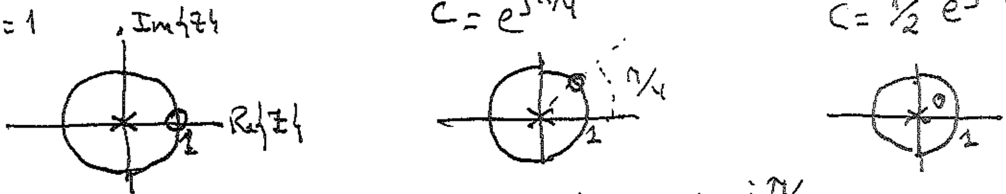
P6.1. Sea un sistema causal con

$$H(z) = 1 - cz^{-1}$$

a) Representar diagrama de polos y ceros para $c=1$; $c=e^{j\pi/4}$; $c=\frac{1}{2}e^{j\pi/4}$

$$H(z) = 1 - \frac{c}{z} = \frac{z-c}{z} \Rightarrow \begin{cases} \text{Cero en } c \\ \text{Polo en } 0 \end{cases}$$

$c=1$ $\qquad$ $c=e^{j\pi/4}$ $\qquad$ $c=\frac{1}{2}e^{j\pi/4}$



Suponiendo ahora (siguientes apartados) $c = \frac{1}{2}e^{j\pi/4}$

b) Hallar la expresión de la respuesta frecuencial del sistema (módulo

c) y fase).

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{e^{j\omega}-c}{e^{j\omega}}$$

$$|H(e^{j\omega})| = \frac{|e^{j\omega}-c|}{|e^{j\omega}|} = \left|e^{j\omega} - \frac{1}{2}e^{j\pi/4}\right| = \sqrt{\left(e^{j\omega}-\frac{1}{2}e^{j\pi/4}\right)\left(e^{-j\omega}-\frac{1}{2}e^{-j\pi/4}\right)} =$$

$$|a| = \sqrt{a \cdot a^*}$$

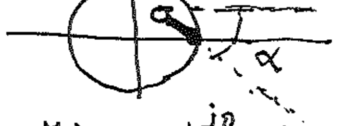
$$= \sqrt{1 - \frac{1}{2}e^{j(\omega-\pi/4)} + \frac{1}{4} - \frac{1}{2}e^{-j(\omega-\pi/4)}} = \sqrt{\frac{5}{4} - \cos\left(\omega - \frac{\pi}{4}\right)}$$

$$\angle H(e^{j\omega}) = \underbrace{\arctan\left(\frac{\text{Parte imag.}}{\text{Parte real}}\right)}_{\text{Fase numerador}} - \underbrace{\omega}_{\text{Fase denominador}} \Rightarrow$$

$$\Rightarrow \angle H(e^{j\omega}) = \arctan\left[\frac{\operatorname{sen}\omega - \frac{1}{2}\operatorname{sen}\frac{\pi}{4}}{\cos\omega - \frac{1}{2}\cos\frac{\pi}{4}}\right] - \omega = \arctan\left[\frac{\operatorname{sen}\omega - \sqrt{2}/4}{\cos\omega - \sqrt{2}/4}\right] - \omega.$$

d) Representar geométricamente $|H(e^{j\omega})|$ y $\angle H(e^{j\omega})$ para $\omega = \begin{cases} 0 \\ \pi/4 \\ \pi/2 \\ -3\pi/4 \end{cases}$ } similar

$\omega = 0$ $\qquad$ $\omega = \pi/4$



Módulo $= \frac{|e^{j0} - c|}{1} \Rightarrow$ distancia entre los 2 vectores

$$\text{Fase} = \alpha - \omega = \alpha = -\frac{\pi}{4}$$

α: Fase de $e^{j\omega} - \frac{1}{2}e^{j\pi/4}$; ω: fase de $e^{j\omega}$

$$|H(e^{j\omega})|_{\omega=\pi/4} = \left|e^{j\omega} - \frac{1}{2}e^{j\pi/4}\right|_{\omega=\frac{\pi}{4}} = \left|e^{j\pi/4}\left(1 - 1/2\right)\right| = \frac{1}{2}$$

$$\angle H(e^{j\omega})\Big|_{\omega=\frac{\pi}{4}} = \underbrace{\frac{\pi}{4}}_{\alpha} - \frac{\pi}{4} = 0$$