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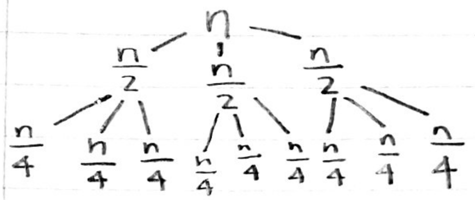
ECE 331

# HW08

4.4-1: $T(n) = 3T(\lfloor \frac{n}{2} \rfloor) + n$

Three Children



$\frac{n}{2^i} = 1 ; n = 2^i$

$\lg n = i$ [height]

$3^i = 3^{\lg n} = \boxed{n^{\lg 3}}$

| level | nodes |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| ⋮ | ⋮ |
| i | $3^i$ |

Fringe Cost: $n^{\lg 3} \times T(1) = \Theta n^{\lg 3}$

Internal Cost: $cn + \frac{3}{2}n + (\frac{3}{2})^2 n + (\frac{3}{2})^{h-1} n$

$\approx n(\frac{3}{2})^{\lg n} = \boxed{n^{\lg 3}}$

PROVE $T(n) = O(n^{\lg 3})$

$T(m) \leq cm^{\lg 3}$ for $m < n$ ; prove $T(n) \leq cn^{\lg 3}$

$T(n) = 3T(\frac{n}{2}) + n \leq 3c(\frac{n}{2})^{\lg 3} + n$

$= \frac{\cancel{3}cn^{\lg 3}}{\cancel{2^{\lg 3}}} + n = cn^{\lg 3} + n$ — FAIL

$T(m) \leq cm^{\lg 3} - dm$ ; $m < n$

$T(n) \leq cn^{\lg 3} - dn \Rightarrow T(n) = 3T(\frac{n}{2}) + n \leq 3c(\frac{n}{2})^{\lg 3} - d(\frac{n}{2}) + n$

$= \cancel{\frac{3cn^{\lg 3}}{2^{\lg 3}}} - \frac{dn}{2} + n \leq \cancel{cn^{\lg 3}} - dn$

$-\frac{dn}{2} + n + dn \leq 0$

$\Rightarrow \frac{dn}{2} + n \leq 0 \Rightarrow n(\frac{d}{2} + 1) \leq 0$

$\frac{d}{2} \leq -1 \Rightarrow \boxed{d \leq -2}$

4.4-2: $T(n) = T\left(\frac{n}{2}\right) + n^2$

$$
\begin{array}{ccc}
n^2 & & n^2 \\
| & = & | \\
\left(\frac{n}{2}\right)^2 & & \frac{n^2}{4} \\
| & & | \\
\left(\frac{n}{4}\right)^2 & & \frac{n^2}{16} \\
\vdots & & \vdots
\end{array}
$$

$\frac{n^2}{4^k} = 1$ ; $n^2 = 4^k$

$n = \sqrt{4^k}$

$n = 2^k$

$\lg n = k$

NODE $\rightarrow 1^k = 1^{\lg n} = 1^{\lg 1} = \boxed{1}$

Fringe Cost: $1 \times T(1) = \Theta(1)$

Internal Cost:

$1 + r + r^2 + \ldots + r^k = \frac{r^{k+1} - 1}{r - 1}$ if $|r| < 1$

$\lim_{k \to \infty} (1 + r + \ldots r^k) = \lim_{k \to \infty} = \frac{1}{1-r}$

$\sum_{k=0}^{\infty} r^k \quad r = \frac{1}{4}$ ; $\frac{1}{1 - \frac{1}{4}} = \frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$

∴ Because we have a finite,

$T(n) = O(n^2)$ // Upper Bound

With Substitution:

$T(n) \leq dn^2$

$T\left(\frac{n}{2}\right) + n^2 = d\left(\frac{n}{2}\right)^2 + n^2$

$\frac{dn^2}{4} + n^2 \leq dn^2$

$\frac{dn^2}{4} + n^2 - dn^2 \leq 0$

$\cancel{n^2}\left(\frac{d}{4} + 1 - d\right) \leq 0 \Rightarrow -\frac{3d}{4} \leq -1$

$\boxed{d \geq \frac{4}{3}}$ ✓

4.5-1

Use the Master Theorum to give tight bounds for

a) $T(n) = 2T(\frac{n}{4}) + 1$

$a = 2$, $b = 4$, $f(n) = 1$

$\log_b a = \log_4 2 = \frac{1}{2}$

$n^{\log_4 2} = n^{1/2}$ & $f(n) = 1 \Rightarrow n^0$

$\therefore n^{\log_b a} > f(n)$ ; $O(n^{1/2 - 1/2})$ $\varepsilon = 0.5$

CASE 1: If $O(n^{\log_b a - \varepsilon}) = f(n)$ ✓

then $T(n) = \Theta(n^{\log_b a})$

$\boxed{\therefore T(n) = \Theta(\sqrt{n})}$

b) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

$a = 2, b = 4, f(n) = \sqrt{n}$

NOW, case 2: If $f(n) = \Theta(n^{\log_b a})$; $n^{\log_4 2} = \sqrt{n}$ ✓

then $T(n) = \Theta(n^{\log_b a} \lg n)$

$\Rightarrow \boxed{T(n) = \Theta(\sqrt{n} \lg n)}$

c) $T(n) = 2T(\frac{n}{4}) + n$ $n^{1/2} + n^{1/2}$

$a = 2$, $b = 4$, $f(n) = n$

case 3: If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ $\varepsilon = \frac{1}{2}$

then $T(n) = \Theta(f(n))$

$n^{\log_4 2 + \frac{1}{2}} = n^{\frac{1}{2} + \frac{1}{2}} = n = f(n)$

$\therefore \boxed{T(n) = \Theta(n)}$

d) $T(n) = 2T(\frac{n}{4}) + n^2$

$a = 2$, $b = 4$, $f(n) = n^2$

case 3: $f(n) = \Omega(n^{\log_b + \varepsilon})$ $\varepsilon = 1.5$

$n^{\log_4 2 + 3/2} = n^{1/2 + 3/2} = n^2$ ✓ $= f(n)$

$\therefore \boxed{T(n) = \Theta(n^2)}$