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# Factors Contributing to Academic Success in Mathematics: A Multi-Level Analysis of NELS-88 Data

## Abstract

Interventions to improve academic success, especially among minorities and those from low socioeconomic backgrounds, have been a controversial topic in recent years. Many state-level programs have been implemented to assure more students are proficient across subject areas, and ultimately that more students graduate from high school. Despite the efforts, some students still struggle to achieve academic proficiency. Therefore, it is imperative that resources are allocated correctly to continue improving the success rate among students. Using a subsample (n=519) of 8th grade mathematics scores from the National Educational Longitudinal Study of 1988, this analysis aims to identify the school-level and student-level factors that contribute to academic success in the area of mathematics while testing for a random effect that individual schools may have on mathematics performance. Our results found that at student-level, time spent on homework has positive correlation with student mathematics performance. The relationship between student's mathematics achievement and socioeconomic status is not the same for student's parents with different education background. Additionally, we found significant interactions between student's minority status and school-level covariates such as the mean socioeconomic status and percent minority. Contrary to general beliefs, gender, school size, teacher and student ratio are not significant factors impact on student mathematics performances. The results could inform policy makers and other stakeholders on where improvement efforts should be focused.

## 1. Introduction

Proliferated studies have been carried out to explore factors impact on student's mathematics achievement in the last three decades. Factors from different levels can lead to poor mathematics performance. At student level, much research has been done to answer questions surrounding the influence of student background on academic performance. Students who come from low socioeconomic backgrounds would enter school behind their peers who come from higher socioeconomic backgrounds which creates disadvantages across topics including counting and number relations (Jordan et al., 2007). Additionally, a publication in 2000 noted gaps in mathematical performance is not only tied to strained financial resources but also to parents' educational background in mathematics. (Starkey and Klein, 2000). Studies also found that there is a positive relationship between students' attitude toward mathematics study and their math performances (Papanastasiou, 2000). Contrary to popular belief, meta-analysis only found negligible amount of gender difference in students' mathematics performance (Hyde et al., 1990).

Research pertaining to school-level attributes are not as plentiful as those pertaining to student-level attributes, but they still provide insight as to which factors influence academic success. Factors such as class size and school size have been studied and resulted in mixed results when evaluating their effect on students' academic success. Project STAR (Student/Teacher Achievement Ratio), an extensive study of which the results ignited many state-level programs to reduce class sizes, found that academic achievement and class size have a negative correlation yet this effect is not as great in higher grade levels. Additionally, Jeremy Finn's analysis of Project STAR found that the benefit of a smaller class size was greater for minority students and students attending inner-city schools. (Finn et al., 2000). Other studies concerning school level factors also reported that school climate[1], school resources (textbooks, computers availability), and number of teachers specialized in mathematics instruction were significantly correlated with school mathematic achievement (Bosker, Kremers, & Lugthart, 1990, Papanastasiou, 2000). Furthermore, cross-level impact has been reported between school environment and student socioeconomic status of the family (Papanastasiou, 2000).

Although a lot of research has been focused on students' academic performance in mathematics and their demographic background, they tend to use a mix linear effect approach to explore the effect of schools and their characteristics as well as non-background related student attributes. The aim of this study is three-fold. First, explore whether there is contextual effect from school level factors. Second, determine whether there are interaction effects within student level and school level factors. Third analyze the presence of cross-level interaction effects from both levels. The results could be used to help stakeholders look for potential improvements from a school-level prospective, such as improving funding, facilities, and teaching staff, or from a student-level perspective, in the form of tutoring programs, or incentive reward programs among others.

## 2. Methodology

### 2.1 Data

The data used for analysis is a subset of a dataset gathered for the purpose of the National Education Longitudinal Study of 1988 (NELS: 88). This study involved a nationally representative sample of eighth-graders first surveyed in the spring of 1988. Students were surveyed on a wide range of topics pertaining to school, work, and home experiences; neighborhood characteristics; educational and occupational aspirations; and the role in education of their parents and peers. Achievements tests in reading, social studies, mathematics, and science were also administered in addition to the student questionnaire. Students' teachers, parents, and school administrators were also surveyed (National Education Longitudinal Study of 1988, 2015). The subsample of data used for this analysis is a three-level data set. Specifically, 519 students (level 1) are nested within 23 randomly samples schools (level 2), which are further nested within 3 different regions (level3). Table 1 presents the layout and the description of the variables included in this study. The response variable for our analysis, mathematics score, is the individual students' scores on the mathematics portion of the achievement exam. This exam included 40 math-related questions to be completed in 30 minutes. It was scored on a right/wrong basis (no "partial credit").

[1] Bosker et al. define "school climate" as percentages of students involved in bullying, disrputing classes, tobacco use, and patterns of students skipping classes

Table 1 Three level of variables

| | |
|---|---|
| Region (level 3) | Region: 1= northeast , 2= north central, 3= south |
| School (Level 2) | Sch_id= school ID numbers<br>Public= whether school is public or private<br>Ratio= class size, measured by the student-teacher ratio<br>Percmin= the percentage of ethnic minority students in the school<br>Scsize= total number students enrolled |
| Student (Level 1) | Sex=indicator variable (0=boy, 1=girl)<br>Ses= socioeconomic status of parents<br>Homework= the number of hours of homework done per week<br>White= student's race. 0= Non-white, 1= white<br>parented:= parents' educational level<br>1=Did not finish H.S 2= H.S. grade or GED<br>3=GT H.S. &LT 4yr degree 4=College graduate<br>5=M.A. or equivalent 6=Ph.D., M.D.,other |

2.2 Analysis strategy and steps

In order to analyze the data, exploratory data analysis was first completed, then a linear mixed model with random effects was fit to accommodate the clustered structure of the dataset. Specifically, the Region is treated as the level 3 cluster and individual School ID within Region is treated as the level 2 cluster. Both level 2 and level 2 clusters were evaluated as random effects. Student-level and school-level covariates, as well as potential interactions, were evaluated as fixed effects. The analysis of this study takes a step up strategy. The analysis steps can be summarized as following:

- Step 1: model 1, random intercepts for schools within regions and schools included.
    - model 1.a, random intercepts for schools within regions omitted.
    - Random intercept for schools is kept
- Step 2: model 2, fixed effects of student-level covariates (interactions with "sex" and "white") are added
    - model 2a: drop insignificant interactions with "sex" and "white"
- Step 3: model 3, fixed effects of school-level covariates (interactions with "public") added to Level 2 model2a.
    - model3a: drop insignificant interactions with "public".
    - model3b: add cross level interactions to model3a ("sex", "white" from student level with "public" from school)
    - model3c: drop insignificant cross level interactions.
    - model3d: drop insignificant interactions.
- Step 4: model4.1, model 4.2 and model 4.3 backward stepwise model selection by dropping insignificant covariates.

## 3. Results

3.1 Descriptive statistics

The data includes 249 boys (48%) and 270 girls(52%). Of the sample, 350 students identified as white (75%). Among 23 schools, 15 are public schools (65%). Among 3 regions, 49.9% were coded as "North Central", 19.3% as "Northeast", and 31.9% as "South". The average math score

for boys and girls was very close with the average for boys being 51.9% and 51.5% for girls. The average math score for those identifying as white was 53.4% and 46.5% for non-white. Girls reported spending an average of 2.05 hours per week homework while boys reported spending an average of 1.9 hours per week. The average math score for those attending a public school was 48.7 vs. 56.5 for those attending a private school. Table 2 presents descriptive statistics of the fixed factors of level 1 and level 2 data. Figure 1 presents boxplot of the average mathematics achievement for 23 schools. The boxplot indicates that there was great variability of mathematics performance among 23 schools.

Table2 Descriptive Statistics for fixed factors*

| School (Level 2) | | | Students (Level 1) | | |
|---|---|---|---|---|---|
| Public | 1=yes | 15(65%) | Sex | 1=girl | 270 (52%) |
| | 0=no | 8(35%) | | 2=boy | 249(48%) |
| Ratio | | 17.35 (4.93) | White | 1=yes | 350(75%) |
| Percmin | | 2.30 (2.04) | | 0=no | 159(25%) |
| Scsize | | 3.13 (1.4) | parented | | 3.29(1.44) |
| | | | Ses | | -0.001(0.88) |
| | | | Homework | | 1.97(1.48) |

* Categorical variables: total numbers and percentage. Continuous variables: mean and standard deviation

Boxplot Math Performance for 23 Schools

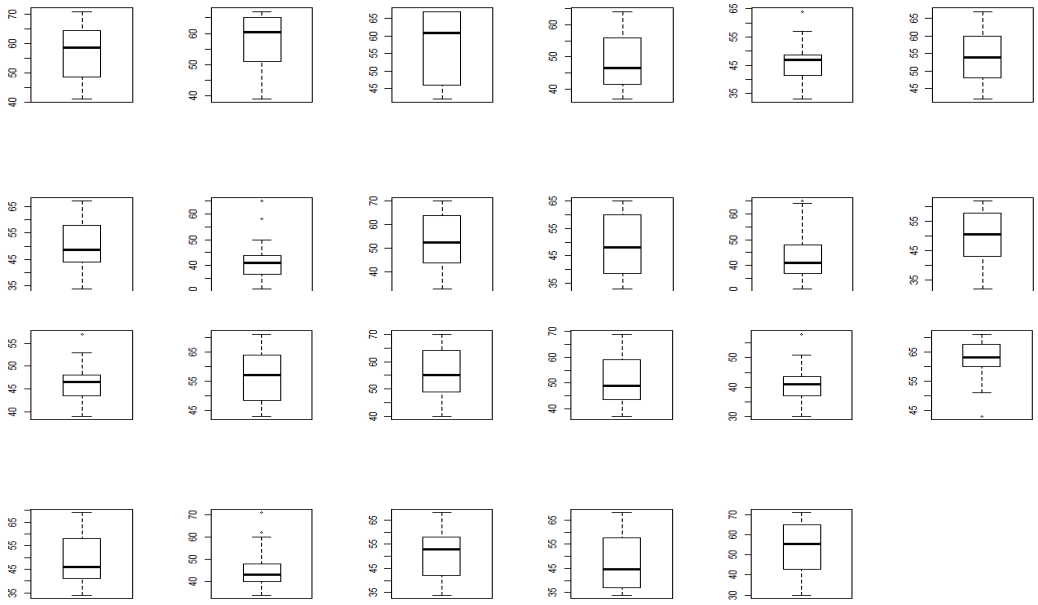

Fig 1. Boxplot for mathematics achievements of 23 schools.

## 3.2 Model selection results

In step1 model selection, we first used the REML estimation to fit the nested Model1 with school nested within region. Then we fit the model1.a by excluding the region from the nesting structure. The likelihood ratio test (p-value=0.147) indicates that there is no random effect from

school nested within region. Therefore we retain the model1.a with only school level random effect. Model1.a $Math_{ij} = \beta_0 + u_j + \varepsilon_{ij}$, where $u_j$ represent the random effect from the school associated with the intercept for school j .

Then in step 2, student level fixed effects were added to the Model1.a. This includes the interactions effects between sex, ses and white with the other covariates. Only the interactions between ses and parented (p-value=0.02), and ses and white (p-value=0.007) were significant with α=0.05 level. So in Model2.a, all insignificant interactions were dropped. Model2.a
$Math_{ij} = \beta_0 + \beta_1 sex + \beta_2 ses + \beta_3 parented + \beta_4 homework + \beta_5 white + \beta_6 ses * parented + \beta_7 ses * white + u_j + \varepsilon_{ij}$
The likelihood ratio test between Model2.a and Model1.2 based on the maximum likelihood estimation suggests we reject the null hypothesis that all effects associated with the student level covariates are equal to zero (p-value<.001).

In step 3 Model3, we added the fixed effects of the school-level covariates to the model2.a. We also included the interaction effects between public and other school level covariates (meanses, ratio,percmin, scsize). The interactions between public and percmin (p-value=0.06) and interactions between public and scsize (p-value=0.05) were significant with α=0.1 level, and therefore were retained in Model3.a. Next, in model 3.b, the cross level interactions were added to the model: sex*public, sex*meanses, sex*scsize, sex*ratio, white*public, white*meanses, white*scsize, white*ratio, public*ses, public*parented, public*homework. Only white*meanses (p-value=0.05) and percmin*white (p-value=0.04) were found significant. After dropping the insignificant cross-level interactions (Model3.c), all insignificant interactions were dropped from the model3.d and resulted in Model3.d.
Model3.d $Math_{ij} = \beta_0 + \beta_1 sex + \beta_2 ses + \beta_3 parented + \beta_4 homework + \beta_5 white + \beta_6 public + \beta_7 meanses + \beta_8 ratio + \beta_9 percmin + \beta_{10} scsize + \beta_{11} parented * ses + \beta_{12} white * meanses + \beta_{13} white * percmin + u_j + \varepsilon_{ij}$

In step 4, a backward stepwise model selection was done by dropping ratio, public and scsize sequentially from model 3.d. The final reduced model is Model4.3. Table 4 presents the final model statistic results.
Model 4.3 $Math_{ij} = \beta_0 + \beta_1 ses + \beta_2 homework + \beta_3 parented + \beta_4 sex + \beta_5 white + \beta_6 meanses + \beta_7 percmin + \beta_8 parented * ses + \beta_9 white * meanses + \beta_{10} white * percmin + u_j + \varepsilon_{ij}$

Table 4 statistic results for final model

| | Value | Std.Error | DF | t-value | p-value |
|---|---|---|---|---|---|
| (Intercept) | 43.729 | 2.924 | 488.000 | 14.954 | 0.000 |
| ses | 1.634 | 1.266 | 488.000 | 1.291 | 0.197 |
| homework | 2.171 | 0.264 | 488.000 | 8.239 | 0.000 |
| parented | 2.499 | 0.587 | 488.000 | 4.258 | 0.000 |
| sex | -0.312 | 0.724 | 488.000 | -0.430 | 0.667 |
| white | -2.664 | 2.162 | 488.000 | -1.232 | 0.219 |
| meanses | -2.623 | 1.725 | 20.000 | -1.520 | 0.144 |
| percmin | -1.661 | 0.459 | 20.000 | -3.621 | 0.002 |

| | | | | | |
|---|---|---|---|---|---|
| ses:parented | -0.634 | 0.300 | 488.000 | -2.109 | 0.036 |
| white:meanses | 5.692 | 1.709 | 488.000 | 3.331 | 0.001 |
| white:percmin | 1.540 | 0.601 | 488.000 | 2.561 | 0.011 |

3.4 Model diagnostics

To check on the assumptions of normality and constant variances for the residuals, the normal Q-Q plot for the residuals and fitted-residual plot was presented in Figure 2. We can see the residuals based on our final model followed an approximately normal distribution, with just a little deviation in the two tails. The residual plot shows no obvious trend or patterns.

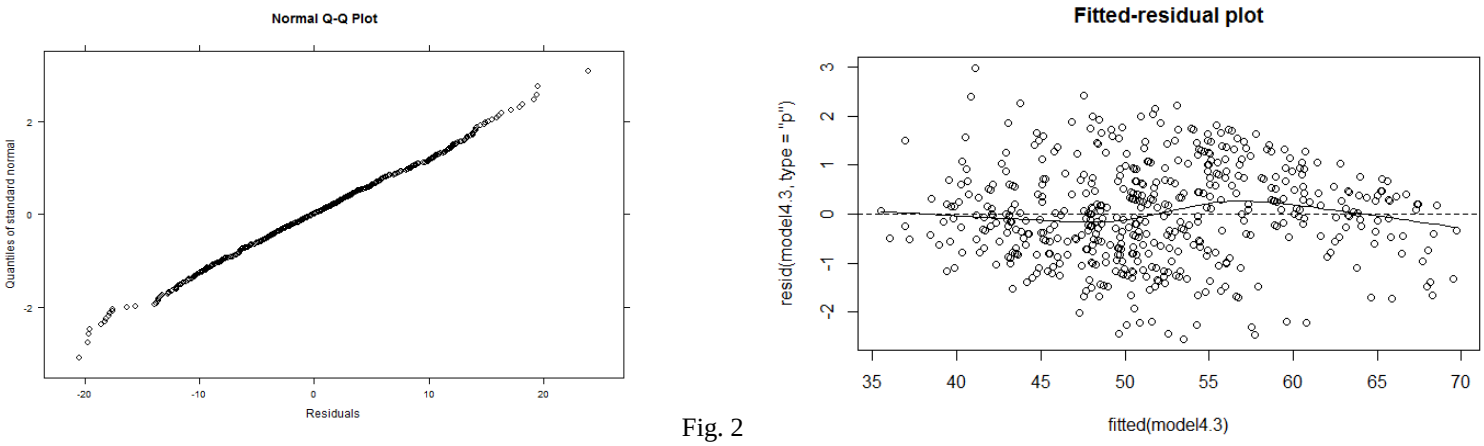


Fig. 2
Normal Q-Q plot and fitted-residual plot for final model (Model4.3)

## 4. Conclusion and Discussion

Although gender difference in academic performance have always been a research focus, consistent with literature, our study did not find significant relationship between mathematic performance and gender, which was consistent with our findings in the exploratory data analysis portion. The time students spend on their homework has a significant positive relationship with their mathematics performance. While holding all other covariates constant, for every hour increase in time spent on homework, our model predicts that the student's math score increases by 2.17. The time a student is willing to spend on homework serves as an indicator for student's motivation and attitude towards mathematics. This finding is also consistent with the literature, which found that there was positive relationship between students' attitude toward mathematics study and their math performances (Papanastasiou, 2000).

Among the student level covariates, we tested for and found interesting interaction effects between student socioeconomic status and their parents' educational level. Though it is generally believed that students from lower socioeconomic background usually experience lower mathematics performance, our findings suggest that the impact form socioeconomic background could be varied based on parent's educational level. Students from lower socioeconomic background but with parents whom have a higher education may still perform well in mathematics and vice versa. Our study also found significant cross level interaction effects between student's race and mean socioeconomic scores of the school, and percentage of minority students of the school. The lower mean socioeconomic scores of the school usually indicate that

the school located in a neighborhood with financially strained families. Since the public school usually depends on tax support for school resources, the lower 'meanses' may suggest that the school has less resources when compared with schools in more affluent areas. Additionally, one expects to see this also be a significant variable as it is a function of student socioeconomic status. Though literature suggested that there is a significant correlation between school resources and school mathematic achievement, our study shows that the minority students were adversely impacted more than their white classmates. With "meanses" increase in 1, our model predicts that the minority students' math scores decrease by 5.69 while keeping all other covariates constant.. Similarly, the minority students' performance can be adversely impacted by the percentage of minority students in the school. Many of the minority students come from non-English speaking background. The higher level percentage of minority students may negatively associate with minority student's English proficiency, as they may be prone to communicate in their native language rather than English. Language incompetency may in turn negatively impact their in-class learning and performance on exams.

Our findings were consistent with much of the preexisting literature that found that students from disadvantaged backgrounds (low parent education, low SES, minority status experience lower levels of performance in mathematics. As one might expect the combination of these variables lead to an even more adverse effect on mathematics performance. Contrary to popular belief, yet in keeping with prior research, gender does not have a significant effect on mathematics scores. Interestingly, the sector (public vs private) of the school also did not prove to be significant. These findings are useful as they confirm intuitive beliefs and prior research as well as point out counterintuitive hypothesis and additional variables that affect students' performance.

Certain limitations of our study should be noted. First, a sample size of 519, while sufficient and less cumbersome to analyze, could be questioned as not being large enough given the original dataset included a sample of 24,599 students. (National Education Longitudinal Study of 1988, 2015) In this subsample there are no students or schools from the "West" region which could result in undercoverage as it is is not a true and complete sample of the US 8th grade student population. Furthermore, much of the student-level data is self-reported which could result in inaccurate responses. Specifically, for the "hours spent on homework" variable, students might be inclined to either over or underestimate the number they report due to various reasons. Our study does have certain strengths as well. First, we aimed to analyze interaction effects as well as if there is a random-effect across the different schools in the three different regions. Additionally, our dataset included micro-level data which can often be difficult to find due to legislation in place to protect the privacy of students. Microlevel data is beneficial as it allows one to observe individual student variables. This analysis could be further extended by analyzing scores from other subject areas. Academic achievement should be evaluated by analyzing scores from all subject areas. A lot of students struggle with mathematics but might be performing well in other subject areas or vice versa. Another insightful extension could be to determine the students whom fall into the categories that we found to be associated with low performance in mathematics (low ses, low parent education, non-white, etc) yet still performed well. Their attributes could be informative, especially if there is commonality among these attributes. Additionally, analyzing the follow-up data could be useful as the NELS study included 4 follow-up surveys, which included information about whether the students graduated from high school, continued studying in college, and their occupational status among others.

**Bibiolography**

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## Appendix

### 1. Dataset Structure

| | sch_id | stuid | math | sex | ses | white | homework | parented | public | ratio | percmin | scsize | region |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 50 | 1 | 0.85 | 1 | 1 | 4 | 0 | 18 | 3 | 3 | 2 |
| 2 | 1 | 2 | 43 | 1 | 0.43 | 1 | 1 | 3 | 0 | 18 | 3 | 3 | 2 |
| 3 | 1 | 4 | 50 | 1 | -0.59 | 0 | 3 | 3 | 0 | 18 | 3 | 3 | 2 |
| 4 | 1 | 11 | 49 | 1 | 1.02 | 1 | 1 | 5 | 0 | 18 | 3 | 3 | 2 |
| 5 | 1 | 12 | 62 | 0 | 0.84 | 1 | 1 | 5 | 0 | 18 | 3 | 3 | 2 |
| 6 | 1 | 13 | 43 | 1 | 1.32 | 1 | 1 | 6 | 0 | 18 | 3 | 3 | 2 |
| 7 | 1 | 18 | 42 | 0 | -0.12 | 1 | 1 | 3 | 0 | 18 | 3 | 3 | 2 |
| 8 | 1 | 22 | 68 | 0 | 0.98 | 1 | 4 | 4 | 0 | 18 | 3 | 3 | 2 |
| 9 | 1 | 23 | 41 | 0 | 0.55 | 0 | 1 | 5 | 0 | 18 | 3 | 3 | 2 |
| 10 | 1 | 24 | 62 | 0 | 1.63 | 1 | 5 | 6 | 0 | 18 | 3 | 3 | 2 |
| 11 | 1 | 25 | 69 | 1 | 1.29 | 1 | 1 | 6 | 0 | 18 | 3 | 3 | 2 |
| 12 | 1 | 26 | 60 | 1 | 1.02 | 1 | 0 | 6 | 0 | 18 | 3 | 3 | 2 |
| 13 | 1 | 27 | 71 | 0 | 0.6 | 1 | 4 | 4 | 0 | 18 | 3 | 3 | 2 |
| 14 | 1 | 28 | 56 | 0 | 1.23 | 1 | 1 | 6 | 0 | 18 | 3 | 3 | 2 |
| 15 | 1 | 32 | 47 | 1 | 0.82 | 0 | 1 | 6 | 0 | 18 | 3 | 3 | 2 |
| 16 | 1 | 33 | 58 | 1 | 0.82 | 1 | 1 | 5 | 0 | 18 | 3 | 3 | 2 |
| 17 | 1 | 34 | 66 | 0 | 0.58 | 1 | 1 | 4 | 0 | 18 | 3 | 3 | 2 |
| 18 | 1 | 36 | 41 | 0 | 1.66 | 0 | 2 | 6 | 0 | 18 | 3 | 3 | 2 |
| 19 | 1 | 39 | 60 | 1 | -0.64 | 0 | 2 | 3 | 0 | 18 | 3 | 3 | 2 |
| 20 | 1 | 42 | 69 | 0 | 1.48 | 1 | 2 | 6 | 0 | 18 | 3 | 3 | 2 |

### 2. R Code

```
library(nlme)
math=read.csv("imm23.csv",header=T)
math$sex=ifelse(math$sex==1,0,1)
names(math)
data=math

# Descriptive
# Level 1 Descriptive Statistics
level1 <- data.frame(sex,white,ses,homework,parented)
summary(level1)
dim(level1)

# Level 2 Descriptive Statistics
level2.agg <- aggregate(data,list(sch_id),FUN=mean)
level2 <-
data.frame(level2.agg$meanses,level2.agg$ratio,level2.agg$percmin,level2.agg$
public,level2.agg$scsize)
summary(level2)
dim(level2)

# boxplot
sch.first12 <- data[data$sch_id <= 12,]
par(mfrow=c(2,6))
```

```
for (i in 1:12)
{boxplot(sch.first12$math[sch.first12$sch_id==i] ~
sch.first12$sch_id[sch.first12$sch_id==i])}
sch.rest11 <- data[data$sch_id >= 12,]
par(mfrow=c(2,6))
for (i in 13:23)
{boxplot(sch.rest11$math[sch.rest11$sch_id==i] ~
sch.rest11$sch_id[sch.rest11$sch_id==i])}

# Model 1. intercept only
model1 <- lme(math ~ 1, random = ~1 | region/schid, data, method = "REML")
summary(model1)
model1a<-lme(math ~ 1,random = ~1 | schid, data = data,method = "REML") #
without random effect
anova(model1, model1a) #LRT test
model1b<-gls(math ~ 1, data=data)
anova(model1a, model1b)

# Model 2. student level covariates ses
model2 <- lme(math ~
sex*white+sex*ses+sex*homework+sex*parented+ses*parented++ses*homework+white*
ses+white*homework+white*parented, random = ~1 | schid, data, method =
"REML")
summary(model2)
model2a<-lme(math ~ sex+ses*white++ses*parented+homework, random = ~1 |
schid, data, method = "REML")
summary(model2a)
#anova model1 and model2 with method ="ML" "LRT test"

model1a.ml <- lme(math ~ 1, random = ~1 | schid, data, method = "ML")
#model2a drop all insignificant interactions
model2a.ml <- lme(math ~ sex+white*ses++ses*parented+homework, random = ~1 |
schid, data, method = "ML")
anova(model1a.ml,model2a.ml)

# Model 3. +school level covariates
model3 <- update(model2a,fixed = ~ ses*white + homework + parented*ses +
sex+meanses*public+ratio*public+percmin*public+public*scsize)
summary(model3)
model3a # drop insignificant interactions school level
model3a <- update(model2a,fixed = ~ ses*white + homework + parented*ses+ sex
+meanses+ratio+percmin*public+public*scsize)
summary(model3a)
#model3b # add cross level interactions
model3b <- update(model2a,fixed = ~ ses*public + homework*public +
parented*ses +
sex*public+white*public+sex*meanses+sex*ratio+sex*percmin+sex*scsize+white*me
anses+white*ratio+white*percmin+white*scsize+percmin*public+public*scsize)
summary(model3b)
#model3c drop insignificant cross level interactions
model3c <- update(model2a,fixed = ~ ses + homework + parented*ses +
sex+white*meanses+ratio+percmin*white+percmin*public+public*scsize)
summary(model3c)
#model3d drop insignificant interactions left
model3d<-update(model2a,fixed = ~ ses + homework + parented*ses +
sex+white*meanses+ratio+percmin*white+scsize+public)
summary(model3d)
#stepwise backward model selection
#drop ratio
model4.1<-update(model3d,fixed = ~ ses + homework + parented*ses +
sex+white*meanses+ percmin*white+scsize+public)
summary(model4.1)
#drop ratio
```

```
model4.2<-update(model3d,fixed = ~ ses + homework + parented*ses +
sex+white*meanses+ percmin*white+scsize)
summary(model4.2)
#drop scsize
model4.3<-update(model4.2,fixed = ~ ses + homework + parented*ses +
sex+white*meanses+percmin*white)
summary(model4.3)

#model diagnostic
library(lattice)
trellis.device(color=F)
qqnorm(model4.3, ~resid(.), plot.it = TRUE,main="Normal Q-Q Plot")
abline(a=0,b=45)
plot(resid(model4.3, type="p") ~ fitted(model4.3),main="Fitted-residual
plot")
abline(h = 0, lty = 2)
lines(lowess(resid(model4.3, type="p") ~ fitted(model4.3)))
```

3. R code with results summary

```
> model1 <- lme(math ~ 1, random = ~1 | region/sch_id, data, method = "REML")
> summary(model1)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3804.571 3821.571 -1898.286

Random effects:
 Formula: ~1 | region
        (Intercept)
StdDev:    2.876611

 Formula: ~1 | sch_id %in% region
        (Intercept) Residual
StdDev:    4.550529 9.011249

Fixed effects: math ~ 1
               Value Std.Error  DF  t-value p-value
(Intercept) 50.84868  1.976696 496 25.72407       0

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.70266883 -0.72876931 -0.02578962  0.70904555  2.70710209

Number of Observations: 519
Number of Groups:
            region sch_id %in% region
                 3                 23
> model1a<-lme(math ~ 1,random = ~1 | sch_id, data = data,method = "REML") #
without random effect
> anova(model1, model1a) #LRT test
        Model df      AIC      BIC    logLik   Test  L.Ratio p-value
model1      1  4 3804.571 3821.571 -1898.286
model1a     2  3 3804.679 3817.429 -1899.339 1 vs 2 2.107114  0.1466
> model1b<-gls(math ~ 1, data=data)
> anova(model1a, model1b)
        Model df      AIC      BIC    logLik   Test  L.Ratio p-value
model1a     1  3 3804.679 3817.429 -1899.339
model1b     2  2 3936.737 3945.237 -1966.368 1 vs 2 134.0582  <.0001
> #model2a drop all insignificant interactions
> model2 <- lme(math ~ sex*white+sex*ses+sex*homework+sex*parented+ses*parent
ed++ses*homework+white*ses+white*homework+white*parented, random = ~1 | sch_i
d, data, method = "REML")
> summary(model2)
```

```
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3663.229 3735.012 -1814.614

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    2.797036 7.999031

Fixed effects: math ~ sex * white + sex * ses + sex * homework + sex * parent
ed +      ses * parented + +ses * homework + white * ses + white *      homew
ork + white * parented
                  Value Std.Error  DF   t-value p-value
(Intercept)    36.69475  4.732977 482  7.752996  0.0000
sex            -7.14655  4.331078 482 -1.650063  0.0996
white           7.23056  4.817160 482  1.501001  0.1340
ses            -0.11253  2.308281 482 -0.048750  0.9611
homework        1.31444  0.727088 482  1.807819  0.0713
parented        3.26047  1.290966 482  2.525604  0.0119
sex:white       2.31247  1.792953 482  1.289754  0.1978
sex:ses        -2.68301  1.940060 482 -1.382949  0.1673
sex:homework    0.66271  0.512710 482  1.292560  0.1968
sex:parented    1.06952  1.136648 482  0.940945  0.3472
ses:parented   -0.77736  0.328902 482 -2.363503  0.0185
ses:homework   -0.09305  0.322613 482 -0.288430  0.7731
white:ses       5.80559  2.156112 482  2.692618  0.0073
white:homework  0.58874  0.730677 482  0.805748  0.4208
white:parented -1.69540  1.317086 482 -1.287236  0.1986
 Correlation:
               (Intr) sex    white  ses    homwrk parntd sx:wht sex:ss sx:hmw
 sx:prn ss:prn ss:hmw wht:ss wht:hm
sex            -0.466

white          -0.783  0.074

ses             0.829 -0.389 -0.673

homework       -0.242  0.024  0.200 -0.114

parented       -0.923  0.440  0.708 -0.802 -0.020

sex:white       0.136 -0.444 -0.142  0.041  0.124 -0.041

sex:ses        -0.457  0.871  0.101 -0.499  0.053  0.459 -0.263

sex:homework    0.121 -0.212 -0.016  0.080 -0.439 -0.003 -0.024 -0.139

sex:parented    0.456 -0.908 -0.050  0.427  0.031 -0.502  0.157 -0.879 -0.013

ses:parented   -0.078 -0.015  0.139 -0.340 -0.010  0.019  0.005  0.023 -0.043
  0.017
ses:homework    0.031 -0.010  0.001 -0.086  0.037 -0.052  0.007 -0.059  0.090
  0.004 -0.234
white:ses      -0.666  0.037  0.834 -0.682  0.096  0.670  0.036  0.104 -0.011
 -0.075 -0.022 -0.046
white:homework  0.186  0.057 -0.233  0.091 -0.835  0.035 -0.129  0.017  0.059
 -0.028  0.080 -0.250 -0.105
white:parented  0.717 -0.032 -0.922  0.660  0.012 -0.763  0.001 -0.084  0.009
  0.054 -0.173  0.072 -0.850 -0.032

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.76782504 -0.71250563 -0.02289382  0.64544937  2.51811643
```

```
Number of Observations: 519
Number of Groups: 23
> model2a<-lme(math ~ sex+ses*white++ses*parented+homework, random = ~1 | sch
_id, data, method = "REML")
> summary(model2a)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3666.229 3708.593 -1823.115

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:     2.84661 7.993804

Fixed effects: math ~ sex + ses * white + +ses * parented + homework
                Value Std.Error  DF   t-value p-value
(Intercept)  37.02800 2.3219117 489 15.947204  0.0000
sex          -0.51201 0.7236818 489 -0.707511  0.4796
ses           0.19536 1.3235013 489  0.147608  0.8827
white         3.51895 1.0695997 489  3.289965  0.0011
parented      2.53716 0.5824598 489  4.355947  0.0000
homework      2.13586 0.2634672 489  8.106741  0.0000
ses:white     3.67301 1.0934249 489  3.359183  0.0008
ses:parented -0.86353 0.3149781 489 -2.741560  0.0063
 Correlation:
             (Intr) sex    ses    white  parntd homwrk ss:wht
sex          -0.189
ses           0.573  0.019
white        -0.425 -0.024 -0.271
parented     -0.839  0.066 -0.459  0.097
homework     -0.202 -0.068 -0.066 -0.036  0.013
ses:white    -0.069 -0.046 -0.311  0.169  0.030 -0.033
ses:parented  0.057 -0.016 -0.467  0.007 -0.230  0.023 -0.334

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-2.628998407 -0.725535842  0.005434865  0.670521928  2.784974976

Number of Observations: 519
Number of Groups: 23
> #anova model1 and model2 with method ="ML" "LRT test"
>
> model1a.ml <- lme(math ~ 1, random = ~1 | sch_id, data, method = "ML")
> #model2a drop all insignificant interactions
> model2a.ml <- lme(math ~ sex+white*ses++ses*parented+homework, random = ~1
| sch_id, data, method = "ML")
> anova(model1a.ml,model2a.ml)
           Model df      AIC      BIC    logLik   Test  L.Ratio p-value
model1a.ml     1  3 3806.776 3819.532 -1900.388
model2a.ml     2 10 3672.162 3714.682 -1826.081 1 vs 2 148.6139  <.0001

> # Model 3. +school level covariates
> model3 <- update(model2a,fixed = ~ ses*white + homework + parented*ses + se
x+meanses*public+ratio*public+percmin*public+public*scsize)
> summary(model3)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC   logLik
  3653.981 3734.134 -1807.99

Random effects:
 Formula: ~1 | sch_id
```

```
        (Intercept) Residual
StdDev:    1.826537 7.992741

Fixed effects: math ~ ses + white + homework + parented + sex + meanses + pub
lic +      ratio + percmin + scsize + ses:white + ses:parented + meanses:publ
ic +      public:ratio + public:percmin + public:scsize
                  Value Std.Error  DF   t-value p-value
(Intercept)    41.14672  8.685866 489  4.737204  0.0000
ses             0.08093  1.340842 489  0.060356  0.9519
white           2.42192  1.163651 489  2.081308  0.0379
homework        2.08338  0.263654 489  7.901938  0.0000
parented        2.54223  0.583687 489  4.355461  0.0000
sex            -0.62470  0.722652 489 -0.864451  0.3878
meanses         0.80509  3.732329  13  0.215708  0.8326
public          3.22993  9.659692  13  0.334372  0.7434
ratio          -0.23249  0.329016  13 -0.706610  0.4923
percmin        -1.77144  0.630382  13 -2.810104  0.0147
scsize          2.21813  0.862822  13  2.570783  0.0233
ses:white       3.78720  1.123490 489  3.370927  0.0008
ses:parented   -0.91952  0.317667 489 -2.894602  0.0040
meanses:public  3.26892  4.945373  13  0.661006  0.5202
public:ratio   -0.04309  0.384211  13 -0.112156  0.9124
public:percmin  1.51383  0.744688  13  2.032833  0.0630
public:scsize  -2.22990  1.056820  13 -2.110014  0.0548
 Correlation:
               (Intr) ses    white  homwrk parntd sex    meanss public ratio
 percmn scsize ss:wht ss:prn mnss:p
ses             0.153

white          -0.117 -0.225

homework       -0.117 -0.066 -0.057

parented       -0.222 -0.445  0.087  0.017

sex            -0.088  0.022 -0.017 -0.060  0.065

meanses        -0.834  0.026 -0.035  0.036 -0.027  0.017

public         -0.834 -0.018 -0.045  0.093 -0.007  0.035  0.758

ratio          -0.899 -0.002 -0.022  0.082 -0.032  0.056  0.846  0.813

percmin        -0.702 -0.072  0.239  0.048 -0.020  0.020  0.526  0.601  0.660

scsize         -0.050 -0.005 -0.058 -0.063  0.068 -0.033 -0.083  0.042 -0.249
 -0.187
ses:white       0.036 -0.328  0.103 -0.024  0.028 -0.048 -0.094 -0.039 -0.048
  0.025  0.003
ses:parented   -0.031 -0.464  0.004  0.033 -0.223 -0.011 -0.025  0.059  0.061
  0.053 -0.036 -0.317
meanses:public  0.629 -0.091 -0.017 -0.010  0.015 -0.009 -0.738 -0.400 -0.633
 -0.404  0.063  0.053  0.073
public:ratio    0.770 -0.011  0.008 -0.088  0.041 -0.043 -0.727 -0.912 -0.859
 -0.569  0.216  0.062 -0.057  0.507
public:percmin  0.572  0.095 -0.046 -0.059  0.018  0.005 -0.435 -0.428 -0.556
 -0.810  0.147 -0.151 -0.020  0.567
public:scsize   0.045  0.017  0.080  0.021 -0.059  0.002  0.067 -0.250  0.198
  0.157 -0.815 -0.001  0.011 -0.310
               pblc:r pblc:p
ses
white
homework
parented
```

```
sex
meanses
public
ratio
percmin
scsize
ses:white
ses:parented
meanses:public
public:ratio
public:percmin  0.441
public:scsize  -0.075 -0.300

Standardized Within-Group Residuals:
          Min            Q1           Med            Q3           Max
-2.5942392211 -0.7028838977 -0.0001821387  0.6687121294  2.7712275065

Number of Observations: 519
Number of Groups: 23
> model3a # drop insignificant interactions school level
Error: object 'model3a' not found
> model3a <- update(model2a,fixed = ~ ses*white + homework + parented*ses+ se
x +meanses+ratio+percmin*public+public*scsize)
> summary(model3a)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3655.251 3727.035 -1810.626

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.600909 7.996711

Fixed effects: math ~ ses + white + homework + parented + sex + meanses + rat
io +      percmin + public + scsize + ses:white + ses:parented + percmin:publ
ic +      public:scsize
                 Value Std.Error  DF   t-value p-value
(Intercept)    39.66960  4.797166 489  8.269382  0.0000
ses             0.18952  1.332362 489  0.142243  0.8869
white           2.47011  1.161617 489  2.126441  0.0340
homework        2.06294  0.261762 489  7.880991  0.0000
parented        2.55760  0.582725 489  4.389037  0.0000
sex            -0.64843  0.720896 489 -0.899484  0.3688
meanses         1.92537  1.894184  15  1.016465  0.3255
ratio          -0.20270  0.141661  15 -1.430909  0.1730
percmin        -1.72300  0.485618  15 -3.548061  0.0029
public          1.94907  3.711038  15  0.525209  0.6071
scsize          2.33460  0.794462  15  2.938594  0.0102
ses:white       3.80609  1.119754 489  3.399039  0.0007
ses:parented   -0.96417  0.314379 489 -3.066900  0.0023
percmin:public  1.29666  0.566744  15  2.287914  0.0371
public:scsize  -2.03106  0.943082  15 -2.153638  0.0479
 Correlation:
               (Intr) ses    white  homwrk parntd sex    meanss ratio  percmn
 public scsize ss:wht ss:prn prcmn:
ses             0.343

white          -0.210 -0.228

homework       -0.111 -0.063 -0.057

parented       -0.455 -0.448  0.086  0.021
```

```
sex            -0.106  0.023 -0.016 -0.064  0.068

meanses        -0.493 -0.049 -0.080 -0.024 -0.002 -0.016

ratio          -0.648 -0.082 -0.048  0.036  0.004  0.050  0.503

percmin        -0.488 -0.119  0.311  0.004  0.002 -0.002  0.122  0.374

public         -0.653 -0.055 -0.095  0.025  0.079 -0.011  0.576  0.254  0.283

scsize         -0.364 -0.008 -0.068 -0.045  0.062 -0.022  0.100 -0.164 -0.080
  0.624
ses:white      -0.034 -0.327  0.103 -0.019  0.025 -0.046 -0.075  0.026  0.082
  0.039 -0.010
ses:parented   -0.036 -0.458  0.007  0.022 -0.222 -0.016 -0.031  0.092  0.048
 -0.004 -0.018 -0.320
percmin:public  0.268  0.183 -0.052 -0.049  0.005  0.024  0.129 -0.232 -0.773
 -0.178  0.084 -0.248 -0.055
public:scsize   0.354 -0.018  0.085  0.030 -0.064  0.001 -0.242  0.135  0.086
 -0.776 -0.886  0.013  0.050 -0.176

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.58792806 -0.70411077 -0.01055393  0.66335512  2.74999897

Number of Observations: 519
Number of Groups: 23
> #model3b # add cross level interactions
> model3b <- update(model2a,fixed = ~ ses*public + homework*public + parented
*ses + sex*public+white*public+sex*meanses+sex*ratio+sex*percmin+sex*scsize+w
hite*meanses+white*ratio+white*percmin+white*scsize+percmin*public+public*scs
ize)
> summary(model3b)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3660.417 3778.031 -1802.209

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.055698 8.109384

Fixed effects: math ~ ses + public + homework + parented + sex + white + mean
ses +      ratio + percmin + scsize + ses:public + public:homework +      ses:
parented + public:sex + public:white + sex:meanses +      sex:ratio + sex:per
cmin + sex:scsize + white:meanses + white:ratio +      white:percmin + white:
scsize + public:percmin + public:scsize
                  Value Std.Error  DF   t-value p-value
(Intercept)     45.63016  7.869874 478  5.798080  0.0000
ses              0.55871  2.137233 478  0.261418  0.7939
public           0.44410  5.143713  15  0.086339  0.9323
homework         2.06387  0.356861 478  5.783388  0.0000
parented         2.57503  0.602423 478  4.274447  0.0000
sex              0.95842  5.285148 478  0.181343  0.8562
white           -8.03526  7.817247 478 -1.027888  0.3045
meanses         -2.62900  3.255109  15 -0.807652  0.4319
ratio           -0.29516  0.257131  15 -1.147896  0.2690
percmin         -2.59040  0.665411  15 -3.892929  0.0014
scsize           2.63133  1.068621  15  2.462361  0.0264
ses:public       1.23766  1.488277 478  0.831606  0.4060
public:homework  0.05669  0.537877 478  0.105404  0.9161
ses:parented    -0.59895  0.365431 478 -1.639021  0.1019
public:sex       1.62365  3.117298 478  0.520850  0.6027
```

```
public:white     3.28879  4.614589 478  0.712694  0.4764
sex:meanses     -0.43375  2.675034 478 -0.162148  0.8713
sex:ratio       -0.13069  0.211952 478 -0.616610  0.5378
sex:percmin      0.31431  0.414789 478  0.757750  0.4490
sex:scsize      -0.32674  0.571198 478 -0.572031  0.5676
white:meanses    7.63314  3.815770 478  2.000419  0.0460
white:ratio      0.25963  0.282149 478  0.920182  0.3579
white:percmin    1.42830  0.677868 478  2.107051  0.0356
white:scsize    -0.27243  0.834805 478 -0.326342  0.7443
public:percmin   1.05119  0.609173  15  1.725596  0.1050
public:scsize   -2.19184  0.901858  15 -2.430354  0.0281
 Correlation:
                (Intr) ses    public homwrk parntd sex    white  meanss ratio
  percmn scsize ss:pbl pblc:h
ses              0.076

public          -0.454 -0.095

homework        -0.113 -0.056  0.082

parented        -0.229 -0.310  0.099  0.039

sex             -0.355  0.025  0.197  0.010 -0.087

white           -0.689  0.049  0.132 -0.039 -0.059  0.069

meanses         -0.491 -0.062  0.617 -0.045 -0.032  0.334  0.396

ratio           -0.728  0.076  0.046  0.059 -0.109  0.439  0.629  0.318

percmin         -0.693 -0.024  0.464 -0.007 -0.013  0.198  0.534  0.423  0.40
2
scsize          -0.452 -0.016  0.124 -0.027  0.043 -0.009  0.225 -0.026  0.08
7  0.131
ses:public       0.018 -0.795  0.141  0.057  0.049 -0.045 -0.012  0.074 -0.08
9  0.012  0.017
public:homework  0.107  0.021 -0.128 -0.669 -0.034  0.045 -0.021  0.003 -0.05
7 -0.027  0.011 -0.076
ses:parented    -0.021 -0.756  0.038  0.013 -0.167  0.015  0.009  0.028  0.02
2  0.050  0.015  0.548  0.004
public:sex       0.270  0.064 -0.197 -0.042  0.048 -0.691 -0.088 -0.334 -0.34
2 -0.096  0.052 -0.030 -0.019
public:white     0.202 -0.052 -0.570  0.097 -0.001 -0.071 -0.438 -0.513 -0.03
4 -0.372  0.302 -0.017 -0.054
sex:meanses      0.278  0.045 -0.200 -0.033  0.053 -0.747 -0.097 -0.435 -0.37
5 -0.104  0.059 -0.035 -0.001
sex:ratio        0.329 -0.070 -0.203 -0.012  0.133 -0.897 -0.091 -0.344 -0.47
2 -0.199  0.059  0.072 -0.035
sex:percmin      0.127  0.047 -0.023  0.019  0.069 -0.547  0.045 -0.111 -0.19
9 -0.259  0.084  0.017 -0.057
sex:scsize       0.011 -0.016  0.039  0.034 -0.069 -0.068  0.039  0.095  0.13
1  0.053 -0.206 -0.001 -0.049
white:meanses    0.309 -0.134 -0.343  0.070  0.072 -0.102 -0.645 -0.655 -0.22
7 -0.356  0.152  0.105 -0.010
white:ratio      0.483 -0.080  0.105  0.016  0.102 -0.094 -0.847 -0.202 -0.73
2 -0.234 -0.047  0.048  0.018
white:percmin    0.506 -0.025 -0.244 -0.027 -0.045  0.086 -0.479 -0.426 -0.24
1 -0.512 -0.243 -0.002  0.080
white:scsize     0.403  0.049  0.199 -0.021  0.040  0.000 -0.390  0.098 -0.24
8 -0.189 -0.673 -0.060  0.027
public:percmin   0.184  0.054 -0.308 -0.005 -0.009 -0.068 -0.037  0.137 -0.06
1 -0.518  0.051 -0.025 -0.042
public:scsize    0.180  0.075 -0.472  0.053 -0.085  0.066  0.123 -0.138  0.14
0  0.018 -0.628 -0.097 -0.052
```

```
                ss:prn pblc:sx pblc:w sx:mns sex:rt sx:prc sx:scs wht:mn wht:
rt wht:pr wht:sc pblc:p
ses

public

homework

parented

sex

white

meanses

ratio

percmin

scsize

ses:public

public:homework

ses:parented

public:sex      -0.087

public:white     0.039 -0.054

sex:meanses     -0.053  0.868  -0.003

sex:ratio        0.029  0.554   0.100  0.665

sex:percmin     -0.063  0.405  -0.068  0.423  0.433

sex:scsize       0.007 -0.340   0.107 -0.270 -0.166 -0.244

white:meanses    0.045  0.020   0.807  0.069  0.144 -0.055  0.033

white:ratio     -0.001  0.139   0.234  0.152  0.109  0.001 -0.093  0.496

white:percmin    0.006 -0.053   0.127 -0.050 -0.055 -0.121 -0.005  0.270  0.2
19
white:scsize    -0.066  0.091  -0.347  0.041 -0.031  0.022 -0.035 -0.150  0.1
88  0.082
public:percmin  -0.077 -0.013   0.276 -0.033  0.060  0.042  0.070 -0.022 -0.0
69 -0.206  0.061
public:scsize    0.000 -0.047  -0.162 -0.028 -0.015 -0.051 -0.075 -0.146 -0.1
79  0.206  0.034 -0.210

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.48802369 -0.65855679  0.01572088  0.64555041  2.97994628

Number of Observations: 519
Number of Groups: 23
> #model3c drop insignificant cross level interactions
> model3c <- update(model2a,fixed = ~ ses + homework + parented*ses + sex+whi
te*meanses+ratio+percmin*white+percmin*public+public*scsize)
> summary(model3c)
Linear mixed-effects model fit by REML
```

```
 Data: data
       AIC      BIC    logLik
  3659.672 3735.642 -1811.836

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.356489  8.05627

Fixed effects: math ~ ses + homework + parented + sex + white + meanses + rat
io +      percmin + public + scsize + ses:parented + white:meanses +      whi
te:percmin + percmin:public + public:scsize
                 Value Std.Error  DF   t-value p-value
(Intercept)    43.59147  5.729652 488  7.608049  0.0000
ses             1.78981  1.267564 488  1.412004  0.1586
homework        2.08865  0.262744 488  7.949383  0.0000
parented        2.55399  0.589064 488  4.335680  0.0000
sex            -0.43548  0.725000 488 -0.600667  0.5483
white          -1.72541  2.291828 488 -0.752852  0.4519
meanses        -2.06845  2.460786  15 -0.840564  0.4138
ratio          -0.20194  0.142060  15 -1.421528  0.1756
percmin        -2.25217  0.559470  15 -4.025533  0.0011
public          1.04647  3.795747  15  0.275695  0.7865
scsize          1.98593  0.785916  15  2.526900  0.0232
ses:parented   -0.69601  0.300946 488 -2.312732  0.0212
white:meanses   4.35866  1.851110 488  2.354621  0.0189
white:percmin   1.23176  0.662228 488  1.860021  0.0635
percmin:public  1.02367  0.589533  15  1.736408  0.1030
public:scsize  -1.68323  0.920930  15 -1.827752  0.0875
 Correlation:
               (Intr) ses    homwrk parntd sex    white  meanss ratio  percmn
 public scsize ss:prn wht:mn wht:pr
ses             0.280

homework       -0.091 -0.071

parented       -0.429 -0.457  0.024

sex            -0.076  0.010 -0.062  0.070

white          -0.560 -0.098 -0.042  0.090 -0.042

meanses        -0.472 -0.079 -0.040 -0.010 -0.053  0.421

ratio          -0.660 -0.063  0.039  0.039  0.049  0.162  0.338

percmin        -0.658 -0.072  0.000  0.043 -0.016  0.604  0.287  0.447

public         -0.699 -0.031  0.026  0.109 -0.013  0.211  0.459  0.347  0.415

scsize         -0.447 -0.008 -0.049  0.078 -0.033  0.219  0.224 -0.076  0.108
  0.653
ses:parented   -0.020 -0.628  0.012 -0.232 -0.037  0.024  0.005  0.078  0.052
 -0.011 -0.022
white:meanses  -0.076  0.047  0.025  0.070  0.048 -0.215 -0.562  0.213  0.021
  0.120 -0.073 -0.105
white:percmin   0.555 -0.009  0.015 -0.059  0.042 -0.862 -0.506 -0.228 -0.550
 -0.311 -0.295  0.001  0.215
percmin:public  0.140  0.085 -0.066 -0.009 -0.014  0.207  0.385 -0.246 -0.489
 -0.152  0.136 -0.092 -0.452 -0.242
public:scsize   0.418 -0.018  0.035 -0.078  0.010 -0.175 -0.309  0.062 -0.076
 -0.779 -0.895  0.056  0.076  0.255
               prcmn:
ses
```

```
        homework
        parented
        sex
        white
        meanses
        ratio
        percmin
        public
        scsize
        ses:parented
        white:meanses
        white:percmin
        percmin:public
        public:scsize  -0.213

        Standardized Within-Group Residuals:
                Min          Q1         Med          Q3         Max
        -2.55140094 -0.70219079 -0.01744149  0.65883548  2.83679521

        Number of Observations: 519
        Number of Groups: 23
> #model3d drop insignificant interactions left
> model3d<-update(model2a,fixed = ~ ses + homework + parented*ses + sex+white*meanses+ratio+perc
min*white+scsize+public)
> summary(model3d)
Linear mixed-effects model fit by REML
 Data: data
     AIC      BIC    logLik
  3662.9 3730.493 -1815.45

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.787481 8.048018

Fixed effects: math ~ ses + homework + parented + sex + white + meanses + ratio +percmin + scsiz
e + public + ses:parented + white:meanses +      white:percmin
                 Value Std.Error  DF   t-value p-value
(Intercept)   45.35230  5.343902 488  8.486739  0.0000
ses            1.58311  1.266595 488  1.249897  0.2119
homework       2.13937  0.263607 488  8.115759  0.0000
parented       2.49596  0.588371 488  4.242143  0.0000
sex           -0.39135  0.726971 488 -0.538337  0.5906
white         -2.55059  2.274668 488 -1.121302  0.2627
meanses       -4.26194  2.376558  17 -1.793326  0.0907
ratio         -0.12460  0.153110  17 -0.813820  0.4270
percmin       -1.84235  0.504146  17 -3.654400  0.0020
scsize         0.76993  0.389948  17  1.974454  0.0648
public        -2.50105  2.255137  17 -1.109047  0.2829
ses:parented  -0.62006  0.300731 488 -2.061841  0.0398
white:meanses  5.48748  1.702120 488  3.223907  0.0013
white:percmin  1.53762  0.654527 488  2.349213  0.0192
 Correlation:
              (Intr) ses    homwrk parntd sex    white  meanss ratio  percmn scsize public ss:pr
n wht:mn
ses            0.283

homework      -0.096 -0.069

parented      -0.425 -0.458  0.023

sex           -0.082  0.011 -0.065  0.070

white         -0.600 -0.109 -0.029  0.086 -0.043
```

```
meanses       -0.549 -0.110 -0.011 -0.025 -0.050  0.346

ratio         -0.757 -0.036  0.021  0.038  0.045  0.214  0.510

percmin       -0.666 -0.026 -0.035  0.031 -0.027  0.803  0.559  0.394

scsize        -0.148 -0.039 -0.046  0.017 -0.050  0.158 -0.108 -0.082  0.005

public        -0.633 -0.027  0.057  0.072 -0.016  0.239  0.690  0.599  0.404 -0.278

ses:parented  -0.020 -0.628  0.010 -0.232 -0.037  0.040  0.043  0.047  0.006  0.046  0.005

white:meanses  0.003  0.087  0.003  0.072  0.046 -0.155 -0.476  0.103 -0.271 -0.068  0.059 -0.15
2
white:percmin  0.576  0.001 -0.001 -0.051  0.041 -0.859 -0.410 -0.291 -0.762 -0.176 -0.321 -0.01
8  0.140

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-2.615770234 -0.693754612 -0.006581683  0.669010276  2.999671140

Number of Observations: 519
Number of Groups: 23
> #stepwise backward model selection
> #drop ratio
> model4.1<-update(model3d,fixed = ~ ses + homework + parented*ses + sex+white*meanses+ percmin*
white+scsize+public)
> summary(model4.1)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3659.645 3723.043 -1814.822

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.817737 8.042936

Fixed effects: math ~ ses + homework + parented + sex + white + meanses + percmin +scsize + publ
ic + ses:parented + white:meanses + white:percmin
                 Value Std.Error  DF   t-value p-value
(Intercept)   42.04264  3.501134 488 12.008290  0.0000
ses            1.54620  1.265324 488  1.221981  0.2223
homework       2.14457  0.263499 488  8.138808  0.0000
parented       2.51384  0.587705 488  4.277394  0.0000
sex           -0.36397  0.725954 488 -0.501367  0.6163
white         -2.13076  2.225011 488 -0.957638  0.3387
meanses       -3.26359  2.054564  18 -1.588459  0.1296
percmin       -1.67660  0.465067  18 -3.605077  0.0020
scsize         0.74472  0.391673  18  1.901373  0.0734
public        -1.40221  1.819888  18 -0.770491  0.4510
ses:parented  -0.60807  0.300316 488 -2.024774  0.0434
white:meanses  5.60780  1.695907 488  3.306668  0.0010
white:percmin  1.37320  0.627916 488  2.186918  0.0292
 Correlation:
              (Intr) ses    homwrk parntd sex    white  meanss percmn scsize public ss:prn wht:m
n
ses            0.391

homework      -0.122 -0.068

parented      -0.605 -0.457  0.022
```

```
sex            -0.073  0.012 -0.066  0.068

white          -0.686 -0.103 -0.034  0.080 -0.054

meanses        -0.290 -0.105 -0.025 -0.051 -0.085  0.282

percmin        -0.612 -0.013 -0.047  0.018 -0.049  0.799  0.452

scsize         -0.323 -0.042 -0.044  0.021 -0.046  0.180 -0.080  0.038

public         -0.344 -0.007  0.055  0.061 -0.053  0.141  0.560  0.229 -0.289

ses:parented    0.025 -0.627  0.009 -0.235 -0.039  0.030  0.022 -0.014  0.049 -0.029

white:meanses   0.123  0.090  0.001  0.069  0.042 -0.182 -0.616 -0.340 -0.060 -0.003 -0.157

white:percmin   0.569 -0.011  0.006 -0.042  0.057 -0.853 -0.317 -0.735 -0.209 -0.190 -0.004  0.17
9

Standardized Within-Group Residuals:
         Min           Q1          Med           Q3          Max
-2.601174571 -0.714901327 -0.007527483  0.664152518  2.984050938

Number of Observations: 519
Number of Groups: 23
> #drop ratio
> model4.2<-update(model3d,fixed = ~ ses + homework + parented*ses + sex+white*meanses+ percmin*
white+scsize)
> summary(model4.2)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3661.268 3720.468 -1816.634

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.772609 8.043281

Fixed effects: math ~ ses + homework + parented + sex + white + meanses + percmin +scsize + ses:
parented + white:meanses + white:percmin
                 Value Std.Error  DF   t-value p-value
(Intercept)   41.13830  3.276884 488 12.554090  0.0000
ses            1.53922  1.264851 488  1.216920  0.2242
homework       2.15494  0.262933 488  8.195762  0.0000
parented       2.54245  0.586427 488  4.335483  0.0000
sex           -0.39508  0.724674 488 -0.545184  0.5859
white         -1.92043  2.196451 488 -0.874334  0.3824
meanses       -2.39444  1.691156  19 -1.415861  0.1730
percmin       -1.60010  0.449971  19 -3.555999  0.0021
scsize         0.65718  0.370703  19  1.772787  0.0923
ses:parented  -0.61553  0.300036 488 -2.051536  0.0407
white:meanses  5.63519  1.690415 488  3.333613  0.0009
white:percmin  1.29343  0.613283 488  2.109023  0.0355
 Correlation:
              (Intr) ses    homwrk parntd sex    white  meanss percmn scsize ss:prn wht:mn
ses            0.416
homework      -0.111 -0.068
parented      -0.625 -0.458  0.019
sex           -0.098  0.012 -0.063  0.072
white         -0.686 -0.104 -0.042  0.072 -0.047
meanses       -0.126 -0.124 -0.068 -0.104 -0.067  0.248
percmin       -0.584 -0.012 -0.061  0.003 -0.038  0.798  0.403
scsize        -0.467 -0.046 -0.029  0.040 -0.065  0.233  0.105  0.114
```

```
ses:parented   0.016 -0.628  0.011 -0.233 -0.041  0.035  0.047 -0.007  0.044
white:meanses  0.130  0.092  0.001  0.069  0.042 -0.183 -0.743 -0.349 -0.064 -0.159
white:percmin  0.546 -0.012  0.016 -0.030  0.048 -0.850 -0.260 -0.725 -0.282 -0.011  0.181

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.62034269 -0.71193602 -0.02309645  0.67888830  2.97065638

Number of Observations: 519
Number of Groups: 23
> #drop scsize
> model4.3<-update(model4.2,fixed = ~ ses + homework + parented*ses + sex+white*meanses+percmin*
white)
> summary(model4.3)
Linear mixed-effects model fit by REML
 Data: data
       AIC      BIC    logLik
  3662.176 3717.172 -1818.088

Random effects:
 Formula: ~1 | sch_id
        (Intercept) Residual
StdDev:    1.951117 8.044638

Fixed effects: math ~ ses + homework + parented + sex + white + meanses + percmin +ses:parented
+ white:meanses + white:percmin
                 Value Std.Error  DF   t-value p-value
(Intercept)   43.72892 2.9242109 488 14.954091  0.0000
ses            1.63443 1.2657523 488  1.291268  0.1972
homework       2.17114 0.2635303 488  8.238684  0.0000
parented       2.49862 0.5867950 488  4.258079  0.0000
sex           -0.31173 0.7244356 488 -0.430306  0.6672
white         -2.66379 2.1624922 488 -1.231815  0.2186
meanses       -2.62319 1.7252191  20 -1.520494  0.1440
percmin       -1.66113 0.4586904  20 -3.621462  0.0017
ses:parented  -0.63359 0.3004633 488 -2.108722  0.0355
white:meanses  5.69242 1.7089141 488  3.331017  0.0009
white:percmin  1.54011 0.6014325 488  2.560728  0.0107
 Correlation:
              (Intr) ses    homwrk parntd sex    white  meanss percmn ss:prn wht:mn
ses            0.441
homework      -0.139 -0.070
parented      -0.682 -0.457  0.020
sex           -0.142  0.009 -0.065  0.074
white         -0.672 -0.093 -0.037  0.066 -0.035
meanses       -0.089 -0.115 -0.065 -0.107 -0.061  0.230
percmin       -0.607 -0.005 -0.058  0.001 -0.032  0.790  0.392
ses:parented   0.041 -0.628  0.014 -0.235 -0.038  0.023  0.040 -0.014
white:meanses  0.115  0.085  0.002  0.072  0.038 -0.176 -0.733 -0.342 -0.152
white:percmin  0.493 -0.028  0.010 -0.023  0.032 -0.844 -0.237 -0.721  0.004  0.171

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-2.54587907 -0.69897271 -0.01665393  0.66569484  2.97343268

Number of Observations: 519
Number of Groups: 23

        >
```