Beispiel 2.1 (Schnitzer)

Messungen an dem Verbraucher mit folgenden rel. Fehler durchgeführt

$$\frac{\Delta U}{U} = -0{,}011 \qquad \frac{\Delta I}{I} = 0{,}02 \qquad \frac{\Delta R}{R} = -0{,}031$$

Relativer Fehler der Verbraucherleistung?

$$P = U \cdot I = I^2 \cdot R = \frac{U^2}{R}$$

Weg ① $P = f(U,I) = U \cdot I$ $\qquad \frac{\partial}{\partial U}(U \cdot I) = I$

$$\Delta P = \frac{\partial f}{\partial U}\Delta U + \frac{\partial f}{\partial I}\Delta I = I \cdot \Delta U + U \cdot \Delta I$$

$$\frac{\Delta P}{P} = \frac{I \cdot \Delta U}{I \cdot U} + \frac{U \cdot \Delta I}{U \cdot I} = \frac{\Delta U}{U} + \frac{\Delta I}{I} = -0{,}011 + 0{,}02 = +0{,}009$$

P ist also 0,9% zu groß gemessen!

$$P_k = P - \Delta P = P\left(1 - \frac{\Delta P}{P}\right) = P \cdot (1 - 0{,}009) = \underline{P \cdot 0{,}991}$$

Weg ②

$$P = f(I,R) = I^2 \cdot R \Rightarrow \Delta P = \frac{\partial f(I,R)}{\partial I} \cdot \Delta I + \frac{\partial f(I,R)}{\partial R}\Delta R$$

$$\frac{\partial f}{\partial I} = 2 \cdot I \cdot R, \qquad \frac{\partial f}{\partial R} = I^2$$

$$\Delta P = 2 \cdot I \cdot R \cdot \Delta I + I^2 \cdot \Delta R \qquad P = I^2 R$$

$$\frac{\Delta P}{P} = 2 \cdot \frac{I \cdot R \cdot \Delta I}{I^2 \cdot R} + \frac{I^2 \Delta R}{I^2 \cdot R} = 2 \cdot \frac{\Delta I}{I} + \frac{\Delta R}{R} = 2 \cdot 0{,}02 + (-0{,}031) = 0{,}009$$

Weg ③

$$P = \frac{U^2}{R} = f(U,R)$$

$$\Delta P = \frac{\partial f(U,R)}{\partial U} \cdot \Delta U + \frac{\partial f(U,R)}{\partial R} = \frac{2 \cdot U}{R} \cdot \Delta U - \frac{U^2}{R^2} \cdot \Delta R$$

$$\frac{\partial}{\partial R}\left(\frac{U^2}{R}\right) = U^2 \cdot \frac{\partial}{\partial R}\left(\frac{1}{R}\right) = U^2 \cdot \frac{\partial}{\partial R}\left(R^{-1}\right) =$$

$$= U^2 \cdot (-1) \cdot R^{-2} = -\frac{U^2}{R^2}$$

$$\left[\frac{\partial}{\partial x} x^n = n \cdot x^{n-1}\right]$$

$$\Delta P = \frac{2 \cdot U}{R} \cdot \Delta U - \frac{U^2}{R^2} \Delta R \qquad P = \frac{U^2}{R} \rightarrow \frac{1}{P} = \frac{R}{U}$$

$$\frac{\Delta P}{P} = \frac{2 \cdot U}{R} \cdot \Delta U \cdot \left(\frac{R}{U^2}\right) - \left(\frac{U^2}{R^2} \cdot \Delta R\right) \cdot \left(\frac{R}{U^2}\right)$$

$$\frac{\Delta P}{P} = 2 \cdot \frac{\Delta U}{U} - \frac{\Delta R}{R} = 2 \cdot (-0{,}011) - (-0{,}031)$$

$$\frac{\Delta P}{P} = 0{,}009 \Rightarrow 0{,}9\,\%$$