[redacted]

ECE331
0$0718

HW07

(1) 4.3-1 : "Show that the solution of $T(n)=T(n-1)+n$ is $O(n^2)$.

Proof by Induction.

Assume $T(m) \le cm^2$, prove $T(n) \le cn^2$

$T(n) = T(n-1) + n \le c(n-1)^2 + n \le cn^2$

$c(n-1)^2 + n = c(n-1)(n-1) + n$

$= c(n^2 - 2n + 1) + n = cn^2 - 2cn + c + n$

$= cn^2 - 2cn + n + c \le cn^2$

$\Rightarrow -2cn + n + c \le 0$

$\Rightarrow n \le 2cn - c$

$\Rightarrow n \le c(2n-1)$

$$\boxed{c \geq \frac{n}{2n-1}}$$

If $c=1$, for any n, $T(n) \le cn^2$ ✓

$1 \geq \frac{n}{2n-1} \Rightarrow 2n-1 \geq n$

$n \geq 1$ ✓

② 4.3-7: $T(n) = 4T(\frac{n}{3}) + n$ is $\Theta(n^{\log_3 4})$

Assume $T(m) \le cm^{\log_3 4}$ fails, so try $T(m) \le cm^{\log_3 4} - dm$ and prove $T(n) \le cn^{\log_3 4} - dn$.

* To show that $T(m) \le cm^{\log_3 4}$ does fail, we try $T(n) \le cn^{\log_3 4}$

$$T(\tfrac{n}{3}) \le c(\tfrac{n}{3})^{\log_3 4} = \frac{cn^{\log_3 4}}{4}$$

$$T(n) = \cancel{4}\frac{cn^{\log_3 4}}{\cancel{4}} + n = cn^{\log_3 4} + n \quad \text{FAILS.}$$

Therefore, we try "$-dn$" b/c $n < n^{\log_3 4}$

$$T(\tfrac{n}{3}) \le c(\tfrac{n}{3})^{\log_3 4} - d(\tfrac{n}{3}) = c\left(\frac{n^{\log_3 4}}{\cancel{3^{\log_3 4}}\,4}\right) - d\left(\frac{n}{3}\right)$$

$$T(n) = 4\left[\frac{cn^{\log_3 4}}{4} - d\left(\frac{n}{3}\right)\right] + n = cn^{\log_3 4} - \frac{4dn}{3} + n \le$$

$$\le cn^{\log_3 4} - dn$$

$$-\frac{4dn}{3} + n \le -dn \Rightarrow -\frac{4dn}{3} + dn + n \le 0$$

$$\cancel{n}\left(-\frac{4d}{3} + d + 1\right) \le 0 \Rightarrow -\frac{d}{3} + 1 \le 0$$

$$-\frac{d}{3} \le -1 \quad \therefore \boxed{d \ge 3}$$