

ECE 340
110317

● HOMEWORK 09

Distributions of the sum of two ind. normals:

Distribution of two ind. Poissons:

1) Student A:

MATH \rightarrow Mean 1000pts & std dev. 100pts

LANG \rightarrow Mean 500pts & std dev. 10pts.

Student B:

MATH \rightarrow Mean 1,200pts & std dev. 20pts

LANG \rightarrow Mean 400pts & std dev. 20pts

★ The college requires above a 1650 in an entrance test (which is the sum of the two ind. normals). ★

Which student is more likely to get in?

$$\begin{array}{l} \text{A } \left\{ \begin{array}{l} \mu = 1,000, \sigma = 100 \\ \mu = 500, \sigma = 10 \end{array} \right. \quad \begin{array}{l} \sigma^2 = 10000 \\ \sigma^2 = 100 \end{array} > \sqrt{10100} \approx 100.5 \\ \text{B } \left\{ \begin{array}{l} \mu = 1200, \sigma = 20 \\ \mu = 400, \sigma = 20 \end{array} \right. \quad \begin{array}{l} \sigma^2 = 400 \\ \sigma^2 = 400 \end{array} > \sqrt{800} \approx 28.2 \end{array}$$

$$\text{A: } P(X \geq 1650) = P\left(Z \geq \frac{1650 - 1500}{100.5}\right) = 1 - P\left(Z \leq \frac{1650 - 1500}{100.5}\right)$$

$$\Rightarrow 1 - P(Z \leq 1.49) = 1 - .9319 = \boxed{0.0681}$$

$$P(Y \geq 1650) = P\left(Z \geq \frac{1650 - 1600}{28.2}\right) = 1 - P\left(Z \leq \frac{1650 - 1600}{28.2}\right)$$

$$\Rightarrow 1 - P(Z \leq 1.77) = 1 - .9616 = \boxed{0.0384}$$

Student A is more likely.

$$2) P(X+Y=k) = \sum_{i=0}^k P(X+Y=k, X=i) = \sum_{i=0}^k P(Y=k-i, X=i) \\ = \sum_{i=0}^k P(Y=k-i, X=i)$$

$$X+Y \sim P(n+\delta)$$

$$\text{So } n=2 \text{ \& } \delta=1 \quad k \geq 2$$

$$P(X+Y \geq 2) = 1 - P(X+Y < 2) = 1 - P(X+Y=0) - P(X+Y=1) \\ = 1 - \frac{(2+1)^0}{0!} e^{-(2+1)} - \frac{(2+1)^1}{1!} e^{-(2+1)} = 1 - e^{-3} - 3e^{-3}$$

$$= 1 - 4e^{-3} = \boxed{0.8008}$$