(1) Let X and Y be the exponential lifetimes of two bulbs with average 2000 hours and 1000 hours respectively. If both bulbs are switched on at the same time, find the expected time until both die, that is, find E(max(X,Y)) with the second method sketched in class: (i) find the average time for the first one to burn; (ii) find the probability that the first one to burn is the one with average lifetime 2000 (iii) use the memoryless property to find the expectation of the time, after the first one burns, that the surviving one takes to burn (you have to consider the two possibilities that the surviving one is the 2000 or the 1000, and each of these possibilities has to be weighted with the probability found in (i)).

(i) Avg. time for one to burn:

$$P[\min(\bar{x},Y) \geq z] = \underbrace{P(\bar{x} \geq z, Y \geq z)}_{\text{independent}} = P(\bar{x} \geq z)(Y \geq z) =$$

ASIDE: $P(\bar{x} \geq z) = e^{-\lambda z}$

$$\lambda = \frac{1}{1000} \ \& \ \frac{1}{2000}$$

$$\therefore P(\bar{x} \geq z)(Y \geq z) = \left(e^{-\frac{z}{1000}}\right)\left(e^{-\frac{z}{2000}}\right)$$

$$\Rightarrow e^{-z\left(\frac{3}{2000}\right)} \Rightarrow \boxed{E(\exp(\bar{x},Y)) = 666.67 \text{ HRS}}$$

$$\frac{2000}{3} = 666.67$$

(ii) First to burn = 2000 Hr Bulb.

$$P(\bar{x} < Y) = \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{1/2000}{3/2000} = \frac{1}{3}$$

We then know the other *must* be 2/3. ✓

$$P(\bar{x} < Y) = 1 - P(x - Y) = 1 - \frac{1}{3} = \frac{2}{3}$$

(iii) Memoryless property:

$$E[\max(\bar{x},Y)] = E(\bar{x},Y) + P(\bar{x} < Y)[E(Y)] + P(Y < \bar{x})[E(\bar{x})]$$

$$= \frac{2000}{3} + \frac{1}{3}(1000) + \frac{2}{3}(2000)$$

$$\Rightarrow \frac{2000}{3} + \frac{1000}{3} + \frac{4000}{3} = \frac{7000}{3} = \boxed{2333.33 \text{ HRS}}$$

(2) Question (2) had some strange characters inserted. It should read thus: Let X, Y, Z be independent exponential lifetimes of bulbs with average lifetimes 3000, 2000 and 1000, respectively. Switch them on at the same time. Find the probability that the first to burn is the 3000 model. (Hint: you are looking for P(X<min(Y,Z)) and you know that min(Y,Z) is also exponential and also you know how to find P(X<W) for two independent exponentials X, W).

We know from the previous problem, the average time for two bulbs was 666.67 hrs. Using this, we can solve for the third bulb w. LT: 3000hrs.

$\lambda_1 = \frac{1}{666.67}$ & $\lambda_2 = \frac{1}{3000}$

$\Rightarrow \frac{1/666.67}{1/666.67 + 1/3000} \cong \boxed{.82}$

exact is: .8181810744