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ECE331

# Homework #4

## 3-3. Ordering by asymptotic growth rates

ORIGINAL ORDER:

$$\begin{array}{cccc}
 \frac{\lg(\lg^* n)}{(\frac{3}{2})^n} & \frac{2^{\lg^* n}}{n^3} & (\sqrt{2})^{\lg n} & \frac{n^2}{\lg(n)!} \\
 \ln \ln n & \lg^* n & \lg^2 n & \frac{n!}{2^{2n}} \\
 \frac{2^{\lg n}}{2^{\lg n}} & \frac{(\lg n)^{\lg n}}{2^{\sqrt{2} \lg n}} & \frac{n \cdot 2^n}{e^n} & \frac{(1 \lg n)!}{n! / \lg n} \\
 \lg^*(\lg n) & & n & \frac{\sqrt{\lg n}}{2^{2n+1}}
 \end{array}$$

Since there are a lot of comparisons, we'd use MERGESORT instead of insertion

Identities:

$$\begin{aligned}
 n \lg \lg n &= (\lg n)^{\lg n} \\
 n^2 &= 4^{\lg n} \\
 n &= 2^{\lg n} \\
 \lg(n!) &= \Theta(n \lg n)
 \end{aligned}$$

$$\begin{aligned}
 2^{\sqrt{2} \lg n} &= n^{\sqrt{2}} \\
 1 &= n^{1/\lg n} \\
 \lg^*(\lg n) &= \lg^* n - 1
 \end{aligned}$$

### STEP ①: GROUP

- Exponentials:  $2^{2n}$ ,  $2^{2n+1}$ ,  $2^n$ ,  $n \cdot 2^n$ ,  $(\frac{3}{2})^n$ ,  $e^n$
- Linear:  $n!$ ,  $1$ ,  $n$ ,  $2^{\lg n}$ ,  $2^{\sqrt{2} \lg n}$ ,  $4^{\lg n}$ ,  $\sqrt{2}^{\lg n}$
- Poly:  $n^2$ ,  $n^3$ ,  $(n+1)!$ ,  $\sqrt{\lg n}$ ,  $n^{1/\lg n}$ ,  $\ln n$ ,  $n \lg \lg n$
- Log:  $\lg(\lg^* n)$ ,  $\lg n$ ,  $\ln \ln n$ ,  $n \lg n$ ,  $\lg \lg n$ ,  $(\lg n)!$ ,  $\lg^2 n$ ,  $\lg(n!)$ ,  $\lg n^{\lg n}$

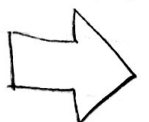
### STEP ②: ORDER BY LINE

Expos:  $2^{2n+1} > 2^{2n} > e^n > n^{2^n} > 2^n > (\frac{3}{2})^n$

LINEAR:  $(n+1)! > n! > 4^{\lg n} > (2^{\lg n} = n)^{\sqrt{2}} > (\sqrt{2})^{\lg n} > 2^{\sqrt{2} \lg n}$

POLY:  $n^3 > n^2$

LOG:  $n \lg \lg n = \lg n^{\lg n} > (\lg n)! > n \lg n = \lg(n!) > \ln \ln n > \lg n = \lg(\lg n) > \lg^2 n > \ln n > \sqrt{\lg n} > \lg(\lg^* n) > n^{1/\lg n}$



STEP ③: ORDER GROUPS TOGETHER

$$2^{2^{n+1}} > 2^{2^n} > (n+1)! > n! > e^n >$$

$$n \cdot 2^n > 2^n > \left(\frac{3}{2}\right)^n > n^{\lg \lg n} = (\lg n)^{\lg n} > (\lg n)! >$$

$$n^3 > n^2 = 4^{\lg n} > n \lg n = \lg(n!) > n = 2^{\lg n} > (\sqrt{2})^{\lg n} >$$

$$2^{\sqrt{2 \lg n}} > \lg^2 n > \ln n > \sqrt{\lg n} > \ln \ln(n)$$

$$2^{\lg^* n} > \lg^*(\lg n) = \lg^* n > \lg(\lg^* n) > 1 = n^{1/\lg n}$$

\*ASIDE: Solved for half of these in class with professor.