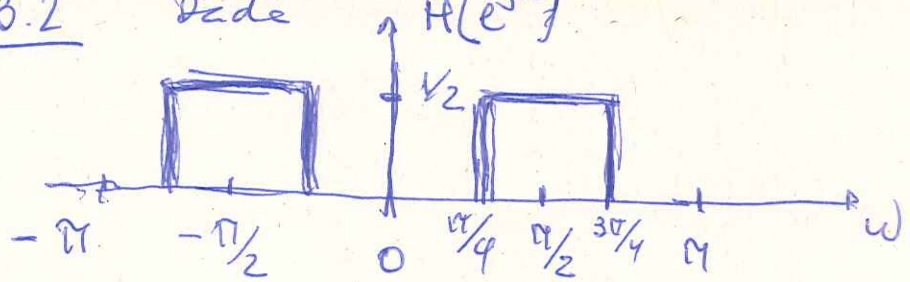
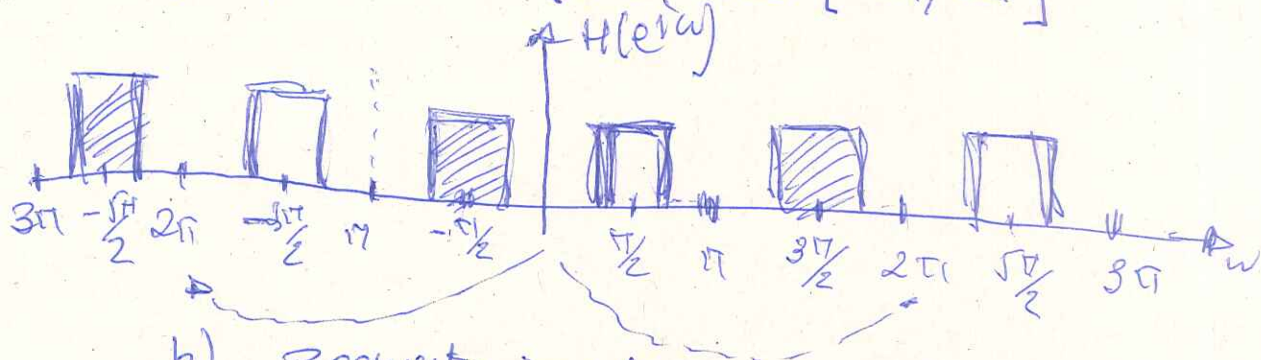
P.3.2 Dada



(fase = 0, $\forall \omega$)

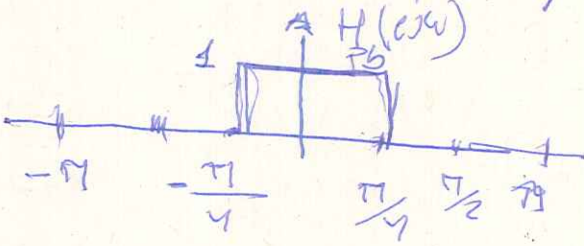
a) Representar $H(e^{j\omega})$ entre $[-3\pi, 3\pi]$



b) Respuesta impulsional del sistema.

NOTA: Sabemos que, entre $[-\pi, \pi]$:

$$h_{pb}[n] = \frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n} \xrightarrow{\mathcal{F}_{DTFT}} H_{pb}(e^{j\omega}) = \Pi\left(\frac{\omega}{\pi/2}\right)$$



b-1) Resolución utilizando la ayuda y las propiedades de la DTFT.

Sabiendo que

$$H(e^{j\omega}) = \frac{1}{2}\Pi\left(\frac{\omega-\pi/2}{\pi/2}\right) + \frac{1}{2}\Pi\left(\frac{\omega+\pi/2}{\pi/2}\right)$$

y que si a $x[n] \longrightarrow X(e^{j\omega})$

$$e^{j\omega_0 n}x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$$ Propiedad de desplaz. en ω.

$$h[n] = \frac{1}{2}e^{j\frac{\pi}{2}n}\frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n} + \frac{1}{2}e^{-j\frac{\pi}{2}n}\frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n} =$$

$$= \frac{1}{2}\frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n}\left[e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right] =$$

$$= \frac{1}{2}\frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n}\cdot 2\cos\frac{\pi}{2}n = \frac{\operatorname{sen}\frac{\pi}{4}n}{\pi n}\cos\frac{\pi}{2}n$$ //

b.2) Resolviéndolo a partir de la definición de la transformada inversa

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} \frac{1}{2} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\frac{\pi}{4}}^{3\pi/4} \frac{1}{2} e^{j\omega n} d\omega =$$

$$= \frac{1}{4\pi}\left[\left.\frac{e^{j\omega n}}{jn}\right|_{-\frac{3\pi}{4}}^{-\pi/4} + \left.\frac{e^{j\omega n}}{jn}\right|_{\pi/4}^{3\pi/4} \right] =$$

$$= \frac{1}{4\pi}\frac{1}{jn}\left\{ \left[e^{-j\frac{\pi}{4}n} - e^{-j\frac{3\pi}{4}n} \right] + \left[e^{j\frac{3\pi}{4}n} - e^{+j\frac{\pi}{4}n} \right] \right\} =$$

$$= \frac{1}{2\pi n}\left\{ \frac{\left[e^{j3\frac{\pi}{4}n} - e^{-j3\frac{\pi}{4}n} \right]}{2j} - \frac{\left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n} \right]}{2j} \right\} =$$

$$= \frac{1}{2\pi n}\left[\operatorname{sen}\frac{3\pi}{4}n - \operatorname{sen}\frac{\pi}{4}n \right]$$

c) Utilizando la relación:

$$\cos a \operatorname{sen} b = \frac{\operatorname{sen}(a+b) - \operatorname{sen}(a-b)}{2}$$

Se demostrará que ambas soluciones son iguales

P.e. de la 1ª se ve inmediato que si $a = \frac{\pi}{2}$ y $b = \frac{\pi}{4}$ se obtiene la 2ª

De donde sale esto

$\operatorname{sen}(a+b) = \operatorname{sen} a \cos b + \cos a \operatorname{sen} b$

$\operatorname{sen}(a-b) = \operatorname{sen} a \cos b - \cos a \operatorname{sen} b$

no

$\cos(a+b) = \cos a \cos b - \operatorname{sen} a \operatorname{sen} b$

$\cos(a-b) = \cos a \cos b + \operatorname{sen} a \operatorname{sen} b$

De estas (restando)

$$\cos a \operatorname{sen} b = \frac{\operatorname{sen}(a+b) - \operatorname{sen}(a-b)}{2}$$