11th August, 2016

# Home Assignment - 2

1. Obtain the Thevenin equivalent circuit at terminal '**a-b**' of the network shown in **Fig. 1**.

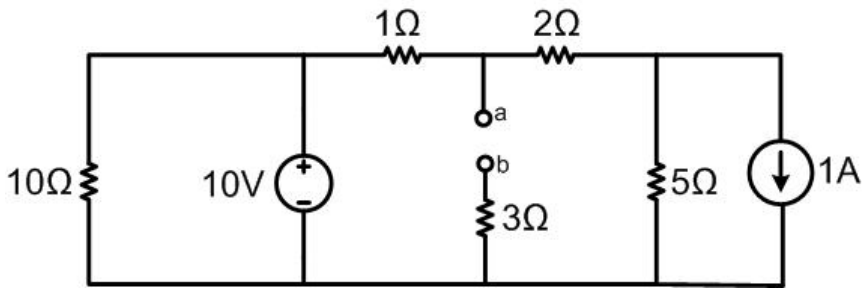



**Fig. 1**

2. Determine the Thevenin equivalent of the network shown in **Fig. 2** across terminals '**a**' and '**b**'.

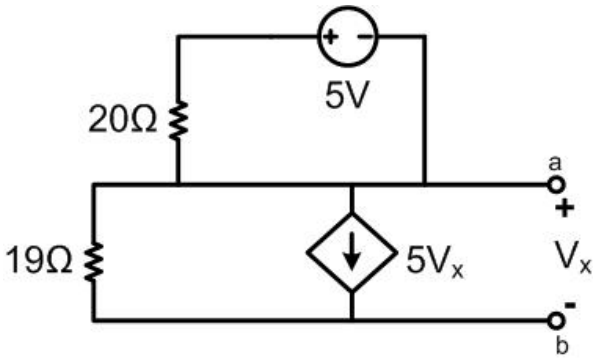



**Fig. 2**

3. For the circuit shown in **Fig. 3**, find '$R_L$' using Thevenin's theorem so that maximum power is transferred to it.

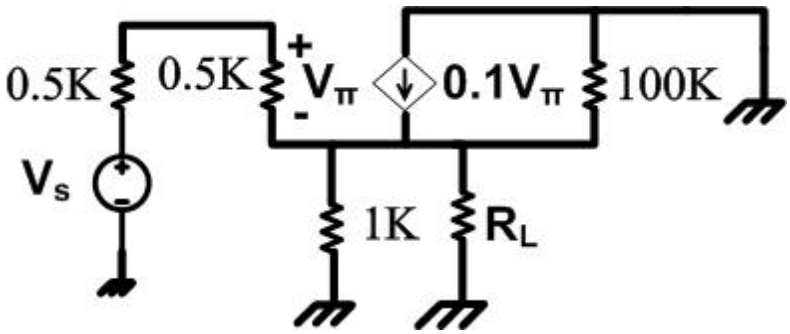



**Fig. 3**

4. For the circuit shown in **Fig. 4,** find the current through **1Ω** resistance using Norton's theorem.

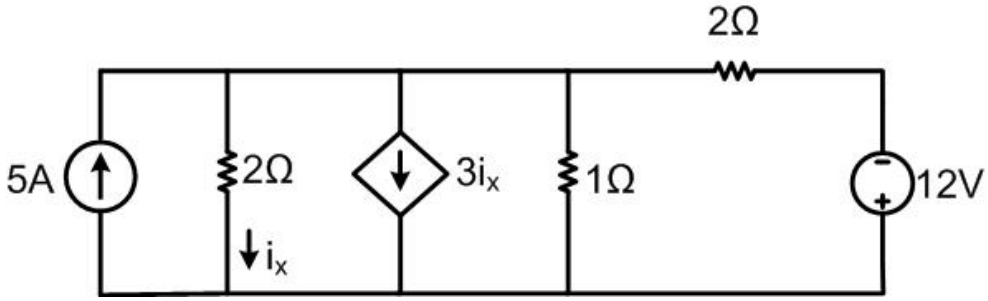



**Fig. 4**

5. Find the value of '$\mathbf{R_L}$' in **Fig. 5** such that maximum power can be transferred to the load '$\mathbf{R_L}$'.

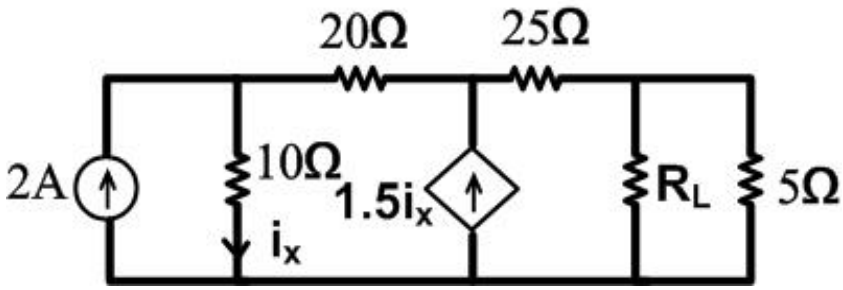



**Fig. 5**

6. For the circuit shown in **Fig. 6,** find current through inductor for **(i)** $t < 0$, **(ii)** $0 < t < 1$ sec, **(iii)** $t=1^-$ sec and **(iv)** $t > 1$ sec.

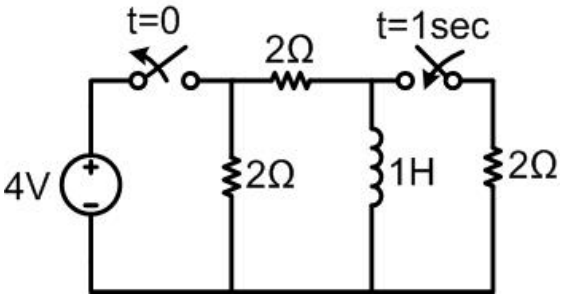



**Fig. 6**

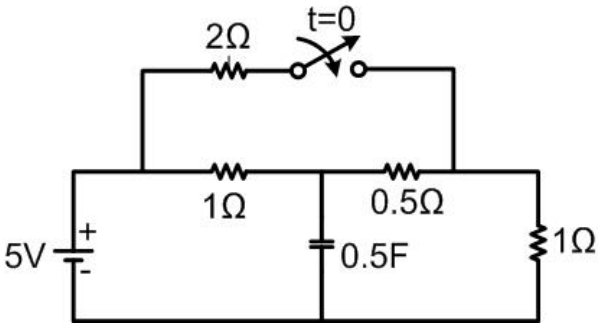
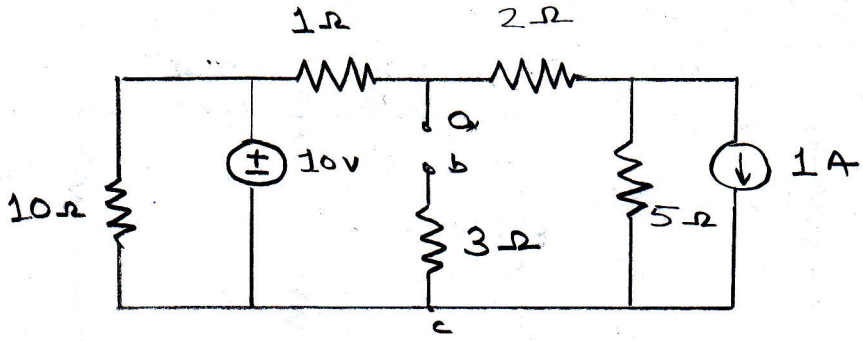
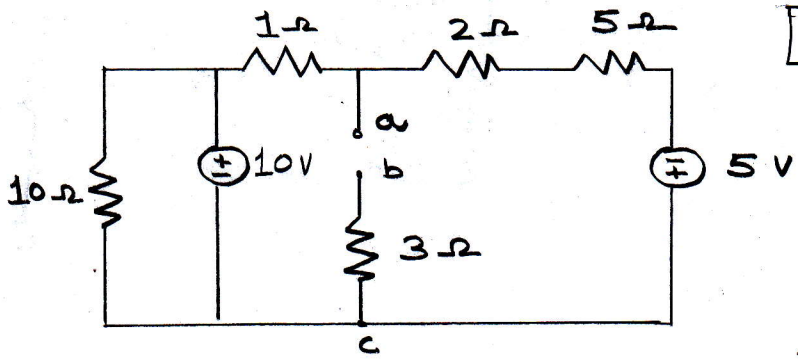



**Fig. 7**

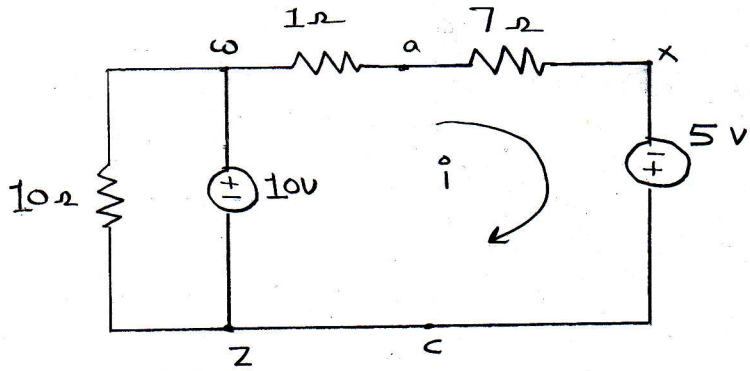
7. The switch shown in **Fig. 7** has been opened for a long time. At $t = 0$, the switch is closed. **(i)** Find the voltage across the capacitor at $t = 0^+$. **(ii)** Find the expression for current through the capacitor and voltage across the capacitor when $t > 0$ and sketch them.




## (i): $V_{TH}$ Calculation:-




[By Source Transformation of 1A Current Source and 5Ω resistance in parallel to it].




Applying KVL across loop (waxczw).

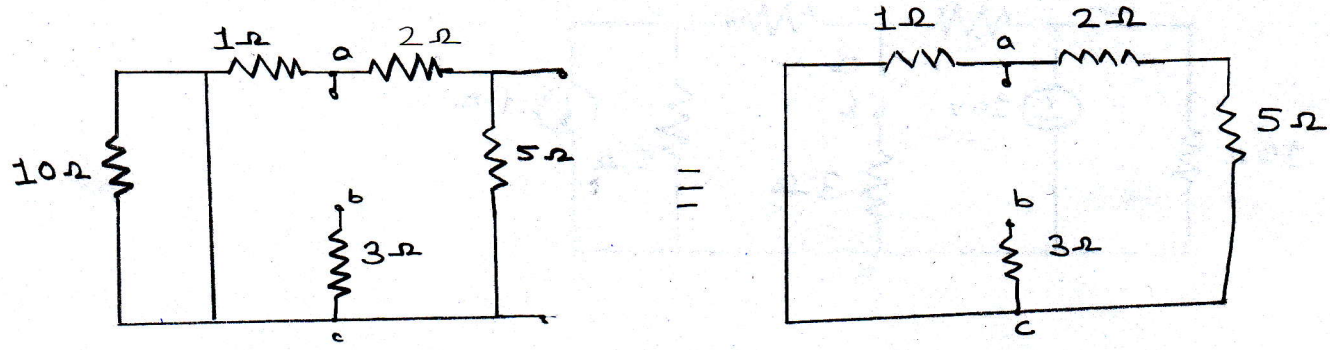
$\Rightarrow \quad i + 7i - 5 - 10 = 0$

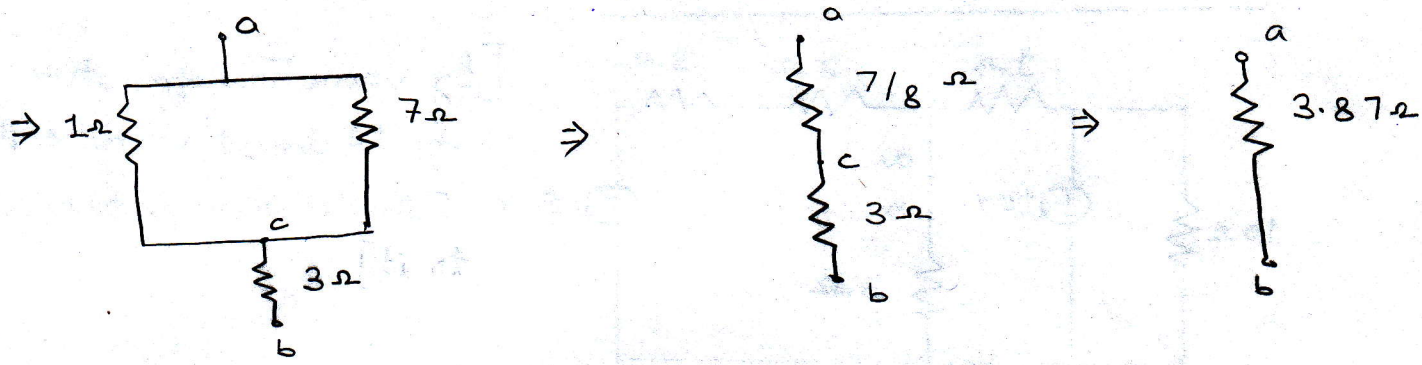
$\Rightarrow \quad 8i = 15 \quad \Rightarrow \quad i = 15/8 \text{ A}$

So, $V_{ac} = 10 - i.1 = 10 - \frac{15}{8}$

$\therefore V_{ab} = V_{ac} = \frac{65}{8} = 8.125 \text{ V.}$

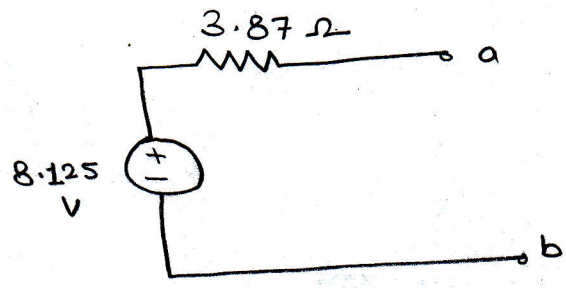
ii) $R_{Th}$ Calculation :-

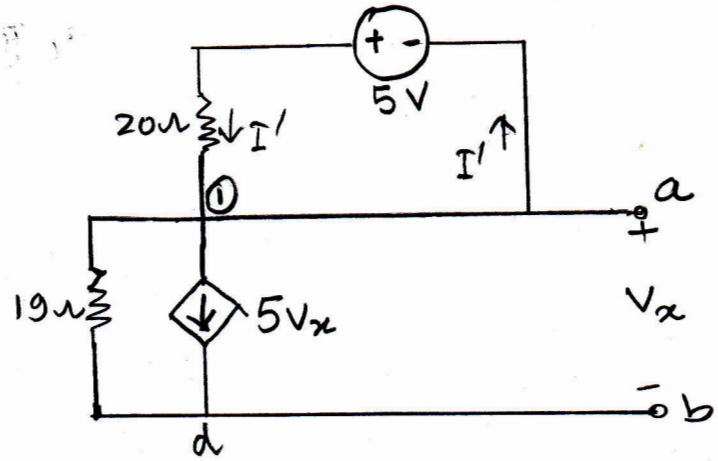



⇒ So $R_{Th} = 3.87\,\Omega$ ; $V_{Th} = 8.125$ V.

⇒ Thevenin equivalent circuit is:-






SOLN. ②

By observing the network

$V_{th} = V_x$

Applying nodal analysis at node (1) we get,

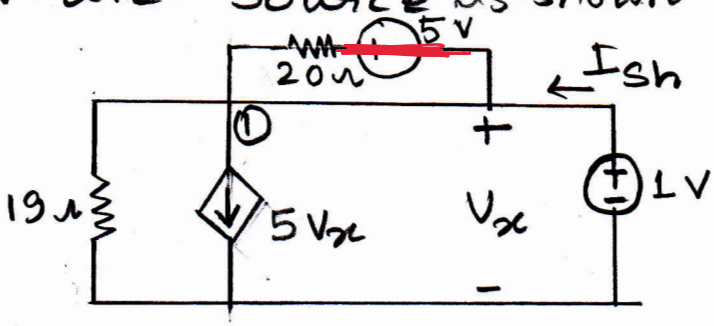
$$5V_x + \frac{V_x}{19} + I' - I' = 0.$$

$$\Rightarrow V_x = 0$$

$$\therefore V_{Th} = 0$$

Since this is a passive network, $R_{Th} \neq 0$

To determine $R_{Th}$ the network is excited by a 1V d.c. source as shown below.




Applying Nodal Analysis at node ①

$$\frac{V_x}{19} + 5V_x - I_{sh} = 0 \quad \text{------} \quad (a)$$

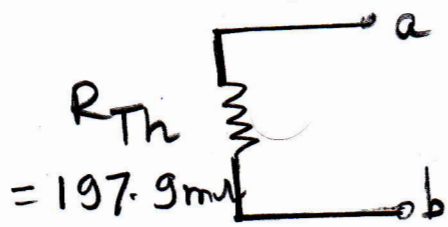
But, $V_x = 1\,V$

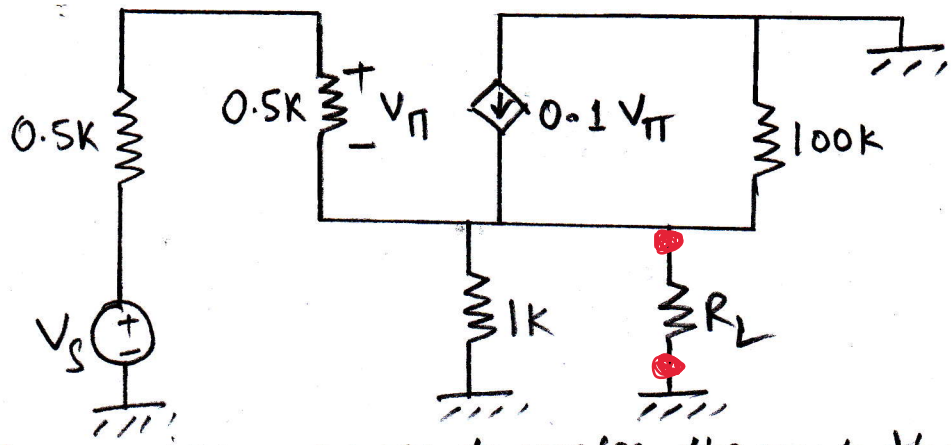
So, eqn (a) reduces to,

$$I_{sh} = 5 + \frac{1}{19} = 5.0526\ A$$

$$\therefore R_{Th} = \frac{V_x}{I_{sh}} = \frac{1}{5.0526} = 197.9\ m\Omega$$

Thevinin equivalent

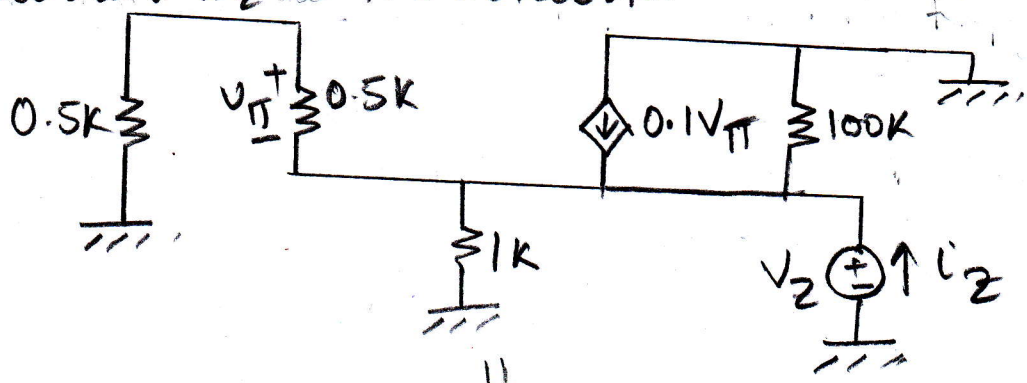




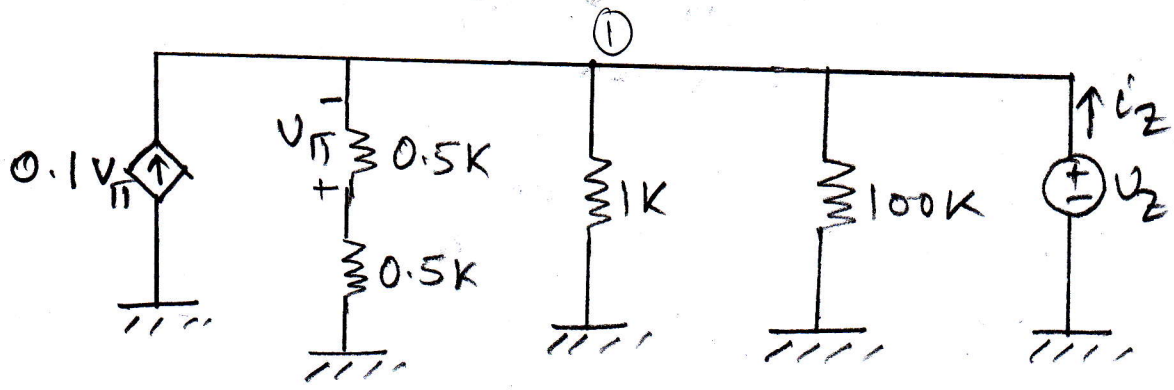

For maximum power transfer through the load $R_L$, $R_L = R_T$

$R_T$ = Thevinin Resistance across the load $R_L$.

$R_T$ can be determined by short circuiting the voltage source $V_s$ and replacing $R_L$ with voltage source $V_z$ which delivers current $I_z$ to the network.




$$R_T = \frac{V_z}{i_z}$$




Applying KCL at node 1:

$$-0.1V_\pi + \frac{V_z}{0.5K+0.5K} + \frac{V_z}{1K} + \frac{V_z}{100K} - i_z = 0$$

$$\Rightarrow -0.1V_\pi + \frac{2V_z}{1K} + \frac{V_z}{100K} - i_z = 0$$

$$\Rightarrow -0.1V_\pi + \frac{201V_z}{100K} - i_z = 0 \quad \text{---(i)}$$

ALSO, $V_\pi = \frac{-0.5V_z}{0.5+0.5} = -0.5V_z$ ---- (ii)

Substituting the value of $V_\pi$ from (ii) into (i) we get,

$$+0.1 \times 0.5 V_2 + \frac{201 V_2}{100K} - i_2 = 0$$

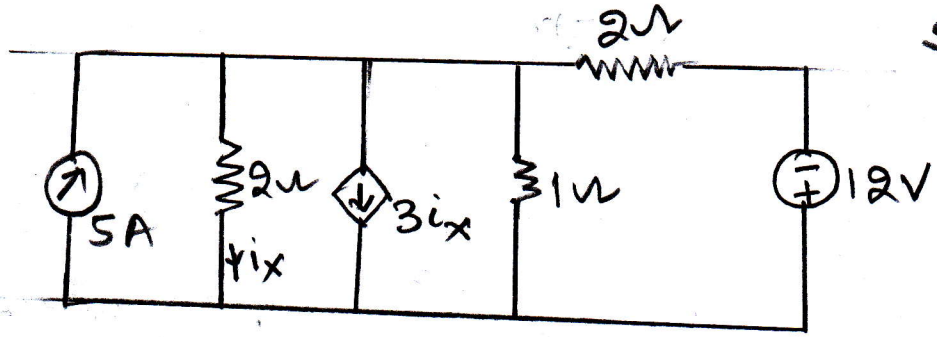
$$\Rightarrow \quad 0.05 V_2 + \frac{201 V_2}{100K} - i_2 = 0$$

$$\Rightarrow \quad V_2 (0.05 + 2.01 \times 10^{-3}) = i_2$$

$$\Rightarrow \quad \frac{V_2}{i_2} = \frac{1}{0.05 + 2.01 \times 10^{-3}}$$

$$R_T = 19.22\ \Omega.$$

$\therefore R_L = 19.22\ \Omega$ for maximum power transfer to the load.

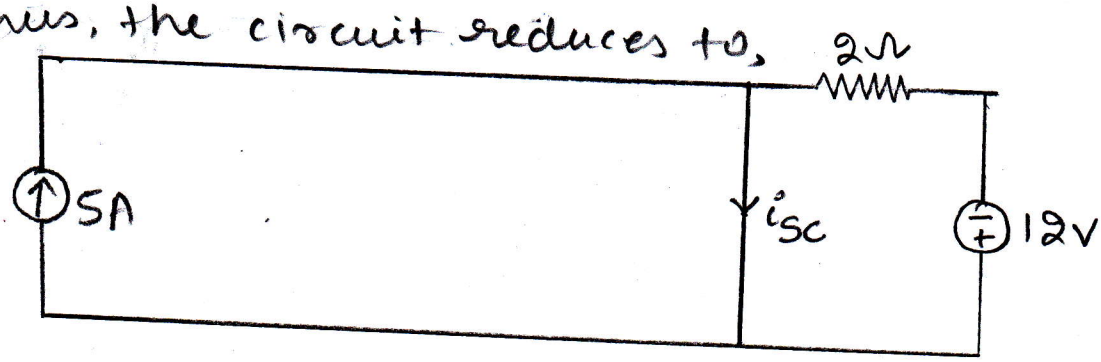



Finding $I_{sc}$: Replace 1Ω by short circuit

So, $i_x = 0$ through 2Ω resistance connected in shunt with the short circuit

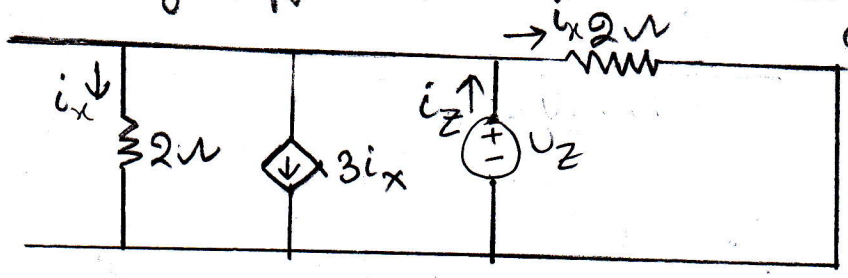
∴ Thus $3i_x = 0$

Thus, the circuit reduces to,




$$i_{sc} = 5 - \frac{12}{2} = -1\,A$$

Finding $R_N$: Replace the 5A current source by open circuit and 12V voltage source by short ckt (Same current $i_x$ will flow through both the 2Ω resistances)




$$i_z = 3i_x + i_x + i_x = 5i_x \qquad \left[\because i_x = \frac{V_z}{2}\right]$$

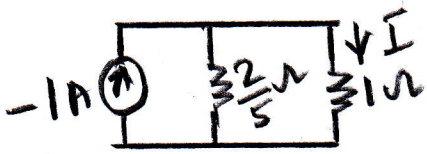
$$U_z = i_x \times 2 = 2i_x = \frac{2}{5} i_z$$

$$R_N = \frac{U_z}{i_z} = \frac{2}{5}\,\Omega$$

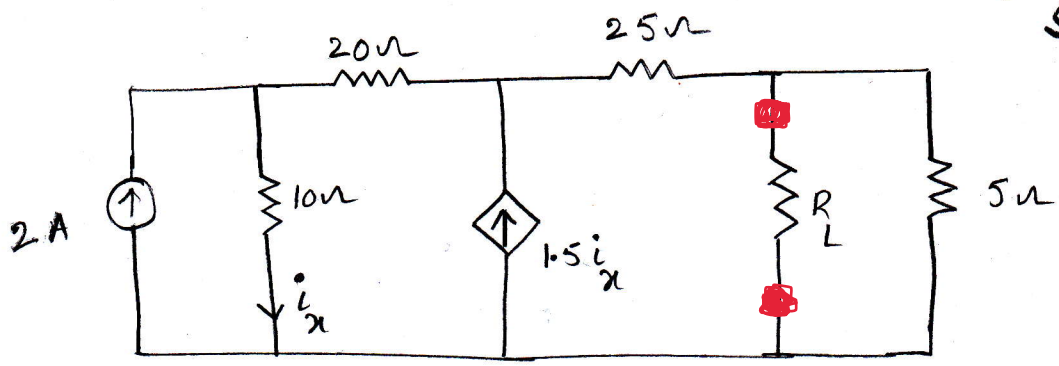
current through 1Ω $= I = i_{sc} \times \dfrac{R_N}{R_N + 1}$

$$= -1 \times \frac{2/5}{2/5 + 1}$$

$$= -2/7\ A.$$

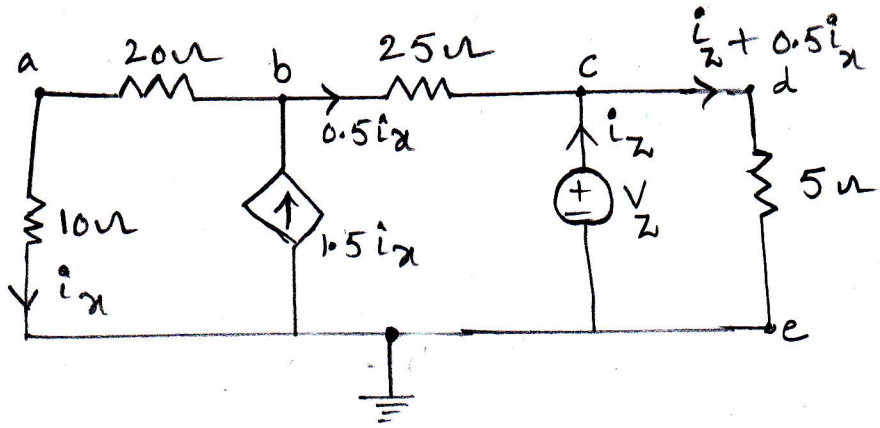



Norton equivalent circuit.




To find $R_{Th}$ we open circuit the '2A' current source and introduce, a voltage source '$V_z$' which produces a current '$i_z$', in place of the load resistor '$R_L$'.

Hence the circuit reduces to,




Apply KVL to the loop, "abcdea"

$$-10 i_x - 20 i_x + 25(0.5 i_x) + 5(i_z + 0.5 i_x) = 0$$

$$\Rightarrow -30 i_x + 12.5 i_x + 5 i_z + 2.5 i_x = 0$$

$$\Rightarrow 5 i_z - 15 i_x = 0$$

$$\Rightarrow i_z = \frac{15 i_x}{5} = 3 i_x \quad \text{——— (1)}$$

From the figure voltage across the 5Ω resistor is,

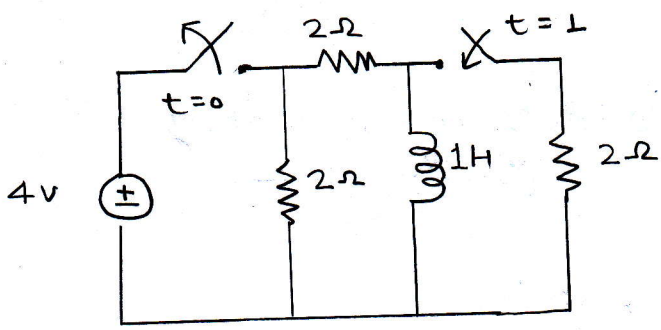
$$V_z = 5(i_z + 0.5 i_x)$$

$$= 5\left(i_z + 0.5\left(\frac{i_z}{3}\right)\right) \quad [\because \text{from (1)}]$$

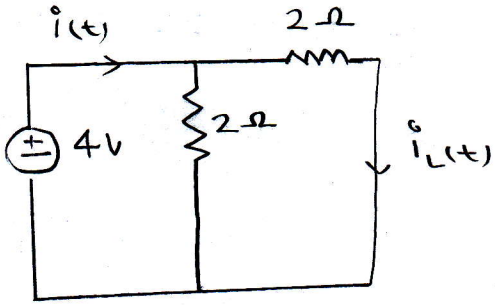
$$= \left(5 + \frac{2.5}{3}\right) i_z = 5.833\ V$$

$$\therefore \boxed{R_{Th} = \frac{V_z}{i_z} = 5.833\,\Omega} \Rightarrow \boxed{R_L = R_{Th}} = 5.833\,\Omega$$

for maximum power Transfer




i) At $t<0$ the inductor becomes short circuit & current through inductor is determined by current through the 2Ω resistance



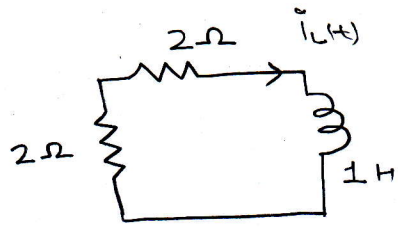

(t<0)

$$i(t) = \frac{4}{(2\|2)} = \frac{4}{1} = 4\text{ A.}$$

$$\therefore i_L(t<0) = 4 \times \frac{2}{2+2} = 2\text{ A.}$$

ii) $\because i_L(t<0) = 2\text{ A} = i_L(t=0^-) = i_L(t=0^+)$

The inductor current will decay with a time constant $(\tau = L/R)$




(0<t<1)

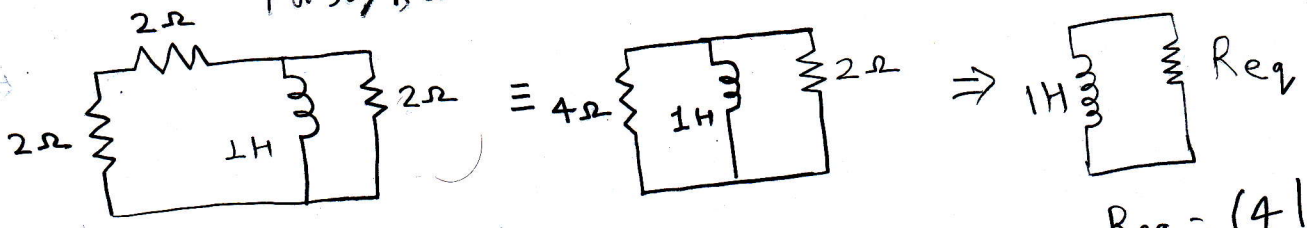
$$\tau = \frac{1}{(2+2)} = \frac{1}{4}\text{ sec.}$$

$$\therefore i_L(t) \text{ for } 0<t<1 = i_L(t=0^+)e^{-t/\tau} = 2e^{-4t}\text{ A.}$$

iii) $i_L(t=1^-) = 2e^{-4} = 0.0366\text{ A}$

iv) $\because i_L(t=1^+) = i_L(t=1^-) = 0.0366\text{ A}$

For $t>1$, the circuit can be redrawn as follows:




(t=1)

$$R_{eq} = (4\|2)$$

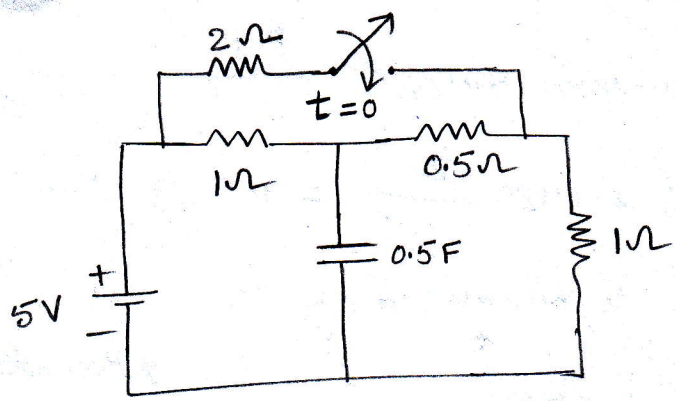
$\Rightarrow$ Time constant with which current will decay. $\left(\tau = \frac{L}{R_{eq}}\right)$

$\tau = \frac{1}{(4\|2)}$

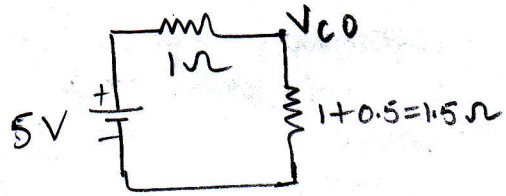
$\Rightarrow \tau = \frac{3}{4}$ sec.

$\therefore i_L(t>1) = i_L(t=1^+)\, e^{-\frac{4}{3}(t-1)}$

$i_L(t>1) = 0.0366\, e^{-\frac{4}{3}(t-1)}$ A.




* Initially, the switch has been opened for a long time. Then we may have the following equivalent circuit under this condition.

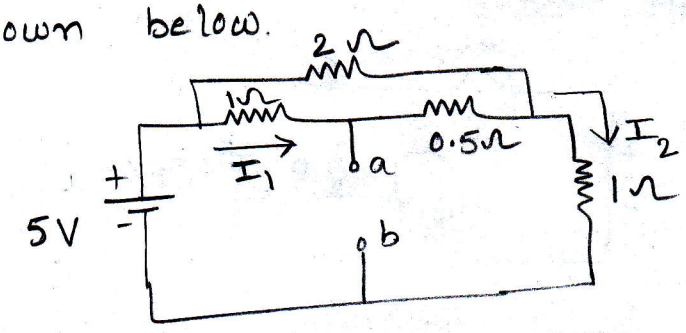



The capacitor will be charged to a voltage of

$$v_{CO} = 5 \times \frac{1.5}{1.5+1} = 5 \times \frac{1.5}{2.5} = 3V \quad \text{—— (1)}$$

* Then the switch is closed at $t=0$.

To find the voltage across the capacitor, first we find the Thevenin's equivalent across the capacitor, as shown below.




Now, from the above figure we can write,

$$I_2 = \frac{5}{1+[2 \parallel (1+0.5)]} = \frac{5}{1+[2 \parallel 1.5]}$$

$$= \frac{5}{1+\frac{3}{3.5}}$$

$$= \frac{5 \times 3.5}{3.5+3} = 2.692 \text{ Amp}$$

Then from current division rule,

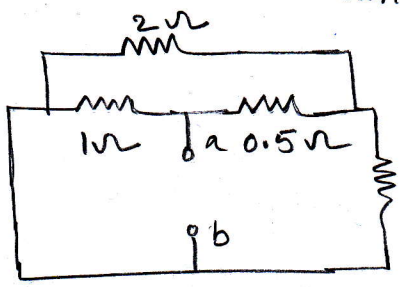
$$I_1 = I_2 \times \frac{2}{2+1+0.5} = 2.692 \times \frac{2}{3.5} = 1.538 \text{ Amp}$$

The voltage across the terminals a-b is

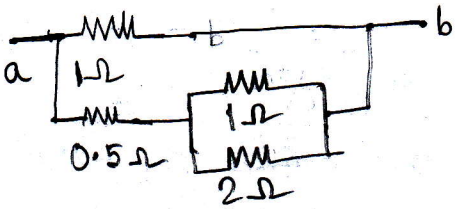
$$V_{th} = V_{ab} = 0.5 \times I_1 + 1 \times I_2$$
$$= (0.5 \times 1.538) + (1 \times 2.692)$$
$$= 0.769 + 2.692 = 3.461 \text{ V}$$

Alternatively,
$V_{th} = V_{ab} = 5 - I_1 \cdot 1$
$= 5 - (1.538 \times 1)$
$= 3.462$ Volt

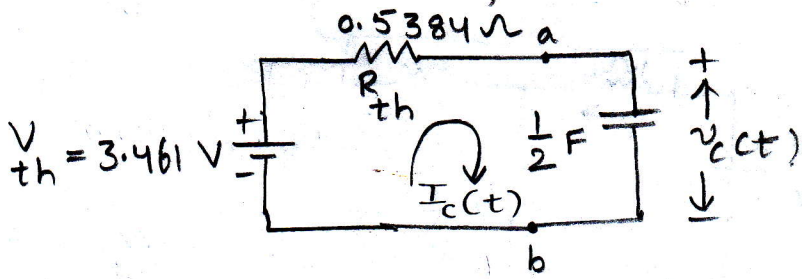
To find Thevenin's resistance, '$R_{th}$' (replacing voltage source by short ckt)




$$\Rightarrow R_{th} = R_{ab} = 1 \parallel [0.5 + 2 \parallel 1]$$
$$= 1 \parallel [0.5 + \tfrac{2}{3}]$$
$$= 1 \parallel [\tfrac{3.5}{3}]$$
$$= \frac{1 \times 3.5/3}{1 + \frac{3.5}{3}}$$
$$= \frac{3.5}{6.5} = 0.5384\,\Omega$$




Now the circuit reduces to,




We have initial voltage across the capacitor, $v_c(0) = 3V$
[∵ from ①]

$V_{final} = 3.461$ V. and $V_{initial} = 3V$ ($\because v_c(0^+)$)

Now, for $t>0$

voltage across the capacitor is,

$$v_c(t) = V_{final} + (V_{initial} - V_{final})\, e^{-t/\tau}$$

where, $\tau =$ time constant

$= R_{th} \times C$

$= 0.5384 \times \frac{1}{2}$

$= 0.2692$ sec

$$\Rightarrow v_c(t) = 3.461 + (3 - 3.461)\, e^{-\frac{t}{0.2692}}$$

$$\therefore v_c(t) = 3.461 - 0.461\, e^{-3.714t} \text{ volt.}$$

Current flowing through the capacitor is,

$$I_c(t) = C\frac{dv_c(t)}{dt} = 0.5 \times [(-0.461) \times -(3.714)]\, e^{-3.714t}$$

Therefore, $= 0.8562\, e^{-3.714t}$ Amp.

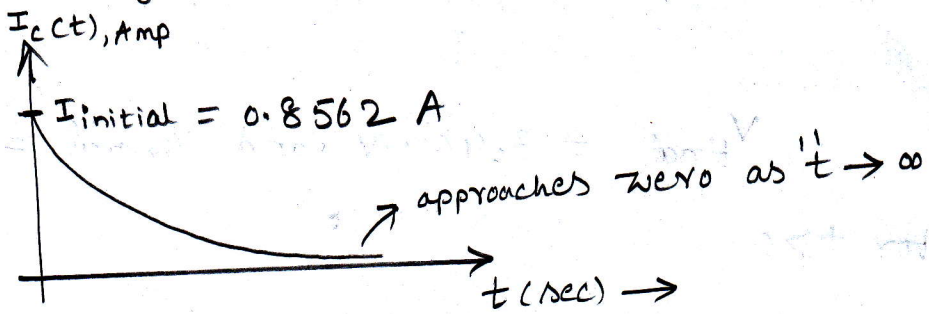
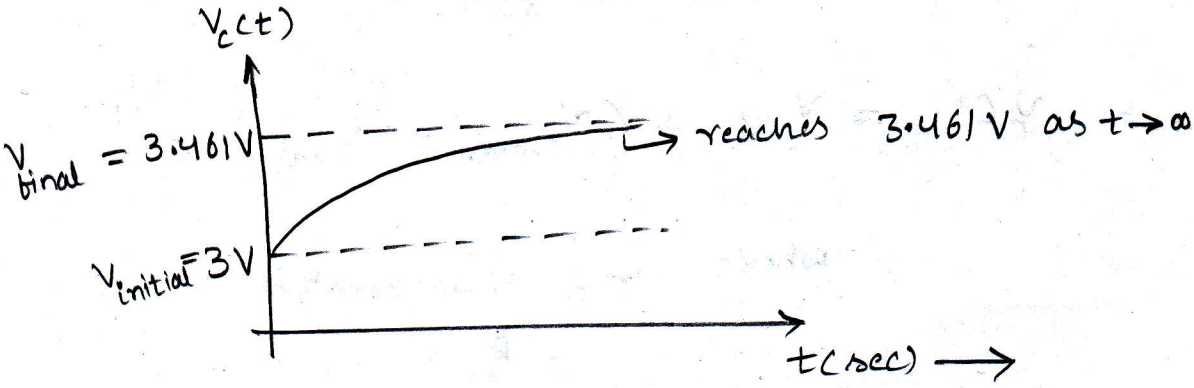
$I_c(t=0^+) = 0.8562$ A

$I_c(t\to\infty) = 0$ A

i) voltage across the capacitor is 3V at $t=0$

ii) for $t>0$,

$$I_c(t) = 0.8562\, e^{-3.714t} \text{ Amp.}$$

$$v_c(t) = 3.461 - 0.461\, e^{-3.714t} \text{ volt.}$$

iii) The voltage and current waveforms (for $t \geq 0$) are