[redacted]

ECE 340
112717

HW11: PROBLEM #1, §5.1, #4, §5.2

1) CHEBYSHEV'S INEQUALITY

ASIDE: If $\bar{X}$ is a random var. w/ mean μ & variance σ^2;
$P(|\bar{X}-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$, for all $c > 0$.

A statistician wants to estimate the mean height h of a population, based on n independent samples $\bar{X}_1, \ldots \bar{X}_n$ chosen uniformly from the entire population.
$M_n = (\bar{X}_1 + \ldots + \bar{X}_n)/n$ & $1.0 \sim \sigma$

(a) How large should n be so that std. dev is at *most* 1cm?

If $M_n = \frac{(\bar{X}_1 + \ldots + \bar{X}_n)}{n}$, then $\sigma_{M_n} = \frac{1}{\sqrt{n}}$

$\boxed{\sigma_{M_n} \leq 0.01 \text{, so AT most, } n \geq 10{,}000}$

(b) How large should n be so Cheb's guarantees that the estimate is within 5cm from h, w. prob. at least 0.99?

$P(|M_n - \overset{\mu}{h}| \leq 0.05) \geq 0.99$
$h = E[M_n]$ $(\mu = E[X])$ ✓
$\sigma^2_{M_n} = \frac{1}{n}$

$\Rightarrow P(|M_n - E[M_n]| \leq 0.05) = 1 - P(|M_n - E[M_n]| \geq 0.05)$
$= 1 - \frac{(\frac{1}{n})}{(0.05)^2}$

$\Rightarrow 1 - \frac{(1/n)}{(0.05)^2} \geq 0.99 \Rightarrow 1 - 0.99 \geq \frac{(1/n)}{(0.05)^2}$

$(1-.99)(0.05)^2 \geq \frac{1}{n} \Rightarrow n \geq \frac{1}{(1-0.99)(0.05)^2} = \boxed{40{,}000}$

(c) $\sigma^2 \leq (b-a)/4$
$\leq (2 - .4)^2/4 \Rightarrow \leq (0.6)^2/4$, $\sigma^2 \leq .09$
$\sigma_{M_n} = 0.3/\sqrt{n}$ & $\sigma_{M_n} \leq 0.01$
$\therefore$ REVISED $\Rightarrow \boxed{n \geq 900}$

$$1 - \frac{(0.09/n)}{(0.05)^2} \geq 0.99 \quad , \quad \frac{0.09/n}{(0.05)^2} \geq 0.99 - 1$$

$$n \geq \frac{0.09}{(1-0.99)(0.05)^2} = \boxed{3600}$$

4) WEAK LAW of LARGE NUMBERS (WLLN)

ASIDE: $P(|M_n - \mu| \geq \epsilon) = P\left(\left|\frac{X_1 + \ldots X_n}{n} - \mu\right| \geq \epsilon\right) \to 0$ as $n \to \infty$

$M_n = S_n/n \quad P(|M_n - f| \geq \epsilon) \leq \delta$

$\delta = 1/4n\epsilon^2$

$\Rightarrow P(|M_n - f| \geq \epsilon) \leq \frac{1}{4n\epsilon^2}$

a) if half of its original (ϵ)

$\Rightarrow P(|M_n - f| \geq \frac{\epsilon}{2}) \leq 1/4n\epsilon^2$

To keep bound $\left(\frac{1}{4n\epsilon^2}\right)$ constant

$\boxed{\text{n needs to be 4 times LARGER.}}$

b) if half its original (δ)

$\frac{\frac{1}{4}}{2} = \frac{1}{4} \times \frac{1}{2}$

$\Rightarrow P(|M_n - f| \geq \epsilon) \leq 1/8n\epsilon^2$

$\boxed{\therefore \text{The sample size would need to be DOUBLED.}}$