29th September, 2016

Home Assignment – 8

1. For the circuit shown in **Fig. 1,** determine the **labeled node voltages**. The NMOS transistors are identical having $\mathbf{V}_t$ = 1 V and $k_n' \frac{W}{L} = 2\ mA/\ V^2$.

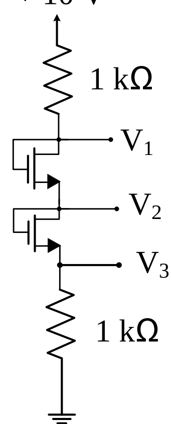


Fig. 1

2. A **Common Source** amplifier is shown in **Fig. 2**.
   (a) Verify that biasing of NMOS is proper ($\mathbf{V_t}$ = 1 V and $k_n' \frac{W}{L} = 2\ mA/\ V^2$).
   (b) Find $\mathbf{g_m}$ and $\mathbf{r_o}$ if $\mathbf{V_A}$ = 100 V.
   (c) Draw the complete small signal equivalent circuit for the amplifier.
   (d) Find $\mathbf{R_{in}}$, $\boldsymbol{v_{gs}} / \boldsymbol{v_{sig}}$,$\boldsymbol{v_o} / \boldsymbol{v_{gs}}$ *and* $\boldsymbol{v_o} / \boldsymbol{v_{sig}}$ and $\mathbf{R_{out}}$.

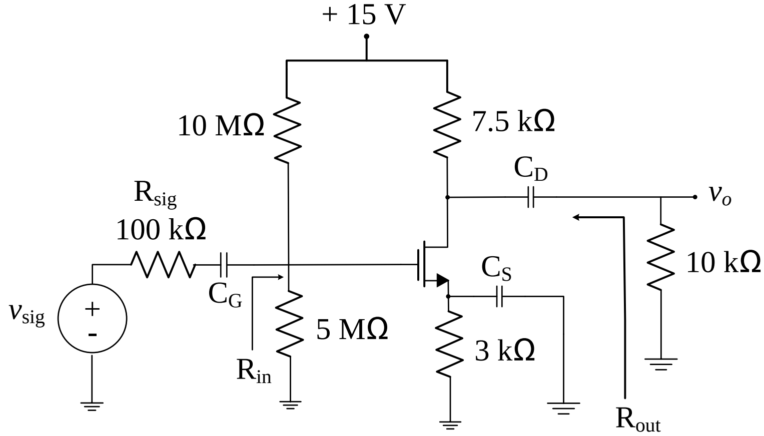


Fig. 2

3. The NMOS transistor shown in **Fig. 3,** has $\mathbf{V_t}$ = 0.9 V and $\mathbf{V_A}$ = 50 V and operates with $\mathbf{V_D}$ = 2 V. What is the **voltage gain** v_0 / v_i ? What do '$\mathbf{V_D}$' and "**gain**" become when '**I**' is increased to 1 mA?

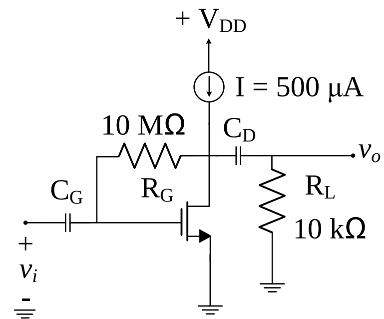


Fig. 3

4. Calculate the overall voltage gain $\mathbf{G_v}$ **($\mathbf{v_o}$ / $\mathbf{v_{sig}}$)** of a common source amplifier, shown in **Fig. 4**, which has g_m = 2mA/V, r_o = 50 kΩ, R_D = 10 kΩ and R_G = 10 MΩ. The amplifier is fed from a signal source with a resistance, $\mathbf{R_{sig}}$ of 0.5 kΩ and amplifier output is coupled to a load resistance, $\mathbf{R_L}$ of 20 kΩ.

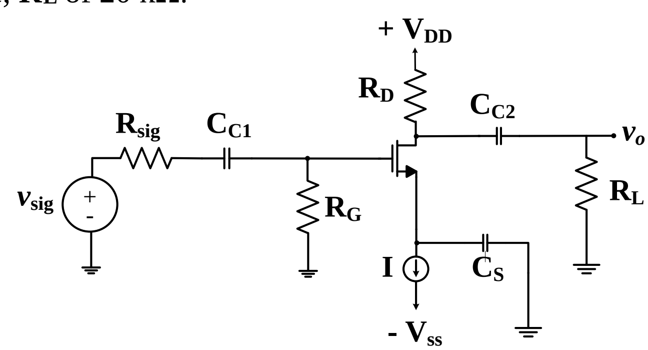


Fig. 4

5. The overall voltage gain of the amplifier shown in **Fig. 5** was measured with resistance $\mathbf{R_s}$ of 1kΩ in place and found to be **-10**. When $\mathbf{R_s}$ is shorted, but the circuit operation remained linear the gain doubled. What must g_m be? What value of $\mathbf{R_s}$ is needed to obtain an overall voltage gain of **-8**?

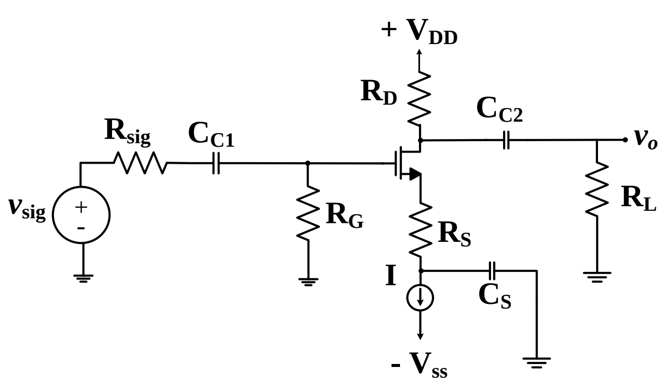
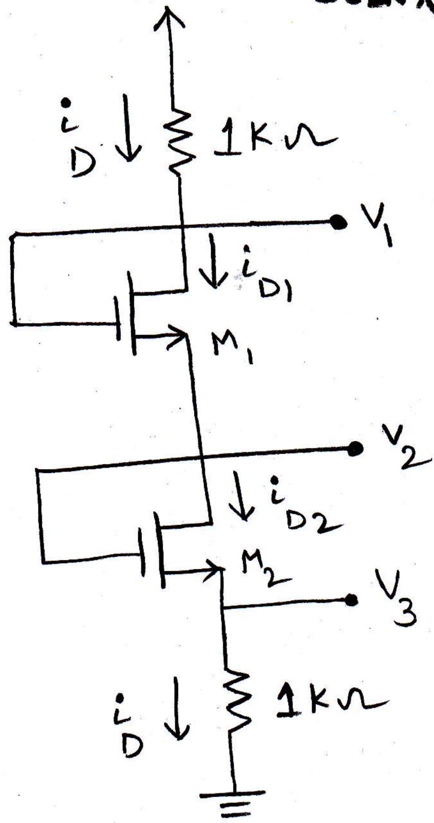


Fig. 5

+10V SOLN. 1 P-1



$V_t = 1V$

$k_n' \frac{W}{L} = 2\ mA/V^2$
$(= K,\ \text{say})$

if we consider both the devices M_1 & M_2 to be identical, then

$$i_{D1} = i_{D2} = i_D \quad \text{--- (1)}$$

where $i_{D1} = \frac{1}{2}K(V_{GS1} - V_t)^2$
$i_{D2} = \frac{1}{2}K(V_{GS2} - V_t)^2$

$$\Rightarrow V_{GS1} = V_{GS2}$$

$$\Rightarrow V_1 - V_2 = V_2 - V_3$$

$$\Rightarrow V_3 = 2V_2 - V_1 \quad \text{--- (2)}$$

But,
$$i_D = \frac{V_3}{1000} = \frac{10 - V_1}{1000} \Rightarrow V_3 = 10 - V_1 \quad \text{--- (3)}$$

from (2) & (3),
$$10 - V_1 = 2V_2 - V_1 \Rightarrow 2V_2 = 10$$

$$\Rightarrow \boxed{V_2 = 5V}$$

from (1), take $i_{D2} = i_D$

$$\Rightarrow \frac{1}{2}K(V_{GS2} - V_t)^2 = \frac{V_3}{1000}$$

$$\Rightarrow \frac{1}{2} \times 2\times10^{-3} \times (V_2 - V_3 - 1)^2 = \frac{V_3}{1000}$$

$$\Rightarrow V_3 = (V_2 - V_3 - 1)^2$$

$$\Rightarrow V_3 = (5 - V_3 - 1)^2 = (4 - V_3)^2$$

$$\Rightarrow V_3 = V_3^2 - 8V_3 + 16$$

$$\Rightarrow V_3^2 - 9V_3 + 16 = 0$$

on solving $V_3 = 6.56\ V\ ;\ 2.44\ V$

i) when $V_3 = 6.56\ V$

from ③,

$$V_1 = 10 - V_3 = 10 - 6.56$$
$$= 3.44\ V$$

but

$$V_{GS1} = V_1 - V_2 = 3.44 - 5$$
$$= -1.56\ V$$

this is not feasible since 'V_{GS1}' must be atleast 'V_t', for i_{D1} to flow.

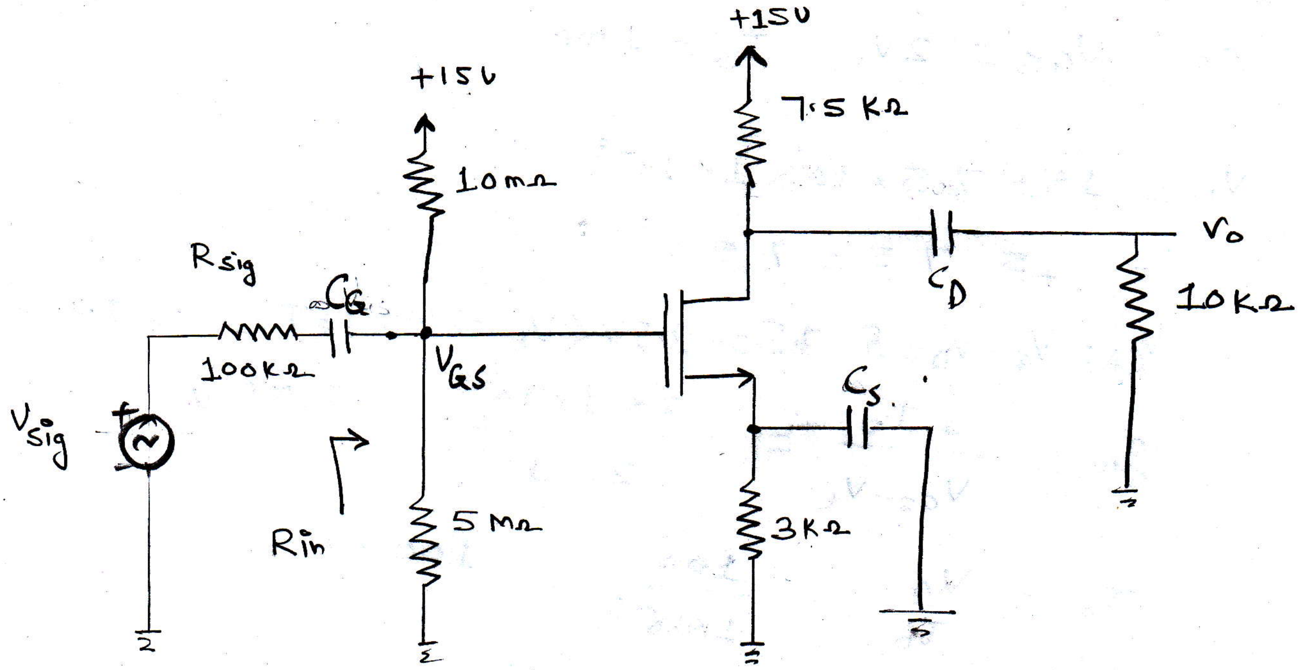
ii) when $V_3 = 2.44\ V$

$$\Rightarrow V_1 = 10 - V_3 \quad (\because \text{from ③})$$
$$= 10 - 2.44 = 7.56\ V$$

which is sufficient to make $V_{GS1} \geq V_t$.

$$\therefore \boxed{V_1 = 7.56\ V\ ;\ V_2 = 5V\ ;\ V_3 = 2.44\ V}$$

SOLN. 2 P-3



a) Given that $V_t = 1\,V$, $k_n' \frac{W}{L} = 2\,mA/V^2$.

$\therefore \quad V_G = 15 \times \dfrac{5 \times 10^6}{10 \times 10^6 + 5 \times 10^6} = 15 \times \dfrac{5}{15} = 5\,V.$

$V_{DD} = 15\,V, \qquad V_{GS} = V_G - V_S$

$\because V_S = 3 I_D \qquad \Rightarrow V_{GS} = 5 - 3 I_D$

$\therefore \quad I_D = \dfrac{k_n'}{2} \times \dfrac{W}{L} (V_{GS} - V_t)^2 = \dfrac{2}{2} \times (5 - 3 I_D - 1)^2$

$\Rightarrow \quad I_D = (4 - 3 I_D)^2 = 16 + 9 I_D^2 - 24 I_D$

$\Rightarrow \quad 9 I_D^2 - 25 I_D + 16 = 0$

$\Rightarrow \quad I_D = 1\,mA \quad$ or $\quad I_D = 1.77\,mA.$

$V_{GS} = 5 - 3 I_D$

$\therefore V_{GS} = 5 - 3 \times 1 \quad$ or $\quad V_{GS} = 5 - 3 \times 1.77$

$V_{GS} = 2V \quad$ or $\quad V_{GS} = -0.31\,V < V_t$

So $V_{GS} = 2V, \quad I_D = 1mA$

$\Rightarrow V_D = 15 - 7.5\times10^3 \times 1\times10^{-3}$

$= 15 - 7.5 = 7.5\ V.$

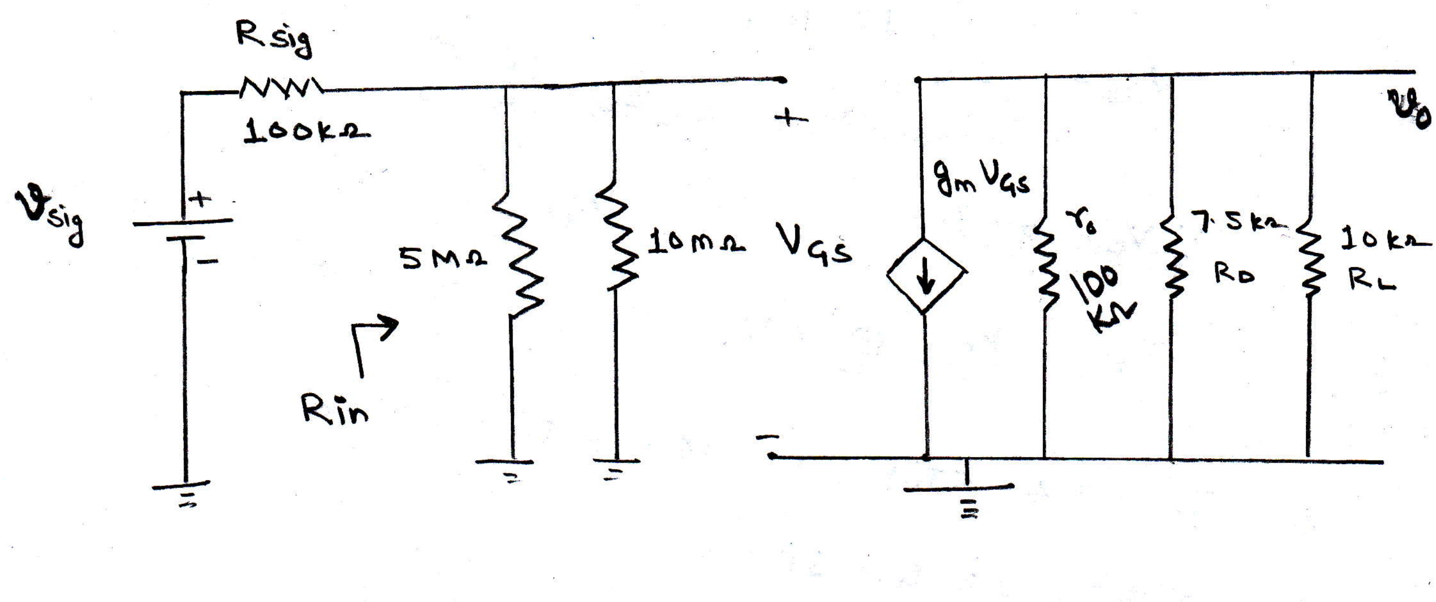
$V_{GD} = V_G - V_D = 5 - 7.5 = -2.5V < V_t \Rightarrow$ MOSFET is in saturation.

b) $\because \; g_m = \dfrac{2I_D}{V_{GS}-V_t} = \dfrac{2\times 1\times10^{-3}}{2-1} = 2\ mA/V.$

$r_o = \dfrac{V_A}{I_D} = \dfrac{100}{1\times10^{-3}} = 100\ k\Omega$

$\left(g_m = \dfrac{\partial I_D}{\partial V_{GS}} = \dfrac{\partial}{\partial V_{GS}}\left[\frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2\right] = k_n'\frac{W}{L}(V_{GS}-V_t) = \dfrac{2I_D}{V_{GS}-V_t} \right)$

c)



d) $R_{in} = 5M\Omega \,||\, 10m\Omega = \dfrac{5m\times10m}{5m+10m} = 3.33m\Omega$

$v_{gs} = v_{sig}\times\dfrac{R_{in}}{R_{in}+R_{sig}} \Rightarrow \dfrac{v_{gs}}{v_{sig}} = \dfrac{3.33\times10^6}{100\times10^3 + 3.33\times10^6}$

$$\Rightarrow \quad \frac{v_{gs}}{v_{sig}} = \frac{3.33}{0.1 + 3.33} = 0.97$$

$$\Rightarrow \because \; v_o = -g_m v_{gs} \, (r_o \,||\, R_D \,||\, R_L).$$

$$\Rightarrow \quad \frac{v_o}{v_{gs}} = -2 \times 10^{-3} \times 4.1 \times 10^{3} = -8.2$$

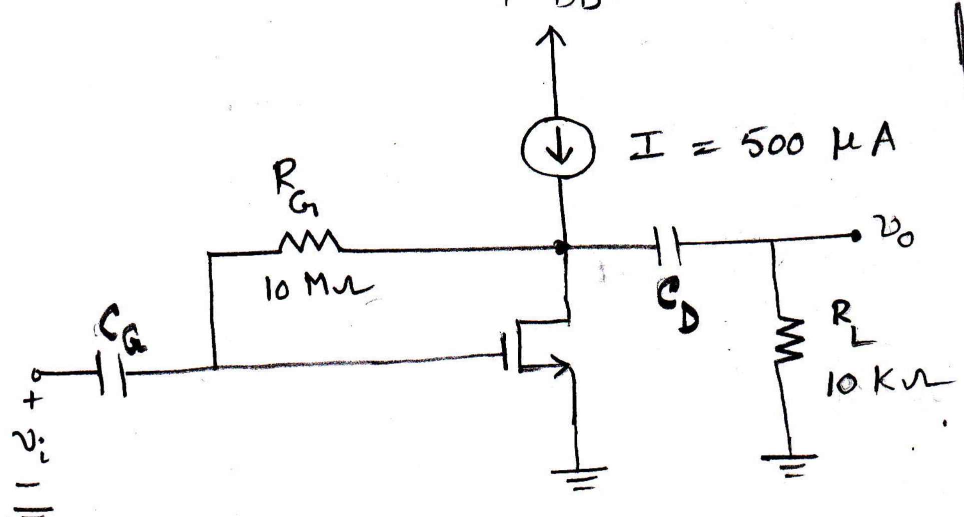
$$\Rightarrow \quad \frac{v_o}{v_{sig}} = \frac{v_o}{v_{gs}} \times \frac{v_{gs}}{v_{sig}} = -8.2 \times 0.97 = -7.95$$

$$R_{out} = r_o \,||\, R_D \,||\, R_L$$

$$= 100K\Omega \,||\, 7.5K\Omega \,||\, 10K\Omega$$

$$= 4.1 \; K\Omega.$$

SOLN ③ P-6



For MOSFET to operate in saturation,

$V_{GS} > V_t$ — (i)

and $V_{DS} > (V_{GS} - V_t)$ — (ii)

$\Rightarrow V_t > V_{GS} - V_{DS}$

$V_t > V_{GD}$

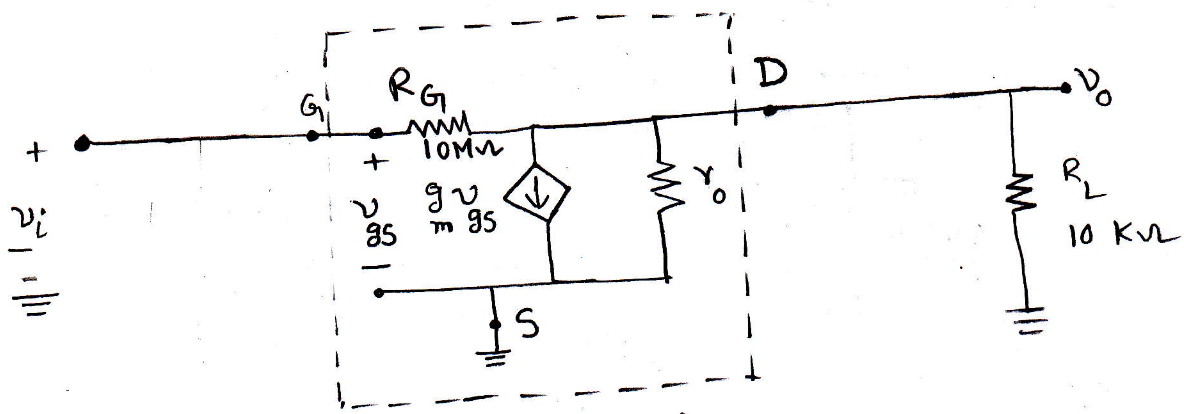
For this ckt $V_{GD} = 0$ as no current flow in R_G for DC analysis,

$V_t = 0.9\,V > (V_{GD} = 0)$

$(V_{GS} = V_{DS} = V_D = 2\,V) > V_t$

given $V_t = 0.9\,V$; $V_A = 50\,V$ with $V_D = 2\,V$

The ac equivalent circuit of the Amplifier is,



MOS Device

The Dotted box has,

$$r_o = \frac{V_A}{I_D} = \frac{50}{0.5 \times 10^{-3}} \qquad \left[\because \underset{\downarrow \text{ DC current}}{I_D} = I = 0.5\,mA\right]$$

$$= 100\ K\Omega$$

$$g_m = \frac{2 I_D}{V_{GS} - V_t} = \frac{2 \times 0.5 \times 10^{-3}}{2 - 0.9} \qquad \left[\because \text{Since "}R_G\text{" is large } V_{GS} = V_{DS} = V_D\right]$$

$$= \frac{10^{-3}}{1.1} = 0.91\ mA/V$$

$$\left[I_D = \frac{k}{2}(V_{GS} - V_t)^2 \Rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = K(V_{GS} - V_t) \Rightarrow g_m = \frac{2 I_D}{V_{GS} - V_t} \right]$$

from the figure,

$$v_o \approx -g_m v_{gs}\left[r_o \parallel R_L\right] \quad \text{as } R_G >> r_o \parallel R_D \parallel R_L$$

$$= -g_m v_i\left[r_o \parallel R_L\right]$$

$$\Rightarrow \frac{v_o}{v_i} = -g_m\left[r_o \parallel R_L\right]$$

$$= -0.91\times 10^{-3}\left[100K \parallel 10K\right]$$

$$= -0.91\times 10^{-3}\times\left[\frac{100\times 10}{100+10}\right]\times 10^{3}$$

$$\Rightarrow A_v = \frac{v_o}{v_i} = -8.27$$

$$\therefore \boxed{\text{ac Voltage gain} = \frac{v_o}{v_i} = -8.27}$$

When $I = 1mA$, (that is 'I' is doubled)

then,

$$\frac{I_{D,new}}{I_{D,old}} = \frac{\left[V_{GS,new}-V_t\right]^2}{\left[V_{GS,old}-V_t\right]^2} = 2$$

$$\Rightarrow V_{GS,new} = V_t + \sqrt{2}\left[V_{GS,old}-V_t\right]$$

$$= 0.9+\sqrt{2}\,(2-0.9)$$

$$= 0.9+1.56 = 2.46\,V = V_{D,new}$$

which implies,

$$\frac{g_{m,new}}{g_{m,old}} = \sqrt{\frac{I_{D,new}}{I_{D,old}}} = \sqrt{2} \quad \left[\begin{array}{l} I_D = \frac{1}{2}K(V_{GS}-V_t)^2 \\ g_m = \frac{\partial I_D}{\partial V_{GS}} = K(V_{GS}-V_t) \\ \therefore g_m \propto \sqrt{I_D} \end{array}\right]$$

$$\Rightarrow g_{m,new} = \sqrt{2}\times 0.91\,mA/V = 1.3\,mA/V$$

Also,

$$\frac{r_{o,new}}{r_{o,old}} = \frac{I_{D,old}}{I_{D,new}} = \frac{1}{2}$$

$$\Rightarrow r_{o,new} = \frac{r_{o,old}}{2} = \frac{100\text{ k}}{2} = 50\text{ k}\Omega$$

Then,

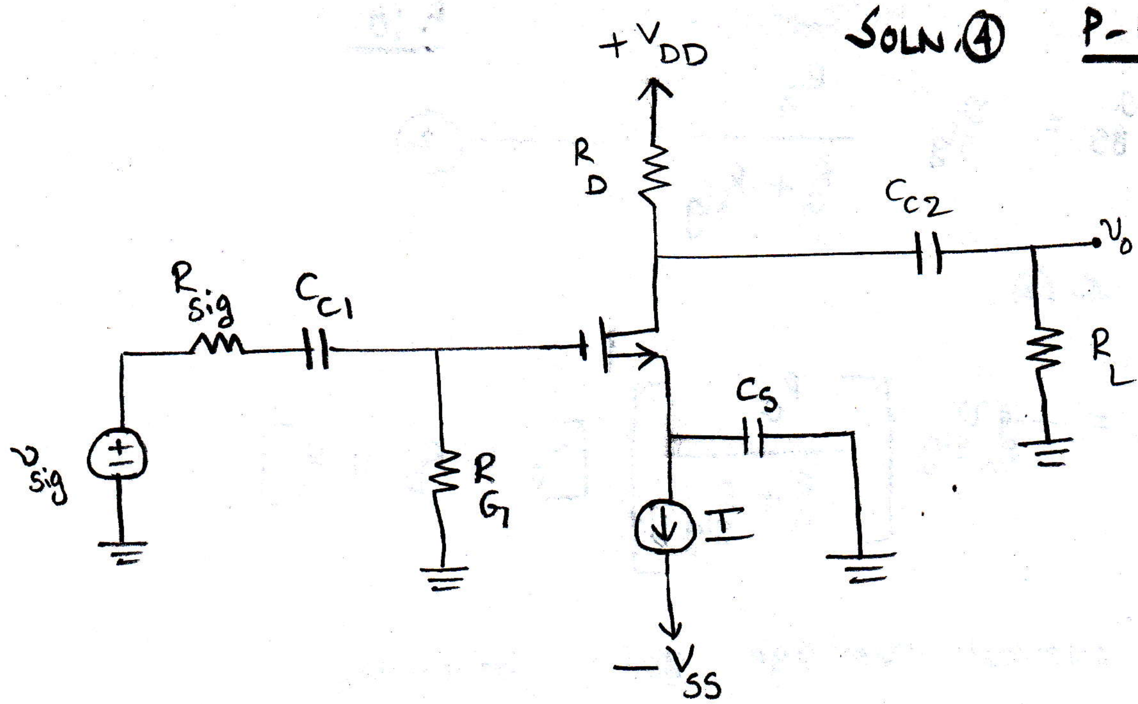
$$A_{v,new} = -g_{m,new}\left[r_{o,new} \parallel R_L\right]$$

$$= -1.3\times10^{-3} \times \frac{50K \times 10K}{50K + 10K}$$

$$= -1.3 \times \frac{500}{60} = -10.833 \text{ V/V}$$

$\therefore$

$$\boxed{\begin{aligned} V_{D,new} &= 2.46\text{ V} \\ A_{v,new} &= -10.833\text{ V/V} \end{aligned}}$$



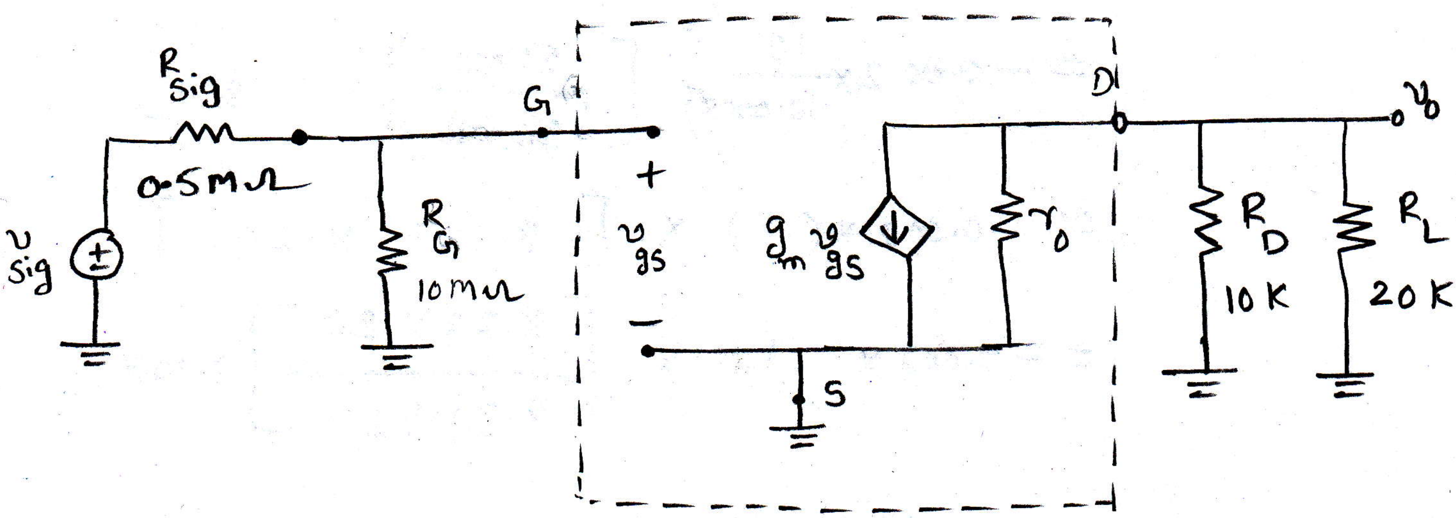
given

$g_m = 2\ mA/V$ $\quad R_D = 10\ K\Omega$

$r_o = 50\ k\Omega$ $\quad R_G = 10\ M\Omega$

$R_{sig} = 0.5\ M\Omega$ $\quad R_L = 20\ K\Omega$

The equivalent circuit of the CS Amplifier is,



mos device

from the figure,

$$v_o = -g_m v_{gs}\left[r_o \parallel R_D \parallel R_L\right] \quad \text{—— (1)}$$

$$v_{gs} = v_{sig} \frac{R_G}{R_G + R_{sig}} \quad \text{———} \quad (2)$$

from (1) & (2),

$$v_o = -g_m v_{sig} \left[\frac{R_G}{R_G + R_{sig}} \right] \left[r_o \parallel R_D \parallel R_L \right]$$

The overall voltage gain becomes,

$$G_v = \frac{v_o}{v_{sig}} = -\frac{g_m R_G}{R_G + R_{sig}} \left[r_o \parallel R_D \parallel R_L \right]$$

$$\Rightarrow G_v = -2 \times 10^{-3} \times \frac{10\,M}{10\,m + 0.5\,k} \left[50K \parallel 10K \parallel 20K \right]$$

$$= -0.002 \times \frac{10}{10.0005} \left\{ \left[\frac{50 \times 10}{50 + 10} \right] K \parallel 20K \right]$$

$$\simeq -0.002 \times (1) \times \left[8.33\,K \parallel 20\,K \right]$$

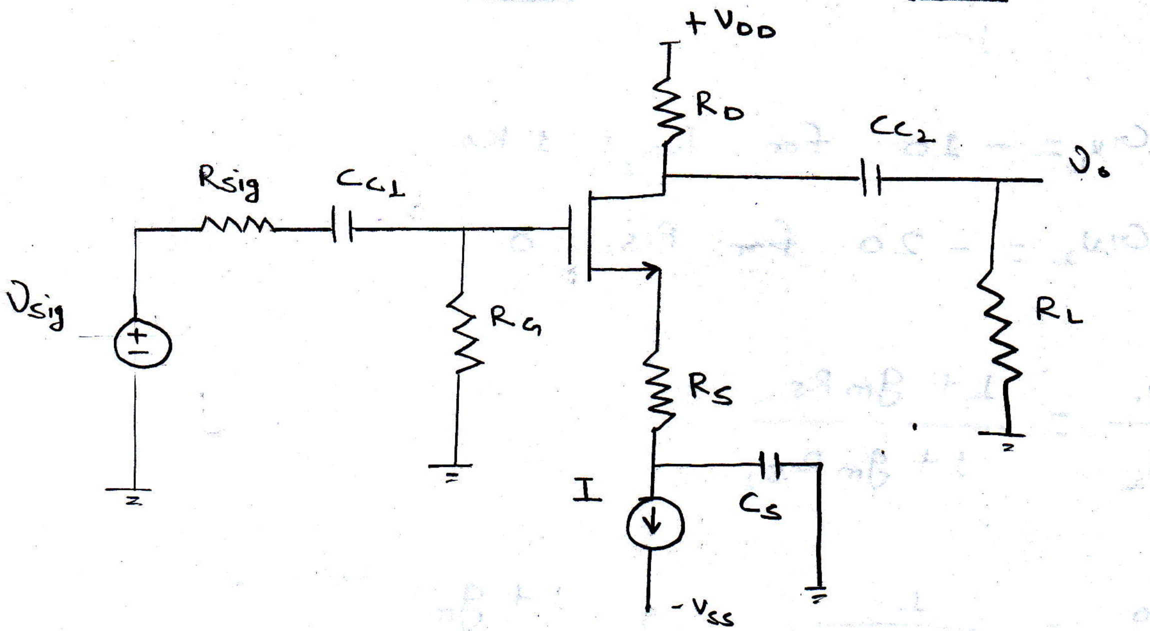
$$= -0.002 \times 1 \times \left[\frac{8.33 \times 20}{8.33 + 20} \right] \times 1000$$

$$= -0.002 \times 1 \times 5.882 \times 1000$$

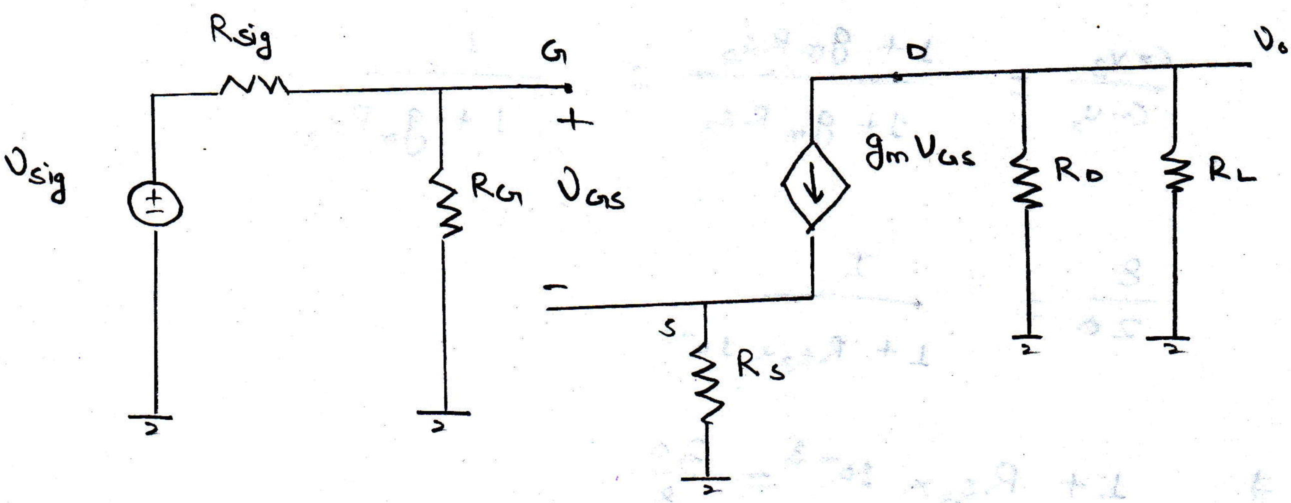
$$= -11.76$$

$$\therefore \boxed{G_v = -11.76\ V/V}$$

SOLN. 5



Equivalent circuit of Amplifier is.



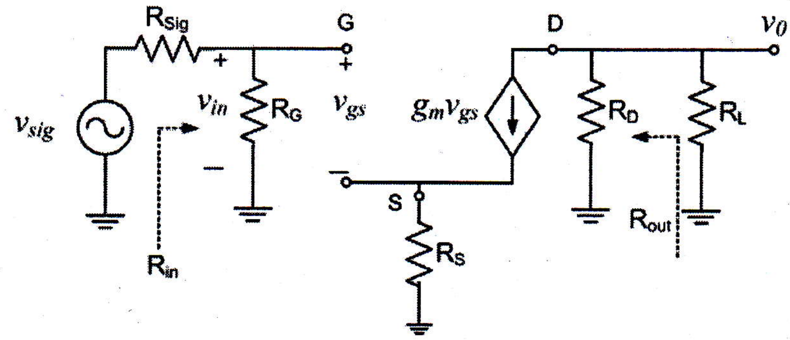
$$\therefore \quad G_V = -\frac{R_G}{R_G + R_{sig}} \times \frac{g_m (R_D \parallel R_L)}{1 + g_m R_S}$$

Small-signal Equivalent Circuit: Neglecting r_o

$$v_{in} = v_{gs} + g_m v_{gs} R_s$$

$$v_o = -g_m v_{gs} (R_D \parallel R_L)$$

Voltage gain $$A_v = \frac{v_o}{v_{in}} = -\frac{g_m (R_D \parallel R_L)}{1 + g_m R_s}$$



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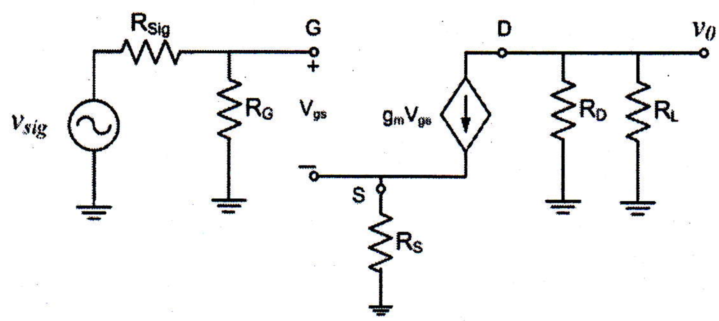
Overall voltage gain

$$v_{in} = \frac{R_G}{R_G + R_{sig}} v_{sig}$$

$$G_v = \frac{v_o}{v_{sig}} = -\frac{R_G}{R_G + R_{sig}} \frac{g_m (R_D \parallel R_L)}{1 + g_m R_s}$$



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$\because$

$\because$ $G_{v_1} = -10$ for $R_{s_1} = 1\,K\Omega$

$G_{v_2} = -20$ for $R_{s_2} = 0.$

$\therefore$ $$\frac{G_{v_1}}{G_{v_2}} = \frac{1+g_m R_{s_2}}{1+g_m R_{s_1}}$$

$$\Rightarrow \frac{10}{20} = \frac{1}{1+g_m\times 10^3} \Rightarrow 1+g_m\times 10^3 = 2$$

$$\Rightarrow g_m = 1\,mA/V.$$

$\Rightarrow$ $\because$ $G_{v_3} = -8.$

$$\Rightarrow \frac{G_{v_3}}{G_{v_2}} = \frac{1+g_m R_{s_2}}{1+g_m R_{s_3}} = \frac{1}{1+g_m R_{s_3}}$$

$$\Rightarrow \frac{8}{20} = \frac{1}{1+R_{s_3}\times 10^{-3}}$$

$$\Rightarrow 1+R_{s_3}\times 10^{-3} = \frac{20}{8}$$

$$\Rightarrow R_{s_3} = 1.5\,k\Omega.$$

$\therefore$ $R_s = 1.5\,K\Omega$ for voltage gain of -8.