P7 Ex Nov. 2019

$$x[n] = \frac{2}{3}\,\frac{\operatorname{sen}\left(\frac{\pi}{6}n\right)}{\pi n}\cdot \operatorname{sen}\left(\frac{\pi}{2}n\right)$$

Determinar energía

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \underset{\substack{\uparrow\\ T.P.}}{=} \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2\,dw$$

Resolución

$$x[n] = \frac{2}{3}\,\frac{\operatorname{sen}\left(\frac{\pi}{6}n\right)}{\pi n}\;\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{2j}$$

Sabiendo por tablas:

$$\frac{\operatorname{sen}\frac{\pi}{a}n}{\pi n} \xrightarrow{\mathcal{F}} \Pi\left(\frac{w}{2\pi/a}\right)$$

$$x[n]e^{jw_0 n} \longrightarrow X(e^{j(w-w_0)})$$

Entonces:

$$X(e^{jw}) = \frac{2}{3}\cdot\frac{1}{2j}\left[\Pi\left(\frac{w-\pi/2}{2\pi/6}\right) - \Pi\left(\frac{w+\pi/2}{2\pi/6}\right)\right]$$

$$E_x = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left\{\frac{1}{3j}\left[\Pi\left(\frac{w-\pi/2}{\pi/3}\right) - \Pi\left(\frac{w+\pi/2}{\pi/3}\right)\right]\right\}\right|^2 dw =$$

$$= \frac{1}{18\pi}\int \underbrace{\qquad\qquad}\,dw = \frac{1}{18\pi}\,2\frac{\pi}{3} = \frac{1}{27}$$