

## Homework #3

2.3-7 (pg 39)

"Describe a  $\Theta(n \lg n)$ -time algorithm that, given a set  $S$  of  $n$  integers and another integer  $x$ , determines whether or not there exist two elements in  $S$  whose sum is exactly  $x$ ."

\*NOTE: when seeing  $\Theta(n \lg n)$ , think of "divide-and-conquer" algorithms.

First, we want to sort the set  $S$  using MERGE-SORT to get it ascending in order. By using for loops that run " $n$ " times, we can apply a BINARY search to every  $S[i]$ . We have a worse case of  $\Theta(n \lg n)$  for the MERGE and  $\Theta(\lg n)$  for BINARY. Finally, because it runs  $n$  times, the total time is  $n * \Theta(\lg n) = \Theta(n \lg n)$ .

### 3 Horner's Rule

$$P(x) = \sum_{k=0}^n a_k x^k = a_0 + x(a_1 + x(a_2 + \dots + x(a_{n-1} + x a_n) \dots))$$

a) "In terms of  $\theta$ -notation, what is the asymptotic running time of this code?"

The algorithm runs in a loop for all elements,  $n$ , and the time taken,  $\theta$ .  
Therefore, the run time is  $\boxed{\theta n}$

b) NAIVE-POLY-EVAL( $A, x$ ) //  $A$  is an array

$y = 0$

for  $i = 1$  to  $A.length$

$m = 1$

for  $j = 1$  to  $i - 1$

$m = m * x$

$y = y + A[i] * m$

Here, we have two for loops, each running at  $n$  times. Therefore, we'd have  $\theta n^2$ , which is slower than Horner's Rule.