3rd August, 2016

Home Assignment 1

1. In the circuit given in **Fig. 1** use nodal analysis to find the value of '***k***' that will cause 'V_y' to be zero.

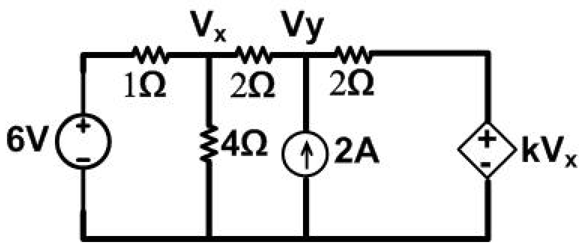


Fig. 1

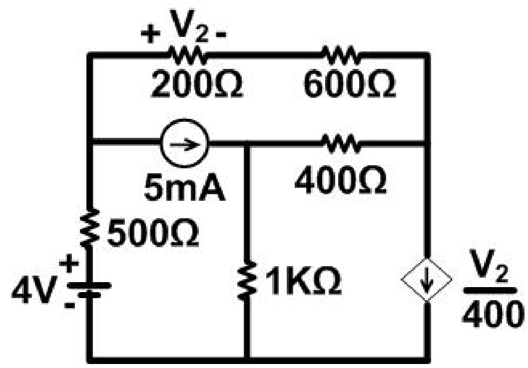


Fig. 2

2. Use mesh analysis on the circuit shown in **Fig. 2** to find the power supplied by the **4V** battery.
3. Use super mesh analysis to determine the voltage drop '**V**' across **5Ω** resistor as shown in **Fig. 3**.

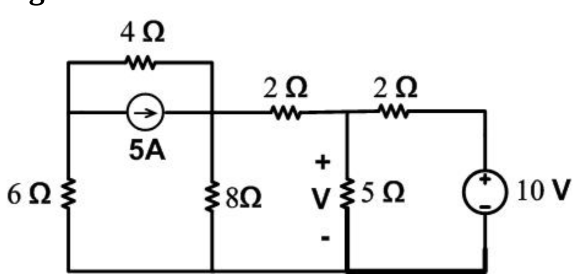


Fig. 3

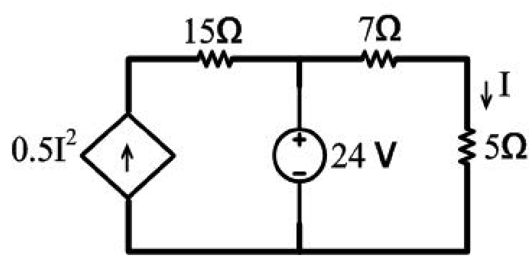


Fig.4

4. In **Fig. 4**, determine the power delivered by the dependent source.

5. In the circuit of the **Fig. 5**, determine the maximum positive current to which the source 'I_x' can be set before any resistor exceeds its power rating.

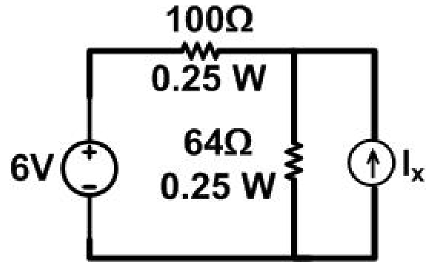


Fig. 5

6. Use the superposition theorem to find '**i**' in **Fig. 6**.

7. For the circuit shown in **Fig. 7,** find '**R'** so that power supplied by both the sources is equal to each other.

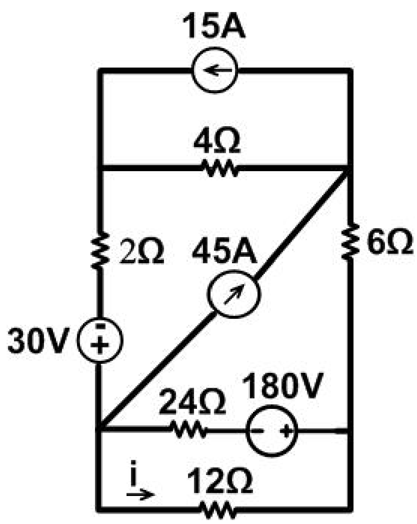


Fig. 6

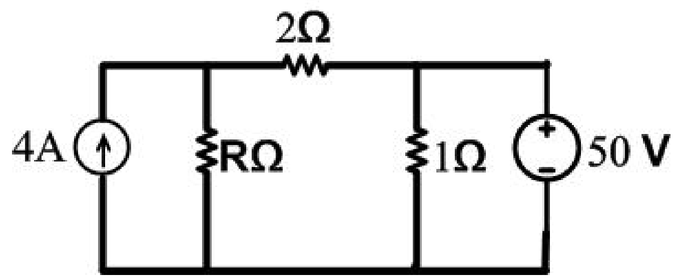
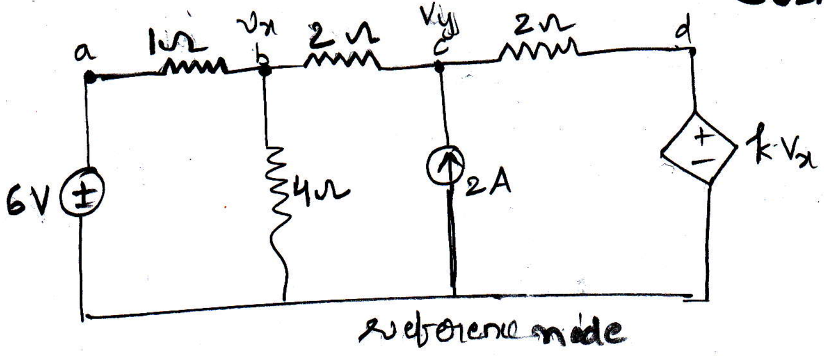


Fig. 7

SOLN. 1.



At nodes 'a' and 'd', we have

$$V_a = 6V \; ; \; V_d = k v_x$$

At nodes 'b' and 'c', we can write that

$$\frac{v_x - 6}{1} + \frac{v_x - v_y}{2} + \frac{v_x}{4} = 0 \quad \text{——— (1)}$$

$$\frac{v_y - v_x}{2} + \frac{v_y - k v_x}{2} = 2 \quad \text{——— (2)}$$

Put $v_y = 0$ for finding 'k'

From (1),

$$\frac{v_x - 6}{1} + \frac{v_x}{2} + \frac{v_x}{4} = 0$$

$$\Rightarrow \; 6 = v_x \left(\frac{7}{4}\right)$$

$$\Rightarrow \; v_x = \frac{24}{7} \text{ V.}$$

From (2),

$$\frac{v_x}{2} + \frac{K v_x}{2} = -2$$

$$\Rightarrow (1+K)\frac{v_x}{2} = -2$$

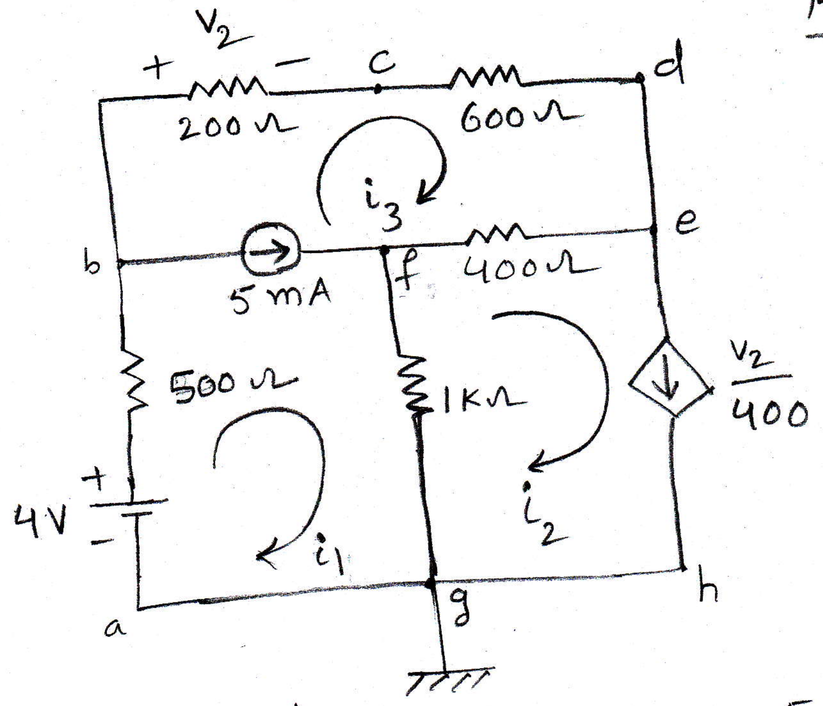
$$\Rightarrow \; 1+K = -\frac{4}{v_x} = \frac{-4}{(24/7)} = -\frac{28}{24}$$

$$\Rightarrow \; K = -1 - \frac{28}{24} = -\frac{52}{24} = -\frac{13}{6}$$

$$\therefore \; \boxed{K = -\frac{13}{6}}$$

Assignment-1

Solution-2



in branch 'bf' $\longrightarrow$ $i_1 - i_3 = \frac{5}{1000}$

$\Rightarrow 1000 i_1 - 1000 i_3 = 5$ ——— (1)

for loop "abcdefga";

$-4 + 500 i_1 + 200 i_3 + 600 i_3 + 400(i_3 - i_2) + 1000(i_1 - i_2) =$

$\Rightarrow -4 + 1500 i_1 - 1400 i_2 + 1200 i_3 = 0$ ——— (2)

and from the figure, $V_2 = 200 i_3$ ——— (3)

in branch "eh" $\longrightarrow$ $i_2 = \frac{V_2}{400} = \frac{200 i_3}{400}$ [∵ from (3)]

$\Rightarrow i_2 = \frac{i_3}{2}$ ——— (4)

from (2) & (4),

$-4 + 1500 i_1 - 1400\left(\frac{i_3}{2}\right) + 1200 i_3 = 0$

$\Rightarrow 1500 i_1 + 500 i_3 = 4$ ——— (5)

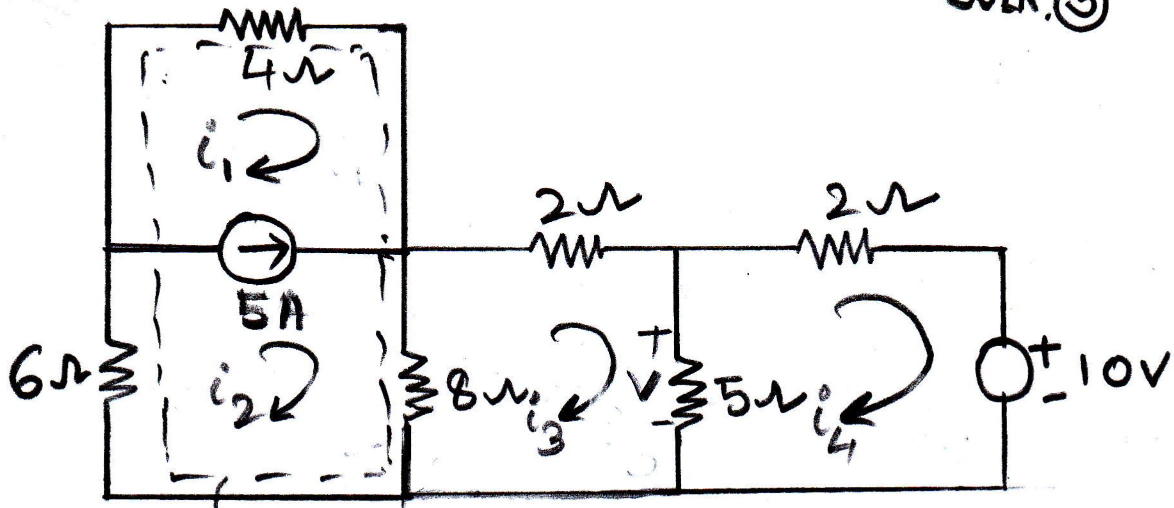
(1) × 1 $\Rightarrow$ $1000 i_1 - 1000 i_3 = 5$

(5) × 2 $\Rightarrow$ $3000 i_1 + 1000 i_3 = 8$

$4000 i_1 = 13 \Rightarrow i_1 = \frac{13}{4000}$ Amp

∴ Power generated by 4V battery is, $P_{gen} = 4 \times \frac{13}{4000}$ W

$= 13$ mW.



Supermesh

Applying KVL to the supermesh.

$$4i_1+8(i_2-i_3)+6i_2=0 \quad \cdots\cdots (1)$$

$$i_2-i_1=5 \quad \cdots\cdots (2)$$

Applying KVL to Mesh 3,

$$2i_3+5(i_3-i_4)+8(i_3-i_2)=0 \quad \cdots\cdots (3)$$

Applying KVL to Mesh 4,

$$2i_4+10+5(i_4-i_3)=0 \quad \cdots\cdots (4)$$

Equations 1, 2, 3, 4 can be rearranged as,

$$4i_1+14i_2-8i_3+0.i_4=0$$

$$-i_1+i_2+0i_3+0i_4=5$$

$$0i_1-8i_2+15i_3-5i_4=0$$

$$0i_1+0i_2-5i_3+7i_4=-10$$

Now, applying cramers rule we can obtain,

$$i_3 = \frac{\begin{vmatrix} 4 & 14 & 0 & 0 \\ -1 & 1 & 5 & 0 \\ 0 & -8 & 0 & -5 \\ 0 & 0 & -10 & 7 \end{vmatrix}}{\begin{vmatrix} 4 & 14 & -8 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -8 & 15 & -5 \\ 0 & 0 & -5 & 7 \end{vmatrix}}$$

$$= \frac{220}{992} \simeq 0.22 \text{ A}$$

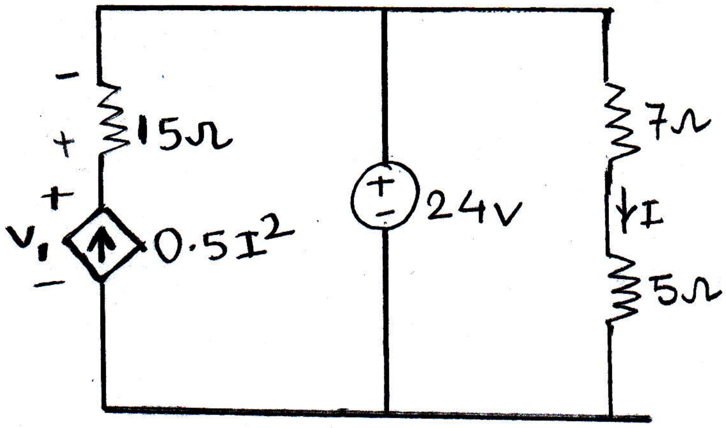
$$i_4 = \frac{\begin{vmatrix} 4 & 14 & -8 & 0 \\ -1 & 1 & 0 & 5 \\ 0 & -8 & 15 & 0 \\ 0 & 0 & -5 & -10 \end{vmatrix}}{\begin{vmatrix} 4 & 14 & -8 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -8 & 15 & -5 \\ 0 & 0 & -5 & 7 \end{vmatrix}}$$

$$= -\frac{1260}{992} \simeq -1.27 \text{ A}$$

$\therefore$ Voltage drop across 5Ω

$= 5(i_3 - i_4) = 7.45$ Volt

SOLN. 4.



$$I = \frac{24}{7+5} = 2A$$

$\therefore$ Strength of the dependent current source $= 0.5 I^2$

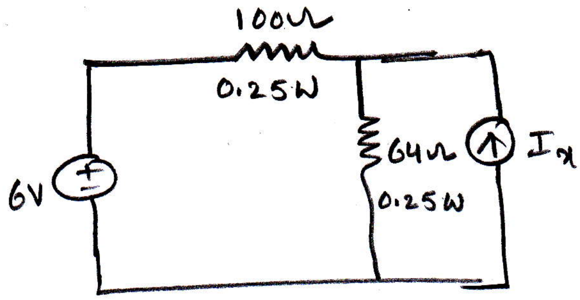
$$= 0.5 \times 2^2$$
$$= 0.5 \times 4$$
$$= 2A$$

$$24 - V_1 + 0.5 I^2 \times 15 = 0$$
$$24 = V_1 - 0.5 I^2 \times 15$$
$$\Rightarrow 24 = V_1 - 30$$
$$\Rightarrow V_1 = 54 \text{ V}$$

Hence, Power delivered by the dependent source

$$= V_1 \times 0.5 I^2$$
$$= 54 \times 2$$
$$= 108 \text{ W}$$

(Note:- Current flows from − to + terminal of the dependent source ⇒ hence, it is delivering power).



Each resistor is rated to a max. of 250mW.

If this value is exceeded by forcing too much current through resistors, excessive heating will occur leading to ckt damage.

Now 6V source cannot be changed

only I_x can be suitably chosen such that max current through resistors is not exceeded.

Based on 250mW power rating,

max current through 100Ω = $i_{100} = \sqrt{\frac{0.25}{100}}$ —(1)

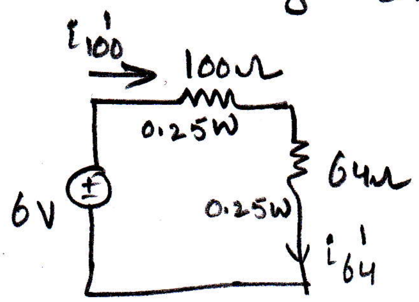
$= 50\,mA$

Similarly current through 64Ω resistor = $i_{64} = 62.5\,mA$

—(2)

We use superposition principle because unlike Nodal and mesh analysis methods, superposition allows us to identify the effect of the current source "I_x" separately

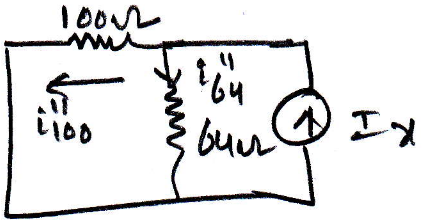
Consider only 6V source in the given ckt



$i'_{100} = \frac{6}{164} = i'_{64} = 36.59\,mA$ —(3)

⇒ 6V source acting alone does not pose any overheating problems for either resistor

For the circuit only with current source I_x present, using current division we have



$$i''_{100} = \frac{I_x \cdot 64}{164} \qquad -(4)$$

$$i''_{64} = \frac{100}{164} I_x \qquad -(5)$$

we note i''_{64} adds to i'_{64} and i''_{100} is opposite in direction to i'_{100}

$\therefore I_x$ can supply contribute upto $62.5 - 36.59$ $\quad -(6)$

$= 25.91\,mA = i''_{64}(max)$

(to 64Ω resistor)

and $50 - (-36.59) = 86.59\,mA$ (to 100Ω resistor) $\quad -(7)$

$= i''_{100}(max)$

($\because$ Note $(-i'_{100}) + i''_{100} \leq 50 \Rightarrow i''_{100} = 50 + i'_{100}$).

Using current division, the 100Ω resistor places the following constraint on I_x: [from (4) & (7)]

$$I_x < 86.59 \times 10^{-3}\left(\frac{164}{64}\right)$$

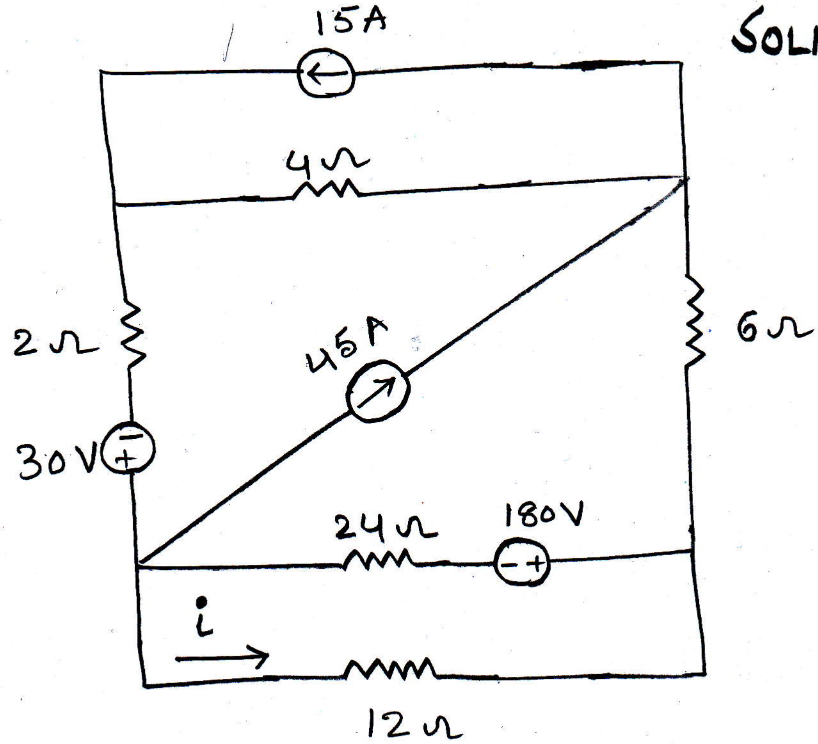
$$< 221.9\,mA$$

and 64Ω resistor requires that [from (5) & (6)]

$$I_x < 25.91 \times 10^{-3}\left(\frac{164}{100}\right)$$

$$< 42.49\,mA$$

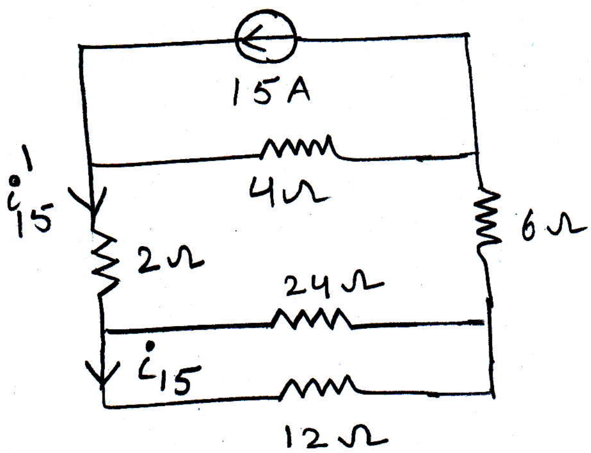
In order to meet both constraints, I_x must be less than $42.49\,mA$.



To apply the Superposition Principle,

i) consider 15A source alone

Then the circuit reduces to



Using the current division rule we have,

$$i'_{15} = \frac{4}{4+[2+6+(24 \parallel 12)]} \times 15A$$

$$= \frac{4}{4+2+6+8} \times 15A$$

$$= \frac{4}{20} \times 15A = 3A$$

Now,

$$i_{15} = \frac{24}{24+12} \times i'_{15}$$

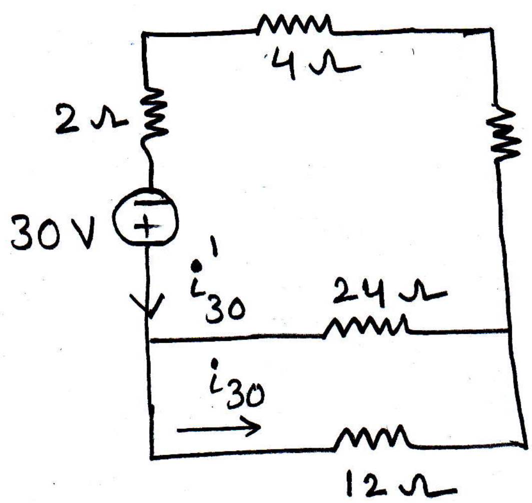
$$= \frac{24}{36} \times 3A$$

$$= 2\ A.$$

$$\therefore \boxed{i_{15} = 2A}$$

ii) consider 30V source alone

Then the circuit reduces to



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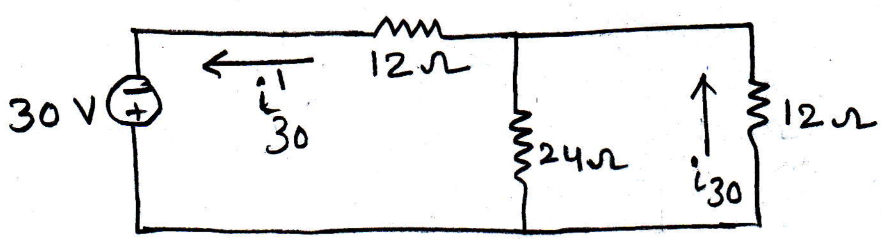


Now i'_{30} can be found as,

$$\Rightarrow i'_{30} = \frac{30}{12 + [24 \parallel 12]}$$

$$= \frac{30}{12+8} = \frac{30}{20} = 1.5A.$$

Now using the current division rule,

$$i_{30} = \frac{24}{24+12} \times i'_{30}$$

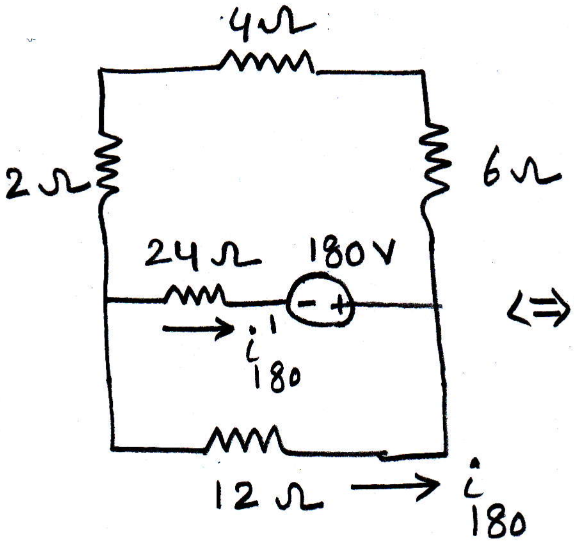
$$= \frac{24}{36} \times 1.5\,A$$

$$= 1\,A$$

$$\therefore \boxed{i_{30} = 1\,A}$$

iii) consider 180V source alone.

Then the circuit reduces to



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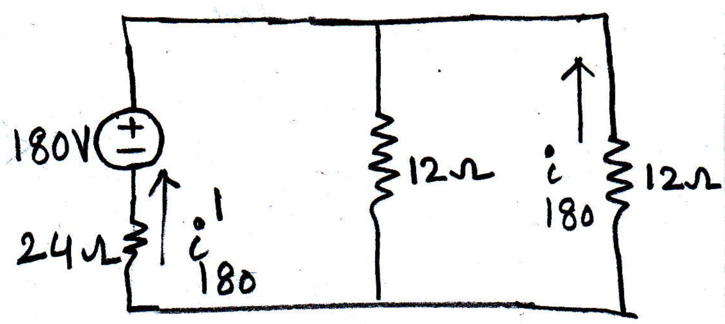


Now i'_{180} can be found as

$$\Rightarrow i'_{180} = \frac{180}{24 + [12 \parallel 12]}$$

$$= \frac{180}{24+6} = \frac{180}{30} = 6\,A$$

Now from current division rule,

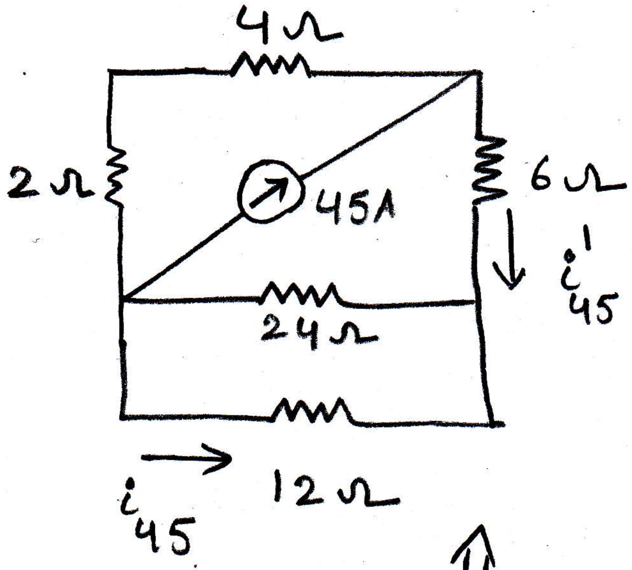
$$i_{180} = -\frac{12}{12+12} \times i'_{180}$$

$$= -\frac{12}{24} \times 6\,A$$

$$= -3A$$

$$\boxed{i_{180} = -3A}$$

iv) consider 45A source alone



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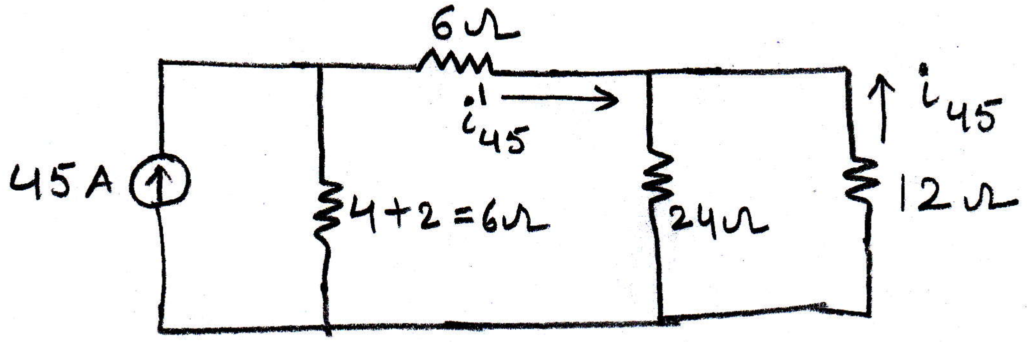


Now from the above circuit

$$i'_{45} = \frac{6}{6+6+[24 \parallel 12]} \times 45A$$

$$\Rightarrow i'_{45} = \frac{6}{6+6+8} \times 45\,A$$

$$= 13.5\,A$$

Now

$$i_{45} = -\frac{24}{24+12} \times i'_{45}$$

$$= -\frac{24}{36} \times 13.5\,A$$

$$= -9\,A$$

$$\boxed{i_{45} = -9\,A}$$

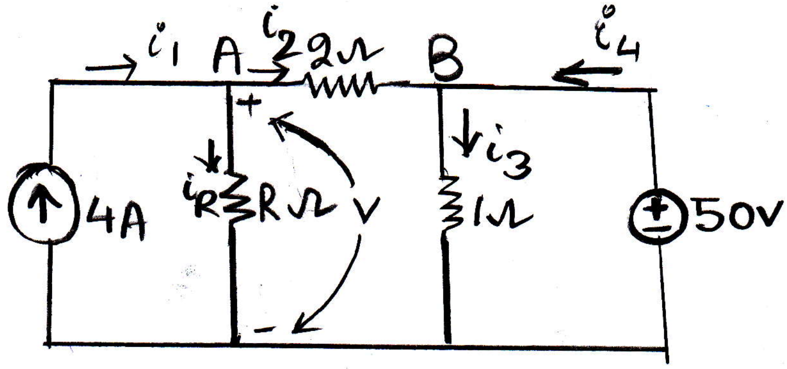
Now the desired current 'i' can be written as,

$$i = i_{15} + i_{30} + i_{180} + i_{45}$$

$$= 2A + 1A - 3A - 9A$$

$$= -9A$$

$$\therefore \boxed{i = -9A}$$



Node A:

$$-4 + \frac{V}{R} + \frac{V-50}{2} = 0$$

Rearranging, $\frac{V}{R} + 0.5V = 29$ ------- (i)

At node B:

$$\frac{50-V}{2} + \frac{50}{1} - i_4 = 0$$

$$\Rightarrow -0.5V + 75 - i_4 = 0$$

$$\Rightarrow 0.5V + i_4 = 75 \quad \text{------ (ii)}$$

Solving (i) and (ii),

$$\frac{V}{R} - i_4 = -46$$

$$\Rightarrow i_4 = \frac{V}{R} + 46 \quad \text{------ (iii)}$$

Since, power flow from both the sources are equal,

$$50 i_4 = 4V \quad \text{---------- (iv)}$$

From (iii) substituting the value of i_4,

$$50 \times \left(\frac{V}{R} + 46\right) = 4V$$

$$\Rightarrow \frac{V}{R} + 46 = \frac{2V}{25} \quad \text{--- (V)}$$

From (i) and (V),

$$\Rightarrow 29 - 0.5V + 46 = \frac{2V}{25}$$

$$\Rightarrow 75 = \frac{2V}{25} + 0.5V$$

$$\Rightarrow 75 = \frac{2 + 12.5}{25} \cdot V$$

$$= \frac{14.5}{25} V$$

$$\Rightarrow V = \frac{75 \times 25}{14.5}$$

$$= 129.3 \quad \text{------} \quad (\text{vi})$$

Substituting the value of V in (i),

$$\frac{129.3}{R} + 0.5 \times 129.3 = 29$$

$$\Rightarrow \frac{129.3}{R} + 64.65 = 29$$

$$\Rightarrow \frac{129.3}{R} = -35.65$$

$$\Rightarrow R = -\frac{129.3}{35.65} = -3.63\ \Omega$$

Since the value of R comes out to be negative, so power supplied by both sources can not be equal to each other for any practical (positive) value of R.