C_p and C_{pk}

MIS 744

Roger Chen

Definition of C_p and C_{pk}

As one of the most representative reflector of quality measurement, variation has been in the center of the topic in quality control and process capability. One of the most influential quality pioneers, Walter A. Shewhart, initiated the first concept of reducing variation in his book *Economical Control of Quality of Manufactured Product*[1]in early 1920s. The book includes manufacturing variability with normal deviation observed by a control chart, which often refers as Shewhart control chart. Following Shewhart's footsteps, Joseph M. Juran scientifically present methods to define specification limits in process capability of C_p and C_{pk} in 1974. Process capability (C_p), covers a broad range of processes and quality control. A general accepted definition of process capability is the ability of a process to produce products according to specified requirements. The process capability of index (C_{pk}) is the measure of process capability that shows how closely a process is able to meet the requirement of its overall specifications. C_{pk} is computed as the ratio of "distance of the process mean from the closest specification limit" to "3 times of standard deviation of the process variation".[2]

$$C_p = \frac{USL-LSL}{6\sigma}$$

$$C_{pk} = \min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right]$$

where

USL refers to upper specification limit
LSL refers to lower specification limit
σ denotes the population standard deviation of the process interest
μ denotes the population mean of the process interest

[1] Shewhart, (1931), *Economical Control of Manufactured Produc*t, republished in 1980 by American Society for Quality Control, Milwaukee, Wisconsin, BookCrafters, Inc. Chelsea, Michigan.
[2] CSSBB Tutorial Series : Lesson 9

The estimator for C_{pk} can also be expressed as the following:

C_{pk} = C_p × (1 – k)

Where

$$k = \frac{|\frac{USL+LSL}{2} - \mu|}{\min[\frac{USL+LSL}{2} - LSL, USL - \frac{USL+LSL}{2}]}$$

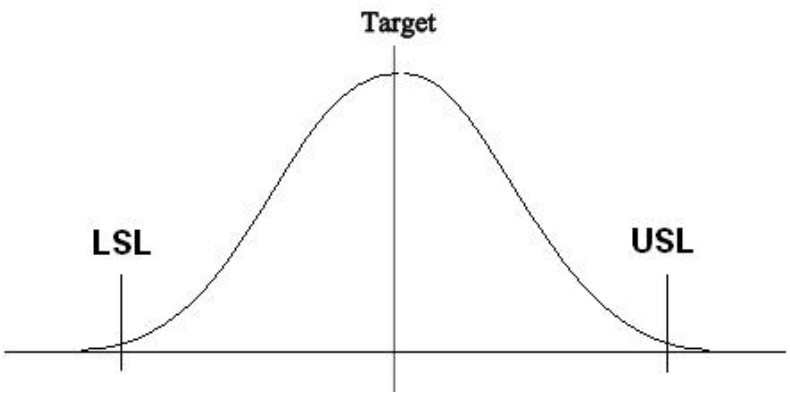

Figure 1. Specification Limit on Normal Distribution

Notice that μ can sometimes be placed with $\bar{\bar{X}}$ because the population mean is usually hard to get, so a sample mean is used instead. C_p indicates the potential of the process to perform, while C_{pk} produces realistic measurement of the ability of the process to meet target specifications. However, a high C_{pk} can give less satisfaction than a lower a C_{pk} in some cases. In figure 2, the right hand side graph of higher C_p and C_{pk} yields lower variation of the distribution yet it's far away from the target mean. The combination of C_p and C_{pk} should be used to determine the overall qualification of the result. On the other hand, if C_p and C_{pk} produces the same value, the process is perfectly centered.

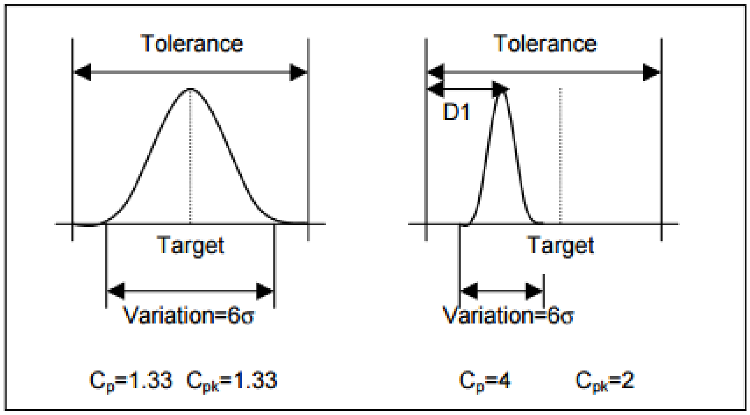

Figure 2. Cp vs. Cpk

Applications of C_p and C_{pk}

To use C_p and C_{pk} to evaluate the process ability, the following conditions must be met:

- The sample size must be large enough
- The distribution of the data is normal and produces constant variance
- The process is analyzed under statistical control

The following are some typical values of C_p and C_{pk}:

- C_{pk} = 1 implies 99.72% of the process is within the specification limits. It's usually refers to "3 sigma level"..
- C_{pk} = 1.33 refers to "4 sigma level", the minimum level of process.
- C_{pk} = 1.67 refers to "5 sigma level", which converts to 232.67 defects per million.
- C_{pk} = 2 refers to "6 sigma level", which converts to 3.4 defects per million, which is the ultimate target of any process.

The following chart produces a glance of expected defect parts per million (ppm) for six processes with C_p values ranging from 1.00 to 2.00.

Table 1. ppm with respect to Cp

Distribution	C_p	Centered	C_{pk}	Expected Defects
± 3.0 sigma	1.00	NO	0.50	66,810 ppm
± 3.0 sigma	1.00	YES	1.00	2,700 ppm
± 4.5 sigma	1.50	NO	1.00	2,700 ppm
± 4.5 sigma	1.50	YES	1.50	3.4 ppm
± 6.0 sigma	2.00	NO	1.50	3.4 ppm
± 6.0 sigma	2.00	YES	2.00	0.0002 ppm

Both C_p and C_{pk} are measurement tools for process capability. The difference between the two is an indicator of how close the average of the process is from the target specification. C_{pk} is more widely used than C_p. Notice that the C_{pk} value can never exceed C_p value.

Demonstration of the Use of C_p and C_{pk}

It's hard to grasp the full concept of the tool without applying it to a practical scenario in real life. The following is an example of the application of C_p and C_{pk}.

Scenario 1[3]:

Food served at a restaurant should be between 38°C and 49°C when it is delivered to the customer. The process used to keep the food at the correct temperature has a process standard

[3] Learn How to Calculate Cp and Cpk – Tutorial, Definition and Example

deviation of 2°C and the mean value for these temperature is 40. What is the process capability of the process?

Solution:

Given the above information, we know that:

USL = 49°C

LSL = 39°C

Standard Deviation = 2°C

Mean = 40

Then apply the formula of C_p:

$C_p = \frac{USL-LSL}{6\sigma} = \frac{49-39}{6*2} = 0.833$

The calculation of C_{pk} is broken into three steps:

Step 1: $\frac{USL-\mu}{3\sigma} = \frac{49-40}{3*2} = 1.5$

Step 2: $\frac{\mu-LSL}{3\sigma} = \frac{40-39}{3*2} = 0.166$

Step 3: $C_{pk} = \min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right] = 0.166$

From the results of C_p and C_{pk}, the minimum level of 4 sigma is not met. Considering the natural of the restaurant business, most work is processed manually instead of machine automation, a low C_p value is accepted. However, small value of C_{pk} should be further investigated since the mean temperature is barely higher than LSL.

Scenario 2 (Mesa Products):

Mesa Products produces solvents and spray cans. The company wants to analyze the process capability of the products on pressure per square inch (psi) of the cans. The sample includes 124 cans.

Solution:

After descriptive analysis of the data, the mean and standard deviation are in the following:

Mean = 30.7177

Standard Deviation = 3.0512

The visual distribution of the can PSI is in the following run chart:

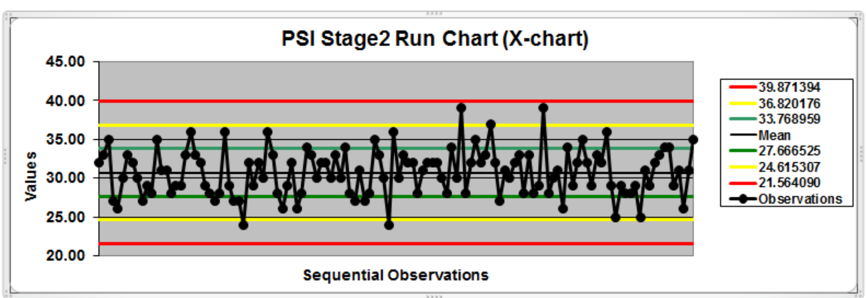

Figure 3. Mesa Product Can PSI Run Chart

Table 2. PSI Raw Data Cutoffs for Various Zones

PSI Data Stage2 Raw Score Cutoffs for Various Zones				
Distance from Mean	**Mean +/− k Standard Deviations**	**Cutoffs**		
+3 Sigma	Mean + 3 SD	39.8714		
+2 Sigma	Mean + 2 SD	36.8202		
+1 Sigma	Mean + 1 SD	33.7690		
Mean	Mean	30.7177		
−1 Sigma	Mean - 1 SD	27.6665		
−2 Sigma	Mean - 2 SD	24.6153		
−3 Sigma	Mean - 3 SD	21.5641		
Distribution of Values within Zones				
Zone		**Count**	**% of Total**	**Cumulative**
Beyond +3 Sigma	Between +3 Sigma and +∞	0	0.00	0
A	Between +2 Sigma and +3 Sigma	3	2.42	3
B	Between +1 Sigma and +2 Sigma	17	13.71	20
C	Between the Mean and +1 Sigma	44	35.48	64
C	Between the Mean and −1 Sigma	43	34.68	107
B	Between −1 Sigma and −2 Sigma	15	12.10	122
A	Between −2 Sigma and −3 Sigma	2	1.61	124
Beyond −3 Sigma	Between −3 Sigma and −∞	0	0.00	124

From the above table, we can derive the specification limits of the data:

USL = 39.8714

LSL = 21.5641

Then apply the formula of C_p:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{39.8714 - 21.5641}{6 * 3.0512} = 1.00$$

To calculate C_{pk}:

Step 1: $\frac{USL-\mu}{3\sigma} = \frac{39.8714-30.7177}{3*3.0512} = 1.00$

Step 2: $\frac{\mu-LSL}{3\sigma} = \frac{30.7177-21.5641}{3*3.0512} = 1.00$

Step 3: $C_{pk} = \min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right] = 1.00$

From the above results of C_p and C_{pk}, the process mean is perfectly centered between the target specification, and $C_p = C_{pk} = 1$ implies a 3 sigma level process.

Summary

As a crucial measurement tool of quality management, process capability has helped many organizations to measure, analyze, and improve their process performance. To better utilize the tool, appropriate assumption should be made prior to the application, and reasonable requirement should be made after the application. Therefore, process capability improvement is a life-long learning lesson involves continuous improvement. As technology improves over the years, 5 sigma level has become the standard across many industries. Although 6 sigma level is still the ideal goal for any quality level, it eventually will be achieved with consistent and continuous improvement on process capability management.

References

Shewhart, W. (1980). *Economic control of quality of manufactured product*. Milwaukee, Wis.: American Society for Quality.

Process Capability (Cp, Cpk) and Process Performance (Pp, Ppk) – What is the Difference? (n.d.). Retrieved from http://www.isixsigma.com/tools-templates/capability-indices-process-capability/process-capability-cp-cpk-and-process-performance-pp-ppk-what-difference/

6.1.6. What is Process Capability? (n.d.). Retrieved December 13, 2015. Retrieved from http://www.itl.nist.gov/div898/handbook/pmc/section1/pmc16.htm

Learn How to Calculate Cp & Cpk -Tutorial, Definition and Example. (n.d.). Retrieved from https://www.easycalculation.com/statistics/learn-cp-cpk-calculator.php

CSSBB Tutorial Series : Lesson 9. (n.d.). Retrieved from https://improveandinnovate.wordpress.com/tag/ppppk/

Cp and Cpk. (n.d.). Oracle Application. Retrieved from https://docs.oracle.com/cd/A60725_05/html/comnls/us/qa/cpcpk.htm

Larsson, A. (2002). *Capability in Machining Systems*. Stockholm.