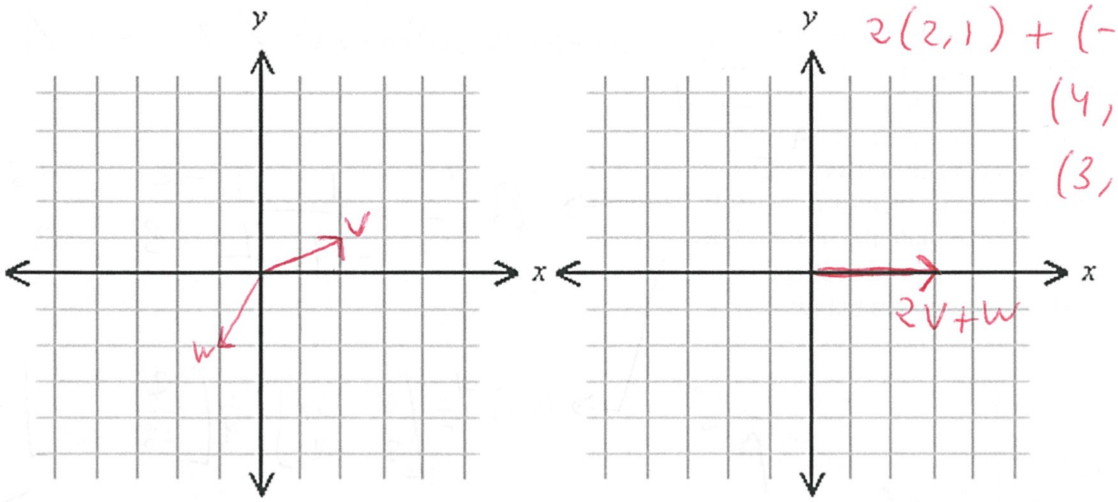
Linear Algebra Practical

1. Given the following traditional Cartesian planes, on the left one, draw the vectors **v** = (2, 1) and **w** = (-1, -2) on it. On the right Cartesian plane, draw the result of adding 2**v**+**w**.

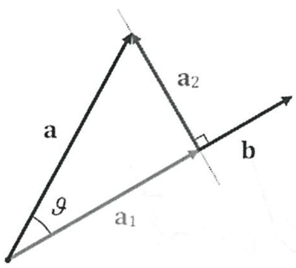


$2(2,1)+(-1,-2)=$
$(4,2)+(-1,-2)=$
$(3,0)$

2. What is the length or 2nd norm (L_2) of vector **z** = (3, 4)?

$$\|z\|=\sqrt{3^2+4^2}=\sqrt{25}=5$$

3. If ϑ is 30 degrees and **a** = (3,4), what is the magnitude of the projection (a_1) of **a** upon **b**?



$$a_1 = \|a\|\cos\theta \simeq 5\cdot 0.866 \simeq 4.33$$

4. Given $\boldsymbol{u}$ = (5,2,3) and v=(1,-1,2), find $u\cdot v$ (the scalar product/inner product/dot product).

$$u\cdot v = 5\cdot 1+2(-1)+3(2) = 5-2+6 = 9$$

5. Let

$$A=\begin{pmatrix}4 & -1\\ 6 & 9\end{pmatrix} \text{ and } B=\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}$$

Find

(i) $A+B$,

$$\begin{pmatrix}4 & -1\\ 6 & 9\end{pmatrix}+\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}=\begin{pmatrix}4 & 2\\ 9 & 7\end{pmatrix}$$

(ii) $2A-B$

$$2\begin{pmatrix}4 & -1\\ 6 & 9\end{pmatrix}-\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}=\begin{pmatrix}8 & -2\\ 12 & 18\end{pmatrix}-\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}=\begin{pmatrix}8 & -5\\ 9 & 20\end{pmatrix}$$

(iii) AB

$$\begin{pmatrix}4 & -1\\ 6 & 9\end{pmatrix}\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}=\begin{pmatrix}-3 & 14\\ 27 & 0\end{pmatrix}$$

(iv) BA

$$\begin{pmatrix}0 & 3\\ 3 & -2\end{pmatrix}\begin{pmatrix}4 & -1\\ 6 & 9\end{pmatrix}=\begin{pmatrix}18 & 27\\ 0 & -21\end{pmatrix}$$

(v) A^T (the transpose of A) $\begin{pmatrix} 4 & 6 \\ -1 & 9 \end{pmatrix} = A^T$

(vi) Det(A) (the determinant of A) Det(A) = 42

(vii) tr(A) (the trace of A) 13

6. Let $A = \begin{pmatrix} 4 & -1 \\ 6 & 9 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix}$

Yes $\begin{pmatrix} 4 & -1 \\ 6 & 9 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 14 \\ 27 & 0 \\ 9 & 0 \end{pmatrix}$

(i) Is AB defined? If it is defined, find it.

(ii) Is BA defined? How come? No, # columns B $\neq$ # rows A

(iii) What is the element A_{22} of A → 9

(iv) What is the result of AI? A

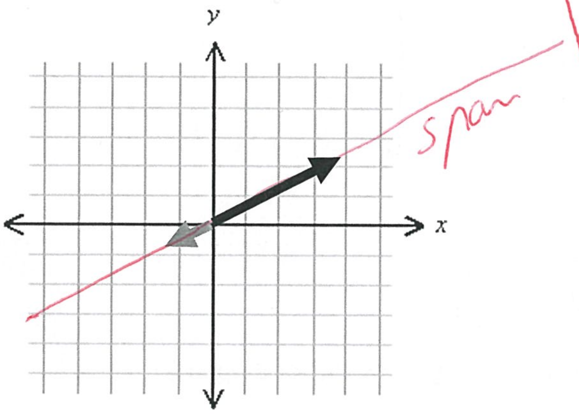
(v) What is the result of BB^{-1}? I

(vi) Calculate the inverse of B, i.e. B^{-1} → $B^{-1} = \frac{1}{-9}\begin{bmatrix} -2 & -3 \\ -3 & 0 \end{bmatrix}$

(vii) Manually calculate the result of BB^{-1}

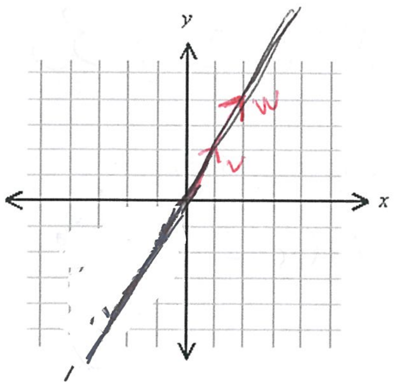
$BB^{-1} = \begin{bmatrix} 0 & 3 \\ 3 & -2 \end{bmatrix} \cdot \frac{1}{9}\begin{bmatrix} -2 & -3 \\ -3 & 0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -9 & 0 \\ 0 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. Draw the span of the vectors below



8. Are the following 2 vectors linearly dependent or independent? (Hint: plot them)

$$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad w = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$



Linearly dependant

9. The vectors i and j below are the basis vectors in some space. Can you draw the vector (1,1) in that basis?