[redacted]

10 2017
ECE 340

HW07

(1) 3.11 Let X and Y be normal random variables w. means 0 and 1, and variances 1 and 4

(a) Find $P(X \leq 1.5)$ and $P(X \leq -1)$

$$P(X \leq 1.5) = P\left(Z \leq \frac{1.5-0}{1}\right) \Rightarrow P(Z \leq 1.5)$$

$$= \boxed{0.9332}$$

$$P(X \leq -1) = P(X \geq 1) \Rightarrow 1 - P(Z \leq 1) = 1 - P\left(Z \leq \frac{1-0}{1}\right)$$

$$1 - P(Z \leq 1) = 1 - 0.8413 = \boxed{0.1587}$$

(b) Find the PDF of $(Y-1)/2 \Rightarrow$ Standard Normal

$\therefore 1 = \mu,\ 2 = \sigma \Rightarrow 4 = \sigma^2$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \Rightarrow \boxed{\frac{1}{2\sqrt{2\pi}} * e^{-\left(\frac{(y-1)^2}{8}\right)}}$$

(c) Find $P(-1 \leq Y \leq 1) = P\left(-1 \leq \frac{Y-1}{2} \leq 0\right)$

- $P(-1 \leq Z \leq 0) = P(0 \leq Z \leq 1)$

$= P(Z \leq 1) - P(Z \leq 0) \Rightarrow .8413 - 0.5$ (from table)

$\therefore \boxed{= 0.3413}$

(2) 3.12: Let X be a normal random variable with zero mean and stnd dev σ. Use the normal tables to compute the events $\{X \geq k\sigma\}$ and $\{|X| \leq k\sigma\}$ for $k = 1, 2, 3$

$\Phi(1) = 0.8413$
$\Phi(2) = 0.9772$
$\Phi(3) = 0.9986$

$k=1 \Rightarrow X \geq \sigma \Rightarrow 1 - 0.8413 = \boxed{0.1587}$

$k=2 \Rightarrow X \geq 2\sigma \Rightarrow 1 - 0.9772 = \boxed{0.0228}$

$k=3 \Rightarrow X \geq 3\sigma \Rightarrow 1 - 0.9986 = \boxed{0.0014}$