Test 2 Cheat Sheet

🔟 Chapter 10: Counting Principles

🔢 10.1 – Counting by Systematic Listing

Purpose: List all possible outcomes when the sample space is small.

Techniques:

- Tables
- Tree diagrams
- Lists

Example:
List all 2-letter codes from {A, B, C} (repetition allowed):

- Possible outcomes: `AA, AB, AC, BA, BB, BC, CA, CB, CC`
- Total: $3 \times 3 = 9$ outcomes

📊 10.2 – Fundamental Counting Principle

Definition: If one event has m outcomes and another has n, the total number of outcomes is:

$$\text{Total outcomes} = m \times n$$

Extended Form:

$$\text{Total outcomes} = n_1 \cdot n_2 \cdot \dots \cdot n_k$$

Example:
Choose 1 shirt (3 options), 1 pant (2 options), and 1 pair of shoes (4 options):

$$3 \times 2 \times 4 = 24 \text{ total outfits}$$

📈 10.3 – Permutations & Combinations

🔁 Permutations (Order Matters)

- Without repetition:

$P(n,r) = \frac{n!}{(n-r)!}$

- Using all items:

$P(n) = n!$

Example:
How many ways to arrange 3 books out of 5 on a shelf?

$P(5,3) = 5!(5-3)! = 1202 = 60$
or
$P(5,3) = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$

🔀 Combinations (Order Doesn't Matter)

$C(n,r) = (nr) = n!r!(n-r)!$
or
$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Example:
How many ways to choose 3 students out of 7?

$C\binom{7}{3} = \frac{7!}{3!4!} = \frac{5040}{6 \cdot 24} = \frac{5040}{144} = 35$

🔺 10.4 – Pascal's Triangle

Each row represents binomial coefficients:

- Row 0: 1
- Row 1: 1 1
- Row 2: 1 2 1
- Row 3: 1 3 3 1
- Row 4: 1 4 6 4 1

Application: Use for expanding binomials.

Example: Expand $(x+y)^4$:

$(x + y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

🚫 10.5 – Counting with "Not" and "Or"

Complement Rule:

$\text{Not A} = \text{Total} - A$

Addition Rule:

- Disjoint:

$(A \cup B) = (A) + (B)(A \cup B) = (A) + (B)$

- Overlapping:

$(A \cup B) = (A) + (B) - (A \cap B)(A \cup B) = (A) + (B) - (A \cap B)$

Example: 40 students: 25 play piano, 18 play violin, 10 play both:

$(P \cup V) = 25 + 18 - 10$
$= 33(P \cup V)$
$= 25 + 18 - 10$
$= 33$

🔢 Chapter 11: Probability

🎲 11.1 – Basic Concepts

Probability Formula:

$P(E) = \frac{\text{favorable outcomes}}{\text{total outcomes}}$

Complement:

$P(\text{Not E}) = 1 - P(E)$

Example:
Probability of drawing a red card from a standard deck:

$P(\text{Red}) = \frac{26}{52} = \frac{1}{2}$

Six people N={A,B,C,N,O,R}N = { A, B, C, N, O, R }N={A,B,C,N,O,R} form a club.

If they choose a president **randomly**, find the **odds against Ryan** (R) becoming president.

✅ **Solution**

There are a total of 666 members.

Ryan has a $\frac{1}{6}$ chance of being chosen, so the chance that **Ryan is *not* chosen** is:

$\frac{5}{6}$

The **odds against Ryan** are the ratio of the number of ways Ryan is *not* chosen to the number of ways he *is* chosen:

$\text{Odds against Ryan} = \frac{5}{1} = 5 : 1$

⚖️ 11.2 – "Not" and "Or" in Probability

Addition Rule:

- Mutually exclusive:

$P(A \cup B) = P(A) + P(B)$

- Overlapping:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example:
$P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.2 :$

$P(A \cup B) = 0.5 + 0.4 - 0.2 = 0.7$

Use the multiplication rule of probability. If A and B are any two events, then the following formula holds.
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P(A and B) =P(A) • P(B ∣ A)

🔗 11.3 – Conditional Probability and "And"

Conditional Probability:

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Independent Events:

$P(A \cap B) = P(A) \cdot P(B)$

Example:
Draw 2 cards with replacement:

$P(\text{Red, then Black}) = \frac{26}{52} \cdot \frac{26}{52} = \frac{1}{4}$

Without replacement:

$P(2 \text{ Reds}) = \frac{26}{52} \cdot \frac{25}{51}$

🧮 11.4 – Binomial Probability

Used when:

- Fixed number of trials n
- Only success/failure outcomes
- Each trial is independent
- Constant success probability p

Formula:

$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$

Where:

- n = total trials
- k = number of successes
- p = probability of success
- $1 - p$ = probability of failure

Example:
Flip a coin 4 times. What's the probability of exactly 2 heads?

- $n = 4$, $k = 2$, $p = 0.5$
- Plug into formula:

$P(2) = \binom{4}{2}(0.5)^2(0.5)^2$
$= 6 \cdot 0.25 \cdot 0.25$
$= 6 \cdot 0.0625$
$= 0.375$

💰 11.5 – Expected Value & Simulation

Expected Value:

$E = \sum(\text{Value} \cdot \text{Probability})$

Example:
Game:

- Win $10 with P = 0.2
- Lose $5 with P = 0.8

Then:

$E = (10)(0.2) + (-5)(0.8)$
$= 2 - 4$
$= -2$

Simulation:

A technique for estimating probabilities using repeated random sampling (real or digital).

💡 Summary Table

Concept	Formula / Rule
Fundamental Principle	$n_1 \cdot n_2 \cdots n_k$
Permutations	$P(n, r) = \frac{n!}{(n-r)!}$
Combinations	$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
Addition Rule	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Complement	$P(\text{Not A}) = 1 - P(A)$
Conditional Probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Concept	Formula / Rule
Binomial Probability	$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$
Expected Value	$E = \sum(\text{value} \cdot P)$