P.6.5 : Para cada uno de los pares $X(z)$, $H(z)$, determinar la ROC y la $Y(z)$

$$X(z) \rightarrow \boxed{H(z)} \rightarrow Y(z) = X(z) \cdot H(z)$$

a) $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}$, ROC: $|z| > \frac{1}{2}$

$H(z) = \dfrac{1}{1-\frac{1}{4}z^{-1}}$, $|z| > \frac{1}{4}$

$$Y(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)} \quad \text{, ROC: } |z| > \frac{1}{2}$$

b) $X(z) = \dfrac{1}{1-2z^{-1}}$, $|z| < 2$

$H(z) = \dfrac{1}{1-\frac{1}{3}z^{-1}}$, $|z| > \frac{1}{3}$

$$Y(z) = \frac{1}{\left(1-2z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} \quad \text{, ROC: } \frac{1}{3} < |z| < 2$$

c) $X(z) = \dfrac{1}{\left(1-\frac{1}{5}z^{-1}\right)\left(1+3z^{-1}\right)}$, $\frac{1}{5} < |z| < 3$

$H(z) = \dfrac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}$, $|z| > \frac{1}{3}$

$$Y(z) = \frac{1}{\left(1-\frac{1}{5}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)} \quad \text{, ROC: } \frac{1}{5} < |z| > \frac{1}{3}$$

$\Downarrow$

ROC: $|z| > \frac{1}{3}$

(desaparece el polo de $X(z)$ en $z=3$)