$

The turbine exit total pressure is therefore $p_{t4} \cdot \pi_t$. We had found $p_{t4} = 1.92$ MPa in Example 4.3, therefore,

$$p_{t5} = 0.01941(1.92\,\mathrm{MPa}) = 37.26\,\mathrm{kPa}$$

The turbine shaft power is the product of the turbine mass flow rate and the total enthalpy drop across the turbine, i.e.,

$$\begin{aligned} \wp_t &= \dot{m}_t c_{pt}(T_{t4} - T_{t5}) \\ &= 51\,\mathrm{kg/s}\,(1156\,\mathrm{J/kg\cdot K})(1390 - 626.7)\,\mathrm{K} \cong 45\,\mathrm{MW} \end{aligned}$$

EXAMPLE 4.5

Consider a turbine nozzle blade row with a hot gas mass flow rate of 50 kg/s and $h_{tg} = 1850$ kJ/kg. The nozzle blades are internally cooled with a coolant mass flow rate of 0.5 kg/s and $h_{tc} = 904$ kJ/kg as the coolant is ejected through the nozzle blades trailing edge. The coolant mixes with the hot gas and causes a reduction in the mixed-out enthalpy of the gas. Calculate the mixed-out total enthalpy after the nozzle. Also for the $c_{p,\,\text{mixed-out}} = 1594$ J/kg · K, calculate the mixed-out total temperature.

SOLUTION

A simple energy balance between the mixed-out state and the hot and cold streams solves this problem, namely,

$$\begin{aligned}\dot{m}_c h_{t,c} + \dot{m}_g h_{t,g} &= (\dot{m}_c + \dot{m}_g) h_{t,\text{mixed-out}} \\ 50.5\,\text{kg/s}\,(h_{t,\text{mixed-out}}) &= 0.5\,\text{kg/s}\,(904\,\text{kJ/kg}) \\ &\quad + 50\,\text{kg/s}\,(1850\,\text{kJ/kg}) \\ &= 92{,}952\,\text{kW}\end{aligned}$$

$$h_{t,\text{mixed-out}} = 92{,}952\,\text{kW}/50.5\,\text{kg/s} = 1840.6\,\text{kJ/kg}$$

$$T_{t,\text{ mixed-out}} = 1840.6\,\text{kJ/kg}/1.594\,\text{kJ/kg}\cdot\text{K} = 1154.7\,\text{K}$$

EXAMPLE 4.6

Consider the internally cooled turbine nozzle blade row of Example 4.5. The hot gas total pressure at the entrance of the nozzle blade is $p_{t4} = 1.92$ MPa, $c_{pg} = 1156$ J/kg · K, and $\gamma_g = 1.33$. The mixed-out total pressure at the exit of the nozzle has suffered 2% loss due to both mixing and frictional losses in the blade row boundary layers. Calculate the entropy change $\Delta s/R$ across the turbine nozzle blade row.

SOLUTION

The hot gas total temperature is h_{tg}/c_{pg}, which for a $c_{pg} = 1156$ J/kg · K is calculated to be $T_{tg} = [1850\text{ kJ/kg}]/[1.156\text{ kJ/kg}\cdot\text{K}] = 1600$ K. This is the same temperature as T_{t4} at the entrance to the nozzle. The coolant total temperature is $T_{tc} = h_{tc}/c_{pc} = 904$ kJ/kg/1.04 kJ/kg · K = 900.4 K. Also, the mixed-out total pressure is 98% of the incoming (hot gas) total pressure, i.e., $p_{t,\text{mixed-out}} = 0.98$ (1.92 MPa) = 1.8816 MPa. We may use the Gibbs equation (assuming constant gas specific heats) to calculate the entropy change, namely,

$$\Delta s/R = \frac{\gamma_g}{\gamma_g - 1}\ln\left(T_{t,\text{mixed-out}}/T_{t4}\right) - \ln\left(p_{t,\text{mixed-out}}/p_{t4}\right)$$

Upon substitution, we get

$$\Delta s/R \cong -1.297$$

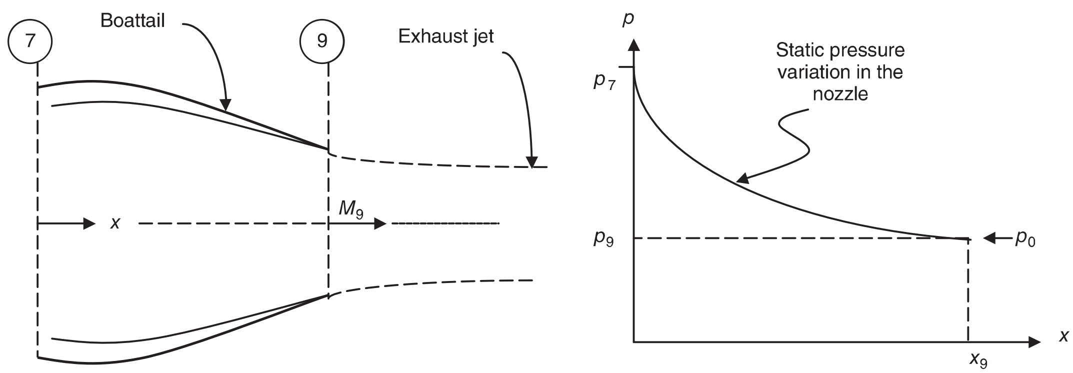
The negative sign of entropy change is due to cooling.

4.3.1.5 The Nozzle. The primary function of an aircraft engine exhaust system is to accelerate the gas efficiently. The nozzle parameter that is of utmost importance in *propulsion* is the gross thrust F_g. The expression we had earlier derived for the gross thrust was

$$F_g = \dot{m}_9 V_9 + (p_9 - p_0)A_9 \tag{4.54}$$

In this equation, the first term on the right-hand side is called the momentum thrust and the second term is called the pressure thrust. It is interesting to note that the nozzles produce a "signature," typically composed of infrared radiation, thermal plume, smoke, and acoustic signatures, which are key design features of a stealth aircraft exhaust system in *addition* to the main propulsion requirement of the gross thrust.

As the fluid accelerates in a nozzle, the static pressure drops and hence *a favorable pressure gradient* environment is produced in the nozzle. This is in contrast to diffuser flows where an *adverse pressure gradient* environment prevails. Therefore, boundary layers are, by and large, well behaved in the nozzle and less cumbersome to treat than the inlet. For a subsonic exit Mach number, that is, $M_9 < 1$, the nozzle expansion process



■ **FIGURE 4.18**
Schematic drawing of a subsonic nozzle with its static pressure distribution

will continue all the way to the ambient pressure p_0. This important result means that in subsonic streams, the static pressure inside and outside of the jet are the same. In fact, there is no mechanism for a pressure jump in a subsonic flow, which is in contrast to the supersonic flows where shock waves and expansion fans allow for static pressure discontinuity. We have depicted a convergent nozzle in Figure 4.18 with its exhaust stream (i.e., a jet) emerging in the ambient gas of static state (0). The outer shape of the nozzle is called a boattail, which affects the *installed performance* of the exhaust system. The external aerodynamics of the nozzle installation belongs to the propulsion system integration studies and does not usually enter the discussions of the *internal performance*, that is, the cycle analysis. However, we need to be aware that our decisions for the internal flow path optimization, for example, the nozzle exit-to-throat area ratio, could have adverse effects on the installed performance that may offset any gains that may have been accrued as a result of the internal optimization.

We have learned in aerodynamics that a convergent duct, as shown in Figure 4.18, causes flow acceleration in a subsonic stream to a maximum Mach number of 1, which can only be reached at the minimum area of the duct, namely at its exit. So we stipulate that for all subsonic jets, that is, $M_9 < 1$, there is a static pressure equilibrium in the exhaust stream and the ambient fluid, that is, $p_9 = p_0$. Let us think of this as the *Rule 1* in nozzle flow.

$$\boxed{\text{Rule 1 :} \quad \text{If} \quad M_{\text{jet}} < 1, \quad \text{then} \quad p_{\text{jet}} = p_{\text{ambient}}}$$

The expression "jet" in the above rule should not be confusing to the reader, as it relates to the flow that emerges from the nozzle. In that context, M_{jet} is the same as M_9, and p_{ambient} is the same as p_0. We recognize that not only it is entirely possible but also desirable for the sonic and supersonic jets to expand to the ambient static pressure as well. Such nozzle flows are called *perfectly expanded*. Actually, the nozzle gross thrust can be maximized if the nozzle flow is perfectly expanded. We state this principle here without proof, but we will address it again, and actually prove it, in the next chapters. Let us think of this as the *Rule 2* in nozzle flow.

$$\boxed{\begin{array}{ll}\text{Rule 2 :} & \text{If } p_{\text{jet}} = p_{\text{ambient}} \\ & \text{then we have a perfectly expanded nozzle which results in } F_{\text{g,max}}\end{array}}$$

Here, the stipulation is only on the *static pressure match* between the jet exit and the ambient static pressure and not *perfect flow* inside the nozzle. A real nozzle flow experiences total pressure loss due to viscous dissipation in the boundary layer as well as shock waves, and yet it is possible for it to *perfectly expand* the gas to the ambient condition. Remembering compressible duct flows in aerodynamics, we are reminded that the exit pressure p_9 is a direct function of the nozzle area ratio $A_{\text{exit}}/A_{\text{throat}}$, which in our notation becomes A_9/A_8, and the nozzle pressure ratio (NPR). Recalling the definition of NPR,

$$\text{NPR} \equiv \frac{p_{t7}}{p_0} \tag{4.55}$$

we will demonstrate a critical value of NPR will result in the choking condition at the nozzle throat, that is, $M_8 = 1.0$, when NPR $\geq$ (NPR)$_{\text{critical}}$. Let us think of this as *Rule 3* in convergent or convergent-divergent nozzle aerodynamics. These rules do not apply if the divergent section of a C–D nozzle becomes a subsonic a diffuser.

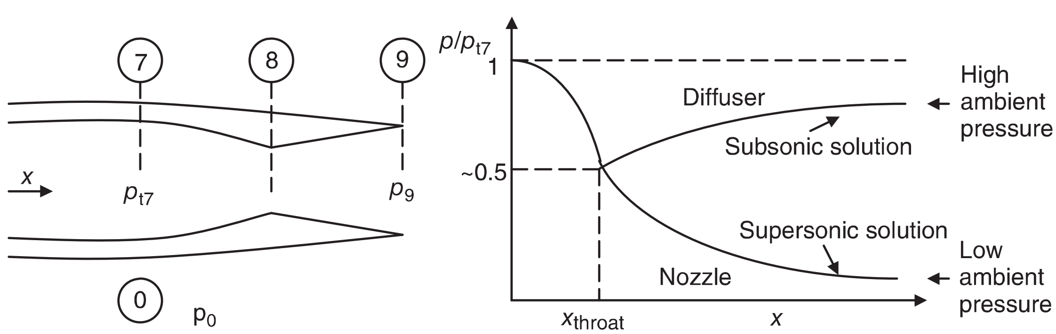
Rule 3 :	If NPR $\geq$ (NPR)$_{\text{critical}}$ then the nozzle throat velocity is sonic (i.e., choked), $M_8 = 1.0$

A schematic drawing of a convergent–divergent supersonic nozzle is shown in Figure 4.19.

The nozzle throat becomes choked at a static pressure of about 50% of the nozzle total pressure, that is, $p_{\text{throat}} \sim 1/2\, p_{t7}$. This fact provides for an important *rule of thumb* in nozzle flows, which is definitely worth remembering. Let us consider this as *Rule 4*.

Rule 4 :	If $\dfrac{p_{t7}}{p_0} \geq\sim 2$ then the nozzle throat can be choked, i.e., $M_8 = 1.0$

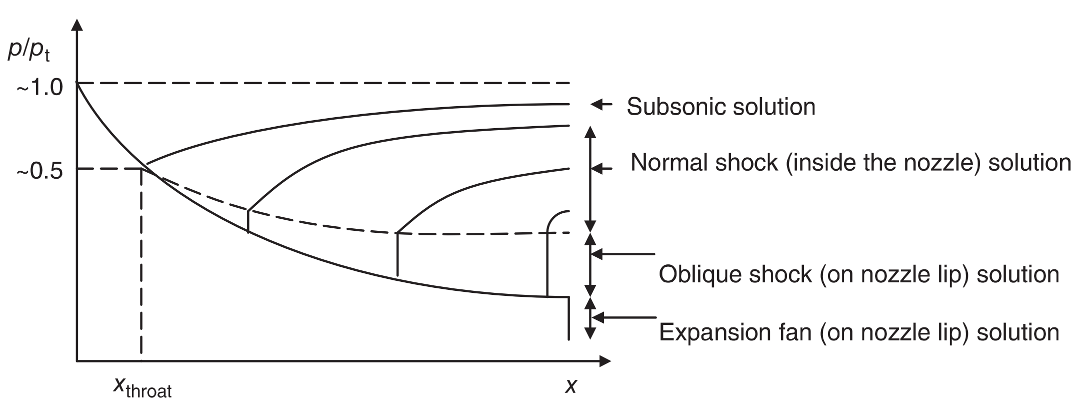
For example, a passage (or a duct) connecting a pressure vessel of about 30 psia pressure to a room of about 15 psia pressure will experience sonic speed at its minimum (orifice) area. As with any rules of thumb, Rule 4 is stated as an *approximation* and the exact value of the critical nozzle pressure ratio depends on the nozzle expansion efficiency, that is, the extent of losses in the nozzle.



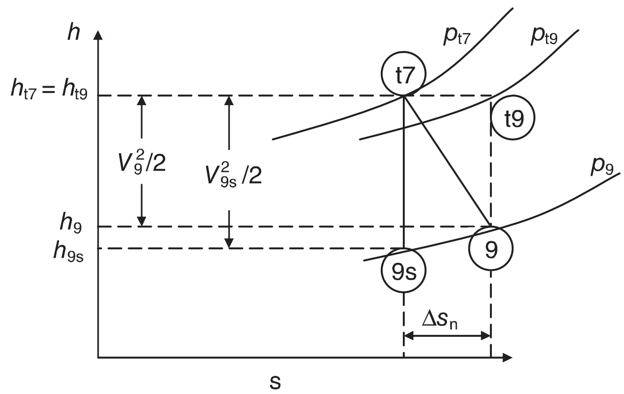
■ **FIGURE 4.19**
A choked convergent–divergent nozzle and the static pressure distribution along the nozzle axis

The two possible solutions in static pressure distribution along the nozzle axis are shown in Figure 4.19. One is a subsonic solution downstream of the nozzle throat and the second is a supersonic flow solution downstream of the throat. The subsonic solution is clearly a result of high backpressure, that is, p_0, and causes a flow deceleration in the divergent duct downstream of the throat. Therefore, the divergent portion of the duct is actually a *diffuser* and not a nozzle. The supersonic solution is a result of low backpressure and, therefore, the flow continues to expand (i.e., accelerates) beyond the throat to supersonic speeds at the exit. Also note that only a perfectly expanded nozzle flow is shown as the supersonic branch of the nozzle flow in Figure 4.19. For ambient pressures in between the two pressures shown in Figure 4.19, there are a host of shock solutions, which occur as an oblique shock at the lip or normal shock inside the nozzle. The range of backpressures for a given nozzle area ratio, which lead to a normal shock solution inside the nozzle is shown in Figure 4.20. The jump in static pressure across a normal shock is shown as an abrupt (i.e., vertical) pressure rise in Figure 4.20, whereas in reality shocks have finite thickness, and, therefore, all the *jump* conditions are then *diffused* or *spread* over the scale of shock thickness. We note the diffusing nature of the flow, that is, the static pressure increasing, after the shock occurs inside the nozzle, which signifies a *diffuser environment* after the shock. For a lower range of backpressures, the oblique shock solution is possible, which indicate an oblique shock is hanging on the nozzle lip. Then we reach a unique nozzle pressure ratio, which leads to perfect expansion, as shown in Figure 4.19 and repeated in Figure 4.20. For all backpressures lower than the perfect expansion, we get the isentropic centered expansion fan solution. All these waves are the necessary mechanisms to satisfy the physical requirement of continuous static pressure across the slipstream in the jet exit flow. Let us call this important gas dynamic *boundary condition* as rule number 5 in nozzle flows. Note that this rule is not unlike the familiar Kutta condition in airfoil aerodynamics, where we prevented a static pressure jump at the sharp tailing edge of an airfoil.

To examine the efficiency of a nozzle in expanding the gas to an exit (static) pressure p_9, we create an enthalpy–entropy diagram, very similar to an inlet. In the inlet studies, we took the ambient static condition 0 and compressed it to the total intake exit condition t2. But since a nozzle may be treated as a reverse-flow diffuser (and vice versa), we take the nozzle inlet gas, at the total state t7, and expand it to the exit static condition 9. This process is shown in Figure 4.21. The analogy in the thermodynamic analysis of a nozzle and a diffuser is helpful in better learning both components. The actual nozzle expansion process is shown by a solid line connecting the gas total state t7 to the exit static state 9.



■ **FIGURE 4.20** **Schematic drawing of possible static pressure distributions inside a choked supersonic nozzle**



■ **FIGURE 4.21** **Enthalpy–entropy diagram of nozzle flow expansion**

As the real nozzle flows may still be treated as adiabatic, the total enthalpy h_t, remains constant in a nozzle.

Rule 5 :	Static pressure must be continuous across the slipstream of a jet exhaust plume.

$$h_{t7} = h_{t9} \tag{4.56}$$

We remember that the inlet flow was also considered to be adiabatic. An ideal exit state 9s is reached isentropically from the total state t7 to the same exit pressure p_9, in Figure 4.21. The vertical gap between a total and static enthalpy states, in an h–s diagram, represents the kinetic energy of the gas, $V^2/2$, which is also shown in Figure 4.21. Due to frictional and shock losses in a nozzle, the flow will suffer a total pressure drop, that is,

$$p_{t9} \prec p_{t7} \tag{4.57}$$

as depicted in Figure 4.21. Again, this behavior is quite analogous to a diffuser. We can immediately define the total pressure ratio across a nozzle as its figure of merit, similar to the total pressure recovery of an inlet, namely,

$$\pi_n \equiv \frac{p_{t9}}{p_{t7}} \tag{4.58}$$

Now we can quantify the entropy rise in an adiabatic nozzle, as a result of total pressure losses, using the combined first and second law of thermodynamics,

$$\frac{\Delta s_n}{R} = -\ln \pi_n \tag{4.59}$$

We may also define a nozzle adiabatic efficiency η_n very similar to the inlet adiabatic efficiency, as

$$\eta_n \equiv \frac{h_{t7} - h_9}{h_{t7} - h_{9s}} = \frac{V_9^2/2}{V_{9s}^2/2} \tag{4.60}$$

Let us interpret the above definition in physical terms. The fraction of an ideal nozzle exit kinetic energy $V_{9s}^2/2$, which is realized in a real nozzle $V_9^2/2$, is called the nozzle adiabatic efficiency. The loss in total pressure in a nozzle manifests itself as a loss of kinetic energy. Consequently, the adiabatic efficiency η_n, which deals with the loss of kinetic energy in a nozzle, has to be related to the nozzle total pressure ratio π_n. Again, in a very similar manner as in the inlet, let us divide the numerator and denominator of Equation 4.60 by h_{t7}, to get

$$\eta_n = \frac{1 - \dfrac{h_9}{h_{t7}}}{1 - \dfrac{h_{9s}}{h_{t7}}} = \frac{1 - \dfrac{h_9}{h_{t7}}}{1 - \left(\dfrac{p_9}{p_{t7}}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1 - \left(\dfrac{p_9}{p_{t9}}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\dfrac{p_9}{p_{t7}}\right)^{\frac{\gamma-1}{\gamma}}} \tag{4.61}$$

where we used $h_{t7} = h_{t9}$ in the numerator and the isentropic relation between the temperature and pressure ratios in both numerator and denominator. It is interesting to note that as the nozzle exit total pressure p_{t9} approaches the value of the nozzle inlet total pressure p_{t7}, the nozzle adiabatic efficiency will approach 1. We may treat Equation 4.61 as containing only one unknown and that is p_{t9}. Therefore, for a given nozzle exit static pressure p_{t9} and the nozzle inlet total pressure p_{t7}, the nozzle adiabatic efficiency η_n will result in a knowledge of the total pressure at the exit of the nozzle, p_{t9}. Now, if we multiply the numerator and denominator of the right-hand side of Equation 4.61 by $(p_{t7}/p_9)^{\frac{\gamma-1}{\gamma}}$, we will reach our goal of relating the two figures of merit in a nozzle, that is,

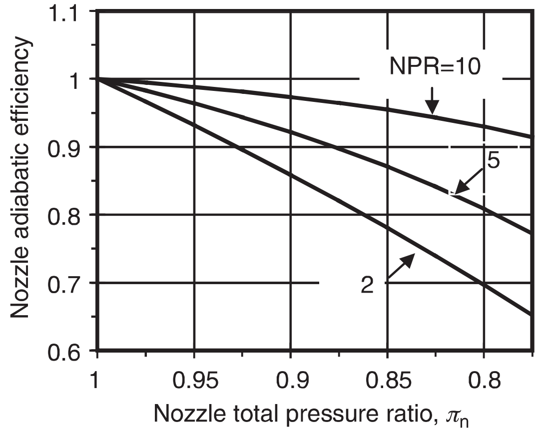
$$\eta_n = \frac{\left(\dfrac{p_{t7}}{p_9}\right)^{\frac{\gamma-1}{\gamma}} - \pi_n^{-\frac{\gamma-1}{\gamma}}}{\left(\dfrac{p_{t7}}{p_9}\right)^{\frac{\gamma-1}{\gamma}} - 1} \tag{4.62}$$

There are three parameters in Equation 4.62. The two figures of merit, η_n and π_n, and the ratio of nozzle inlet total pressure to the nozzle exit static pressure, p_{t7}/p_9. The last parameter is a known quantity, as we know the nozzle inlet total pressure p_{t7} from the upstream component analysis (in a turbojet, it is the turbine, $p_{t7} = p_{t5}$), and the nozzle exit pressure p_9 is a direct function of the nozzle area ratio A_9/A_8. To express Equation 4.62 in terms of the NPR, we may split p_{t7}/p_9 into

$$\frac{p_{t7}}{p_9} = \frac{p_{t7}}{p_0}\frac{p_0}{p_9} = \text{NPR} \cdot \left(\frac{p_0}{p_9}\right) \tag{4.63}$$

Now, substituting the above expression in Equation 4.62, we get

$$\eta_n = \frac{\left\{\text{NPR}\left(\dfrac{p_0}{p_9}\right)\right\}^{\frac{\gamma-1}{\gamma}} - \pi_n^{-\frac{\gamma-1}{\gamma}}}{\left\{\text{NPR}\left(\dfrac{p_0}{p_9}\right)\right\}^{\frac{\gamma-1}{\gamma}} - 1} \tag{4.64}$$



■ **FIGURE 4.22**
Two figures of merit in a nozzle plotted as a function of NPR (γ = 1.33)

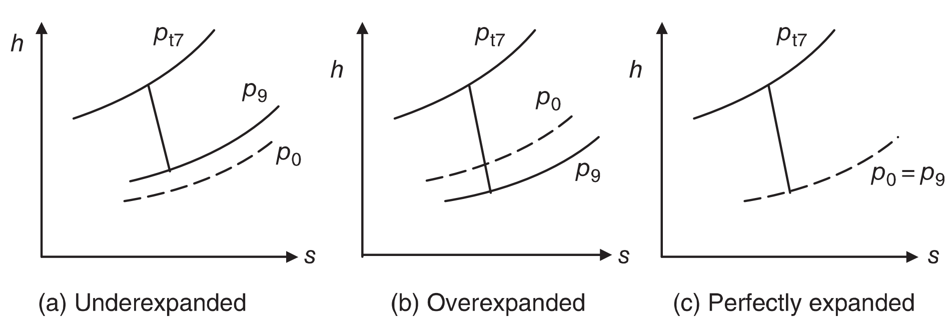
A plot of Equation 4.64 is shown in Figure 4.22 for a perfectly expanded nozzle, that is, $p_9 = p_0$.

It is comforting to note that in Equation 4.64, the nozzle adiabatic efficiency will approach 1, as the nozzle total pressure ratio approaches 1. The parameter p_0/p_9 represents a measure of mismatch between the nozzle exit static pressure and the ambient static pressure. For $p_9 > p_0$, the flow is considered to be *underexpanded*. For $p_9 < p_0$, the flow is defined as *overexpanded*. In the under-expanded scenario, the nozzle area ratio is not adequate, that is, not large enough, to expand the gas to the desired ambient static pressure. In the overexpanded nozzle flow case, the nozzle area ratio is too large for perfect expansion. As noted earlier, a perfectly expanded nozzle will have $p_0 = p_9$, therefore, the above equation is further simplified to

$$\eta_n = \frac{\{\text{NPR}\}^{\frac{\gamma-1}{\gamma}} - \pi_n^{-\frac{\gamma-1}{\gamma}}}{\{\text{NPR}\}^{\frac{\gamma-1}{\gamma}} - 1} \tag{4.65}$$

for a perfectly expanded nozzle. It is instructive to show the three cases of nozzle expansion, that is, under-, over-, and perfectly expanded cases, on an *h–s* diagram (Figure 4.23).

In Figure 4.23a, the nozzle area ratio is smaller than necessary for perfect expansion and that explains the high exit pressure p_9. However, an expansion fan at the nozzle



■ **FIGURE 4.23**
The *h–s* diagram for the three possible nozzle expansions

exit plane will adjust the pressure down to the ambient level, after the jet leaves the nozzle, in accordance to Rule 5. After all, the slipstream cannot take a (static) pressure jump. In Figure 4.23b, the nozzle area ratio is too large for this altitude, which explains a lower-than-ambient static pressure at the nozzle exit. To adjust the low exit pressure up to the ambient level, a shock wave is formed on the nozzle exit lip, which for mild overexpansion will be an oblique shock. Higher levels of over expansion will strengthen the shock to a normal position and, eventually, cause the shock to enter the nozzle. Figure 4.23c shows a perfect expansion scenario, which implies the nozzle area ratio is perfectly matched to the altitude requirements of ambient pressure. Again, as noted in Figure 4.23, all nozzle expansions are depicted as irreversible processes with an associated entropy rise; therefore, *perfect expansion* is not to be mistaken as *perfect (isentropic) flow*.

In summary, we learned that:

- the primary function of a nozzle is to accelerate the gas efficiently
- the gross thrust parameter F_g signifies nozzle's contribution to the thrust production
- the gross thrust reaches a maximum when the nozzle is perfectly expanded; that is, $p_9 = p_0$
- real nozzle flows may still be considered as adiabatic
- a nozzle pressure ratio (NPR) that causes a Mach-1 flow at the throat (i.e., choking condition) is called the *critical nozzle pressure ratio*, and as a rule of thumb, we may remember an $(\text{NPR})_{\text{crit}}$ of ~2
- there are two efficiency parameters that quantify losses or the degree of irreversibility in a nozzle and they are related
- nozzle losses manifest themselves as the total pressure loss
- all subsonic exhaust streams have $p_{\text{jet}} = p_{\text{ambient}}$
- a perfect nozzle expansion means that the nozzle exit (static) pressure and the ambient pressure are equal
- an imperfect nozzle expansion is caused by a mismatch between the nozzle area ratio and the altitude of operation
- underexpansion is caused by smaller-than-necessary nozzle area ratio, leading to $p_9 > p_0$
- overexpansion is caused by larger-than-necessary nozzle area ratio, leading to $p_9 < p_0$.

EXAMPLE 4.7

Consider a convergent–divergent nozzle with a pressure ratio NPR = 10. The gas properties are $\gamma = 1.33$ and $c_p = 1{,}156\ \text{J/kg}\cdot\text{K}$ and remain constant in the nozzle. The nozzle adiabatic efficiency is $\eta_n = 0.94$. Calculate

(a) nozzle total pressure ratio π_n

(b) nozzle area ratio A_9/A_8 for a perfectly expanded nozzle

(c) nozzle exit Mach number M_9 (perfectly expanded)