23rd August, 2016

Home Assignment – 4

1. Using the node voltage method, determine the voltages **V_1 (t)** and **V_2 (t)** in the circuit shown in **Fig. 1.**

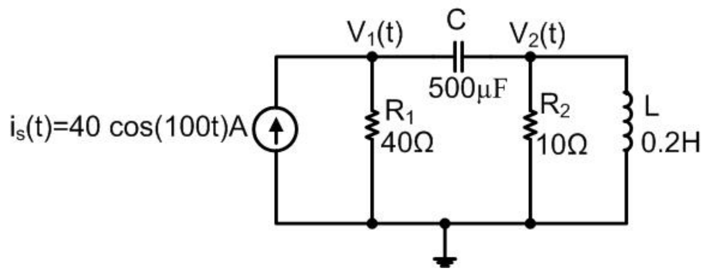


Fig. 1

2. Using the superposition theorem, find the voltage **V_1(t)** in the circuit shown in **Fig. 2**

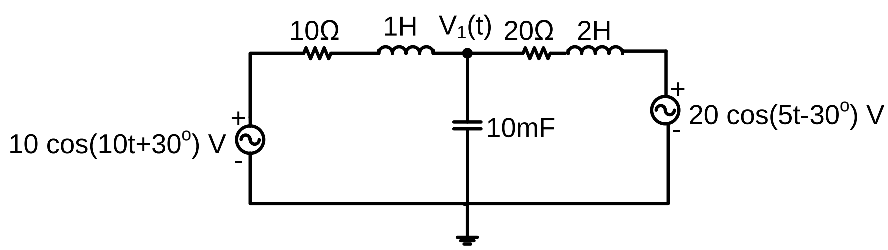


Fig. 2

3. Find the **average power** and **reactive power**, in the network shown in **Fig. 3.**

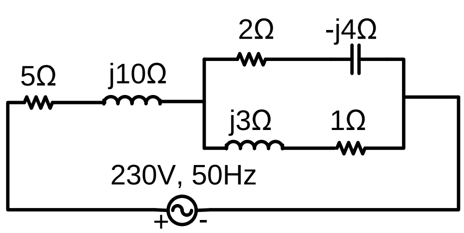


Fig. 3

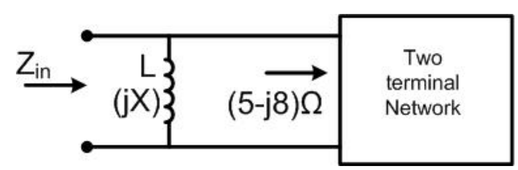


Fig. 4

4. The two terminals network shown in **Fig. 4**, has an input impedance of (**5-j8**) **Ω**. At a frequency of 4 krad/s, what inductance should be placed in parallel with the network to cause the input impedance to:
   a) have zero reactance?
   b) have a magnitude of 4 **Ω**?

5. Using the loop current method, find the current '**I**' in the circuit shown in **Fig. 5**. Assume ω= 10,000 rad/s.

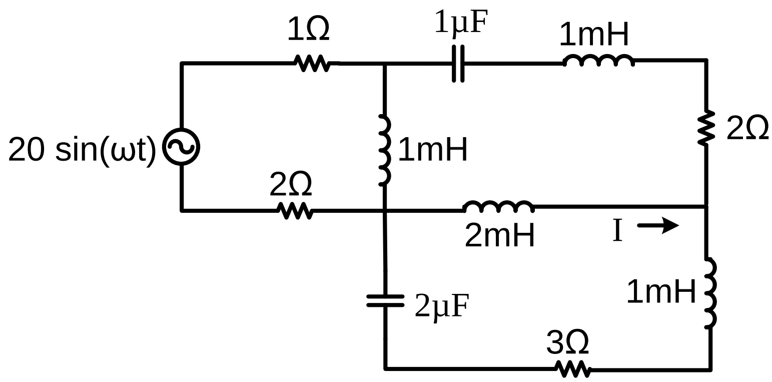


Fig. 5

6. For the circuit shown in **Fig. 6**, $v_1 = 10\angle 0°$ V and $i_1 = 10\angle 90°$ mA at ω = 10^6 rad/s. If the circuit is in steady state, find the current '***i*(t)**' through the inductor.

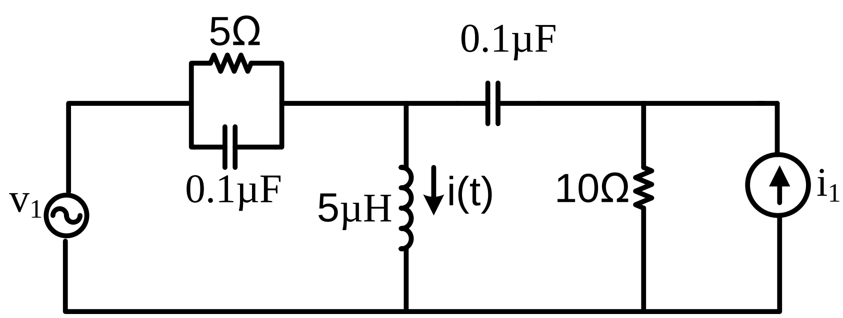
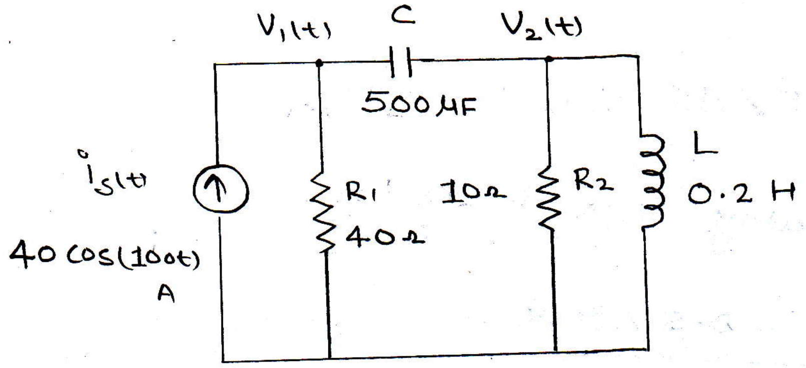


Fig. 6

7. In a series R-L-C circuit derive the expressions for frequency (a) at which the magnitude of voltage across the capacitor (**V**$_c$) is maximum and (b) at which the magnitude of voltage across the inductor (**V**$_L$) is maximum.

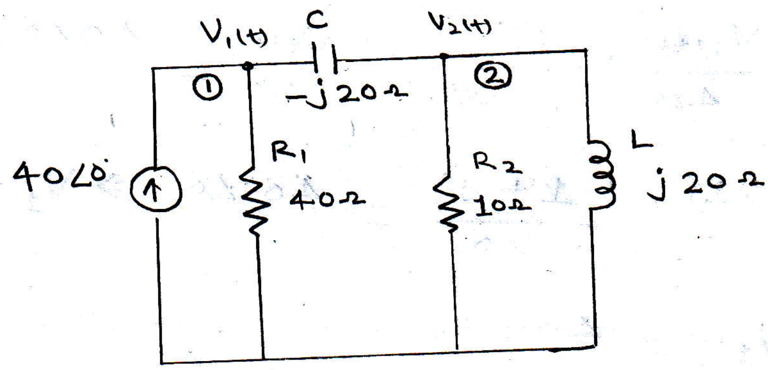
Soln. 1



$\because \omega = 100$ rad/s. $\therefore X_L = \omega L = 100 \times 0.2 = 20\,\Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{10^6}{100 \times 500} = 20\,\Omega.$

$\therefore Z_L = jX_L$; $Z_C = -jX_C$



⇒ Applying node voltage method at node 1 :-

$$-40\angle 0° + \frac{V_1(t)}{40} + \frac{V_1(t) - V_2(t)}{-j20} = 0 \quad \text{— i)}$$

⇒ Applying node voltage method at node 2 :-

$$\Rightarrow \frac{V_2(t)}{10} + \frac{V_2(t)}{j20} + \frac{V_2(t) - V_1(t)}{-j20} = 0$$

$$\Rightarrow \frac{V_2(t)}{10} + \frac{V_2(t)}{j20} = \frac{V_2(t) - V_1(t)}{j20} \quad \text{— ii)}$$

$$\Rightarrow \frac{V_2(t)}{10} + \cancel{\frac{V_2(t)}{j20}} = \cancel{\frac{V_2(t)}{j20}} - \frac{V_1(t)}{j20}$$

$$\Rightarrow \frac{V_2(t)}{10} = -\frac{1}{j20} V_1(t) \quad \Rightarrow V_2(t) = j0.5\, V_1(t)$$

$\Rightarrow \quad V_2(t) = 0.5\angle 90^\circ \; V_1(t) \quad$ — iii)

$\Rightarrow$ on Substituting $V_2(t)$ in eq. i)

$$\Rightarrow \frac{V_1(t)}{40} + \frac{V_1(t) - 0.5\angle 90^\circ V_1(t)}{-j20} = 40\angle 0^\circ$$

$$\Rightarrow \frac{V_1(t)}{40} + \frac{jV_1(t)}{20} - \frac{j\,0.5\;V_1(t)}{-j20} = 40\angle 0^\circ$$

$$\Rightarrow \frac{V_1(t)}{40} + \frac{jV_1(t)}{20} + \frac{V_1(t)}{40} = 40\angle 0^\circ \Rightarrow \frac{V_1(t)}{20} + \frac{jV_1(t)}{20} = 40\angle 0^\circ.$$

$$\Rightarrow V_1(t)\frac{(1+j)}{20} = 40\angle 0^\circ \quad \Rightarrow V_1(t) = \frac{800\angle 0^\circ}{(1+j)}$$

$$\Rightarrow V_1(t) = 400(1-j) = 565.68\angle -45^\circ \text{ Volt}$$

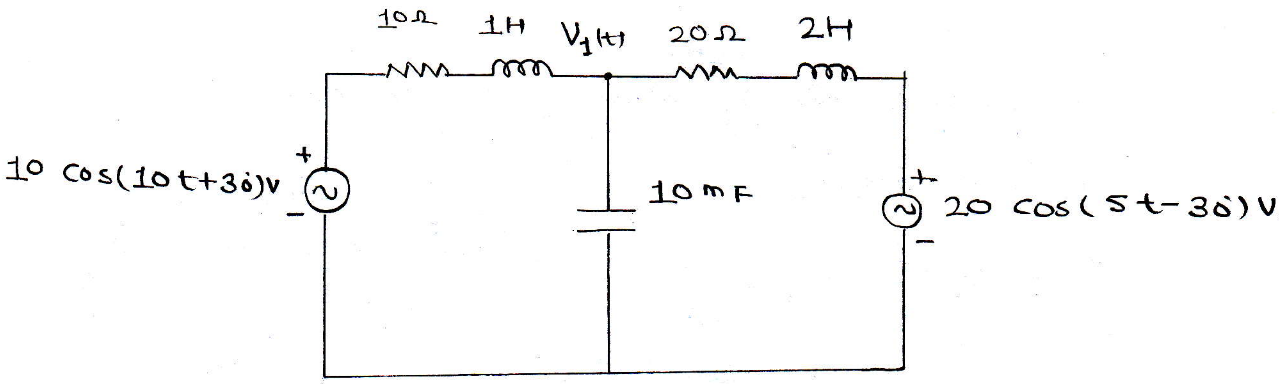
From eq. iii), $V_2(t) = 0.5\angle 90^\circ V_1(t) = 0.5 \times 565.68\ \angle 90^\circ - 45^\circ$

$V_2(t) = 282.84\angle 45^\circ$ Volt

$$\Rightarrow V_1(t) = 565.68\cos(\omega t - 45^\circ) \text{ V.}$$

$$V_2(t) = 282.84\cos(\omega t + 45^\circ) \text{ V.}$$

SOLN. 2



Applying Superposition Theorm :-

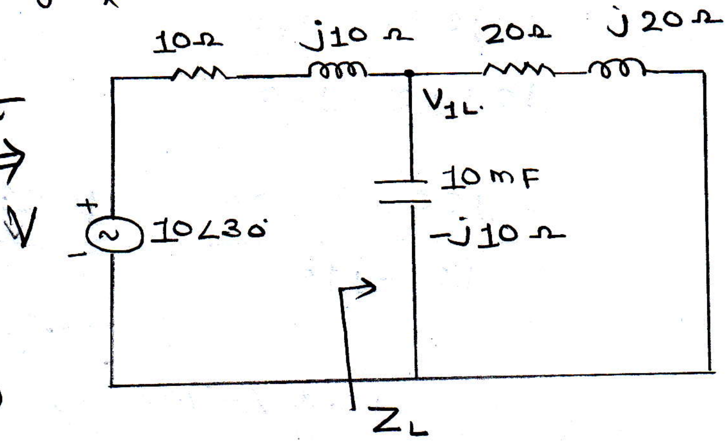
i) Take left voltage source only, right voltage source is shorted.

$\because \omega = 10$ rad/s

$X_L = j\omega L$

$X_C = \dfrac{-j}{\omega C}$

The equivalent circuit ⇒



$$\Rightarrow Z_L = (20+j20) \parallel (-j10)$$

$$= \frac{(20+j20)\times(-j10)}{(20+j20-j10)} = \frac{200-j200}{(20+j10)} = \frac{20(1-j)}{(2+j)}$$

$$\Rightarrow Z_L = \frac{20(1-j)(2-j)}{5} = 4[2-j-2j-1] = 4(1-3j) = 4-12j$$

By voltage division

$$\Rightarrow V_{1L} = \frac{Z_L}{Z_L+10+j10} \times 10\angle 30^\circ = \frac{(4-j12)}{4-j12+10+j10} \times 10\angle 30^\circ$$

$$\Rightarrow V_{1L} = \frac{12.65\angle -71.57^\circ \times 10\angle 30^\circ}{14-j2} = \frac{126.5\angle -41.57^\circ}{14.142\angle -8.13^\circ}$$

$$V_{1L} = 8.944\angle -33.44^\circ$$

$$V_{1L}(t) = 8.944 \cos(10t-33.44^\circ)\ \text{V}$$

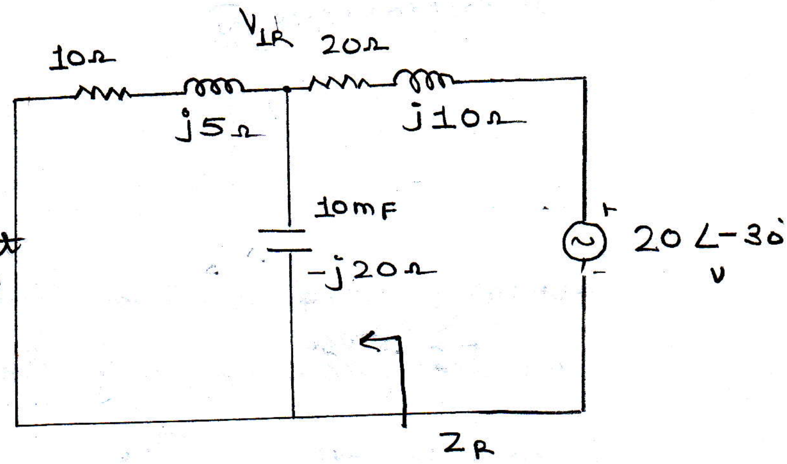
ii) Now take right voltage source, shorted out left voltage source.

$\omega = 5$ rad/s

$X_L = \omega L \, ; \quad X_c = \dfrac{1}{\omega c}$

Equivalent Circuit

$\Rightarrow Z_R = (10+j5) \parallel (-j20) \Rightarrow$



$$= \frac{(10+j5)\times(-j20)}{10+j5-j20}$$

$$= \frac{-j200+100}{10-j15} = \frac{(100-j200)(10+j15)}{100+225} = \frac{1}{325}\left[1000-j2000+j1500+3000\right]$$

$$= \frac{1}{325}\left[4000-j500\right] = 12.3 - j1.536$$

$= 12.4 \angle -7.125^\circ$.

$$\therefore V_{1R} = \frac{12.4\angle -7.12^\circ}{12.3 - j1.536 + 20 + j10} \times 20\angle -30^\circ$$

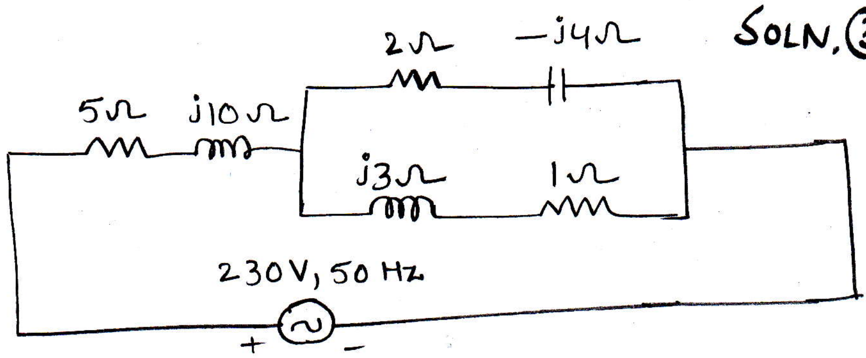
$$= \frac{248\angle -37.12^\circ}{33.38\angle 14.67} = 7.43 \angle -51.8^\circ \text{ V}.$$

$V_{1R}(t) = 7.43 \cos(5t - 51.8^\circ)$ V.

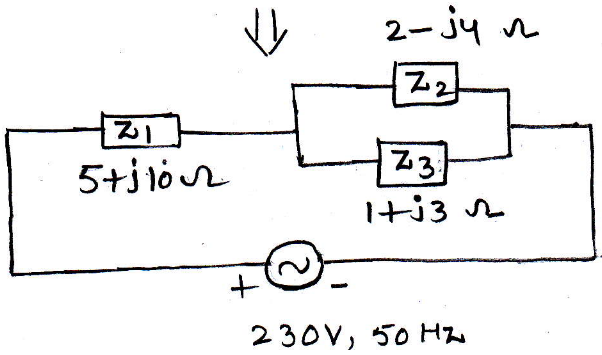
$\therefore$ Net $V_1(t) = V_{1L}(t) + V_{1R}(t)$

$= 8.944 \cos(10t - 33.44^\circ) + 7.43 \cos(5t - 51.8^\circ)$ V.

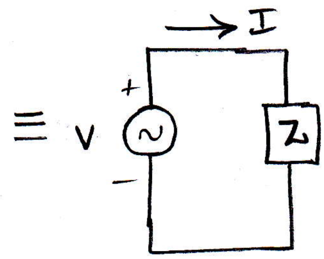
SOLN. 3



⇓



≡



Here,

$$Z = Z_1 + (Z_2 \parallel Z_3) = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= 5 + j10 + \frac{(2-j4)(1+j3)}{2-j4+1+j3}$$

$$= 5+10j + \frac{14+2j}{3-j} = 5+10j + \frac{(14+2j)(3+j)}{10}$$

$$= 5+10j + \frac{1}{10}[42+6j+14j-2]$$

$$= 5+10j+4+2j = 9+j12\ \Omega$$

& $V_{rms} = 230$ Volts

Then,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{9+j12} = \frac{230(9-j12)}{225} = 9.2 - j12.27$$

$$= 15.33 \angle -53.13^\circ \text{ Amp}$$

which implies that $\Rightarrow \theta = 53.13^\circ$

∴ Average Power, $P_{avg} = V_{rms} I_{rms} \cos\theta = 230 \times 15.33 \times \cos 53.13^\circ$

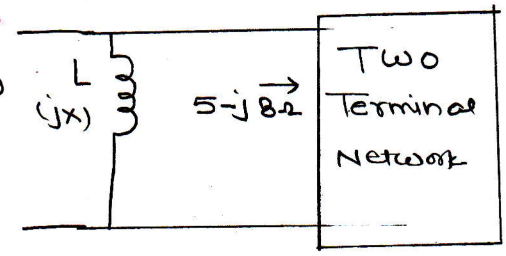
$$= 2115.54 \text{ W}$$

Reactive Power, $P_{react} = V_{rms} I_{rms} \sin\theta = 230 \times 15.33 \times \sin 53.13^\circ$

$$= 2820.72 \text{ W}$$

SOLN. 4

$\therefore \omega = 4k$ rad/s.



$$Z_{in} = jX \| (5-j8)$$

a) $\therefore Z_{in} = \dfrac{(jX)\times(5-j8)}{5-j8+jX}$

$$= \frac{8X+j5X}{5+j(X-8)} = \frac{8X+j5X}{5+j(X-8)} \times \frac{5-j(X-8)}{5-j(X-8)}$$

$$= \frac{1}{25+(X-8)^2}\left[40X+j25X-j8X(X-8)+5X(X-8)\right] = R_{in}+jX_{in}$$

$\because$ Now, $X_{in}=0$ for zero reactance in input impedance.

$\Rightarrow 25X - 8X(X-8) = 0 \Rightarrow 25 = 8(X-8)$ $[\because X \neq 0]$

$\Rightarrow X = 8 + \dfrac{25}{8} = 11.125\ \Omega.$

$L = \dfrac{X}{\omega} = \dfrac{11.125}{4000}\ H = 2.781\ mH$

b) $\therefore |Z_{in}| = 4$ $\left[\because Z_{in} = \dfrac{8X+j5X}{5+j(X-8)}\right]$

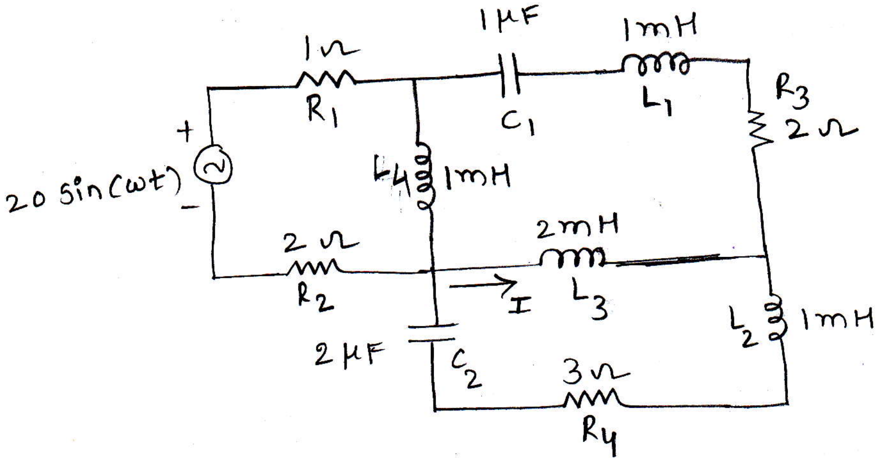
$\Rightarrow \dfrac{64X^2+25X^2}{25+(X-8)^2} = 4^2$

$\Rightarrow \dfrac{89X^2}{X^2-16X+89} = 16 \Rightarrow 73X^2+256X-1424=0$

$\Rightarrow X = 2.9986, -6.505$

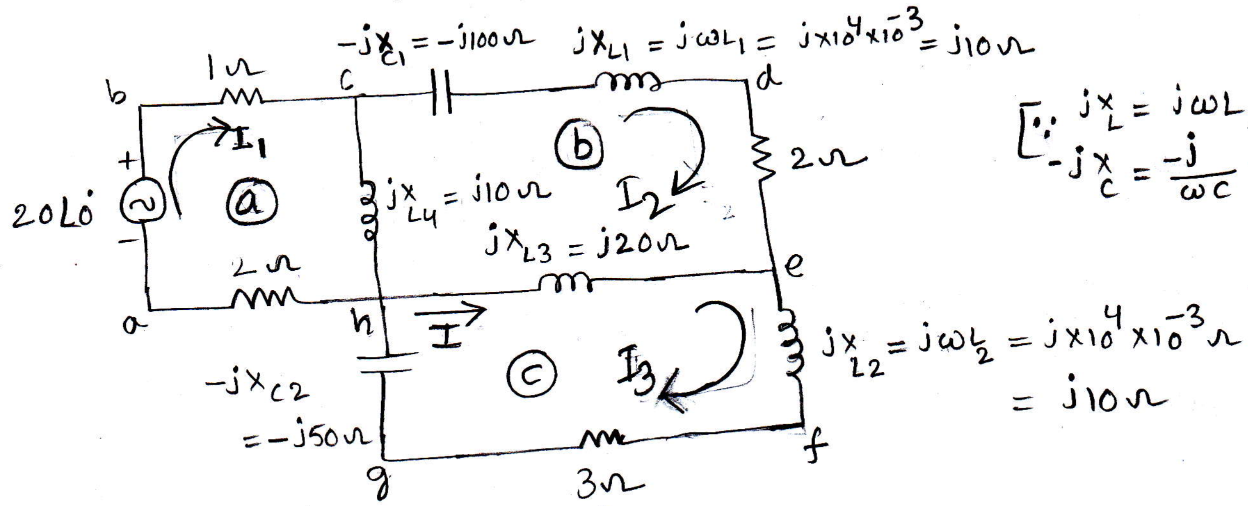
$\Rightarrow X = 2.9986\ \Omega$ ($\because$ Inductance has to be placed parallel to the network, so only positive X is taken)

$\Rightarrow L = \dfrac{X}{\omega} = 0.7496\ mH$



ω = 10,000 rad/s

By converting capacitors and inductors into their equivalent impedances, we get the following circuit



$$\left[\because\ jX_L = j\omega L,\ \ -jX_C = \frac{-j}{\omega C}\right]$$

Now writing the loop equations,

Applying KVL in loop abcha gives,

$$-20 + I_1 + j10(I_1 - I_2) + 2I_1 = 0$$

$$\Rightarrow (3+10j)I_1 - j10I_2 = 20 \quad \text{—— (1)}$$

Applying KVL in loop cdehc gives,

$$(-j100 + j10 + 2)I_2 + j20(I_2 - I_3) + j10(I_2 - I_1) = 0$$

$$\Rightarrow -j10I_1 + (2 - j60)I_2 - j20I_3 = 0 \quad \text{—— (2)}$$

Applying KVL in loop efghe gives,

$$(j10 + 3 - j50)I_3 + j20(I_3 - I_2) = 0$$

$$\Rightarrow -j20I_2 + (3 - j20)I_3 = 0 \quad \text{—— (3)}$$

$$\Rightarrow I_2 = \left(\frac{3-j20}{j20}\right)I_3 = (-1 - j0.15)I_3 \quad \text{—— (4)}$$

from eqn ① we can write that,

$$I_1 = \frac{20 + j10\,I_2}{3+10j} \quad \text{— ⑤}$$

Putting ⑤ in ②,

$$-j10\left(\frac{20+j10\,I_2}{3+10j}\right) + (2-j60)I_2 - j20\,I_3 = 0$$

$$\Rightarrow I_2\left[2-j60+\frac{100}{3+10j}\right] - j20\,I_3 = \frac{200j}{3+10j}$$

$$\Rightarrow I_2(2-j60+2.752-j9.174) - j20\,I_3 = 18.35+j5.5$$

$$\Rightarrow I_2(4.752-j69.174) - 20j\,I_3 = 18.35+j5.5 \quad \text{— ⑥}$$

Putting ④ into ⑥,

$$\Rightarrow -20j\,I_3 + (4.752-j69.174)(-1-j0.15)I_3 = 18.35+j5.5$$

$$\Rightarrow -20j\,I_3 + (-15.128+j68.46)I_3 = 18.35+j5.5$$

$$\Rightarrow I_3 = \frac{18.35+j5.5}{-15.128+j48.46} = -4.293\times10^{-3} - j0.377\text{ A.}$$

Then from ④,

$$I_2 = (-1-j0.15)\times(-4.293\times10^{-3}-j0.377)$$

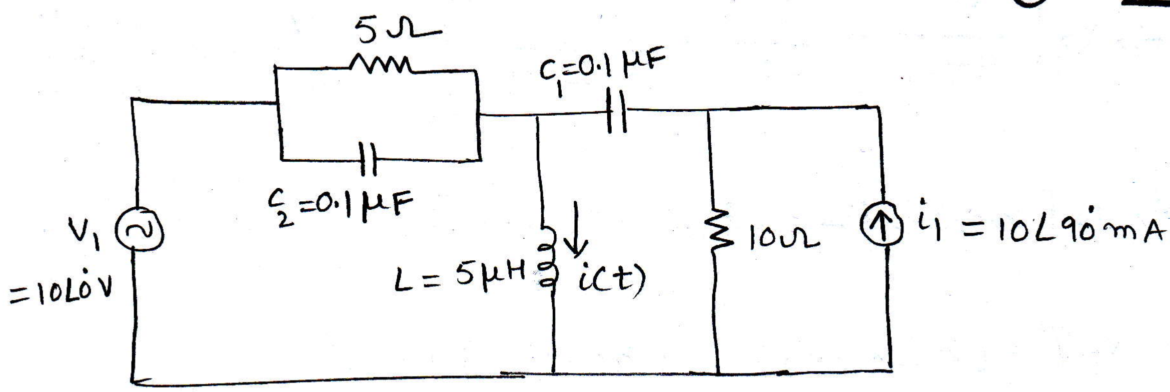
$$= -0.0523 + j\,0.378\text{ A.}$$

Now the desired current is,

$$I = I_3 - I_2 = -4.293\times10^{-3} - j0.377 + 0.0523 - j0.378$$

$$= 0.048 - j0.755 = 0.756\angle -86.36^\circ\text{ A.}$$

$$\therefore \boxed{I = 0.756\angle -86.36^\circ\text{ A}}$$

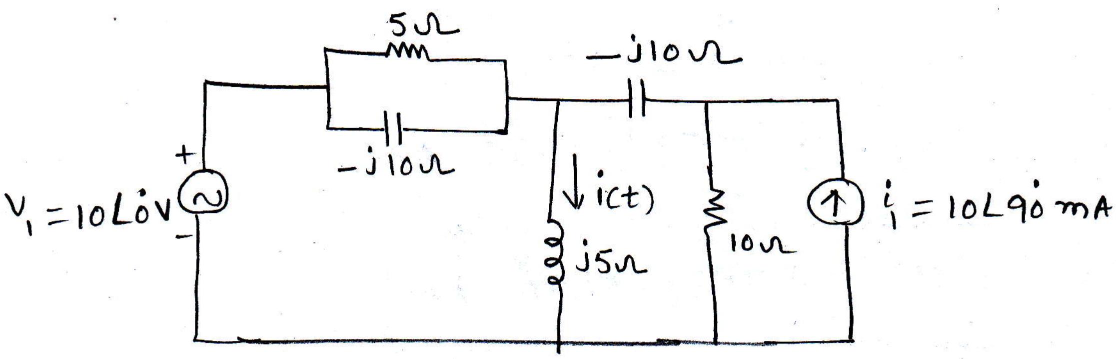


$$\omega = 10^6 \text{ rad/s}$$

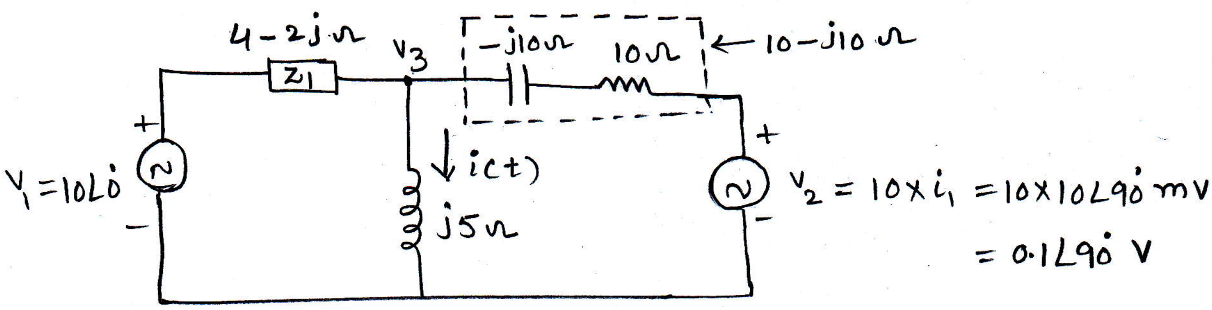
$$X_L = \omega L = 10^6 \times 5\times 10^{-6} = 5\,\Omega$$

$$X_{C_i} = \frac{1}{\omega C_i} = \frac{1}{10^6 \times 0.1\times 10^{-6}} = 10\,\Omega \quad (\because i = 1, 2)$$

Then the circuit reduces to,



⇓ (from Source Transformation Theorem)



Here $Z_1 = 5 \parallel (-j10) = \dfrac{5\times -10j}{5-10j} = 4-2j\,\Omega$

Using the Nodal Analysis for the above circuit, we write that

$$\frac{V_3 - 10\angle 0°}{4-2j} + \frac{V_3}{5j} + \frac{V_3 - 0.1\angle 90°}{10-10j} = 0$$

$$\Rightarrow V_3\left[\frac{1}{4-2j}+\frac{1}{5j}+\frac{1}{10-j10}\right]=\frac{10\angle 0^\circ}{4-j2}+\frac{0.1\angle 90^\circ}{10-j10}$$

$$\Rightarrow V_3[0.2+0.1j-0.2j+0.05+0.05j]=.2+j1-5\times10^{-3}+j5\times10^{-3}$$

$$\Rightarrow V_3(0.25-0.05j)=1.995+j\,1.005$$

$$\Rightarrow V_3=\frac{1.995+j1.005}{0.25-0.05j}=6.9+j5.4 \text{ volts}$$

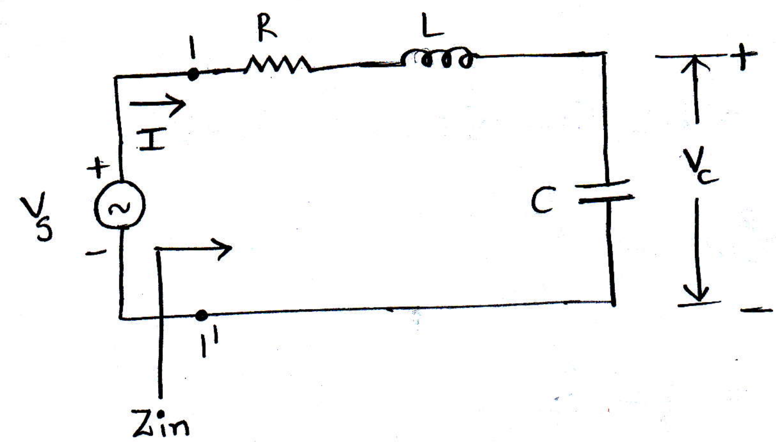
Now,

$$i(t)=\frac{V_3}{j5}=\frac{6.9+j5.4}{j5}$$

$$=1.08-j1.38$$

$$=1.753\angle-51.95^\circ \text{ Amp}$$

$$\therefore \boxed{i(t)=1.753\angle-51.95^\circ \text{ Amp}}$$



impedance looking at the terminals 1-1' is,

$$Z_{in} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$$\Rightarrow |Z_{in}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

from the figure,

$$|I| = \frac{|V_S|}{|Z_{in}|} = \frac{|V_S|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(a) voltage across the capacitor is,

$$|V_c| = |jX_c| \cdot |I| = \frac{|V_S|}{\omega C\left[\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\right]}$$

$$= \frac{|V_S|}{\sqrt{R^2 \omega^2 C^2 + (\omega^2 LC - 1)^2}}$$

$|V_c|$ is maximum when, $\left.\frac{d|V_c|}{d\omega}\right|_{\omega = \omega_c} = 0$

$$\Rightarrow \left.\frac{d|V_c|}{d\omega}\right|_{\omega=\omega_c} = -\frac{1}{2} \cdot \frac{|V_S|\left[2\omega_c R^2 C^2 + 2(\omega_c^2 LC - 1)\cdot 2\omega_c LC\right]}{\left[R^2 \omega_c^2 C^2 + (\omega_c^2 LC - 1)^2\right]^{3/2}} = 0$$

$$\Rightarrow 2\omega_c R^2 C^2 + 2(\omega_c^2 LC - 1)\cdot 2\omega_c LC = 0$$

$$\Rightarrow 2\omega_c R^2 C^2 = 4\omega_c LC(1 - \omega_c^2 LC)$$

$$\Rightarrow R^2C = 2L(1-\omega_c^2 LC) \quad [\because \omega_c C \neq 0]$$

$$\Rightarrow 1-\omega_c^2 LC = \frac{R^2C}{2L}$$

$$\Rightarrow \omega_c^2 LC = 1 - \frac{R^2C}{2L}$$

$$\Rightarrow \omega_c^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\Rightarrow \omega_c = 2\pi f_c = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\therefore \boxed{f_c = \frac{1}{2\pi}\left[\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}\right] \text{ Hz.}}$$

(b) voltage across the inductor,

$$|V_L| = |jX_L|\cdot|I| = \frac{\omega L\cdot|V_S|}{\sqrt{R^2+(\omega L - \frac{1}{\omega C})^2}}$$

$|V_L|$ is maximum when $\left.\frac{d|V_L|}{d\omega}\right|_{\omega=\omega_L} = 0$

$$\Rightarrow \left.\frac{d|V_L|}{d\omega}\right|_{\omega=\omega_L} = \frac{L\cdot|V_S|}{\sqrt{R^2+(\omega_L L - \frac{1}{\omega_L C})^2}} - \frac{\omega_L L|V_S|\cdot 2\left[\omega_L L - \frac{1}{\omega_L C}\right]\left[L+\frac{1}{\omega_L^2 C}\right]}{2\left\{R^2+(\omega_L L - \frac{1}{\omega_L C})^2\right\}^{3/2}}$$

$$\Rightarrow \frac{L|V_S|}{\sqrt{R^2+(\omega_L L - \frac{1}{\omega_L C})^2}} = \frac{\omega_L L|V_S|\left[\omega_L L - \frac{1}{\omega_L C}\right]\left[L+\frac{1}{\omega_L^2 C}\right]}{\left[R^2+(\omega_L L - \frac{1}{\omega_L C})^2\right]^{3/2}}$$

$$\Rightarrow \quad R^2 + \left(\omega_L L - \frac{1}{\omega_L C}\right)^2 = \left(\omega_L L - \frac{1}{\omega_L C}\right)\left(\omega_L L + \frac{1}{\omega_L C}\right)$$

$$\Rightarrow R^2 + \omega_L^2 L^2 + \frac{1}{\omega_L^2 C^2} - \frac{2L}{C} = \omega_L^2 L^2 - \frac{1}{\omega_L^2 C^2}$$

$$\Rightarrow -R^2 + \frac{2L}{C} = \frac{2}{\omega_L^2 C^2}$$

$$\Rightarrow \omega_L^2 C^2 = \frac{2}{\frac{2L}{C} - R^2}$$

$$\Rightarrow \omega_L^2 = \frac{1}{LC - \frac{R^2 C^2}{2}}$$

$$\Rightarrow \omega_L = 2\pi f_L = \frac{1}{\sqrt{LC - \frac{R^2 C^2}{2}}}$$

$$\boxed{\therefore \; f_L = \frac{1}{2\pi\sqrt{LC - \frac{R^2 C^2}{2}}} \; \text{Hz}}$$