Orbital Simulations of the Haumea Family

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ABSTRACT

The Haumea collisional family is a group of trans-Neptunian objects with similar orbital characteristics, and/or water-ice, and thought to have originated from the dwarf planet Haumea (2003 EL_{61}). In this report, we created 8 hypothetical clones for each of the 18 objects within their accepted uncertainties of their orbital parameters and conducted simulations over 4.5 Gyrs to analyse the orbital evolution of these objects. We find that the Haumea family are largely a stable group that did not show any signs of orbital variation. Clones of four objects, 2005 CB_{79}, Haumea (2003 EL_{61}), 2003 HZ_{199} and 2003 UZ_{117} were found to escape the Solar system. Haumea (2003 EL_{61}) was found with certainty to escape via a close encounter with Neptune after approximately 1.25 Gyrs. All clones of 2005 CB_{79} were found to escape, but the time and exact cause is unknown and requires further study. Clones of 2003 HZ_{199} and 2003 UZ_{117} both had clones that escaped and remained in a stable orbit. Larger simulations of these two objects would be required to determine if they escape the Solar system or remain in stable orbit.

Key words: Simulations – Haumea Family

1 INTRODUCTION

Haumea (2003 EL_{61}) was discovered by Brown et al. (2005) along with its first satellite in the Edgeworth-Kuiper Belt (EKB) which was calculated to have a mass of approximately 28.6% of that of the Pluto-Charon system, a density of $\approx 2.6 gcm^{-3}$ and a rotation period of $\sim 4hr$ (Rabinowitz et al. 2005). A second satellite of Haumea was found in 2006 (Brown et al. 2006; Barkume et al. 2006), suggesting that both of the satellites formed due to an impact with Haumea. Near-infrared reflectance spectroscopy of the satellites in the EKB showed six (including Haumea and its two satellites) non-methane objects with strong water ice absorption features similar to Haumea (Brown et al. 2007).

The five objects that carry features similar Haumea were 1995 SM_{55}, 1996 TO_{66}, 2002 TX_{300}, 2003 OP_{32} and 2005 RR_{43} (Brown et al. 2007). Brown et al. (2007) also observed that these five Kuiper Belt Objects (KBO), along with their similar features, have a small dynamical range in which they orbit within the EKB. These observations suggest that the five KBOs were the result of ejected debris from the mantle of proto-Haumea after a collision with another object, creating a collisional family (Brown et al. 2007). Simulations (Brown et al. 2007) show that the five KBOs appear to be captured in the 12:7 mean-motion resonance with Neptune (except TO_{66} which exhibits a 19:11 resonance) that Haumea also occupies. It strongly suggests that Haumea was initially part of a tight cluster of objects (Brown et al. 2007) and that the impact on Haumea caused it to be captured in the 12:7 resonance along with the debris ejected from its mantle and increasing their eccentricity.

Two more family members, 2003 UZ_{117} and 1999 OY_3 (exhibits a 7:4 resonance), were confirmed by Ragozzine & Brown (2007) due to their similar characteristics to six confirmed members based on observational and dynamical evidence. 2005 CB_{79} was added to the Haumea family due to similar dynamics and comparable high fractions of pure ice water on its surface (Schaller & Brown 2008) compared to Haumea. Snodgrass et al. (2010) further confirmed 2005 CB_{79} and added 2003 SQ_{317} due to water ice detections by NIR spectroscopy. Trujillo et al. (2011) added 2009 YE_7 as a family member due to high fractions of water ice and orbital properties similar to the other family members. The 11 family members have a faster rotation with respect to other trans-Neptunian objects (TNOs) (Thirouin et al. 2016) and observations show that the smaller members rotate slower than the larger members, concluding that proto-Haumea might have been a fast rotator. Proudfoot & Ragozzine (2019) concluded that seven new dynamically confirmed family members were added to the Haumea family as a result of high probabilities of likely membership from simulations. The seven family members were 2005 UQ_{513}, 2010 VK_{201}, 2015 AJ_{281}, 2008 AP_{129}, 2014 LO_{28}, 2014 HZ_{199}, and 2014 QW_{441} (Proudfoot & Ragozzine 2019).

All of the objects in the Haumea family can be identified due to water ice on their surfaces and similar orbital properties of i $\approx$ 22° - 31°, a $\approx$ $40 - 47AU,\ and\ e \approx 0.06 - 0.2$ (Ragozzine & Brown 2007; Trujillo et al. 2011; Vilenius et al. 2019).

The discovery and confirmation of the two members, 2003 UZ_{117} and 1999 OY_3, concluded that the age of the Haumea family to be around $3.5 \pm 2\,Gyr(1\,\sigma)$ (Ragozzine & Brown 2007), showing that the family is ancient and likely primordial, and that the formation of the family shows consistency with formation at the beginning of the Solar system. However, Levison et al. (2008) suggested that the formation of the family was the result of the collision of two scattered-disk objects (SDOs) with highly eccentric and unstable orbits from N-body simulations. This increases the probability of collision as there was a larger population of objects in the scattered

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disk in the early Solar system and the dynamically hot orbits of the two SDOs caused the family members to occupy stable orbits due to the damping of the orbital energy on collision (Levison et al. 2008). Furthermore, Schlichting & Sari (2009) and Ćuk et al. (2013) proposed that, instead of the two previous postulations, the Haumea family formed under different circumstances. Haumea was subjected to a giant impact which ejected debris to form a single satellite, large enough to undergo tidal evolution which increased its semi-major axis (Schlichting & Sari 2009; Ćuk et al. 2013) which collides with an unbound KBO ejecting debris and creating the family members with a velocity dispersion ~ $190 ms^{-1}$ in agreement with observed velocity dispersion ~ $140 ms^{-1}$. To remain a feasible suggestion, Schlichting & Sari (2009) states that the creation of the family members had to have occurred after the dynamical excitation of the EKB to establish the dynamical coherence of the family members.

Early N-body simulations by Horner et al. (2004) analysed the changing flux of 32 centaurs that occupy the four major planets. EKB objects and TNOs occupy their orbits for a brief amount of time before being transferred to a different orbital class (Horner et al. 2004). The high collision rate of objects greater than 4m in diameter and disruption of 1km diameter objects is ~ 1 Gyr imply that a high collision rate within the EKB suggests that a new Centaur once every ~ 125 yr seems reasonable to conserve the steady-state of the population (Horner et al. 2004). Simulations regarding the Haumea family indicate that satellites and collisions fragments can occur due to a graze and merge collision between almost equal mass objects (Leinhardt et al. 2010) rather than a catastrophic collision event. (Leinhardt et al. 2010) through simulations show that the properties of Haumea, its family members and satellites can occur due to the graze and merge collision, which is further confirmed by the Outer Solar System Origins Survey (OSSOS) (Pike et al. 2019). Further simulations by Lykawka et al. (2012) show that massive TNOs and family members, and their gravitational influence is negligible on the evolution of the Haumea family, but increased the amount of particles that were ejected by 2 – 4% due to their influence. It was found that larger fragments of a collision stay close to the location of that impact, matching the observational data of the Haumea family members (Lykawka et al. 2012) and that the fragments of the collision were dispersed widely enough to cover the entire region that the Haumea family members occupied. Lykawka et al. (2012) stated that collisions such as those that formed the Haumea family were common rather than rare in the outer Solar system during the beginning of the Haumea family. Observational constraints on the family formation consist of a group of clustered objects with similar spectral signatures, velocity dispersion and total mass lower than expected but consistent size distribution in regards to the collision, and an elongated, fast rotating parent object(s) (Carry et al. 2012). Due to Haumeas fast rotation, it was proposed that rotational fission might have caused the creation of the Haumea family (Ortiz et al. 2012). Ortiz et al. (2012) created four sets of simulations, all consisting of rotational fission, under different conditions. Simulations provided evidence that rotational fission was more probable to occur than high-energy collision scenarios and could explain the formation of Haumeas satellites and the velocity dispersion of the family members (Ortiz et al. 2012).

Revision on the literature by Campo Bagatin et al. (2016) found that Ragozzine & Brown (2009) justified the coherence of the family members and not for Haumea, suggesting that Haumea and the formation of its family members may be independent of each other. An collisional event between two objects in the scenarios covered would have a probability of $0.1 - 0.3 \times 10^{-3}$ during that late heavy bombardment (LHB) phase and smaller than 0.15×10^{-4} post LHB (Campo

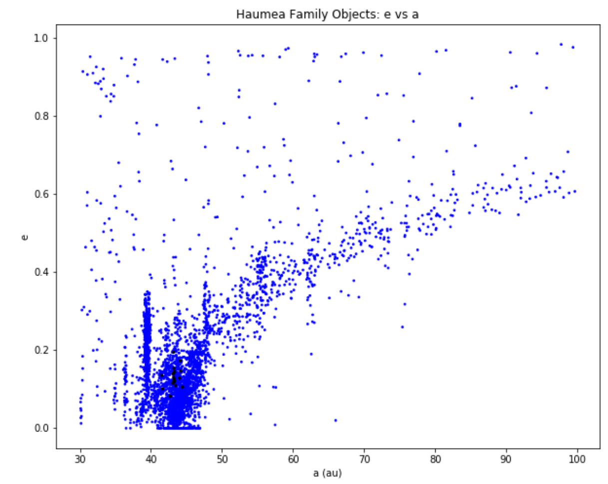

Figure 1. Semi-major axis vs eccentricity of all KBOs with a<=100au and the 18 Haumea family objects in black

Bagatin et al. 2016). Campo Bagatin et al. (2016) dismissed all formation scenarios at the time due to low probability of past and present day values with the exception of rotational fission (Ortiz et al. 2012) during the LHB phase as it presents a more probable scenario. Either current understanding of the EKB undergoes revision or the formation of the Haumea system was a low-probability outcome (Campo Bagatin et al. 2016). Simulations by Proudfoot & Ragozzine (2019) further dismissed and confirmed that all current formation hypothesis of the Haumea are unlikely, including rotational fission. A new hypothesis called the "tip of the iceberg" (Proudfoot & Ragozzine 2019) was postulated, however, the hypothesis was not considered seriously with more investigation, simulations, and geophysical and collisional modelling needed to support it.

Simulations by Volk & Malhotra (2012) generated 800 test particles within the uncertainties of the calculated orbital parameters for the Haumea family orbit and integrated them over 1 Gyr. It was found that ~ 3% reach Haumeas current eccentricity in 500 Myr and ~ 6% reach it in 1 Gyr, indicating a ~ 95% confidence that the Haumea family is older than 1 Gyr (Volk & Malhotra 2012); reaching the same conclusion as Ragozzine & Brown (2007) who reached a ~ 90% confidence that the the family is older than 1 Gyr, with an age estimate of 3.5 ± 2 Gyr. Volk & Malhotra (2012) found that the non-resonant family members had a velocity of $\pm 50 - 100 ms^{-1}$ of their current velocity at the time of collision. Lykawka et al. (2012) conducted simulations with 1600 particles where most survive in the trans-Neptunian belt over 4 Gyr, with ~ 1.9-2.5% being lost after the last 1 Gyr, after evolving to a steady-state configuration for the first 1 Gyr, resulting in the final distributions remaining similar to their initial conditions. Surviving family members tend to remain concentrated around the location of the initial impact which caused the generation of the family (Lykawka et al. 2012).

The aim of this report is to investigate the orbital dynamics of the current 18 objects of the Haumea family as listed above and in Figs. 1 and 2. Simulations of the clones of the Haumea family objects were integrated over 4.5 Gyr which were analysed to examine the orbits of the Haumea family and the gravitational influence from the Solar system's four gas giants on the orbits. Methods are shown in Section 2 with the results, discussion shown in Section 3 and recommendations and improvements for further research discussed in sub-section 3.6.

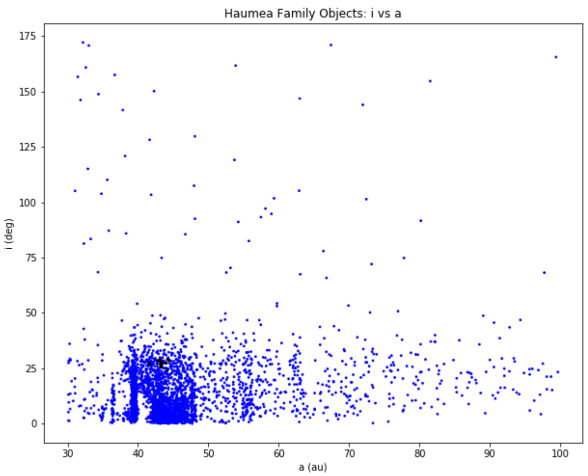

Figure 2. Inclination vs semi-major axis of all KOBs with a<=100au and the 18 Haumea family objects in black

2 METHODS

Initial data for the x, y and z positions and velocities, as well as accepted uncertainties of these parameters of the 18 Haumea family objects at the date of 01/01/2000 was obtained using the NASA Horizons Database [1] (Giorgini et al. 1996). Eight clones of each object were made from the accepted uncertainties of the values from the lower to upper limits of these values, from clone 1 to clone 8, respectively, with clones 4 and 5 being either side of the accepted orbital parameters for each object. Separate programs made for each object with the same conditions present for all simulations except Haumea family objects clones. The programs only included the Sun and the four gas giants in the system along with the eight clones of each object as other massive TNOs was found to be negligible on the evolution of the Haumea family (Lykawka et al. 2012). Simulations were conducted using REBOUND (Rein & Liu 2012) in Python with the orbits integrated using the mercurius integrator (Rein et al. 2019) which is a hybrid integrator using IAS15 (Rein & Spiegel 2014) and WHFAST (Rein & Tamayo 2015) integrators. The time step used was 1/30th of the orbital period of the inner most planet being Jupiter. Simulations were run over 4.5 Gyr with x, y, z positions, semi-major axis, inclination and eccentricity, recorded every 25,000 years. Simulations were developed locally, and conducted using the HPC at the University of Southern Queensland (USQ).

Data obtained from simulations were used to calculate the heliocentric distance of each clone of each object from the centre-of-mass (COM). The heliocentric distance, along with the eccentricity and inclination of the clones of each object were plotted against each other, showing the variation in orbital parameters between clones. Clones that reached 100au were classed as escaped despite eccentricity being <1, as when a clone reached that heliocentric distance, it was on a very unstable orbit and escaped not long after. All graphs are shown in Appendix A. The graphed heliocentric distance was compared with x, y, z values to determine which (if any) clones had a close encounter (within 3 Hill radii) with any of the four gas giants.

[1] Accessed 02/12/2020, https://ssd.jpl.nasa.gov/horizons.cgi

3 RESULTS AND DISCUSSION

Section 3.1 presents results of stable objects, sections 3.2, 3.3, 3.4 and 3.5 present results of clones of objects that escape the Solar system, and section 3.6 discusses results.

3.1 Stable Objects

Clones of 2015 AJ_{281}, 2008 AP_{129}, 2014 LO_{28}, 2003 OP_{32}, 1999 OY_3, 2014 QW_{441}, 2005 RR_{43}, 1995 SM_{55}, 2003 SQ_{317}, 1996 TO_{66}, 2002 TX_{300}, 2005 UQ_{513}, 2010 VK_{201} and 2009 YE_7 all maintained stable orbits throughout simulations, with only very minor increases and decreases in heliocentric distance, eccentricity and inclination, but remaining similar to initial conditions, which is shown by clones of 2015 AJ_{281} in Figure 3. These clones are representative of the Haumea family in general as they consist of 14 of the 18 members and are stable despite residing in one of the most dynamically hot regions in the Edgeworth-Kuiper belt.

Clones of objects 2015 AJ_{281} and 2003 SQ_{317} showed the most change in orbital parameters out of the stable objects. 2015 AJ_{281} notably had a approximate difference of 0.035 in the final eccentricities between clones 1 and 8. The lower limits on the parameters deviated more from initial conditions than the upper limits for all parameters, presenting a clear trend which could allow the orbit of the object with accepted values to placed between clones 4 and 5, but with exact parameters unknown. Clones of 2003 SQ_{317} showed the greatest deviation from initial values out of the stable objects, particularly with eccentricity. Clones 1 and 8 maintained similar initial conditions after 4.5 Gyrs while clones 4 and 5 (clones closest to the accepted parameters of the object) deviated more from initial conditions in opposite directions for all parameters. Analysis of data produced from clones of 2003 SQ_{317} present no clear trend in orbital evolution over 4.5 Gyrs.

3.2 2005 CB_{79}

2005 CB_{79} was the most dynamically hot object of the Haumea family with all clones escaping the Solar system after 1 Myrs. No close encounters were recorded from simulations as output from the simulations was recorded every 25,000 years. Clones 1, 4, 5, 6, 7 and 8 appear to have escaped from a close encounter with Neptune in Figure 4, while clones 2 and 3 possibly escaped from close encounters with Saturn or Uranus. Orbital parameters of all clones were altered drastically from initial conditions shown in their eccentricities and inclination. It is likely that a majority of the clones had experienced more than one close encounter with the four gas giants. Times that clones escaped the Solar system are not shown due to inaccuracies resulting from the infrequent, 25,000 year recording of orbital positions over the 1 Myrs. More accurate and frequent recording of clones over 1 Myrs is needed to properly understand the orbital evolution of 2005 CB_{79}.

3.3 Haumea (2003 EL_{61})

All clones of Haumea escaped the Solar system after approximately 1.162×10^9 years. Clones 3, 4 and 8 were all recorded to have had close encounter with Neptune from data obtained. These were the only clones recorded due to the orbits of the clones only being recorded every 25,000 years. Clone 3 experienced a close encounter with Neptune at 4.02625×10^8 years, before it escaped the Solar system at 4.1175×10^8 years. Clone 3 must have had another close encounter with Neptune during the 9.125×10^6 years from the recorded

Table 1. Times (in years) that each of the clones of Haumea, HZ199 and UZ117 escaped (greater than 100au).

Object	Clone 1	Clone 2	Clone 3	Clone 4	Clone 5	Clone 6	Clone 7	Clone 8
Haumea	1.16185×10^9	5.1255×10^8	4.06175×10^8	2.04×10^8	1.73175×10^8	6.735×10^7	5.4775×10^8	1.12395×10^9
HZ199	3.6876×10^9	1.36675×10^9	NA	1.443325×10^9	NA	NA	3.2965×10^9	2.961325×10^9
UZ117	3.08325×10^9	4.185325×10^9	7.148×10^8	NA	1.476225×10^9	NA	2.5654×10^9	NA

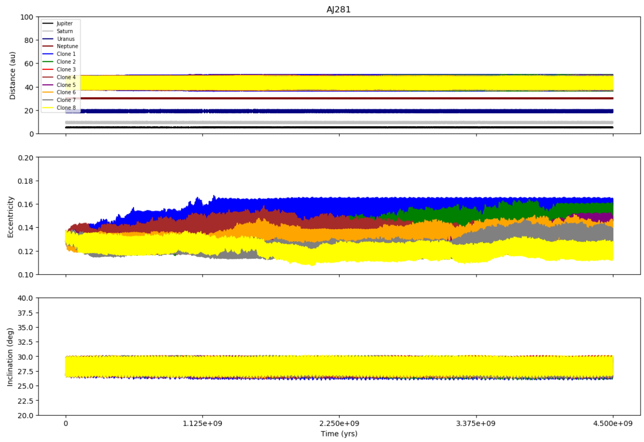

Figure 3. Heliocentric distance, eccentricity and inclination of eight clones of object 2015 AJ_{281} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

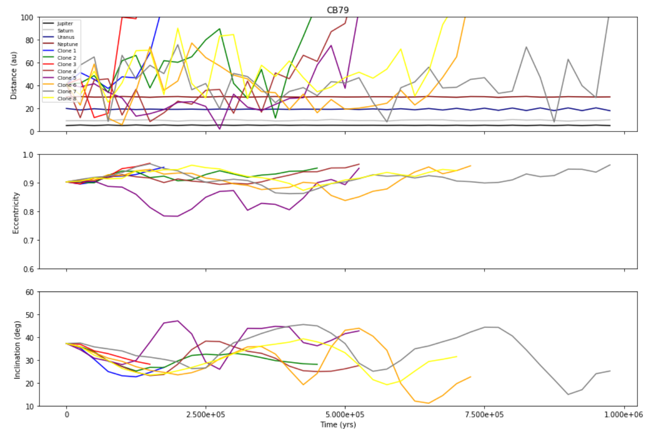

Figure 4. Heliocentric distance, eccentricity and inclination of eight clones of object 2005 CB_{79} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC. All clones escape the Solar system after 1 Myrs.

encounter to escaping, which can be inferred from Figure 5, and encounters prior to the recorded encounter could have also occurred. Clone 4 had a close encounter with Neptune at 1.59625×10^8 years which was well before it escaped the Solar system. It most likely could have had another close encounter with Neptune, and possibly Uranus, before it was ejected. Clone 8 experienced a close encounter with Neptune at 1.111275×10^9 years, relatively not long before it escaped. All clones escaped the Solar system due to close encounters with Neptune, with the times that they escaped shown in Table 1 . Clones 1, 4 and 8 had possibility a close encounter with Uranus, but did not cause them to escape.

The inclination of all of the clones tended to reduce from initial conditions before the clones escaped the Solar system. The close encounters that the clones with would have experienced from Neptune would have dampened their orbits, reducing their inclination as Neptune's inclination is relatively low. Eccentricities slowly increased due to gravitational influence from Neptune, causing them to more volatile and increasing the likelihood of ejection from a close encounter.

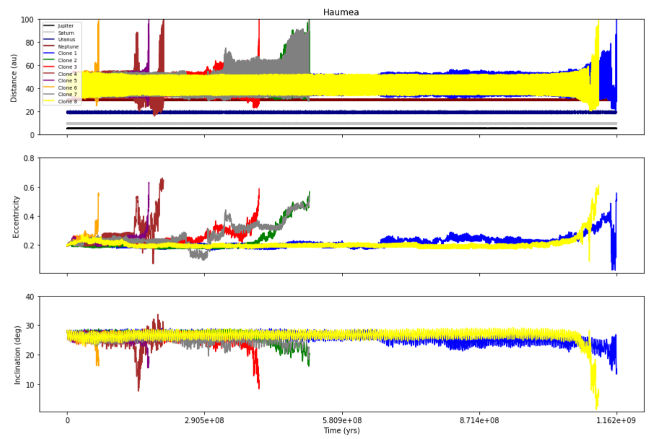

Figure 5. Heliocentric distance, eccentricity and inclination of eight clones of object Haumea (2003 EL_{61}) against time. Data obtained using REBOUND with simulations conducted using the USQ HPC. All clones escape the Solar system after $1.162e09$ yrs.

3.4 2003 HZ_{199}

Simulations of 2003 HZ_{199} clones resulted in 5 of the 8 clones escaping the Solar system as shown in Figure A2. Only 1 clone was recorded to have a close encounter (due to frequency of recording) with Neptune which was clone 7 at 3.270375×10^9 years. This encounter most likely caused clone 7 to escape, however, the recorded data not distinguishable in Figure A2 suggests a close encounter with Jupiter or Saturn was a likely cause of escape, with the close encounter with Neptune only altering its orbit. More accurate simulations with higher frequency of recorded outputs and analysis would be required to treat it more seriously. It, along with clones 1, 2, 4 and 8 escaped while clones 3, 5 and 6 maintained relatively stable orbits throughout the 4.5 Gyrs. All clones escaped via close encounters with Neptune, except with the possibility of clone 7 being ejected by Jupiter or Saturn, but exact times are unknown due to lower frequency of recorded output. The times that the clones at which they escaped are shown in Table 1.

The eccentricity and inclination of the clones that remain in the Solar system remain similar to initial conditions, with clone 6 resulting in a slightly reduced eccentricity. The clones that escaped had eccentricities and inclinations altered similar to those of the Haumea clones before escaping. Object 2003 HZ_{199} would require more analysis and simulations due to multiple possibilities of ejection and the possibility of remaining in a stable orbit in the Solar system.

3.5 2003 UZ_{117}

Simulations of 2003 UZ_{117} clones resulted in 5 of the 8 clones escaping the Solar system. Clones 1, 2 and 7 were recorded to have close encounters with Neptune at 3.050025×10^9, 4.1371×10^9 and 2.480575×10^9, respectively. Clone 7 shows possibility of a close encounter with Uranus and clone 5 shows possibility of a close encounter with Uranus and Neptune in A13, however, data from simulations do not show evidence of these encounters. More accurate and larger simulations would be required to further discuss these possibilities. Clones 3 and 5 were not recorded to have close encounters, but Figure A13 shows that close encounters did occur within one of the 25,000 year intervals. The times that clones escaped are shown in Table 1. Clones 4, 6 and 8 maintained stable orbits with only minor deviations to initial conditions. Results do not conclude on a single outcome of 2003 UZ_{117} from clones over 4.5 Gyrs, further study would be needed to analyse the orbit more accurately.

3.6 Discussion

Volk & Malhotra (2012) and Lykawka et al. (2012) conducted similar but more accurate and larger simulations which found that about 20-45% escaped due to close encounters with Neptune and 25-40%, respectively. Results from this report show that approximately 11-22% of objects escaped. This variation in results when compared to the literature was most likely due to the fact that simulations conducted in this report were on a smaller scale, with less clones and less frequent recording of positions of the objects orbital positions throughout the simulations.

A more complete and accurate examination of these objects could have been done by including the accepted orbital parameters for each object in the simulations as this report did not include the orbits of the objects with their accepted parameters. Their orbital evolution over 4.5 Gyr can be roughly estimated from the graphs in Appendix A as it includes clones within the lower and upper range on the accepted orbital values. Examination of the possibility of mean-motion resonances (MMR) of objects was not included in the data analysis due to time constraints. Examination of the clones with stable orbits was not considered.

The inclusion of more clones of each object in further research endeavours would produce a clearer picture of the evolution of the Haumea family, as well as backwards integration of the objects. A more frequent collection of data from simulations would provide a more accurate indication on the time that clones experienced a close encounter. The inclusion of other TNOs in the same region as the Haumea family could potentially result in orbital changes of the stable objects of the Haumea family as only the four gas giants were the only other objects included in these simulations, however, this would require more research and simulation time. A more detailed examination of the clones on stable orbits, but with slightly altered eccentricity and inclination after 4.5 Gyr could be examined in more detail to determine what caused those changes as it was excluded from this report.

4 CONCLUSION

In this report, 8 clones of each of the 18 Haumea family objects were made with data from the NASA Horizons Database [2] (Giorgini et al. 1996), and simulated using REBOUND (Rein & Liu 2012) from 01/01/2000 over 4.5 Gyrs in a system consisting of the Sun and the four Solar system gas giants. Orbital parameters of the clones was obtained every 25,000 years which was used to calculate the heliocentric distance of clones, to determine if any clones experienced close encounters with the four gas giants and how clones that escaped the Solar system were influenced.

Graphs from data obtained were created and analysed along with raw data to determine the orbital evolution of clones of the 18 Haumea family objects. Results showed that the Haumea family is a relatively stable cluster of TNOs, with only the clones of 4 objects escaping the Solar system. All clones of objects 2005 CB_{79} and 2003 EL_{61} all escaped the Solar system, and 5 clones of 2003 HZ_{199} and 2003 UZ_{117} escaped, with most due to close encounters with Neptune. However, data and figure suggest that a few of clones of those 4 objects may have had close encounters prior to escaping the Solar system not just with Neptune. The other 14 Haumea family objects are found to remain stable in the Solar system for 4.5 Gyrs.

Further study and larger simulations of the 4 objects that showed clones escaping the Solar system are required to obtain a more detailed and accurate analysis on the evolution of these objects. Close encounters that the 4 objects experience need to be analysed in more detail from data from simulations to be certain of encounters proposed and recorded in this paper.

[2] Accessed 02/12/2020, https://ssd.jpl.nasa.gov/horizons.cgi

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DATA AVAILABILITY

Data and programs used in this paper are available on github at: Loch64/ResearchProject

REFERENCES

Barkume K. M., Brown M. E., Schaller E. L., 2006, The Astrophysical Journal, 640, L87
Brown M. E., et al., 2005, The Astrophysical Journal, 632, L45
Brown M. E., et al., 2006, The Astrophysical Journal, 639, L43
Brown M. E., Barkume K. M., Ragozzine D., Schaller E. L., 2007, Nature, 446, 294
Campo Bagatin A., Benavidez P. G., Ortiz J. L., Gil-Hutton R., 2016, Monthly Notices of the Royal Astronomical Society, 461, 2060
Carry B., Snodgrass C., Lacerda P., Hainaut O., Dumas C., 2012, Astronomy and Astrophysics, 544, 137
Ćuk M., Ragozzine D., Nesvorný D., 2013, Astronomical Journal, 146, 89
Giorgini J. D., et al., 1996, JPL's On-Line Solar System Data Service, https://ui.adsabs.harvard.edu/abs/1996DPS....28.2504G/abstract
Horner J., Evans N. W., Bailey M. E., 2004, Monthly Notices of the Royal Astronomical Society, 354, 798
Leinhardt Z. M., Marcus R. A., Stewart S. T., 2010, Astrophysical Journal, 714, 1789
Levison H. F., Alessandro M., Vokrouhlicky D., Bottke W., 2008, Astronomical Journal, 136, 1079
Lykawka P. S., Horner J., Mukai T., Nakamura A. M., 2012, Monthly Notices of the Royal Astronomical Society, 421, 1331

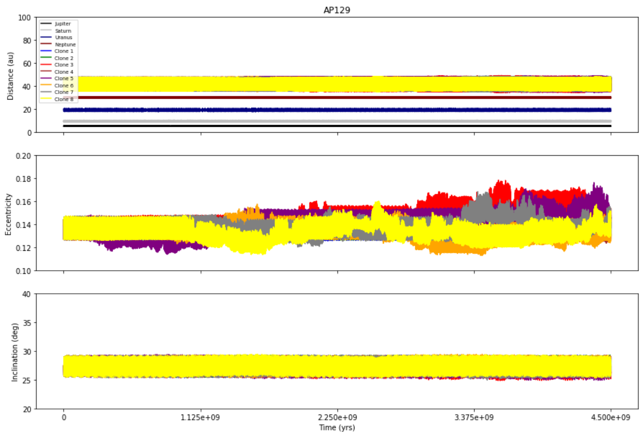

Figure A1. Heliocentric distance, eccentricity and inclination of eight clones of object 2008 AP_{129} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

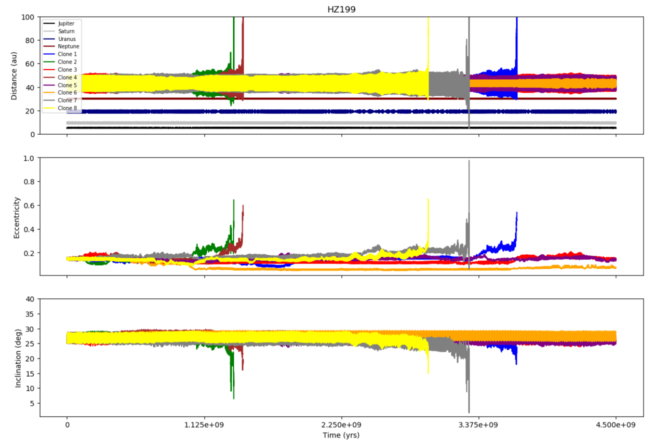

Figure A2. Heliocentric distance, eccentricity and inclination of eight clones of object 2014 HZ_{199} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC. Clones 1, 2, 4, 7 and 8 escape the Solar system after $3.6876e9$ yrs, with clones 3, 5 and 6 remaining in the Solar system.

Ortiz J. L., et al., 2012, Monthly Notices of the Royal Astronomical Society, 419, 2315

Pike R. E., et al., 2019, Nature Astronomy, 4, 89

Proudfoot B. C. N., Ragozzine D., 2019, The Astronomical Journal, 157, 230

Rabinowitz D. L., Barkume K., Brown M. E., Roe H., Schwartz M., Tourtellotte S., Trujillo C., 2005, The Astrophysical Journal, 639, 1238

Ragozzine D., Brown M. E., 2007, The Astronomical Journal, 134, 2160

Ragozzine D., Brown M. E., 2009, Astronomical Journal, 137, 4766

Rein H., Liu S. F., 2012, Astronomy and Astrophysics, 537, 128

Rein H., Spiegel D. S., 2014, Monthly Notices of the Royal Astronomical Society, 446, 1424

Rein H., Tamayo D., 2015, Monthly Notices of the Royal Astronomical Society, 452, 376

Rein H., et al., 2019, Monthly Notices of the Royal Astronomical Society, 485, 5490

Schaller E. L., Brown M. E., 2008, The Astrophysical Journal, 684, L107

Schlichting H. E., Sari R., 2009, Astrophysical Journal, 700, 1242

Snodgrass C., Carry B., Dumas C., Hainaut O., 2010, Astronomy and Astrophysics, 511

Thirouin A., Sheppard S. S., Noll K. S., Moskovitz N. A., Ortiz J. L., Doressoundiram A., 2016, The Astronomical Journal, 151, 148

Trujillo C. A., Sheppard S. S., Schaller E. L., 2011, Astrophysical Journal, 730, 105

Vilenius E., et al., 2019, Astronomy and Astrophysics, 618

Volk K., Malhotra R., 2012, Icarus, 221, 106

APPENDIX A: GRAPHS OF HAUMEA FAMILY OBJECTS

The follow are graphs of clones of the 18 Haumea family objects. The graphs are limited to when a clone of an object reaches 100au as at 100au, the object has escaped the Solar system.

This paper has been typeset from a TEX/LATEX file prepared by the author.

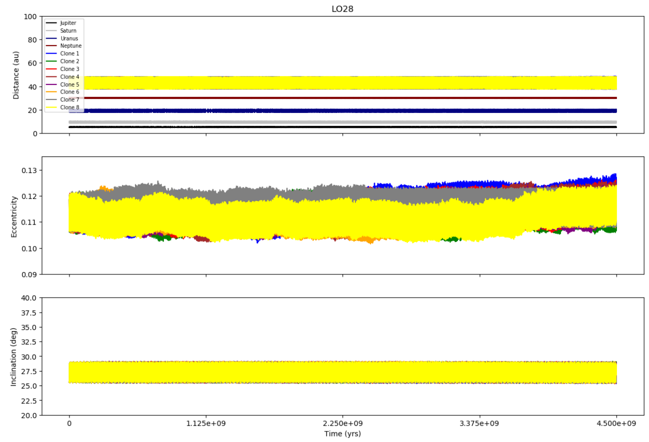

Figure A3. Heliocentric distance, eccentricity and inclination of eight clones of object 2014 LO_{28} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

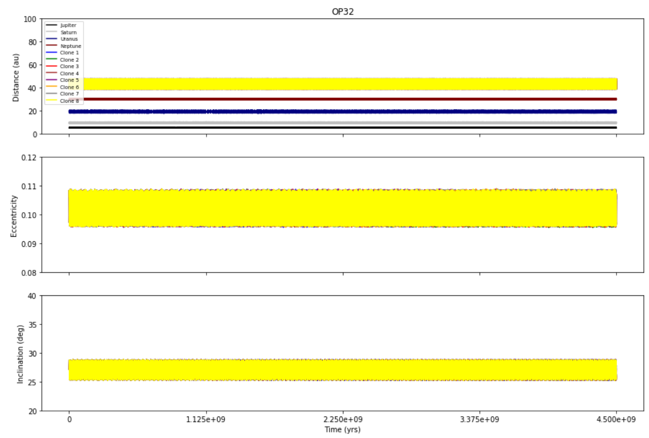

Figure A4. Heliocentric distance, eccentricity and inclination of eight clones of object 2003 OP_{32} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

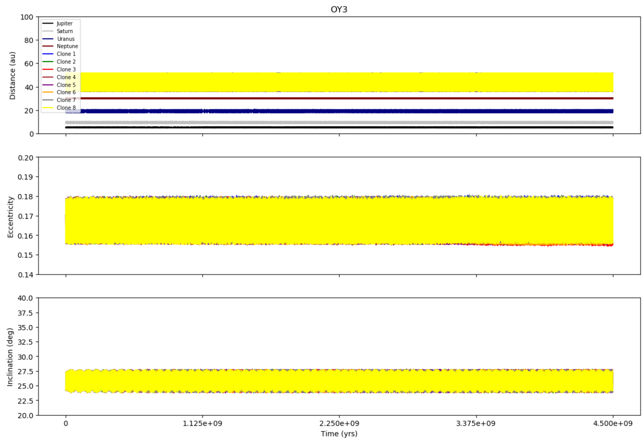

Figure A5. Heliocentric distance, eccentricity and inclination of eight clones of object 1999 OY_3 against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

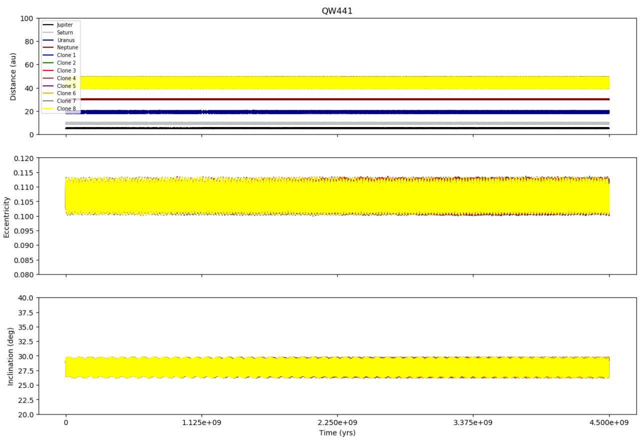

Figure A6. Heliocentric distance, eccentricity and inclination of eight clones of object 2014 QW_{441} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

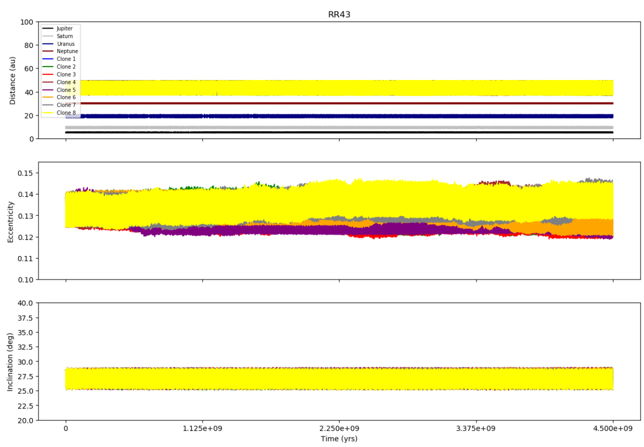

Figure A7. Heliocentric distance, eccentricity and inclination of eight clones of object 2005 RR_{43} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

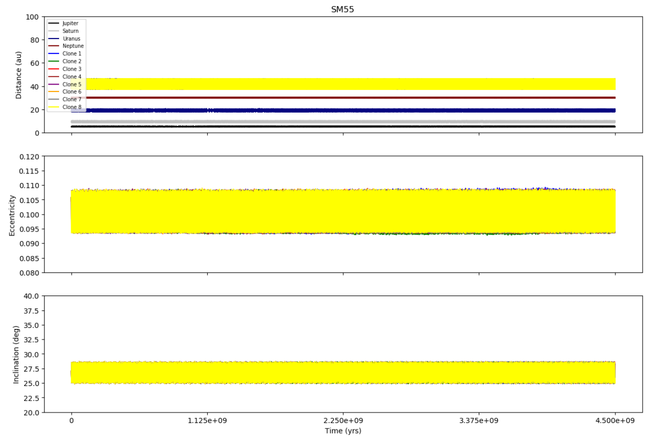

Figure A8. Heliocentric distance, eccentricity and inclination of eight clones of object 1995 SM_{55} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

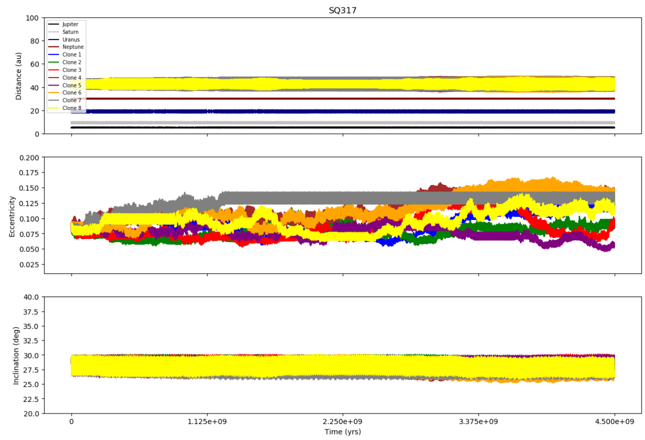

Figure A9. Heliocentric distance, eccentricity and inclination of eight clones of object 2003 SQ_{317} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

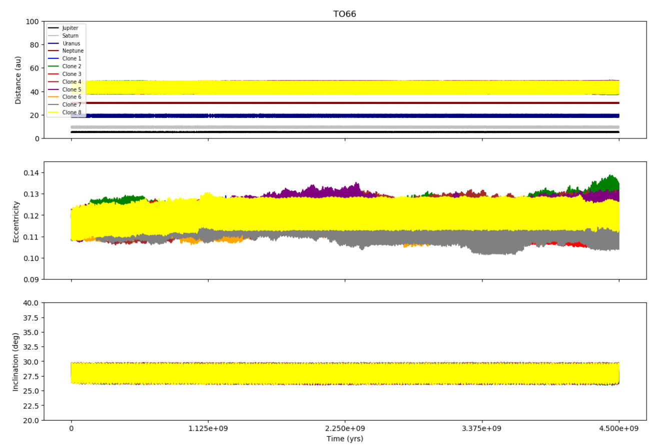

Figure A10. Heliocentric distance, eccentricity and inclination of eight clones of object 1996 TO_{66} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

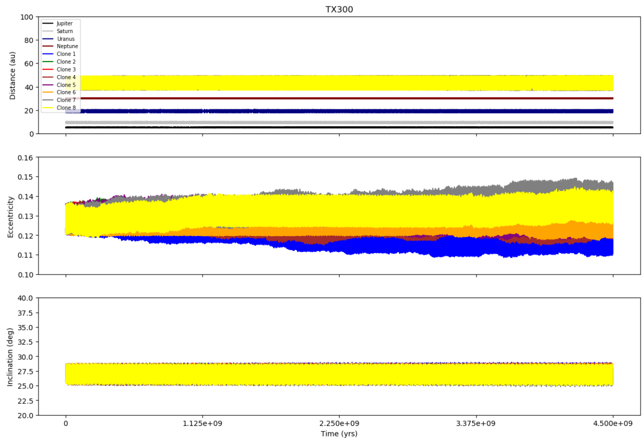

Figure A11. Heliocentric distance, eccentricity and inclination of eight clones of object 2002 TX_{300} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

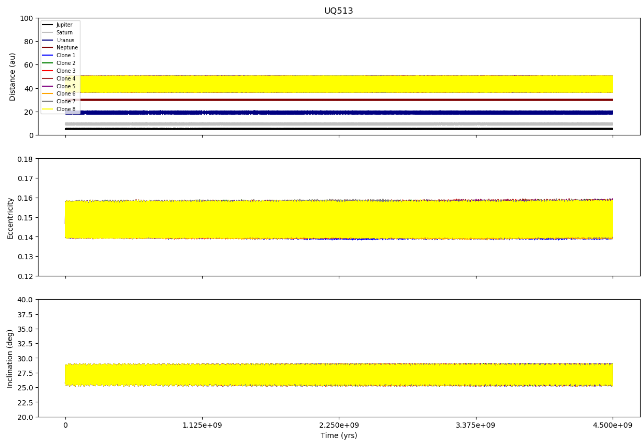

Figure A12. Heliocentric distance, eccentricity and inclination of eight clones of object 2005 UQ_{513} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

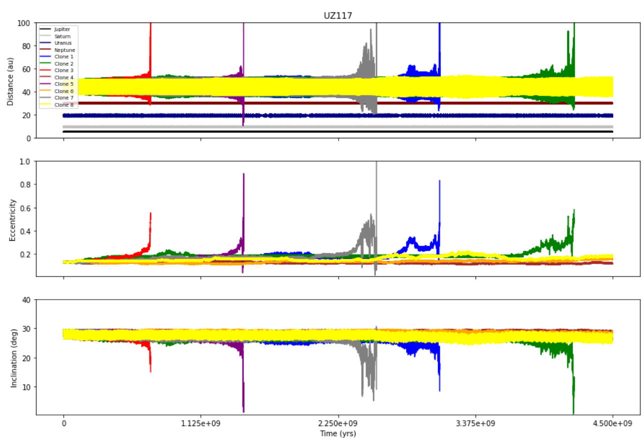

Figure A13. Heliocentric distance, eccentricity and inclination of eight clones of object 2003 UZ_{117} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC. Clones 1, 2, 3, 5 and 7 escape after 4.185325×10^{9} years, while clones 4, 6 and 8 remain in a stable orbit.

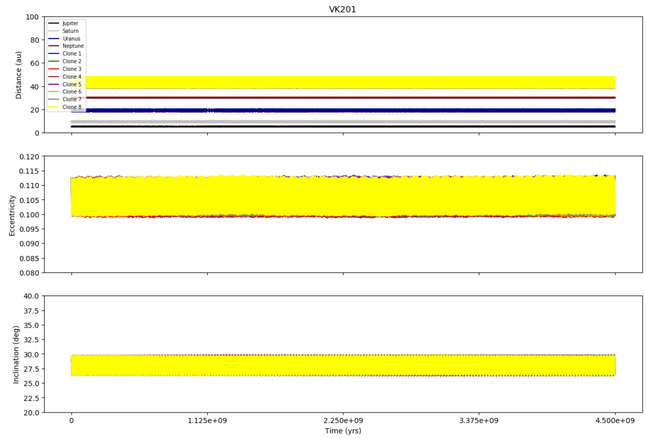

Figure A14. Heliocentric distance, eccentricity and inclination of eight clones of object 2010 VK_{201} against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.

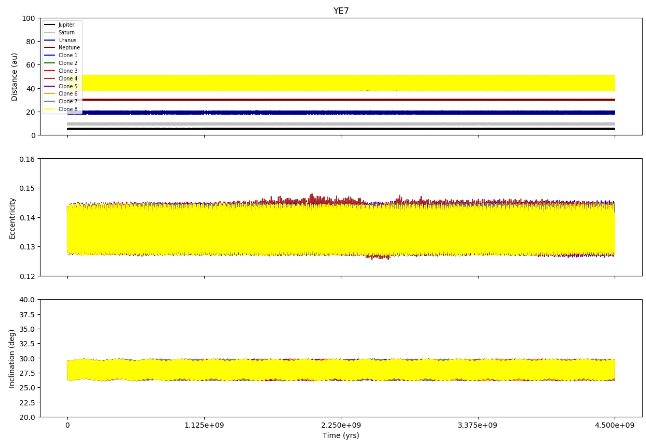

Figure A15. Heliocentric distance, eccentricity and inclination of eight clones of object 2009 YE_7 against time. Data obtained using REBOUND with simulations conducted using the USQ HPC.