D. Implementation

From all the measurement values x_1,y_1,z_1 to x_n,y_n,z_n, build the design matrix D.
Using the example file `mag.txt` with n = 25599, this produces a huge matrix of 10 rows x 25599 columns.

$$D = \begin{bmatrix} x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ y_1^2 & y_2^2 & y_3^2 & \cdots & y_n^2 \\ z_1^2 & z_2^2 & z_3^2 & \cdots & z_n^2 \\ 2y_1z_1 & 2y_2z_2 & 2y_3z_3 & \cdots & 2y_nz_n \\ 2x_1z_1 & 2x_2z_2 & 2x_3z_3 & \cdots & 2x_nz_n \\ 2x_1y_1 & 2x_2y_2 & 2x_3y_3 & \cdots & 2x_ny_n \\ 2x_1 & 2x_2 & 2x_3 & \cdots & 2x_n \\ 2y_1 & 2y_2 & 2y_3 & \cdots & 2y_n \\ 2z_1 & 2z_2 & 2z_3 & \cdots & 2z_n \\ 1 & 1 & 1 & & 1 \end{bmatrix}$$

Simplified implementation example:

```
CStdioFile readFile;
CString strFilePath = "C:\\MagCal\\mag.txt";
int nlines = 0;
char buf[120];
readFile.Open(strFilePath, CFile::modeRead);

// count number of lines
while(readFile.ReadString(buf, 100) != NULL)
   nlines++;

// come back to beginning of file
readFile.SeekToBegin();

// allocate memory for design matrix D
double* D = new double[10*nlines];

int i;
double x, y, z;
for( i = 0; i < nlines; i++)
{
    readFile.ReadString(buf, 100);
    sscanf(buf, "%lf\t%lf\t%lf", &x, &y, &z);
    D[i*10] = x * x;
    D[i*10+1] = y * y;
    D[i*10+2] = z * z;
```

```
    D[i*10+3] = 2.0 * y * z;
    D[i*10+4] = 2.0 * x * z;
    D[i*10+5] = 2.0 * x * y;
    D[i*10+6] = 2.0 * x;
    D[i*10+7] = 2.0 * y;
    D[i*10+8] = 2.0 * z;
    D[i*10+9] = 1.0;
}
readFile.Close();
```

Now create the 10x10 symmetric matrix S by multiplying D by its own transpose D^T:

$D*D^T = S$ [10 x nlines] * [nlines x 10] = [10 x 10]

[m x k] * [k x n] = [m x n]

[A] * $[B]^T$ = [C]

This is done with a single call to the function DGEMM.

```
// allocate memory for matrix S
double* S = new double[10*10];

// configure DGEMM parameters
char transa = 'N';
char transb = 'T';  // use the transpose of the right-hand matrix B
int m = 10;
int k = nlines;
int n = 10;
double alpha = 1.0;
double beta = 0.0;
int lda = 10;  // number of rows of left-hand matrix A
int ldb = 10;  // number of rows of right-hand matrix B (before transposition)
int ldc = 10;  // number of rows of result matrix C

DGEMM(&transa, &transb, &m, &n, &k, &alpha, D, &lda, D, &ldb, &beta, S, &ldc);

// discard matrix D
delete [] D;
```

For the example file `mag.txt`, we have S[10x10] = D[10x25599] * D^T[25599x10].
The resulting matrix S appears as attachment below.

Now split the 10x10 *S* matrix in 4 smaller matrices as follows:

$$S = \begin{bmatrix} S_{11}(6x6) & S_{12}(6x4) \\ S_{12}^T(4x6) & S_{22}(4x4) \end{bmatrix}$$

Because *S* is a symmetric matrix, S_{12}^T is the transpose of S_{12}.

```
// create S11   6x6
double* S11 = new double[6*6];
for(i = 0; i < 6; i++)
```

```
{
    for(j = 0; j < 6; j++)
        S11[6*j+i] = S[10*j+i];
}

// create S12  6x4
double* S12 = new double[6*4];
for(i = 0; i < 6; i++)
{
    for(j = 0; j < 4; j++)
        S12[6*j+i] = S[10*j+i+60];
}

// create S12t  4x6
double* S12t = new double[4*6];
for(i = 0; i < 4; i++)
{
    for(j = 0; j < 6; j++)
        S12t[4*j+i] = S[10*j+i+6];
}

// create S22 4x4
double* S22 = new double[4*4];
for(i = 0; i < 4; i++)
{
    for(j = 0; j < 4; j++)
        S22[4*j+i] = S[10*j+i+66];
}
```

Calculate $S22^{-1}$, the Moore-Penrose pseudo-inverse of the square matrix *S22*.

Note : the pseudo-inverse of a square matrix is identical to its inverse, except if the matrix is singular, which should not occur unless you have a really bad or incomplete sample of measurements.

```
 // allocate memory for the pseudo-inverse S22p
double* S22_1 = new double[4*4];

// initially set S22_1 to identity matrix
for(i = 0; i < 4; i++)
{
    for(j = 0; j < 4; j++)
    {
        if(i == j)
            S22_1[i*4+j] = 1.0;
        else
            S22_1[i*4+j] = 0.0;
    }
}

// configure the parameters for querying the optimal DGELSS workspace
int info;
int ldwork = -1;
double *work = new double[1];
int nrhs = 4;
int lda = 4;
int ldb = 4;
double* singz = new double[4];
double rcond = -1.0;
int irank = -1;
```

```
// query the optimal workspace
DGELSS(&m, &n, &nrhs, S22, &lda, S22_1, &ldb, singz, &rcond, &irank, work, &ldwork,
&info);

// allocate optimal workspace
ldwork = work[0];
delete [] work;
work = new double[ldwork];

// working call to DGELSS
DGELSS(&m, &n, &nrhs, S22, &lda, S22_1, &ldb, singz, &rcond, &irank, work, &ldwork,
&info);

delete [] work;
delete [] singz;
```

Calculate $SS = S_{11} - S_{12} * S_{22}^{-1} * S_{12}^{T}$

First calculate $S22a = S_{22}^{-1} * S_{12}^{T}$ (4x6 = 4x4 * 4x6) ($m_{x}n = m_{x}k * k_{x}n$)

```
double* S22a = new double[4*6];
transb = 'N';
m = 4;
n = 6;
k = 4;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, S22_1, &lda,  S12t, &ldb, &beta, S22a,
&ldc);
```

Then calculate $S22b = S_{12} * S22a$ (6x6 = 6x4 * 4x6) ($m_{x}n = m_{x}k * k_{x}n$)

```
double* S22b = new double[6*6];
m = 6;
n = 6;
k = 4;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, S12, &lda, S22a, &ldb, &beta, S22b, &ldc);
```

Finallly calculate $SS = S_{11} - S22b$

```
double* SS = new double[6*6];
for(i = 0; i < 36; i++)
    SS[i] = S11[i]- S22b[i];
```

Setup the constraint matrix C

```
double* C = new double[6*6];

C[0] = -1.0; C[6] = 1.0; C[12] = 1.0; C[18] = 0.0;  C[24] = 0.0;  C[30] = 0.0;
C[1] = 1.0;  C[7] = -1.0; C[13] = 1.0; C[19] = 0.0;  C[25] = 0.0;  C[31] = 0.0;
C[2] = 1.0;  C[8] = 1.0; C[14] = -1.0; C[20] = 0.0;  C[26] = 0.0;  C[32] = 0.0;
C[3] = 0.0;  C[9] = 0.0;  C[15] = 0.0;  C[21] = -4.0; C[27] = 0.0;  C[33] = 0.0;
C[4] = 0.0;  C[10] = 0.0; C[16] = 0.0;  C[22] = 0.0;  C[28] = -4.0; C[34] = 0.0;
C[5] = 0.0;  C[11] = 0.0; C[17] = 0.0;  C[23] = 0.0;  C[29] = 0.0;  C[35] = -4.0;
```

Invert matrix C in place

```
double *ipiv = new double[6];
work = new double[1];
lwork = -1;
m = 6;
DGETRF(&m, &m, C, &m, ipiv, &info);

// query optimal DGRTRI workspace
DGETRI(&m, C, &m, ipiv, work, &lwork, &info);

lwork = work[0];
delete [] work;
work = new double[lwork];

// working DGETRI call
DGETRI(&m, C, &m, ipiv, work, &lwork, &info);
delete [] work;
delete [] ipiv;
```

Calculate *E* = *C* * *SS* (6x6 = 6x6 * 6x6) ($m_x n = m_x k * k_x n$)

```
double* E = new double[6*6];
m = 6;
n = 6;
k = 6;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, C, &lda,  SS, &ldb, &beta, E, &ldc);
```

Calculate eigenvalues wr(6x1) and eigenvectors vr(6x6) of matrix *E*

```
char jobvl = 'N';
char jobvr = 'V';
n = 6;
lda = 6;
double* wr = new double[6];
double* wi = new double[6];
double* vl = NULL;
int ldvl = 6 ;
double* vr = new double[6*6];
int ldvr = 6;
work = new double[1];
lwork = -1;
DGEEV(&jobvl, &jobvr, &n, E, &lda, wr, wi, vl, &ldvl, vr, &ldvr, work, &lwork, &info);
```

```
lwork = work[0];
delete [] work;
work = new double[lwork];
DGEEV(&jobvl, &jobvr, &n, E, &lda, wr, wi, vl, &ldvl, vr, &ldvr, work, &lwork, &info);
delete [] wi;
```

Find the zero-based position of the only positive eigenvalue. The associated eigenvector will be in the corresponding column of matrix vr(6x6).

```
int index = 0;
double maxval = wr[0];
for(i = 1; i < 6; i++)
{
    if(wr[i] > maxval)
    {
        maxval = wr[i];
        index = i;
    }
}
```

Extract the associated eigenvector *v1*

```
v1 = new double[6];

v1[0] = vr[6*index];
v1[1] = vr[6*index+1];
v1[2] = vr[6*index+2];
v1[3] = vr[6*index+3];
v1[4] = vr[6*index+4];
v1[5] = vr[6*index+5];

delete [] wr;
delete [] vr;
```

Check sign of eigenvector *v1*

```
if(v1[0] < 0.0)
 {
  v1[0] = -v1[0];
  v1[1] = -v1[1];
  v1[2] = -v1[2];
  v1[3] = -v1[3];
  v1[4] = -v1[4];
  v1[5] = -v1[5];
 }
```

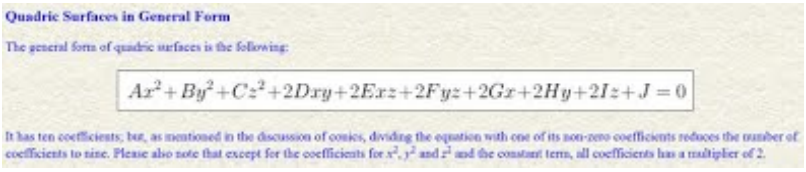
Calculate *v2* = *S22a* * *v1* (4x1 = 4x6 * 6x1) ($m_{x}n = m_{x}k * k_{x}n$)

```
v2 = new double[4];
m = 4;
n = 1;
k = 6;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, S22a, &lda,  v1, &ldb, &beta, v2, &ldc);
```

Setup vector *v*

```
v = new double[10];
v[0] = v1[0];
v[1] = v1[1];
v[2] = v1[2];
v[3] = v1[3];
v[4] = v1[4];
v[5] = v1[5];
v[6] = -v2[0];
v[7] = -v2[1];
v[8] = -v2[2];
v[9] = -v2[3];

delete [] v1;
delete [] v2;
```

At this point, we have found the general equation of the fitted ellipsoid:



Quadric Surfaces in General Form

The general form of quadric surfaces is the following:

$$Ax^2+By^2+Cz^2+2Dxy+2Exz+2Fyz+2Gx+2Hy+2Iz+J=0$$

It has ten coefficients; but, as mentioned in the discussion of conics, dividing the equation with one of its non-zero coefficients reduces the number of coefficients to nine. Please also note that except for the coefficients for x^2, y^2 and z^2 and the constant term, all coefficients has a multiplier of 2.

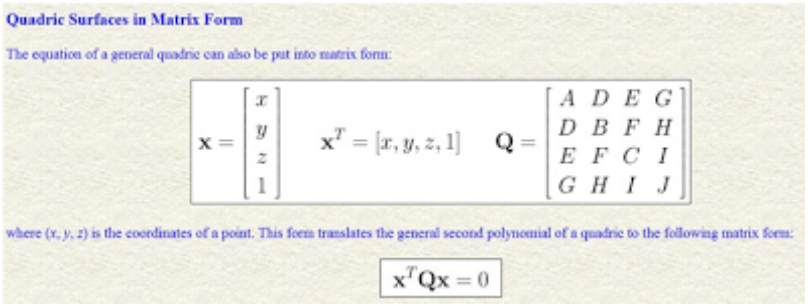
Source : http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/geometry/simple.html

where:

A = v[0] - term in x^2
B = v[1] - term in y^2
C = v[2] - term in z^2
D = v[5] - term in xy
E = v[4] - term in xz
F = v[3] - term in yz
G = v[6] - term in x
H = v[7] - term in y
I = v[8] - term in z
J = v[9] - constant term

Note the different order of terms in xy, xz and yz between this general equation and the design matrix D above, which explains why we find the order v[5], v[4], v[3] instead of v[3], v[4], v[5].

The general equation of the ellipsoid can also be put in matrix form:



Quadric Surfaces in Matrix Form

The equation of a general quadric can also be put into matrix form:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \mathbf{x}^T = [x, y, z, 1] \qquad \mathbf{Q} = \begin{bmatrix} A & D & E & G \\ D & B & F & H \\ E & F & C & I \\ G & H & I & J \end{bmatrix}$$

where (x, y, z) is the coordinates of a point. This form translates the general second polynomial of a quadric to the following matrix form:

$$\mathbf{x}^T\mathbf{Q}\mathbf{x} = 0$$

Source : http://www.cs.mtu.edu/~shene/COURSES/cs3621/NOTES/geometry/simple.html

If we define:

$$Q = \begin{bmatrix} A & D & E \\ D & B & F \\ E & F & C \end{bmatrix} \qquad U = \begin{bmatrix} G \\ H \\ I \end{bmatrix}$$

then the center of the ellipsoid can be calculated as the vector $B = -Q^{-1}U$. The center of the ellipsoid represents the combined bias.

Setup symmetric matrix *Q* (3x3)

```
Q = new double[3*3];

Q[0] = v[0];
Q[1] = v[5];
Q[2] = v[4];
Q[3] = v[5];
Q[4] = v[1];
Q[5] = v[3];
Q[6] = v[4];
Q[7] = v[3];
Q[8] = v[2];
```

Setup vector *U*

```
U = new double[3];
U[0] = v[6];
U[1] = v[7];
U[2] = v[8];
```

Calculate matrix Q^{-1}, the inverse of matrix *Q*

```
double* Q_1 = new double[3*3];

for(i = 0; i < 9; i++)
    Q_1[i] = Q[i];

ipiv = new double[3];
work = new double[1];
lwork = -1;
m = 3;
DGETRF(&m, &m, Q_1, &m, ipiv, &info);
```

```
DGETRI(&m, Q_1, &m, ipiv, work, &lwork, &info);
lwork = work[0];
delete [] work;
work = new double[lwork];
DGETRI(&m, Q_1, &m, ipiv, work, &lwork, &info);
delete [] work;
delete [] ipiv;
```

Calculate $B = Q^{-1} * U$ (3x1 = 3x3 * 3x1) ($m \times n = m \times k * k \times n$)

```
double* B = new double[3];
m = 3;
n = 1;
k = 3;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, Q_1, &lda, U, &ldb, &beta, B, &ldc);
```

Calculate combined bias

```
B[0] = -B[0];     // x-axis combined bias
B[1] = -B[1];     // y-axis combined bias
B[2] = -B[2];     // z-axis combined bias
```

It can be shown that : (see the page 'Geometric interpretation')

$$A^{-1} = \frac{H_M}{\sqrt{B^T Q B - J}} Q^{1/2}$$

where H_M is the norm of the field provided by the user.

Calculate *btqb* = $B^T * Q * B$

First calculate $QB = Q * B$ (3x1 = 3x3 * 3x1) ($m \times n = m \times k * k \times n$)

```
double* QB = new double[3];
m = 3;
n = 1;
k = 3;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, Q, &lda, B, &ldb, &beta, QB, &ldc);
```

Then calculate $btqb = B^T * QB$ (1x1 = 1x3 * 3x1) ($m \times n = m \times k * k \times n$)

```
double btqb;
m = 1;
n = 1;
k = 3;
lda = m;
ldb = k;
ldc = m;

DGEMM(&transa, &transb, &m, &n, &k, &alpha, B, &lda, QB, &ldb, &beta, &btqb, &ldc);
```

Calculate *hmb = sqrt(btqb - J).*

```
double J = v[9];
double hmb = sqrt(btqb - J);
```

Calculate *SQ*, the square root of matrix *Q*

```
jobvl = 'N';
jobvr = 'V';
n = 3;
lda = 3;
wr = new double[3];
wi = new double[3];
vl = NULL;
ldvl = 3;
vr = new double[9];
ldvr = 3;
work = new double[1];
lwork = -1;
DGEEV(&jobvl, &jobvr, &n, Q, &lda, wr, wi, vl, &ldvl, vr, &ldvr, work, &lwork, &info);
lwork = work[0];
delete [] work;
work = new double[lwork];
DGEEV(&jobvl, &jobvr, &n, Q, &lda, wr, wi, vl, &ldvl, vr, &ldvr, work, &lwork, &info);

double* Dz = new double[3*3];
for(i = 0; i < 9; i++)
    Dz[i] = 0.0;
Dz[0] = sqrt(wr[0]);
Dz[4] = sqrt(wr[1]);
Dz[8] = sqrt(wr[2]);

double* vdz = new double[3*3];
m = 3;
n = 3;
k = 3;
lda = m;
ldb = k;
ldc = m;
DGEMM(&transa, &transb, &m, &n, &k, &alpha, vr, &lda, Dz, &ldb, &beta, vdz, &ldc);



// invert matrix vr
ipiv = new double[3];
work = new double[1];
lwork = -1;
```

```
m = 3;
DGETRF(&m, &m, vr, &m, ipiv, &info);
DGETRI(&m, vr, &m, ipiv, work, &lwork, &info);
lwork = work[0];
delete [] work;
work = new double[lwork];
DGETRI(&m, vr, &m, ipiv, work, &lwork, &info);
delete [] work;
delete [] ipiv;

double* SQ = new double[3*3];


m = 3;
n = 3;
k = 3;
lda = m;
ldb = k;
ldc = m;
DGEMM(&transa, &transb, &m, &n, &k, &alpha, vdz, &lda, vr, &ldb, &beta, SQ, &ldc);
```

Calculate A^{-1}

```
double hm;
double* A_1 = new double[3*3];

for(i = 0; i < 9; i++)
    A_1[i] = SQ[i] * hm / hmb;
```

Calculate *A* to permit comparison with MagCal

```
double* A = new double[3*3];

for(i = 0; i < 9; i++)
    A[i] = A_1[i]

ipiv = new double[3];
work = new double[1];
lwork = -1;
m = 3;
DGETRF(&m, &m, A, &m, ipiv, &info);
DGETRI(&m, A, &m, ipiv, work, &lwork, &info);
lwork = work[0];
delete [] work;
work = new double[lwork];
DGETRI(&m, A, &m, ipiv, work, &lwork, &info);
delete [] work;
delete [] ipiv;
```

S.txt (1k) Merlin Oz, 10 Sep 2011, v.1

Comments

You do not have permission to add comments.