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ECE331

Homework #4

3-3. Ordering by asymptotic growth rates

ORIGINAL ORDER:

$\cancel{\lg(\lg^* n)}$ $\cancel{2^{\lg^* n}}$ $\cancel{(\sqrt{2})^{\lg n}}$ $\cancel{n^2}$ $\cancel{n!}$ $\cancel{(\lg n)!}$

$\cancel{(\frac{3}{2})^n}$ $\cancel{n^3}$ $\lg^2 n$ $\cancel{\lg(n!)}$ $\cancel{2^{2^n}}$ $n^{1/\lg n}$

$\cancel{\ln \ln n}$ $\lg^* n$ $\cancel{n \cdot 2^n}$ $\cancel{n^{\lg \lg n}}$ $\cancel{\ln n}$ $\cancel{1}$

$\cancel{2^{\lg n}}$ $\cancel{(\lg n)^{\lg n}}$ $\cancel{e^n}$ $\cancel{4^{\lg n}}$ $\cancel{(n+1)!}$ $\cancel{\sqrt{\lg n}}$

$\lg^*(\lg n)$ $\cancel{2^{\sqrt{2 \lg n}}}$ $\cancel{n}$ $\cancel{2^n}$ $\cancel{n \lg n}$ $\cancel{2^{2^{n+1}}}$

Since there are a lot of comparisons, we'd use MERGESORT instead of insertion

Identities: $n^{\lg \lg n} = (\lg n)^{\lg n}$ $\quad 2^{\sqrt{2 \lg n}} = n^{\sqrt{2/\lg n}}$

$n^2 = 4^{\lg n}$ $\quad 1 = n^{1/\lg n}$

$n = 2^{\lg n}$ $\quad \lg^*(\lg n) = \lg^* n - 1$

$\lg(n!) = \Theta(n \lg n)$

STEP ① : GROUP

- Exponentials ; 2^{2^n}, $2^{2^{n+1}}$, 2^n, $n2^n$, $(\frac{3}{2})^n$, e^n
- Linear ; $n!$, 1, n, $\cancel{2^{\lg n}}$, $2^{\sqrt{2 \lg n}}$, $\cancel{4^{\lg n}}$, $\cancel{\sqrt{2}^{\lg n}}$
- Poly ; n^2, n^3, $(n+1)!$
- Log ; $\lg(\lg^* n)$, $\sqrt{\lg n}$, $n^{1/\lg n}$, $\ln n$, $n^{\lg \lg n}$, $\lg n$, $\ln \ln n$, $n \lg n$, $\lg \lg n$, $(\lg n)!$, $\lg^2 n$, $\lg(n!)$, $\lg n^{\lg n}$

STEP ② ; ORDER BY LINE

Expos ; $2^{2^{n+1}} > 2^{2^n} > e^n > n2^n > 2^n > (\frac{3}{2})^n$

LINEAR ; $(n+1)! > n! > 4^{\lg n} > (2^{\lg n} = n) > (\sqrt{2})^{\lg n} > 2^{\sqrt{2 \lg n}}$

POLY ; $n^3 > n^2$

LOG ; $n^{\lg \lg n} = \lg n^{\lg n} > (\lg n)! > n \lg n = \lg(n!) > \ln \ln n > \lg n = \lg^*(\lg n) >$

$\lg^2 n > \ln n > \sqrt{\lg n} > n^{1/\lg n}$

$\lg(\lg^* n) > n^{1/\lg n}$

STEP (3): ORDER GROUPS TOGETHER

$2^{2^{n+1}} > 2^{2^n} > (n+1)! > n! > e^n >$

$n*2^n > 2^n > \left(\frac{3}{2}\right)^n > n^{\lg\lg n} = (\lg n)^{\lg n} > (\lg n)! >$

$n^3 > n^2 = 4^{\lg n} > n\lg n = \lg(n!) > n = 2^{\lg n} > (\sqrt{2})^{\lg n} >$

$2^{\sqrt{2\lg n}} > \lg^2 n > \ln n > \sqrt{\lg n} > \ln\ln(n)$

$2^{\lg^* n} > \lg^*(\lg n) = \lg^* n > \lg(\lg^* n) > 1 = n^{1/\lg n}$

*ASIDE: Solved for half of these in class with professor.