Planar Jet

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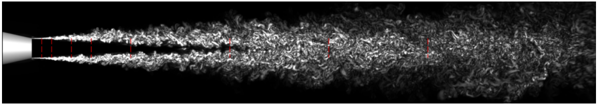
Free shear flows

In free shear flows

- 'Free' – refers to the lack of constraining (solid) boundaries in the flow
    - Hence, pressure is atmospheric almost everywhere
- 'Shear' – indicates that the flows are dominated by velocity gradients

Prominent examples are jets and wakes

- Jet formed when new fluid is injected at high momentum into a stationary fluid
    - The two fluids may not be the same (e.g., water jet in air)
    - Two fluids may be same, but be of different densities (e.g., heated jets)



- Wake is formed behind a solid body after the flow crosses over it

Brès et al., AIAA Paper 2012-2965

Canonical jet flows: planar and round

Both are 2-dimensional flows on an average, and have similar physics

- Planar jets emerge from long narrow slots
  - They have negligible variation of averaged flow quantities in span-wise direction, especially away from the slot ends
- Round jets are axisymmetric on an average; i.e., there is no variation of averaged flow quantities in the azimuthal direction

Planar jet[a]

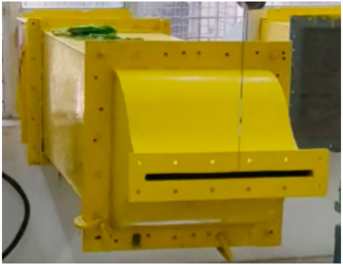
Round jet[b]

[a] http://research.ae.utexas.edu/FloImLab/planar.html

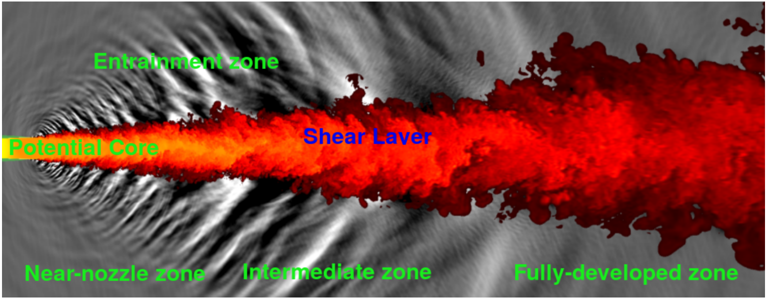
[b] www.blueflametech.com

Parameters of Planar Jet Experiment

- Slot width, $w = 44$ cm, slot thickness, $d = 18$ mm
- Nozzle exit velocity, U_j is about 50 m/s (will vary among experiments)
- Assume unheated stagnation condition
- Jet exit Reynolds numbers 0.69×10^5 to 1.15×10^5
- Measure mean streamwise velocity field U w/ pitot tube on centerplane



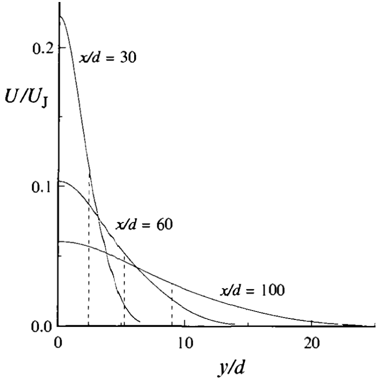
Jet Regions



- Potential core: exit velocity prevails (negligible viscous effects)
- Entrainment zone: zone of small but predominantly inward flow
- Shear layer: mixing of two streams occurs
- Fully-developed zone: flow reaches statistical equilibrium
- Near-nozzle zone: potential core exists

Brès et al., AIAA Paper 2012-2965

Jet characteristics in fully-developed zone



U: Mean streamwise velocity (also denoted by U)

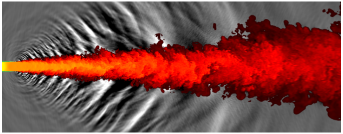
U_j: Nozzle exit velocity

d: Nozzle exit thickness

- Study time-averaged (mean) streamwise velocity, denoted $U(x, y)$
- Velocity decays on centerline, but grows at outer regions
- Jet widens as it slows down
- Profiles appear to be similar (just scaled differently)

Hussein et al., JFM, 1994

Concept of self-similarity

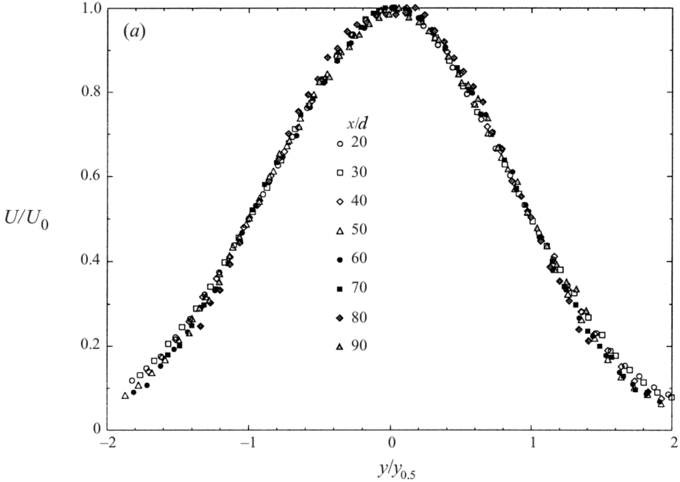


- In fully-developed region, initial (near-nozzle) condition is ‘forgotten’
- Thus, there is no characteristic dimension in the streamwise direction
- Another way of seeing this is that the flow is in ‘equilibrium’
- In such a situation, it is natural to expect the flow to be self-similar
- I.e., if the velocity and lateral coordinate are properly scaled, then the profiles should ‘collapse’ irrespective of the actual streamwise position
- A natural choice for normalizing y is the jet half-width $y_{0.5}(x)$,

$$U(x, y = y_{0.5}) = \frac{1}{2} U(x, y = 0)$$

- Velocity is to be normalized by centerline velocity $U_0(x) := U(x, y = 0)$

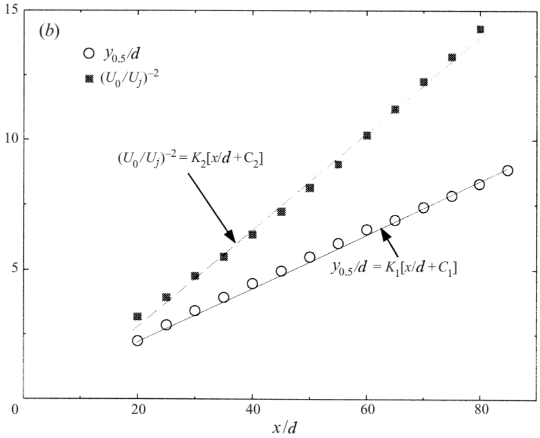
Similarity of fully-developed zone



Self-similarity for data in $x/d = 20$ to 90

Gordeyev & Thomas, JFM, 2000

Axial variation of scales: $U_0(x)$ and $y_{0.5}$



$$\frac{dy_{0.5}}{dx} = S,$$
$$\frac{U_0(x)}{U_j} = \frac{B}{\sqrt{(x - x_0)/d}}.$$

S: Spread rate,
x_0: Virtual origin,
B: Empirical constant

Empirical parameters S and B are more or less independent of Reynolds no.

N.B.: For large x, we have $U_0^2(x)y_{0.5}(x) \approx$ constant

Gordeyev & Thomas, JFM, 2000

Momentum conservation

Mean momentum flux per unit slot width is defined as

$$\dot{P}(x) := \int_{-\infty}^{\infty} \rho U^2 dy$$

Show that the experimental observations on the streamwise variation of $U_0(x)$ and $y_{0.5}$ are consistent with the conclusion that the momentum flux is conserved in the streamwise direction

$$\frac{d\dot{P}}{dx} = 0$$

Mass and energy flux

Show using the similarity function f introduced earlier that

$$\text{Mass flux:} \quad \dot{m}(x) := \int_{-\infty}^{\infty} \rho U dy \sim x^{0.5},$$

$$\text{Energy flux:} \quad \dot{E}(x) := \int_{-\infty}^{\infty} \rho U^3 dy \sim x^{-0.5}.$$

$\sim$ denotes 'varies as'

Experimental Objectives

- To understand the characteristics of the planar jet
- To analyze its mean streamwise velocity
- To characterize the streamwise variation of centerline velocity and jet half width
- To verify self-similarity
- To verify conservation of streamwise momentum flux
- To verify proposed variation of streamwise mass and energy flux

Report and final presentation must include

Apart from the things required to be proven that are mentioned in the previous slides, your report and final presentation must include the following:

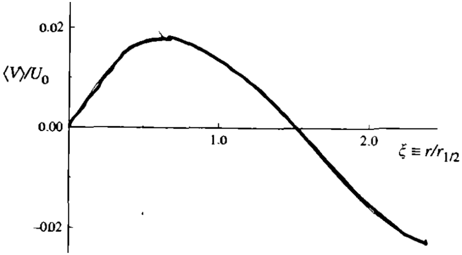
- Detailed documentation of experimental conditions (including ambient)
- Estimate of the potential core length of the jet (from plots of centerline velocity vs. x)
- Comparison with literature, with proper bibliography

Caution:

- Never show length dimensions in absolute units – always non-dimensionalize with slot thickness, d, or similarity variable
- Never show velocities in absolute units – always non-dimensionalize with exit velocity, U_j, or similarity velocity

Additional learning

Mean lateral velocity, V, in self-similar region



Once the mean streamwise velocity is known in the self-similar region (i.e., the function f), the mean lateral velocity may be found by integrating the mean mass conservation equation

We note that

- $V \ll U$
- There is inward flow (-ve V) in the outer entrainment zone

Hussein et al., JFM, 1994

Momentum conservation: Boundary layer approximation

Following is a sketch of the proof for streamwise momentum conservation; you must be familiar with the derivation for the final exam

- Jet spreads slowly, and mean streamwise velocity dominates (true for boundary layers and most shear flows)
- Reynolds number is high
- It can be shown that the (boundary layer) equations are

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \nu\frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{uv}}{\partial y},$$

$$\frac{P}{\rho} = \frac{P_0}{\rho} - \overline{v^2}.$$

$\overline{uv}$ is lateral Reynolds shear stress, ν is kinematic viscosity, P_0 is ambient pressure, ρ is mean density

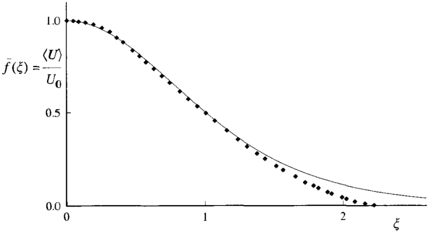
Self-similar profile

Let us denote the lateral similarity parameter as

$$\xi := y/y_{0.5}$$

We define the similarity function

$$f(\xi) := \frac{U(x,y)}{U_0(x)}$$



Pope, *Turbulent flows*, CUP, 2000

Theoretical variation of similarity parameters (Self learning)

From similarity arguments (borne out by experiments), lateral Reynolds shear stress is

$$\overline{uv} = U_0^2 g(\xi).$$

Then, show that the momentum and mass conservation equations together give the following

$$\frac{y_{0.5}}{2\left(x - x_0\right)} \left\{ f^2 + f' \int_0^{\xi} f(\eta) d\eta \right\} = g'.$$

Hence, conclude that $y_{0.5} \sim x$.