

5) Density: $f(t) = \lambda e^{-\lambda t}$ what is $f_{x+y}(t)$?

$$\begin{aligned} f_{x+y}(z) &= \int_0^z f_x(x) \cdot f_y(z-x) dx \\ &= \int_0^z \lambda e^{-\lambda x} \cdot \lambda e^{-\lambda(z-x)} dx \\ &= \lambda^2 e^{-\lambda z} \int_0^z e^0 dx \Rightarrow \lambda^2 e^{-\lambda z} \int_0^z dx \\ &= \lambda^2 e^{-\lambda z} \Big|_0^z = \boxed{z \lambda^2 e^{-\lambda z}} \end{aligned}$$