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Minimizing rumor influence in multiplex online social networks based on human individual and social behaviors☆

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ABSTRACT

With the growing popularity of online social networks, an environment has been set up that can spread rumors in a faster and wider manner than ever before, which can have widespread repercussions on society. Nowadays, individuals are joining multiple online social networks and rumors simultaneously propagating amongst them, thereby creating a new dimension to the problem of rumor propagation. Motivated by these facts, this paper attempts to address the rumor influence minimization in multiplex online social networks. In this work, we consider modeling the propagation process of such fictitious information as a significant step toward minimizing its influence. Thus, we analyze the individual and social behaviors in social networks; subsequently, we propose a novel rumor diffusion model, named the HISBmodel. In this model, we propose a formulation of an individual behavior towards a rumor analog to damped harmonic motion. Following this, the opinions of individuals in the propagation process are incorporated. Furthermore, the rules of rumor transmission between individuals in multiplex networks are incorporated by considering individual and social behaviors. Further, we present the HISBmodel propagation process that describes the spread of rumors in multiplex online social networks. Based on this model, we propose a truth campaign strategy in minimizing the influence of rumors in multiplex online social networks from the perspective of network inference and by exploiting the survival theory. This strategy selects the most influential nodes as soon as the rumor is detected and launches a truth campaign to raise awareness against it, so as to prevent the influence of rumors. Accordingly, we propose a greedy algorithm based on the likelihood principle, which guarantees an approximation within 63% of the optimal solution. Systematically, experiments have been conducted on real single networks crawled from Twitter, Facebook, and Slashdot as well as on multiplex networks of real online social networks (Facebook, Twitter, and YouTube). First, the results indicate the HISBmodel can reproduce all the trends of real-world rumor propagation more realistically than the models presented in the literature. Moreover, the simulations illustrate that the proposed model highlights the impact of human factors accurately in accordance with the literature. Second, compared to the methods in the literature, the experiments prove the efficiency of our strategy in minimizing the influence of rumors in the cases of single network and

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multiplex social network propagation. The results prove that the proposed method can capture the dynamic propagation process of the rumor and select the target nodes more accurately in order to minimize the influence of rumors.

1. Introduction

The last decade has witnessed the emergence of multiple online platforms that facilitate users to earn profits through advertisements. Hence, the search for new audiences has led to the creation of "click-bait" titles to arouse individuals' curiosity. Rumors, as instrumentally relevant information statements, represent the ideal "click-bait" titles to attract more readers. Moreover, online social networks (OSNs) facilitate rapid and broad distribution of information and create the perfect environments to spread these contents for attracting new audiences. By exploiting the attractiveness of rumors and the broad reach of OSNs, rumors can propagate faster and widely than factual information [2]. As a result, titles such as "Cure Cancer Naturally" or "Vaccines are Dangerous" spread widely in OSNs, and several users believe these claims. For instance, online rumors about vaccines have bolstered the influence of the anti-vaccine movement, convincing more parents about the danger of vaccines. As a consequence, measles epidemic broke out[1] in recent years; this disease has been the cause of the death of up to 90% of the native American population in the years that followed contact with Europeans in the fifteenth century [3]. Evidently, such false information presented online can quickly change public opinion [4], create social panic [5], and even cause deaths. Thus, rumors in OSNs have become a significant threat to society and should be halted before they can spread.

Modeling the rumors propagation process is a vital step in minimizing their adverse effects. A rumor propagation model should allow researchers to (1) highlight the significant factors involved in this process; (2) devise strategies to limit the spread of rumors, and (3) test these methods to ensure they can be applied to real-world situations. However, the variety and complexity observed in human behaviors and interaction beyond our imagination make it highly challenging for researchers [6] to model this phenomenon. Moreover, the large size and complex topology of OSNs make the conception of an accurate rumor propagation model even more challenging.

The problem of the spread of rumors in OSNs has been addressed by several studies; however, only a few works [7] have investigated this problem in multiplex OSNs. We have classified the works of literature into macroscopic and microscopic approaches. On the one hand, macroscopic approaches are based primarily on epidemic models [8]. However, they neglect the variety of human beings, which is the primary challenge in accurately modeling a phenomenon such as the propagation of rumors. Moreover, the works in this category emphasize the effects of the studied factors on the spreading of rumors, but do not elaborate on the implications of these findings for developing rumor influence minimization (RIM) strategies. On the other hand, microscopic approaches have devoted most of their attention to the social interactions in the spreading process, i.e., "who influenced whom." The Independent Cascades (IC) and the Linear Threshold (LT) models [9] are the most common models in this category. However, the shortcomings of these models have been explicitly recognized in their simple designing assumptions, such as excluding several aspects of human characteristics. Furthermore, some scholars [10,11] have provided evidence that rumors propagate according to a different trend than factual information; this has not been considered by the IC or LT models. Finally, most of the approaches [12–20] addressing the RIM do not consider the multiplex structure of OSNs and cannot apply it directly to the problem of rumor propagation in multiplex OSNs. Therefore, this work proposes a rumor propagation model to describe this phenomenon and overcome the challenges mentioned above as well as a solution to minimize its influence in OSNs.

This work proposes an approach that is parallel to the studies discussed above and concerned more with how individuals spread rumors. Therefore, we take a step backward to analyze and understand how the behavior of individuals, as well as their social interactions in OSNs, impact the dissemination of rumors. On the basis of this analysis, we propose a novel rumor propagation model named the HISBmodel that considers individual and social behaviors in multiplex OSNs. In this model, we first introduce our schema of multiplex OSNs, where a distinction has been made between the individuals and their accounts in OSNs. Subsequently, we propose a formulation of individual behavior towards a rumor analog to damped harmonic motion. Additionally, we incorporate the opinions of individuals in the propagation process. Furthermore, we establish rules of rumor transmission between individuals in each layer of the multiplex networks. As a result, we present the HISBmodel propagation process, where new metrics are introduced to accurately assess the impact of a rumor spreading through multiplex OSNs. Based on this model, we propose a truth campaign strategy (TCS) for the RIM problem. The TCS selects the most influential nodes and then initiates a truth campaign to raise individuals' awareness and prevent the spread of the rumor. The solution to the problem is formulated from the perspective of the network inference problem by exploiting the survival theory. Accordingly, we propose a greedy algorithm that selects nodes as candidates for the TCS based on the likelihood principle, which guarantees an approximation within 63% of the optimal solution. Experiments are performed

[1] https://www.cdc.gov/measles/cases-outbreaks.html.

on real datasets to evaluate the performance of our model and the effectiveness of the proposed RIM strategy. These experiments testify that the model depicts the evolution of rumor propagation realistically and proves it can highlight the impact of human factors accurately, as proved in the previous studies. Moreover, the experiments are conducted to exhibit the efficiency of our strategy in minimizing the influence of the rumor.

In summary, the contributions of this paper compared to the prior version [1] are as follows:

- We propose a novel rumor propagation model, the HISBmodel. Unlike the models presented in the literature, the proposed model considers the human individual and social behaviors and, thereby, allows to describe the propagation of rumors realistically. We demonstrate that the HISBmodel reproduces all the trends of real-world rumor propagation more precisely than the classical models. Moreover, according to previous studies, HISBmodel highlights the impact of human factors on rumor spreading accurately.
- We introduce a truth campaign strategy (TCS) for rumor influence minimization. Here, the most influential nodes are selected based on individual and social behaviors and most likely to spread the truth campaign in order to minimize the influence of the rumor. The solution to the problem is formulated from the perspective of network inference by exploiting the survival theory. The TCS exhibits better performance in minimizing the influence of the rumor in the cases of single and multiplex OSNs compared to the previous works.
- We show an outstanding performance of the proposed solution. In the best-case, the TCS achieves an average performance of 69% reduction in the number of people most likely to be infected by the rumor. In the worst-case scenario, our method can reduce the impact of the rumor by 31% on an average compared to 17% of the second-best results of the methods of the literature. Besides, the HISBmodel can help us test multiple RIM strategies, such as blocking nodes strategies, blocking links strategies, or truth campaign strategies, which classical models have failed to do.

The remainder of this paper is organized as follows: Section 2 expands on the related work as well as the preliminary knowledge. The HISBmodel rumor propagation model is described in Section 3. Section 4 presents the proposed rumor influence minimization strategy. Section 5 displays the conducted experiment results. Finally, this paper's discussions and future work are presented in Section 6.

2. Related work and preliminary knowledge

2.1. Related work

In the last few years, a growing interest has been seen in rumor propagation in OSNs, and different approaches have been proposed to investigate it. By carefully scrutinizing the existing literature, we categorize the works into those adopting macroscopic and microscopic approaches.

2.1.1. Macroscopic approaches

The first category is mainly based on Epidemic models [8] to investigate the problem of rumor propagation. However, since these models have been specially designed for word-of-mouth propagation, several works [21,22] have considered the topological characteristics of networks to adapt these models to the context of OSN propagation. Recent research has focused on the role of the human individual and social behaviors; the studies investigated the effect of different human factors in the propagation process of rumors. Several works have focused on studying individual human behaviors, such as forgetting and remembering [23–26], hesitating [27], incomplete reading [28], and the education of individuals [29]. Other works focus on the effect of human social behaviors, such as the trust mechanism [30], the herd mentality [31], and the role of social reinforcement [32] in the spreading of online behaviors. These works investigate the impact of different factors in different type of networks such “scale-free networks” [33] and “small-world networks” [34].

Nevertheless, these models are macroscopic approaches; they consider that all users have indistinguishable characteristics with the same ability to influence others. Moreover, the variety and complexity observed in human behaviors and human interaction beyond our imagination are neglected, which makes it challenging to model a phenomena such as the spreading of rumors. Additionally, the spreading of rumors and epidemics is different since individuals have the choice here to quit the rumor spreading process at any time they want to; in epidemics, however, the infection process is relatively passive. Finally, due to its macroscopicity approach to investigate the problem, it is challenging to investigate the RIM problem with the Epidemic model.

2.1.2. Microscopic approaches

The microscopic approaches attracted more attention with respect to individual interactions, such as “who influenced whom.” Over the years, several models have been proposed in the microscopic approach; examples include the following: (1) Probabilistic models such as the well-known independent cascade (IC) and the linear threshold (LT) models [9] or Galam’s model [4]; (2) nature-inspired models such as the energy model [11]; (3) probabilistic models of inference [35]; and, (4) state transition models [36]. Nevertheless, the IC and LT models are well-known models of this category, and the propagation process of these models have been well detailed by Jalili and Perc [37]. These models serve as the milestone for different information diffusion problems in general, such as information maximization problems and RIM in particular. They were first introduced by [9], and several strategies have been proposed to minimize the spread of rumors and improvements been made to these models. Some works [13–17] investigated blocking nodes or link strategies to limit the spread of undesirable information. Other researchers have proposed a truth (good) campaign that can fight the false (bad) campaign [7,12,18–20,38]. Accordingly, several improvements to the IC [18] and LT [19] models have been proposed for multi-campaign propagation models.

Research in this direction focuses on designing strategies to limit the spread rumors, where little attention has been given to the propagation model. Furthermore, these models still lack in some significant details as they have failed to capture some aspects, such as the spreading of individuals' opinions. Finally, the strategies have been postulated on a closed world assumption; however, individuals often join several OSNs, where the rumors are propagated across multiple networks simultaneously. Thus, the proposed RIM approaches cannot be applied directly in multiplex OSN propagation; furthermore, to the best of our knowledge, few there are the works [7] who investigated this problem in multiplex OSN.

2.2. Preliminary knowledge

For better clarity and readability, this section briefly explains the concepts of a damped harmonic motion and survival theory, which will be used in our modeling assumptions.

2.2.1. Damped harmonic motion

In mechanics and physics, a damped harmonic motion is a type of periodic motion or oscillatory motion that is often described as a ball linked to a horizontal spring on a table. The immobile state is the equilibrium state. When the ball is moved from this state, it stretches the spring. Therefore, the restoring force is directly proportional to the displacement. The ball moves back and forth with frequency ω. The damped harmonic motion experiences friction, where the dissipating forces eventually dampen the motion with parameter β to the point the oscillation no longer occurs. Thus, if a system moves under the combined influence of a linear restoring force and a resisting force, the motion is described by the equation [39]

$$x(t) = A_0 e^{-\beta t} \cos(\omega t + \delta), \tag{1}$$

where $x(t)$ is the position of the system at time t, β the damping parameter, ω the characteristic frequency of the motion, δ the phase of the motion determining the starting point at $t = 0$, and A_0 the amplitude of the oscillation.

2.2.2. Survival theory

The survival theory is a mathematical tool widely exploited in the domain of epidemics to estimate the likelihoods of the occurrence of an event after observation time. This tool was first adopted in problem of information diffusion by Gomez-Rodriguez and Leskovec [35], where they proposed an information diffusion model from the perspective of network inference. They exploited the survival theory to develop general additive and multiplicative models under which the network inference problems could be solved efficiently by exploiting their convexity. The survival function is given as follows [40]

$$S(t) = Pr(T > t) = 1 - F(t), \tag{2}$$

where T is a continuous random variable representing the occurrence time of an event of interest; t is a specified constant. Considering the event is the infection of individuals with a virus, the survival function represents the probability that individuals survive the infection after the observation deadline t. When $F(t)$ is the cumulative distribution function, then the probability density function can be given as follows

$$f(t) = \frac{dF(t)}{dt} = -S'(t). \tag{3}$$

An alternative characterization of the distribution of T is given by the hazard function or instantaneous rate of occurrence of the event $h(t)$, which is defined as

$$h(t) = \lim_{dt \to 0} \frac{\Pr\{t \leqslant T < t + dt | T \geqslant t\}}{dt}, \tag{4}$$

the numerator of Eq. (4) is a conditional probability that the event will occur during the interval $[t, t + dt]$ given that it has not occurred before time t. Simplifying this expression, $h(t)$ can be written as

$$\begin{aligned}
h(t) &= \lim_{dt \to 0} \frac{\Pr(t \leqslant T < t + dt | T \geqslant t)}{dt}, \\
&= \lim_{dt \to 0} \frac{\Pr(t \leqslant T < t + dt)}{\Pr(T > t)dt}, \\
&= \lim_{dt \to 0} \frac{F(t + dt) - F(t)}{S(t)dt}, \\
&= \frac{1}{S(t)} \lim_{dt \to 0} \frac{F(t + dt) - F(t)}{dt}.
\end{aligned}$$

Therefore, we obtain

$$h(t) = \frac{f(t)}{S(t)} = -\frac{S'(t)}{S(t)}. \tag{5}$$

As Eq. (5) shows a relationship between the survival function and the hazard rate, where knowing the value of $h(t)$ can make it easier to deduce $S(t)$, Eq. (2) can be used to estimate the cumulative distribution function as follows

$$S(t) = e^{-\int_0^t h(\tau)d\tau}, \tag{6}$$

$$F(t) = 1 - e^{-\int_0^t h(\tau)d\tau}. \tag{7}$$

In this context, the studied event is the rumor infection, where the hazard rate function is defined for the proposed strategy.

3. Proposed rumor propagation model

Considering the predominant role of the rumor propagation model in the problem of limiting the influence rumors in OSNs, the primary goal of this section is to present a propagation model that can reproduce a realistic trend of this phenomenon and provide significant indicators to assess the impact of the rumor to effectively understand the diffusion process and reduce its influence. The variety that exists in human nature makes their decision-making ability pertaining to spreading information unpredictable, which is the primary challenge to model such a complex phenomenon. Hence, it is significant to consider the impact of human individual and social behaviors in the spreading process of the rumors. Thus, we proposed a rumor propagation model that is based on an analysis of the users' behaviors and their social interactions in multiplex OSN known as the HISBmodel. Unlike the models proposed in the literature, this model focuses on how individuals propagate a rumor in an OSN rather than how this information is spread. Therefore, the model attempts to answer the following question: "When does an individual spread a rumor? When does an individual accept rumors? In which OSN does this individual spread the rumors?". In this model, we introduce an individual behavior toward a rumor formulation that is analogous to a damped harmonic motion. Subsequently, we present an integration of the opinions of individuals in this process and consider the social influence. Additionally, we establish the rules of human social interaction between individuals and emphasize identifying the layer of the network in which the individual will spread the rumor. As a result, we describe the propagation process of rumors based on the HSIBmodel, which has been inspired from a real scenario in multiplex OSNs. Furthermore, the model allows us to present new metrics to evaluate the impact of the spreading of a rumor; these metrics accurately reflect the state of propagation of the rumor to evaluate its impact.

3.1. Multiplex online social networks representation

In literature, an OSN is generally considered as a directed or undirected graph $G = (V, E)$, where the set of nodes V represents the users; the set of edges E denotes the relationship among individuals. However, with the diversity in OSNs, individuals usually join several OSNs simultaneously and can, furthermore, maintain several accounts. Therefore, the information is no longer limited to being spread over a single network; in fact, it is available in the multiplex structure of OSNs. Thus, based on this idea and the previous works presented in the literature [41,42], we define multiplex OSNs.

Definition 1. Multiplex OSNs with n networks is a set $\mathbb{G}^n = (I, G^n)$, where $I = (V, C)$ is the set of individuals represented in the center of Fig. 1; for each individual, $i \in I$ is represented by a node $v \in V$ and a set of characteristics $c \in C$. The characteristics of an individual define their response to a rumor. It is further defined in the following section (see Section 3.2). The set $G^n = \{G_1 = (V, E_1),\ G_2 = (V, E_2), \ldots, G_n = (V, E_n)\}$ is a set of n graphs, where $G_i = (V, E_i)$ is a directed graph representing an OSN; for example, in Fig. 1, $G^3 = \{G_1 = (V, E_1),\ G_2 = (V, E_2), G_3 = (V, E_3)\}$ are Instagram, Twitter, and Facebook networks, respectively, represented by a directed graph. Without the loss of generality, we consider that each network of the multiplex has the same number of nodes. If a node $v \in G_i$ does not belong to G_j, we can add this node to G_j as an isolated node, which has been presented in black in Fig. 1.

3.2. Individual behavior toward a rumor formulation

This part presents an analysis of the individuals' behavior towards a rumor based on the work of the literature and has led us to the following description. At the beginning of the propagation process, a rumor is usually perceived as eye-catching and, even if it has not been verified, people are seen to be attracted to it. The individual's attraction to the rumor is initially high and, subsequently, exhibits a gradual downtrend [10,11]. Moreover, most rumors have specific time-lines and can become extinct if officially refuted or people lose interest in them. Additionally, authors in [29] have pointed out the significant effect of individual education. Therefore, the individual's background knowledge (IBK) about the rumor is defined as the ability of an individual to evaluate the trustworthiness of a rumor; it, hence, plays a crucial role. Thus, the greater the IBK about the rumor, the faster the loss of interest in a rumor will be. However, due to the hesitating mechanism (HM), an individual can eventually possess a latent time before spreading the rumor. This is relatively related to the degree of doubt of individuals on the revived rumor [27]. Additionally, a rumor can be eclipsed by other information in OSNs, and individuals can cease and restart transmitting the rumor due to the forgetting-remembering (FR) factor. This factor has been

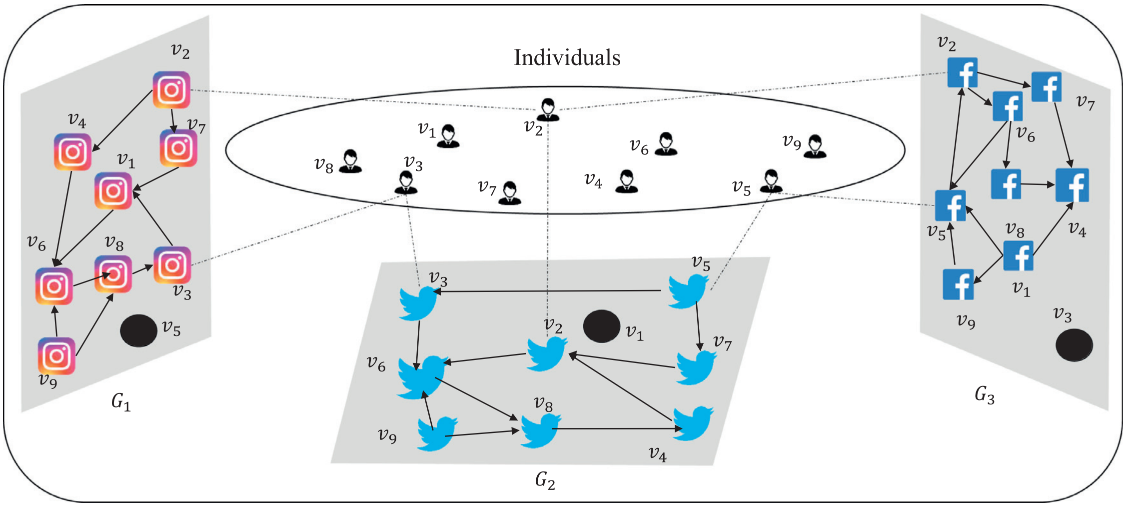
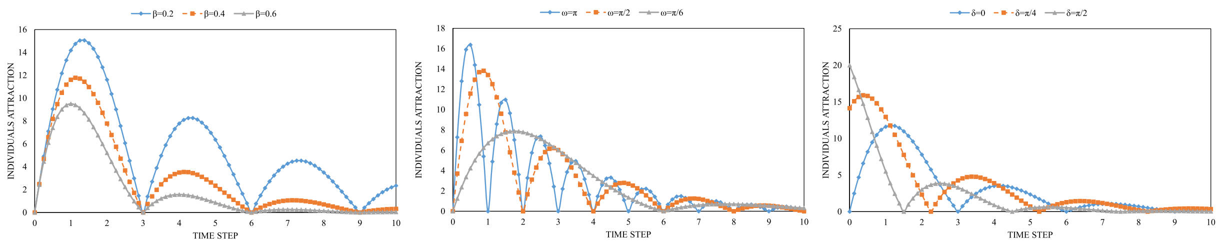


Fig. 1. multiplex of three (3) online social networks $\mathbb{G}^3 = (I, G^3)$. The set of individuals are represented in the center of the figure, where each individual can have accounts in different OSNs presented in black dash lines; for better clarity, we have only illustrated for nodes v_2, v_3, and v_5. $G^3 = \{G_1 = (V, E_1), G_2 = (V, E_2), G_3 = (V, E_3)\}$ are respectively Instagram, Twitter, and Facebook networks represented by a directed graph. Without the loss of generality, we consider that each network of the multiplex has the same number of nodes. Therefore, the nodes presented in black are isolated nodes that are added to each layer since an individual does not have an account in this OSN.



(a) Individual's rumor attraction with different values of individuals' background knowledge factor β, where $\omega = \pi/3$ and $\delta = 0$. It shows that an individual tends to quickly lose interest in a rumor for high values of β compared to lower values.

(b) Individual's rumor attraction with different values of the forgetting-remembering factor ω, where $\beta = 0.4$ and $\delta = 0$. It shows the periodic trend of the forgetting and remembering phase of an individual.

(c) Individual's rumor attraction with different values of the hesitating factor δ, where $\omega = \pi/3$ and $\beta = 0.4$. It shows the latent time before spreading the rumor.

Fig. 2. Individual's attraction to the rumor with various values of ω, β **and** δ. This figure shows how human factors impact the trend of the attraction of an individual toward a rumor.

studied by Zhao et al. [23–26] in various works; we have associated it in this work with individuals' addiction to OSNs. We claim that the more time spent on an OSN, the higher are the chances of remembering the rumor.

While analyzing the individual behavior in OSNs, we were inspired by a model of physics that fit the description of the behaviors. We found the analogy that the attraction of an individual to a rumor is similar to an oscillating system when displaced from the equilibrium position. Similar to the attraction of an individual to a rumor, the amplitude of the motion is high in the beginning and subsequently decreases depending on the damping parameter. The damping parameter in this case presents the IBK about a rumor (see Fig. 2(a)). The latent time an individual has before spreading the rumor due to the HM factor is analogous to the phase of the system (see Fig. 2(c)). Finally, the FR factor is analogized as a system oscillating around its equilibrium position; the oscillation frequency of the system represents the degree of the user's addiction to OSNs (see Fig. 2(b)). This parameter represents the periodicity with which the individual switches between the forgetting and remembering phases. We can define the individual's attraction to a rumor as:

$$A(t) = A_{int} e^{-\beta t} \cos(\omega t + \delta), \tag{8}$$

where $A(t)$ is the attraction of the individual to the rumor at time t, A_{int} the initial attraction to the rumor, β represents the IBK, the FR factor ω the period of forgetting and remembering, and δ the degree of trust in the source of the rumor. In order to accommodate the proposed formulation in actual scenarios, we set $\delta' = \pi/2 + \delta$. Thus, the latent time an individual has before spreading the rumor increases with δ. Finally, for non-negative values of the individual's attraction, we consider $A(t) = |A(t)|$. The individual's attraction to the rumor is presented as follows:

$$A(t) = A_{int} e^{-\beta t} |\sin(\omega t + \delta)|. \tag{9}$$

To prove the consistency of our model with the works available in the literature, we need to show that our model fits the individual behavior description provided before (summarized in Proposition 1).

Proposition 1. *There is an increasing dependence of the impact of the rumor in the network with the FR and HM factors while a decreasing one on the relation between the IBK factor and the impact of the rumor.*

Proof. To verify the above proposition, we assume that the probability of an individual spreading a rumor increasingly depends on their attraction to the rumor. Thus, we estimate $\phi(\omega, \beta, \delta)$, which is the primitive function of $A(t)$ and represents an individual's attraction toward a rumor from the time they heard of the rumor till the loss of interest. This is given as follows

$$\phi(\omega,\beta,\delta)=\int_0^\infty A_{int}e^{-\beta t}|\sin(\omega t+\delta)|dt \qquad \text{for} \quad \beta>0. \tag{10}$$

We notice that

$$|\sin(\omega t+\delta)|=\begin{cases}\sin(\omega t+\delta), & t\in\left[\frac{2k\pi-\delta}{\omega},\frac{(2k+1)\pi-\delta}{\omega}\right]\\ -\sin(\omega t+\delta), & t\in\left[\frac{(2k+1)\pi+\delta}{\omega},\frac{2k\pi-\delta}{\omega}\right]\end{cases}\text{for} \quad \omega\neq 0. \tag{11}$$

Accordingly, we obtain for $\forall\omega,\ \beta,\ \delta>0$ □

$$\begin{aligned}\phi(\omega,\beta,\delta)&=A_{int}\sum_{k=0}^{\infty}(-1)^k\int_{k\pi-\delta/\omega}^{(k+1)\pi-\delta/\omega}e^{-\beta t}\sin(\omega t+\delta)dt,\\ &=A_{int}\sum_{k=0}^{\infty}(-1)^k-\frac{e^{-\beta t}(\beta\sin(\omega t+\delta)+\omega\cos(\omega t+\delta))}{\beta^2+\omega^2},\\ &=A_{int}\sum_{k=0}^{\infty}\frac{\omega e^{-\frac{\beta(k+1)\pi-\delta}{\omega}}}{\beta^2+\omega^2}+\frac{\omega e^{-\frac{\beta k\pi-\delta}{\omega}}}{\beta^2+\omega^2}.\end{aligned}$$

Finally, we obtain

$$\phi(\omega,\beta,\delta)=A_{int}\frac{\omega e^{\frac{\delta}{\omega}}\left(e^{\frac{-\beta\pi}{\omega}}+1\right)}{(\beta^2+\omega^2)\left(1-e^{-\frac{\beta\pi}{\omega}}\right)}. \tag{12}$$

By taking partial derivatives of ϕ with respect to ω, β, and δ, we obtain

$$\phi_\omega=A_{int}\frac{\left(\left(-1+\frac{\beta^2}{\omega^2}+\frac{\delta}{\omega}\left(1+\frac{\beta^2}{\omega^2}\right)\right)\left(\mathrm{e}^{\frac{2\pi\beta}{\omega}}-1\right)+2\pi\frac{\beta}{\omega}\left(1+\frac{\beta^2}{\omega^2}\right)\mathrm{e}^{\frac{\pi\beta}{\omega}}\right)}{a^{-1}\omega^{-2}\left(\omega^2+\beta^2\right)^2\left(\mathrm{e}^{\frac{\pi\beta}{\omega}}-1\right)^2\mathrm{e}^{\frac{\delta}{\omega}}}, \tag{13}$$

$$\phi_\beta=A_{int}\frac{2a\mathrm{e}^{\frac{\delta}{\omega}}\left(\omega\beta\left(\mathrm{e}^{\frac{2\pi\beta}{\omega}}-1\right)+\pi\left(\beta^2+\omega^2\right)\mathrm{e}^{\frac{\pi\beta}{\omega}}\right)}{\left(\beta^2+\omega^2\right)^2\left(\mathrm{e}^{\frac{\pi\beta}{\omega}}-1\right)^2}, \tag{14}$$

$$\phi_\delta=A_{int}\frac{a\left(\mathrm{e}^{\frac{\pi\beta}{\omega}}+1\right)\mathrm{e}^{-\frac{d}{\omega}}}{\left(\omega^2+b^2\right)\left(\mathrm{e}^{\frac{\pi\beta}{\omega}}-1\right)}. \tag{15}$$

Then our study of partial derivatives ϕ_ω, ϕ_δ, and ϕ_β highlights the impact of the factors ω, δ, and β. It is evident that $\phi_\delta\geq 0$, $\forall\omega$, β, and $\delta>0$. Subsequently, we analyze the conditions where $\phi_\omega>0$ and $\phi_\beta<0$.

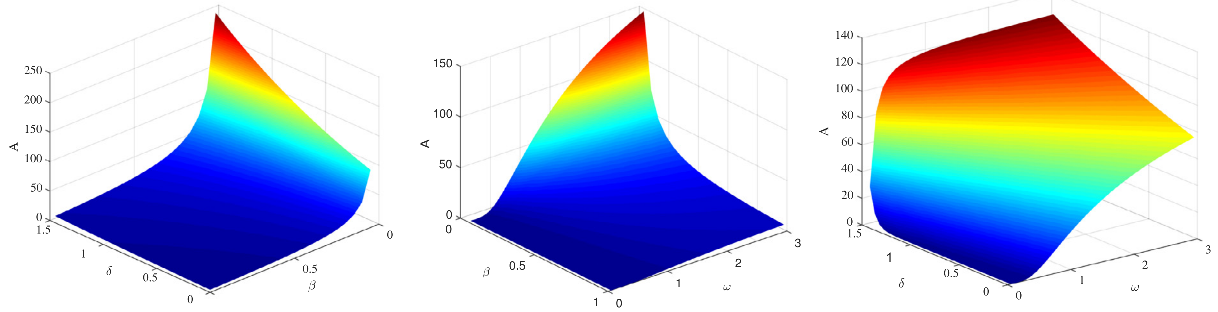
The analysis of ϕ_ω reveals that

1. $\forall\omega,\ \beta,\ \delta>0$, if $\beta>\omega$; then $(\mathrm{e}^{\frac{2\pi\beta}{\omega}}-1)>0$; therefore,

$$\forall\omega,\beta,\delta>0, \quad \text{if} \quad \beta>\omega \quad \Longrightarrow \quad \phi_\omega>0. \tag{16}$$

2. $\forall\omega,\ \beta,\ \delta>0$; if $\beta<\omega$, then $(\mathrm{e}^{\frac{2\pi\beta}{\omega}}-1)<0$. To prove $\phi_\omega>0$, we need to demonstrate that

$$\forall\omega,\beta,\delta>0 \quad if \quad \beta<\omega \quad \Longrightarrow$$
$$\left(\left(-1+\frac{\beta^2}{\omega^2}+\frac{\delta}{\omega}\left(1+\frac{\beta^2}{\omega^2}\right)\right)\left(\mathrm{e}^{\frac{2\pi\beta}{\omega}}-1\right)<2\pi\frac{\beta}{\omega}\left(1+\frac{\beta^2}{\omega^2}\right)\mathrm{e}^{\frac{\pi\beta}{\omega}}\right).$$



(a) Plot of $\phi(\omega = \frac{\pi}{2}, \beta, \delta)$. It shows the impact of β and δ of the trend of the function ϕ.
(b) Plot of $\phi(\omega, \beta, \delta = 0)$. It shows the impact of ω and β of the trend of the function ϕ.
(c) Plot of $\phi(\omega, \beta = 0.2, \delta)$. It shows the impact of ω and δ of the trend of the function ϕ.

Fig. 3. Plot of $\phi(\omega, \beta, \delta)$. This figure shows how the human factors influence the trend of the function ϕ.

Thus, we set $x = \beta/\omega$ for $x \in [0, 1]$, and $y = \delta/\omega$. Furthermore, we assume $f(x) = (-1 + x^2 + \frac{\beta x}{\delta}(1 + x^2))(e^{2\pi x} - 1) + 2\pi x(1 + x^2)e^{\pi x}$. The study f shows $f' > 0$, which implies that f is an increasing function $\forall x \in [0, 1]$ and $\forall \beta, \delta > 0$. Besides, we have $f(0) = 0$ and $f(1) > 0$; we can say that f is a positive function $\forall x \in [0, 1]$. In the light of this demonstration, it can be concluded

$$\forall \omega, \beta, \delta > 0, \quad \text{if} \quad \beta < \omega \quad \Longrightarrow \quad \phi_\omega > 0. \tag{17}$$

Moreover, we have $\phi_\beta < 0$ for $\beta > \omega$ and $\forall \omega, \beta, \delta > 0$. However, for $\beta < \omega$ and $\forall \omega, \beta, \delta > 0$ we have to prove that $\omega\beta(e^{\frac{2\pi\beta}{\omega}} - 1) + \pi(\beta^2 + \omega^2)e^{\frac{\pi\beta}{\omega}} > 0$. Therefore, we set $x = \beta/\omega$ for $x \in [0, 1]$; then, we assumed $f(x) = x(e^{2\pi x} - 1) + \pi(x^2 + 1)e^{\pi x}$. The study f shows that $f' > 0$ where f is an increasing function $\forall x \in [0, 1]$. Besides, we have $f(0) = 0$ and $f(1) > 0$ by which we can conclude that f is a positive function $\forall x \in [0, 1]$. In the light of this demonstration, it can be concluded that if $\beta < \omega$, then $\phi_\beta < 0$ and $\forall \omega, \beta, \delta > 0$.

The study of $\frac{\partial\phi(\omega,\beta,\delta)}{\partial\omega}$, $\frac{\partial\phi(\omega,\beta,\delta)}{\partial\beta}$, and $\frac{\partial\phi(\omega,\beta,\delta)}{\partial\delta}$ shows the following: first, from (16) and (17), we confirm that ϕ_ω is positive for $\forall \omega, \beta, \delta > 0$. From this, we can conclude that ϕ is an increasing function of ω. Second, the analysis shows that ϕ_β is negative for $\forall \omega, \beta, \delta > 0$. Based on this, we can say that ϕ is a decreasing function of β. Since ϕ_δ is positive, ϕ can be considered as an increasing function of δ. Fig. 3 affirms that the peak value of $\phi(\omega, \beta, \delta)$ is attained for lower values of β and higher values of ω and δ. Moreover, we can see that β has a greater impact on the decreasing trend, which is in accordance with the assumption made regarding the importance of the IBK in the spreading process. As a result, we can confirm Proposition 1.

3.3. Individual opinion formulation

During the propagation of a rumor, individuals identify the same rumor differently; this variation manifests and spreads in OSNs differently. Individuals can confirm, refute, query, or discuss matters of interest. Rumor belief classification has attracted several researchers, and various works have proposed approaches for automatic rumor belief classification [43,44]. The existing rumor propagation models did not consider this aspect; its significant role in phenomenons such as rumor propagation has been proven by Bredereck and Elkind [45]. Generally, rumors are classified into four classes: supporting, denying, questioning, or neutral. In this work, we adopt an additive model to estimate B_v, the opinion of an individual v on the rumor. A range of values are associated with each opinion, where $B_v \in [-\infty, 0]$ represents a response in terms of denial, $B_v \in [0, 10]$ a neutral opinion, $B_v \in [10, 20]$ a questioning attitude, and $B_v \in [20, \infty]$ a supporting opinion. This formulation is based on the effect of the herd behavior on the propagation of rumors in [31], showing that most people exhibit a herd mentality that can make people blindly follow others and borrow their opinions. However, when individuals receive the same information more than once, it may not affect them as much as it did initially due to information redundancy [32]. Therefore, we define the opinion of an individual v as follows:

$$B_v(t) = \sum_{u \in \mathbb{N}^v} \sum_{j=1}^{n} \frac{B_u(t-1)}{j}, \quad \text{for} \quad t > 0, \tag{18}$$

where $\mathbb{N}^v$ is the set of neighbors of the individual v and n indicates the number of times v received the rumor from a single neighbor.

3.4. Rumor transmission rules

This section establishes rules on how the rumor can be transmitted in multiplex OSNs, where the attention is given to human interactions and the emphasis on answering the following queries: When will the rumor be sent? When will it be accepted? In which layer of the multiplex will it be sent? Thus, we define the rumor transmission rules between two pairs of nodes inspired by the work of [11,14,15]. The proposed rules are evaluated in the following three steps: network selecting probability, rumor sending probability, and rumor acceptance probability. Given multiplex OSNs with n networks $\mathbb{G}^n = (I, G^n)$, we have defined the rumor transmission probability between two pairs of nodes, u and v in the kth layer of $\mathbb{G}^n$ as follows

$$p^k_{u,v}(t) = p^k_u \cdot p^{send}_u(t) \cdot p^{acc}_{v,u}. \tag{19}$$

First, the network selection probability estimates the probability of node u, sending a rumor in the kth layer of the multiplex $\mathbb{G}^n$. Node degree plays an instrumental role in many processes occurring on or related to complex networks [37]. We consider that a node's in-degree represents the authority of an individual in an OSN [9]. We assume that the probability that an individual will send a rumor in the kth layer of $\mathbb{G}^n$ increasingly depends on its in-degree. Therefore, the probability a node u will select the kth network in $\mathbb{G}^n$ is defined as follows:

$$p^k_u = \frac{d^k_{in}(u)}{\sum_{i=1}^n d^i_{in}(u)}, \tag{20}$$

where $d^i_{in}(u)$ refers to the in-degree of the node u in the ith layer of $\mathbb{G}^n$.

Then, the sending probability estimates the chance a user sends a rumor to his neighbors. This probability depends strongly on the human factors (IBK, FR, and HM). The more the user is attracted to the rumor, the higher are the chances of sending it. Consequently, we define this sending probability as $A(t)/A_{int}$, where the rumor sending probability of a node u at time t is

$$p^{send}_u(t) = e^{-\beta t}|\sin(\omega t + \delta)|. \tag{21}$$

Acceptance probability evaluates the chance individuals accept a rumor from their neighbor. We assume that nodes with higher in-degree have a greater ability (authority) to influence other nodes [9]. However, they cannot be easily influenced; this is known as the celebrity effect. Accordingly, we define a balanced weighted probability after considering the impact of the influence of both the sender u and the receiver v in the kth layer of $\mathbb{G}^n$ as follows:

$$p^{acc}_{v,u} = \frac{1}{1 + d^k_{in}(v)/d^k_{in}(u)} \cdot P, \tag{22}$$

where P is a probability parameter set in the propagation process.

3.5. HISBmodel propagation process

Based on the above analysis and analogy, this section presents the rumor propagation process based on the HISBmodel in multiplex OSNs. Let us consider a population of N individuals joining several OSNs represented by multiplex $\mathbb{G}^n = (I, G^n)$. At time $t = 0$, a set of individuals spread the rumor in different layers of $\mathbb{G}^n$ with different beliefs randomly assigned to each node v; the rest of nodes are ignorant. During this process, if an ignorant accepts the rumor based on the probability of Eq. (19), they then become spreaders and obey the behavior toward the rumor, as presented in Eq. (9). Each time a spreader accepts a rumor, his opinion of the rumor will be updated according to Eq. (18). An individual can receive more than one rumor. However, they can transmit each accepted rumor only once. When a spreader's interest in the rumor fades, they become stifle and can no longer participate in the propagation process. Finally, the spreading process ends when the rumor popularity deteriorates. This is defined as $R(t) \simeq 0$, which can further be defined as follows

$$R(t) = \sum_{i=1}^{n} R_i(t) \quad \text{where} \quad R_i(t) = \sum_{v \in V} A_v(t) \cdot d^i_{in}(v), \tag{23}$$

where $R_i(t)$ is the accumulative attraction of all the individuals in each layer considering their authority.

The rumor popularity is a new metric proposed by our model and illustrates the evolution of the rumor by considering the impact of each individual. Since the individuals possess different authorities in the network, this metric can provide an accurate state of the propagation of the rumor. Apart from the classical metrics provided by the models in the literature, such as the evolution of the number of spreaders and the final size of the rumor (number of infected individuals in the final stage), the HISBmodel provides new metrics for more precise evaluation of the evolution of the rumor and to better understand this phenomenon. This model highlights the evolution of the opinions of individuals; therefore, we can track the evolution of the number of individuals who have a positive opinion about the rumor. Individuals with a negative opinion are believed not to contribute to the spreading of the influence of rumor and contrarily considered to participate in reducing its impact. We evaluate the effect of the rumor by considering the number of infected individuals without a negative opinion; this cohort represents the believers of the rumor. This new metric is at the origin of our motivation to propose a strategy to minimize the influence of the rumors. Our goal is not only to limit the spreading of rumors but also maximize the number of people who do not believe in the rumor. The proposed strategy is presented in the following section.

4. Rumor influence minimization strategy formulation

This section presents our strategy to minimize the influence of the rumor in multiplex OSNs. First, we introduce the problem of rumor influence minimization in multiplex OSNs. Subsequently, we describe our solution to the problem from the perspective of a network inference by exploiting the survival theory. Finally, theoretical proofs are presented to illustrate the performance of the proposed solution.

4.1. Problem formulation

To overcome the adverse effect of rumors, previous studies [14–17] have proposed various node blocking strategies to limit spreading of rumors. However, these strategies have raised several concerns; for instance, authors in [46] have highlighted that if the blocking period exceeds a certain threshold, then the satisfaction of an individual in an OSN is reduced. We distinguish a few works investigating the time blocking of users in these strategies such as [14,15]. Moreover, the blocking nodes or contents in online social networks strategies are considered as a violation of the freedom of expression in many countries.[2] Therefore, instead of excluding individuals from these strategies, we suggest implying the users in this process as the primary actors in minimizing the influence of the rumor. Against this backdrop, we consider the strategy of launching an anti-rumor campaign [7,18–20] to raise the awareness of individuals for preventing the adoption of rumors and further limiting its influence. In their practical application, we have located several websites such as Snoops,[3] Emergent,[4] and the French website Haoxbuster[5] that can track rumors in different OSNs and share the facts on them. Several studies have shown that false information propagates faster than factual information [2,47] since people can interact more often with false rumors. A study [47] investigated the impact of Snopes on the propagation of rumors on Facebook and found a significant decrease in the rate of spreading of rumors after having been detected by this website.

Although individuals are joining several OSNs and forming multiplex structure of OSNs, most studies dealing with rumor detection problems in OSNs [43,44] are conducted in a single network. Generally, Twitter has become the data source par excellence for the collection and analysis of rumors. Therefore, we assume that RIM strategies could only be performed in one network due to the lack of information in other networks. Furthermore, since the rumors are detected after a period of propagation, it is challenging to stop its spreading after already infecting a proportion of the individuals. Therefore, considering the above hypothesis, we define our problem as follows: considering multiplex OSNs, $\mathbb{G}^n = (I, G^n)$. We assume that a rumor is detected in the *ith* layer of the network at time t_{det}. The objective is to select k individuals to launch the truth campaign to minimize the number of individuals who believe the rumor. To accurately select the candidates for the proposed strategy, we have exploited the survival theory to analyze the likelihood of nodes getting infected. This has been discussed in detail previously.

4.2. Proposed solution

We formulate the solution as follows: after a rumor is detected, we consider the population V to be divided into two sets $V = V_{B+} \cup V_{B-}$, where V_{B-} is a set of individuals who have negative opinions and V_{B+} the rest of the population. As mentioned earlier, our goal is to raise the awareness amongst individuals by selecting k most influential nodes from V_{B+} to propagate a negative opinion about the rumor in order to prevent the individuals from adopting the rumor. Accordingly, we propose an additive survival model where the probability of the node u getting activated is the sum of the propagation probabilities represented by Eq. (19). Therefore, the hazard rate of the nodes getting infected by a node v is as follows:

$$h_v(t) = \sum_{i=1}^{n} \sum_{u \in \mathbb{N}_i^v} p^i_{u,v}(t) = p_v^{send}(t) \sum_{i=1}^{n} \sum_{u \in \mathbb{N}_i^v} p_v^i p_{v,u}^{acc}. \tag{24}$$

By substituting $h(t)$ in Eq. (7) yields the cumulative distribution function

$$\begin{aligned} F_v(t) &= 1 - e^{-\int_0^t p_v^{send}(\tau) \sum_{i=1}^{n} \sum_{u \in \mathbb{N}_i^v} p_v^i p_{v,u}^{acc} d\tau} \\ &= 1 - \prod_{i=1}^{n} \prod_{u \in \mathbb{N}^v} e^{-p_v^i p_{v,u}^{acc} \int_0^t p_v^{send}(\tau) d\tau}. \end{aligned} \tag{25}$$

[2] https://bit.ly/2PZ4JFb.
[3] https://www.snopes.com/.
[4] http://www.emergent.info/.
[5] http://www.hoaxbuster.com/.

Then, the likelihood function of the nodes getting infected by v is given as follow

$$f_v(t) = \frac{dF_v(t)}{dt} = \sum_{i=1}^{n}\sum_{u\in\mathbb{N}^v} p_v^i p_{v,u}^{acc} p_v^{send}(t) \prod_{i=1}^{n}\prod_{w\in\mathbb{N}^v} e^{-p_v^i p_{v,w}^{acc}\int_0^t p_v^{send}(\tau)d\tau}. \tag{26}$$

From Eq. (26), we can generalize the likelihood function of any number of nodes getting infected given as

$$f_V(t) = \prod_{v\in V:A_v(t)>0}\sum_{i=1}^{n}\sum_{u\in\mathbb{N}^v} p_v^i p_{v,u}^{acc} p_v^{send}(t) \prod_{i=1}^{n}\prod_{w\in\mathbb{N}^v} e^{-p_v^i p_{v,w}^{acc}\int_0^t p_v^{send}(\tau)d\tau}. \tag{27}$$

A greedy algorithm is designed based on Eq. (27) presented in Algorithm. 1. The objective of this algorithm is to maximize the likelihood of nodes getting infected by the truth campaign by selecting k nodes from V_{B+} to spread the denying opinion about the rumor. Similarly, the objective function can be written as the minimization of the likelihood of the nodes getting infected by the believers of the rumor. The objective function is given as follows

$$\max_I f_{V_{B-}}(t_{det}) \quad \text{or} \quad \min_I f_{V_{B+}}(t_{det}). \tag{28}$$

Algorithm 1: Truth campaign strategy.

Input: $\mathbb{G}^n = (I, G^n), V_{B-}, V_{B+}, k.$
for $i \leftarrow 1 : k$ **do**
  $u = \arg\max_{v\in V_{B+}} \left[f_{V_{B-}\cup\{v\}}(t_{det}) - f_{V_{B-}}(t_{det})\right];$
  $V_{B+} = V_{B+}\setminus u;$
  $V_{B-} = V_{B-}\cup\{u\};$
Output: V_{B-}

4.3. Analysis of the approximation ratio of the TCS algorithm

This section discusses the approximation ratio of the proposed algorithm. The RIM problem has been proven to be an NP-hard problem, as the RIM problem on a single network is a particular case of RIM on multiplex; hence, this last problem is NP-hard as well [41]. Thus, we need to prove that the proposed algorithm guarantees an approximation ratio from the optimal solution. Therefore, given multiplex OSNs $\mathbb{G}^n = (I, G^n)$ and, for further clarity, we note $\sigma(A) = f_A(t)$ the likelihood function of the nodes getting infected by a set $A \in I$ at time t. The submodularity of the functions $\sigma(.)$ presents an excellent way to obtain an approximation of the algorithm for our problem within a factor of $(1-1/e)$. We can say $\sigma(.)$ is submodular if it satisfies the following condition:

$$\sigma(A) + \sigma(B) \geqslant \sigma(A\cup B) + \sigma(A\cap B), \tag{29}$$

where $A,\ B\subset I,\ A\subseteq B$, and $v\notin B$. In other words, σ is submodular if it has the diminishing marginal return property.

Using the following lemmas, we can prove the submodularity property of the functions $\sigma(.)$.

Lemma 1. $\forall A, B\subset I, A\subset B \Longleftrightarrow \sigma(A\cup B) = \sigma(B)$

Proof. We have $\forall A,\ B\subset I$,

$$\sigma(A\cup B) = f_{A\cup B}(t) = \prod_{v\in A\cup B:A_v(t)>0} f_v(t) \tag{30}$$

Since $\forall v\in A \Rightarrow v\in B$

$$\sigma(A\cup B) = f_{A\cup B}(t) = \prod_{v\in B:A_v(t)>0} f_v(t) \tag{31}$$

Therefore

$$\sigma(A\cup B) = f_B(t) = \sigma(B). \tag{32}$$

□

Lemma 2. $\forall A, B\subset V, A\subset B \Longleftrightarrow \sigma(A\cap B) = \sigma(A)$

Proof. We have $\forall A,\ B\subset I$,

$$\sigma(A\cap B) = f_{A\cap B}(t) = \prod_{v\in A\cap B:A_v(t)>0} f_v(t) \tag{33}$$

Since $A \subset B$ then we have

$$\forall v \in A \Rightarrow \quad v \in B$$
$$\forall v \in A \cap B \quad \text{then} \quad v \in A$$
$$\Longrightarrow A \cap B = A$$

$$\sigma(A \cap B) = f_{A \cap B}(t) = \prod_{v \in A : A_v(t) > 0} f_v(t) \tag{34}$$

Therefore

$$\sigma(A \cap B) = f_A(t) = \sigma(A). \tag{35}$$

□

Proposition 2. *The proposed algorithm guarantees an approximation ratio from the optimal solution within a factor of* $(1 - 1/e)$.

Proof. The proof of this claim can be obtained from the submodularity and monoticity of the objective function. Therefore, in the following, this propriety of the objective function will be presented based on Lemmas 1 and 2. It is given that A, $B \subset I$, $A \subseteq B$, and $v \notin B$. Then,

$$\sigma(A) + \sigma(B) \geqslant \sigma(A \cup B) + \sigma(A \cap B). \tag{36}$$

According to Lemmas 1 and 2, we have

$$\sigma(A) + \sigma(B) \geqslant \sigma(B) + \sigma(A). \tag{37}$$

Therefore, we can say that Eq. (29) holds. Moreover, we have $\forall A$, B, $C \subseteq I$ and $A \subseteq B$ and $B \subseteq C$; then,

$$\sigma(B) = f_B(t) = \prod_{v \in B : A_v(t) > 0} f_v(t). \tag{38}$$

Since $A \subseteq B$, then

$$\sigma(B) = f_B(t) = \prod_{v \in A : A_v(t) > 0} f_v(t) + X = \sigma(A) + X. \tag{39}$$

Similarly, we can obtain

$$\sigma(C) = f_C(t) = \prod_{v \in B : A_v(t) > 0} f_v(t) + X' = \sigma(B) + X' = \sigma(A) + X + X', \tag{40}$$

where X ans X' are constants number. Therefore, we can claim that the following equation holds

$$\sigma(A) \leqslant \sigma(B) \leqslant \sigma(C), \tag{41}$$

thus, the function $\sigma(.)$ is monotone. Upon the above, we prove that the objective function is submodular and monotone; hence, the proposed algorithm can guarantee an approximation ratio from the optimal solution within the factor of $(1 - 1/e)$. □

4.4. Time complexity analysis

Proposition 3. *The time complexity of the proposed greedy algorithm is* $O(nkN|\bar{E}|)$, *where* $k << |\bar{E}|$.

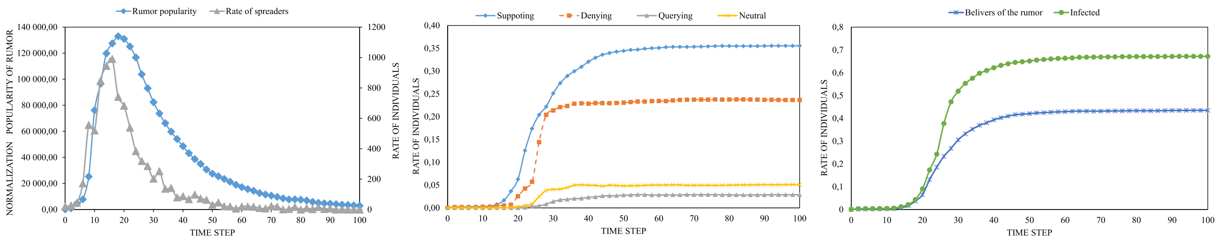
Proof. We consider the proposed algorithm is performed in multiplex $\mathbb{G}^n = (I, G^n)$, where $|V| = N$ is the number of individuals and $|\bar{E}|$ the number of connections in the network. The proposed algorithm enumerates all individuals in the set V_{B^+} and selects the node with the highest marginal likelihood function f. Consequently, to compute this function, the algorithm has to visit every node in V_{B^+} and its connection at most once. This results in the time complexity of $O(|V_B^+| + |\bar{E}| * n) = O(|\bar{E}| * n)$. Subsequently, to select the candidate node u for each k iteration, we need to repeat the estimation of f for each node in V_{B^+}. By considering the worst case scenario where all the nodes are infected and included in the set V_{B^+}, we obtain the time complexity for each k iteration as $O(N)$ times. We conclude after combining these factors that the total time complexity for our proposed algorithm is $O(nkN|\bar{E}|)$. □

5. Experiments

In this section, the experiments have been conducted to highlight the performance of the HISBmodel and the efficiency of the TCS. First, we present the comparisons between the rumor propagation models in the literature with the proposed

Table 1
Data sets description.

#	#Layer	OSN names	Type	#Nodes	#Edge	$\langle k \rangle$	Diameter	⟨ clustering coefficient ⟩
1	1	FaceBook	Undirected	4039	88,234	21.84	8	0.6055
2	1	Twitter	Directed	4,546	280,846	39.53	7	0.5653
3	1	Slashdot	Directed	77,360	905,468	54.55	10	0.0555
4	3	FaceBook	Undirected	663	1,185	1.78		
		Twitter	Directed	5540	62,841	11.52		
		YouTube	Undirected	5,702	84,647	14.84		



(a) **The evolution of the rumor popularity and the rate of individuals attracted to the rumor.** The rumor popularity provides a more accurate overview about the propagation of the rumor compared to the number of individuals attracted by the rumor (individuals spreading the rumor) since the rumor popularity is estimated according to the attraction of individuals to the rumor as well as their impact in OSN.

(b) **The evolution of the rate of infected individuals according to their opinions.** It shows that the HISBmodel provides insights about the opinions of individuals during the propagation process of a rumor.

(c) **The evolution of the rate of infected individuals as well as the number of rumor believers.** This figure illustrates the difference between these two measurements since an infected individual by the rumor can have a negative opinion about it, in which it will contribute to diminishing the impact of the rumor.

Fig. 4. The trend of the propagation of a rumor with the HISBmodel. It shows the new measurements that the HISBmodel has provided for better understand the propagation process and limit the influence of the rumor.

model. Subsequently, we study the influence of the human factor (IBK, FR, and HM) on the rumor spreading under our model. Doing this, we aim to show the features of the HISBmodel. Second, we evaluate the performance of the proposed RIM strategy compared to other strategies. Three datasets were retrieved from Facebook, Twitter, and Slashdot [48] and a multiplex of three OSNs (Facebook, Twitter, and YouTube) [49] have been employed in these experiments and detailed in Table 1. Moreover, a dataset of the collection of Twitter rumors spreading during politically sensitive events has been employed in these experiments. This dataset contains 458 rumors about "the Charlie Hebdo attack" events and 284 rumors about "Ferguson attack." All the experiments[6] in this section have been implemented using MATLAB 2017b and performed on a server (with Intel Xeon processors (34 GHz) and 16 GB memory) with a running Linux operating system.

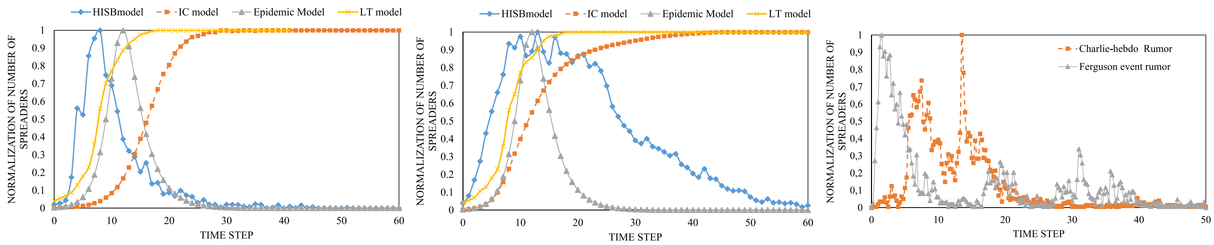
5.1. HISBmodel performance

Our literature review did not yield any study that compares the accuracy of a rumor propagation model to reproduce this phenomenon. Moreover, to the best of our knowledge, a formal method for verifying this characteristic of the information diffusion model does not exist. Therefore, we propose two sets of experiments to highlight the performance of the HISBmodel. In the first part, an overview of the propagation process with the HISBmodel will be presented to illustrate the evolutionary trend of the HISBmodel compared to the classical models and real rumor propagation models. Subsequently, we demonstrate that the impact of the IBK, FR, and HM factors on the rumor spreading in our model are in accordance with the works in the literature.

5.1.1. HISBmodel propagation process overview

In this part of the experiments, we discuss the propagation process of the proposed model. Fig. 4 presents an overview of the simulation the propagation of a rumor on Twitter. For the initial parameters, the human factors were assigned with a random uniform distribution $\beta \in [0.2, 1.2]$, $\omega \in [\pi/12, \pi]$, and $\delta \in [\pi/24, \pi/2]$. Following this, ten (10) nodes were chosen randomly with positive and negative opinions as a set of initial spreaders. Fig. 4(a) illustrates the evolution of the popularity of the rumor and the number of spreaders. As observed, both the charts have an upward trend until they peak and then decrease. However, it can be observed that the peak of these graphs is not reached simultaneously. In other words, the peak of the popularity of the rumor does not refer to the highest rate of numbers or spreaders but instead to the attractiveness

[6] The source code of the HISBmodel is available at URL: https://github.com/adilo231/HISBmodel.git.



(a) **Comparison of the rumor propagation trend between the HISBmodel, the IC model, the LT model, and the Epidemic model on Twitter.** (b) **Comparison of the rumor propagation trend between the HISBmodel, the IC model, the LT model, and the Epidemic model on Facebook.** (c) **An illustration of real-world rumor propagation trends on Twitter.**

Fig. 5. Comparison between the HISBmodel, the traditional models, and real rumors propagation.

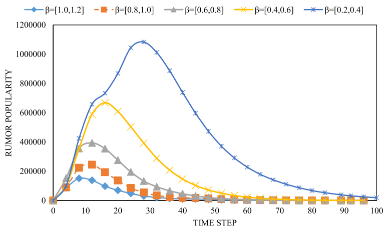
of the population to the rumor. Moreover, we have illustrated in Fig. 4(b) that the HISBmodel helps us display the evolution of the proportion of opinions of the individuals toward the rumor (supporting, denying, querying, and being neutral). Furthermore, Fig. 4(c) displays the ratios of the infected individuals and those who believe in the rumor. These individuals are infected users with a supporting, neutral, or querying reaction. We believe that individuals with negative opinions do not contribute to spreading the influence of the rumor; in fact, they reduce the impact of the rumor, and this fact has been exploited by our strategy for minimizing the influence of rumors.

To highlight the efficiency of our model compared to the models in the previous studies, we conducted simulations for the HISBmodel, the IC model [14,15], the LT model, and the Epidemic model on Twitter and Facebook [48], where the aim was to compare the trends of the rumor propagation. Therefore, the classical SIR model was chosen as the baseline for the Epidemic models. According to Zhao et al. [25], the probability of an individual becoming a spreader is 0.8 and the probability of an individual of becoming a stifler 0.2. For the IC model, we select the model of [14,15] as the baseline; this is an improved IC model that considers individual tendency. The probability of sending the rumor from node u at time t is formulated as $P_u^{send}(t) = \frac{p_0}{\log(10+t)}$, where p_0 is the initial sending probability. The acceptance probability of a node v is formulated as $P_v^{acc} = 1/d_v$ according to [9], where d_v represents the connection degree of the node v. The result is displayed in Fig. 5(a) and (b), where the evolution of the normalized density of spreaders is presented.

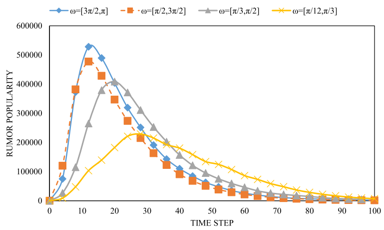
Furthermore, the literature has proved that the evolution of the rumor propagation in an OSN has a rising and falling pattern. This pattern does not present a stable or straightforward shape; it grows in a fast way and fades in a slow fluctuating way [10,11]. To confirm this assumptions, we have displayed in Fig. 5(b) the trends of two real-world rumors propagating on Twitter [50] regarding the “Charlie Hebdo attacks”[7] and “Ferguson Attacks.[8]” These results were obtained after we subdivided the propagation time of the rumor into equal time intervals and counted the number of spreaders in each interval. Since the two rumors have a different number of the spreaders, we normalized the results by dividing them by the maximum value. Moreover, these results were found to be similar to a trend of two real-world rumors spread in Sina Weibo presented in [11].

The experimental results showed the following: first, the epidemic model describes the spreading of rumors goes through the stages of growth and decline in the same way and in a spiky tendency. Second, the IC Model and the LT model simulate only the rising stage of the propagation due to the assumption that once the individuals are activated (infected), they remain in this state. Lastly, the proposed model represents the spread of the rumor with rapid growth and a slow decline, fluctuating in two stages. The HISBmodel and the Epidemic model describe the propagation of the rumor in a similar manner; however, a big difference between these models can be highlighted. Contrary to other models, the HISBmodel has been specially designed to describe the propagation of rumors and centers around how individuals spread rumors in an OSN based on the studies about the role of human factors in the dissemination of rumors in OSNs. The proposed model proposes new metrics such as rumor popularity, the number of rumor believers, and the evolution of the opinion of individuals, and gives us a better insight about the propagation of the rumors for a better assessment of this process. Moreover, the proposed model allows us to test different RIM strategies such as the blocking node strategy or the truth campaign strategy (see Section 5.2) where, for instance, the LT and IC fail to do the same without being updated. Moreover, a few studies have investigated the RIM problem with the Epidemic model due to its macroscopicity approach to investigate the problem. We found that although the HISBmodel presents more complexity than the Epidemic model, the proposed model presents a better approach to describe this process, better features to evaluate the phenomenon, and propose a strategy to minimize its influence. Consequently, the proposed model depicts the pattern of rumor propagation more realistically compared to the IC model, the LT model, and the Epidemic models. These results provide additional support to the performance and the reliability of our model to reproduce the propagation of rumor.

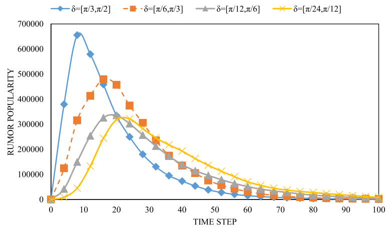
[7] https://en.wikipedia.org/wiki/CharlieHebdoshooting.
[8] https://en.wikipedia.org/wiki/Fergusonunrest.



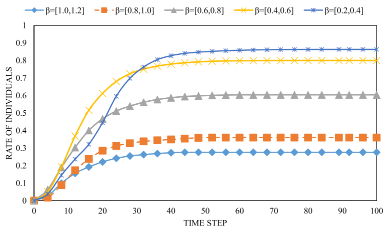
(a) The impact of β on the evolution of the rumor popularity.



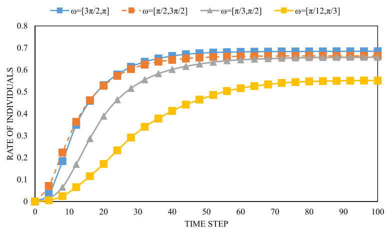
(c) The impact of ω on the evolution of the rumor popularity.



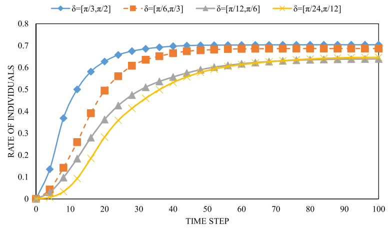
(e) The impact of δ on the evolution of the rumor popularity.



(b) The impact of β on the evolution rate of the infected individuals.

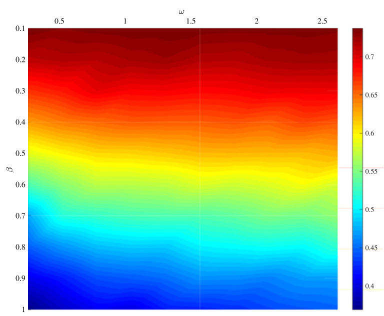


(d) The impact of ω on the evolution rate of the infected individuals.

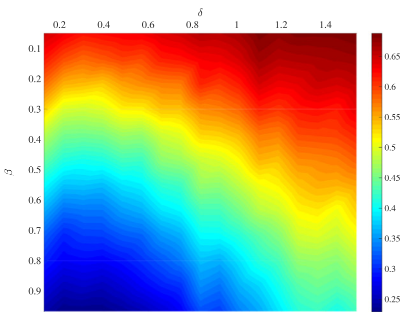


(f) The impact of δ on the evolution rate of the infected individuals.

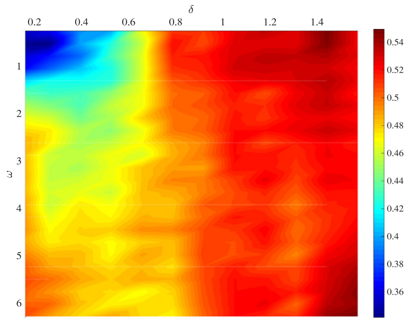
Fig. 6. The impact of the human factors β, ω, and δ on the propagation of the rumor. It illustrates the effect of these factors on the popularity of the rumor and the ratio of the infected individuals.



(a) **The influence of β and ω on the final size of the propagation of a rumor.**



(b) **The influence of β and δ on the final size of the propagation of a rumor.**



(c) **The influence of δ and ω on the final size of the propagation of a rumor.**

Fig. 7. The influence of the human factors on the final size of the propagation of a rumor.

5.1.2. Impact of the human factors on the propagation of rumors

This section illustrates the impact of the IBK, FR, and HM on rumor spreading on Twitter. Therefore, we conducted three experiments and studied the evolution of the propagation process by varying one factor and fixing the other two. The values of ω, β, and δ were assigned with random uniform distribution values based on the studied interval. We selected ten (10) initial spreaders who were chosen randomly; subsequently, we ran the simulations 500 times to avoid randomness.

Fig. 6 displays the results of these experiments and illustrates the impact of these factors on the popularity of the rumor as well as the ratio of the infected individuals. As shown in Fig. 6(a), (c) and (e), the popularity of the rumor increased and reached its peak point; this can be explained by the fact that spreaders were infecting other individuals. We observed a decreasing phase until it reached zero, indicating the cessation of the propagation process of the rumor until it died out. For the ratio of the infected individuals, we found an increasing trend in Fig. 6(b), (d) and (f) until it approached a steady state. Overall, the results show that these factors have a significant impact on (1) the popularity of the rumor, (2) the speed of propagation, and (3) the ratio of the infected individuals. Moreover, we found that the IBK has a more significant impact on the propagation of the rumor compared to the factors ω and δ. For low values of β, the rumor popularity was found to reache the highest value (see Fig. 6(a)) with a larger number of infected individuals and longer propagation time Fig. 6(b). Contrarily, for high values of β, we can observe the lowest value in the popularity of the rumor as well as a smaller number of infected individuals and shorter propagation time. Compered to the other two factors, it was found that β affects the propagation of rumors more significantly than ω and δ (see Fig. 6(c)–(f)). This observation is confirmed by Fig. 7(a) and Fig. 7(b), where a significant impact of β on the rumor final size can be witnessed compared to ω and δ, which is in accordance with Proposition 1. Nevertheless, it can be seen that β does not influence the propagation speed unlike ω and δ. We can also see that for low values, ω and δ of the propagation speed are higher compared to the high values with a wider spread of the rumor. Additionally, Fig. 7(c) reveals that ω and δ have an equivalent effect on the rumor's final size. These results are in accordance with Proposition 1 as well as findings of the works on the impact of the IBK [29], FR [23–26] and HM [27] on the propagation of rumors. These results prove the rationality and correctness of the theoretical assumptions of the proposed model.

5.2. Performance of the proposed strategy

This section aims to evaluate the performance of the proposed influence minimization strategy under our model on the four networks listed in Table 1. The algorithms presented for comparison are as follows:

1. **Natural propagation** (NP) is displayed in the results to illustrate the difference between these algorithms.
2. **Classic greedy algorithm** (CGA) is a basic algorithm that chooses nodes based on the decreasing order of in-degree nodes. This algorithm is selected as a baseline to illustrate the impact of the proposed strategies.
3. **DRIMUX** of [14,15] is a blocking node strategy that selects nodes that are most likely to be infected based on the likelihood principle exploiting the survival theory.
4. **Link removing strategy** (LBS) proposed by [13] selects and removes the relevant link between nodes in the network to minimize the propagation of the rumor.
5. **Positive campaign strategy** (PCS) of [12] a truth campaign strategy algorithm is based on the positive campaign to minimize the influence of the propagation of indecipherable information.
6. **LCAC** of [7] a truth campaign strategy proposed in multiplex structures of OSNs. The objective is to select the least number of nodes to launch an anti-rumor campaign that can target a larger number of overlapping nodes in multiplex OSNs.
7. **Truth campaign strategy** (TCS) is the proposed rumor influence minimization strategy.

Most of the approaches listed above were tested on the classical model (IC or LT); to ensure a uniform environment, the methods were tested on the HISBmodel. It is worth mentioning that a few works investigated the RIM in multiplex OSNs, therefore, it is not suitable to perform the comparison only in multiplex OSNs. Moreover, since our solution has been designed for multiplex OSNs, performing it in a single network is a special case of this solution. Thus, we compare the performance of the TCS to other strategies in a single OSN propagation; subsequently, the experiments are performed in multiplex OSNs.

5.2.1. Experiment setup

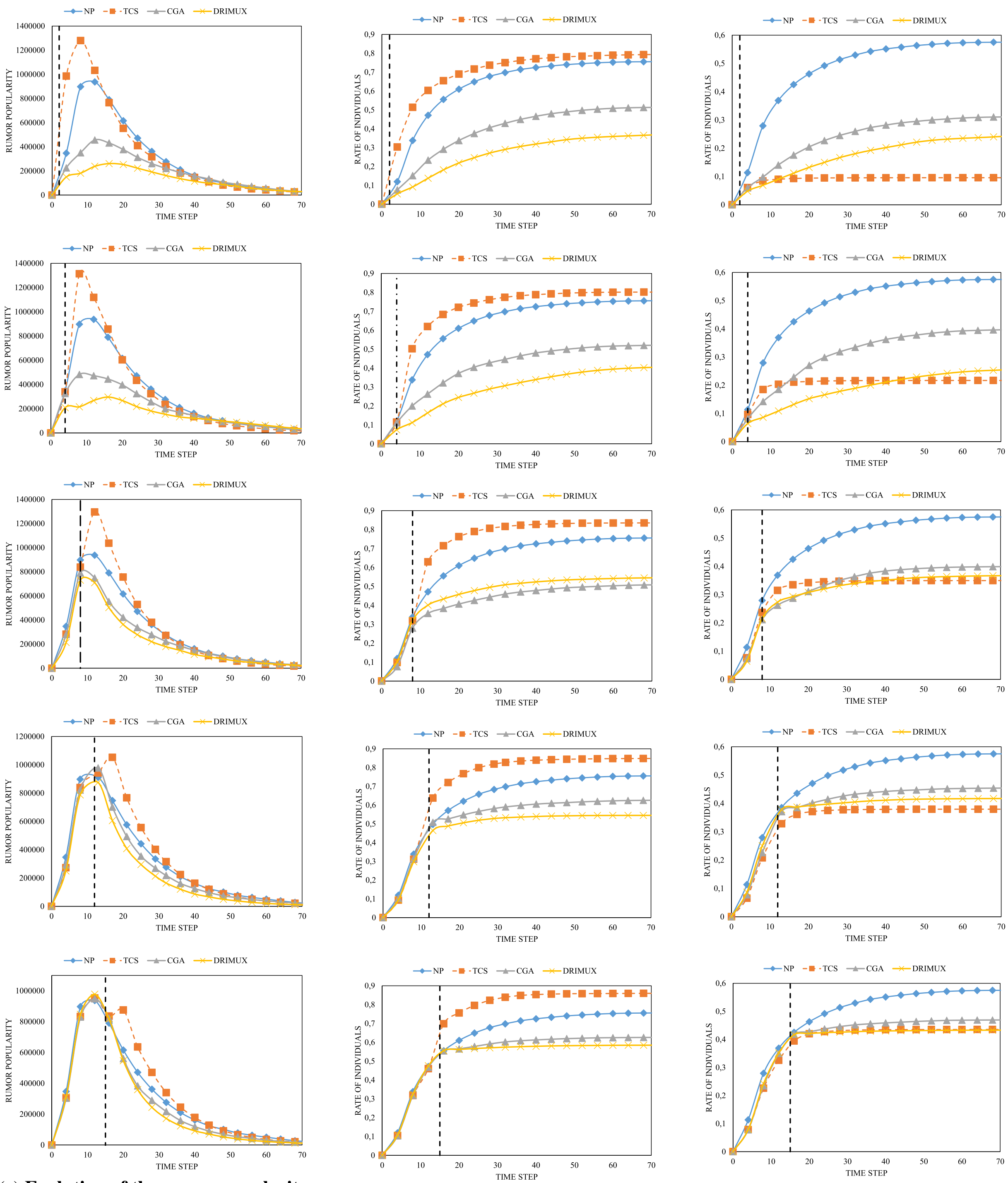
For more clarity, we note the ratio of the nodes selected as targets as RNSasT and the final ratio of individuals who believe the rumor to be FRB. Note that the LBS strategy aims to remove links between the nodes; therefore, the number of links to be removed is set as $|E'| = RNSasT * \langle k \rangle$, where $\langle k \rangle$ is the average degree of nodes in the networks. Inspired by a real-world scenario, we assume that after a particular time $t = t_{det}$ of propagation, the rumor is detected, and the RIM strategies begin. For the case of multiplex OSNs, we assume that the rumor is detected on Twitter, and the RIM strategies are performed only in this layer. The ratio of the rumormonger is set as $0.2\%N$ with different opinions chosen randomly. The individual parameters are assigned with a uniform random distribution to provide a homogeneous population avoiding any experimental noise in the results of the algorithms, where $\omega \in [\pi/12,\ \pi]$, $\beta \in [0.2,\ 1.2]$, and $\delta \in [\pi/24,\ \pi/2]$. We run simulations for different scenarios to test the efficiency of the algorithms in different situations. These experiments vary with the detection time of the rumor $t_{det} = \{2, 4, 8, 12, 15\}$. The detection time is a way to represent the performance of the rumor detection methods, where $t_{det} = \{2, 4\}$ indicate early-stage detection, $t_{det} = \{8, 12\}$ middle-stage detection, and $t_{det} = \{15\}$ late-stage detection. Then, we vary the RNSasT $k = \{5\%, 10\%, 15\%, 20\%\}$ to illustrate the different budgets available for the strategies. RNSasT can be used to represent the threshold to maintain a positive experience of the users in the OSNs. Each simulation is repeated 500 times to avoid randomness. The results of the experiments are presented in Table 2, illustrating the FRB values for each scenario.

5.2.2. Results analysis and interpretation

Before interpreting our results, it is necessary to illustrate the trends of the rumor propagation after the introduction of different RIM. To avoid redundant results, we select an algorithm to represent each strategy. DRIMUX has been selected for the blocking nodes strategy; the TCS represents the truth campaign strategy; and we select CGA as a baseline. We present in Fig. 8 the result of the comparison of three algorithms (CGA, DRIMUX, and TCS) on Twitter for $k = 15\%$ for different detection times. From left to right, Fig. 8 displays the performance of the three strategies in reducing the influence of the rumor. Fig. 8(a) represents the evolution of rumor popularity and Fig. 8(b) shows the trend of the ratio of infected individuals. Fig. 8(c) illustrates the evolution of the ratio of rumor believers, which is the most important metric that reflects an accurate influence of the rumor. Obviously, the rate of the rumor believers decreases considerably after the strategies are introduced (see Fig. 8(c)). Accordingly, the TCS presents the best results among these strategies since the nodes that are most likely to influence others are selected to spread a truth campaign, followed by the DRIMUX algorithm that blocks the susceptible individual from getting infected. It can be seen that after the introduction of the strategies in the early-stages, the CGA

Table 2
The final ratio of the believers of the rumor after the introduction of the rumor propagation strategies in different scenarios. "\" Algorithms can not be performed in this type of data set.

Detection time	Strateries	Twitter				Facebook				Slashdot				Multiplex			
		5%	10%	15%	20%	5%	10%	15%	20%	5%	10%	15%	20%	5%	10%	15%	20%
2	NP	0.5752	0.5752	0.5752	0.5752	0.6374	0.6374	0.6374	0.6374	0.5274	0.5274	0.5274	0.5274	0.6317	0.6317	0.6317	0.6317
	CGA	0.4798	0.4145	0.3127	0.2214	0.4675	0.2840	0.1448	0.0954	0.3604	0.3017	0.2624	0.2333	0.6239	0.6298	0.6272	0.6145
	DRIMUX	0.4451	0.3651	0.2537	0.1714	0.4335	0.2125	0.1015	0.0860	0.2008	0.1757	0.1478	0.1265	0.5242	0.5177	0.5171	0.5170
	LBS	0.3171	0.1715	0.1499	0.0963	0.3582	0.2529	0.2535	0.1848	0.2807	0.2057	0.1784	0.1651	0.5437	0.5433	0.5383	0.5188
	PTC	0.3200	0.2568	0.2200	0.1880	0.2878	0.1877	0.1339	0.1095	0.1763	0.1303	0.1315	0.1226	0.1171	0.0942	0.0870	0.0747
	LCAC	/	/	/	/	/	/	/	/	/	/	/	/	0.0448	0.0269	0.0246	0.0235
	TCS	**0.1266**	**0.1223**	**0.0957**	**0.0952**	**0.1201**	**0.1002**	**0.0896**	**0.0832**	**0.1700**	**0.1487**	**0.1329**	**0.1297**	**0.0148**	**0.0099**	**0.0098**	**0.0078**
4	NP	0.5752	0.5752	0.5752	0.5752	0.6374	0.6374	0.6374	0.6374	0.5274	0.5274	0.5274	0.5274	0.6317	0.6317	0.6317	0.6317
	CGA	0.4974	0.4326	0.3636	0.3208	0.4721	0.3413	0.2139	0.1533	0.4618	0.4215	0.3887	0.3587	0.6170	0.6167	0.6140	0.6197
	DRIMUX	0.4671	0.4139	0.2572	0.2116	0.3905	0.2751	0.1556	0.1331	0.3718	0.3397	0.3030	0.2818	0.5296	0.5235	0.5171	0.5136
	LBS	0.4312	0.3391	0.3072	0.2708	0.3417	0.3263	0.2821	0.2503	0.3577	0.3336	0.3158	0.2998	0.5609	0.5584	0.5496	0.5347
	PTC	0.4339	0.4057	0.3625	0.3200	0.3269	0.2374	0.2052	0.1687	0.3759	0.3186	0.3668	0.2690	0.1171	0.0942	0.0870	0.0747
	LCAC	/	/	/	/	/	/	/	/	/	/	/	/	0.1107	0.0591	0.0537	0.0527
	TCS	**0.2454**	**0.2383**	**0.2169**	**0.1891**	**0.1728**	**0.1582**	**0.1327**	**0.1269**	**0.3096**	**0.2798**	**0.2712**	**0.2550**	**0.0464**	**0.0380**	**0.0300**	**0.0252**
8	NP	0.5752	0.5752	0.5752	0.5752	0.6374	0.6374	0.6374	0.6374	0.5274	0.5274	0.5274	0.5274	0.6317	0.6317	0.6317	0.6317
	CGA	0.5342	0.4537	0.3999	0.3687	0.4896	0.3615	0.3036	0.2357	0.4934	0.4665	0.4411	0.4321	0.5879	0.5880	0.5865	0.5914
	DRIMUX	0.4821	0.4315	0.3677	0.2614	0.4466	0.3565	0.2360	**0.1924**	0.4227	0.4017	0.3850	0.3727	0.5652	0.5626	0.5625	0.5597
	LBS	0.5088	0.5025	0.4198	0.4074	0.4077	0.3619	0.3227	0.3243	0.3807	0.3717	0.3650	0.3607	0.5714	0.5595	0.5347	0.5301
	PTC	0.4492	0.4255	0.4086	0.3970	0.3998	0.3478	0.3412	0.3033	0.4105	0.3817	0.4021	0.3255	0.4220	0.3600	0.3269	0.3199
	LCAC	/	/	/	/	/	/	/	/	/	/	/	/	0.3550	0.3029	0.2191	0.1435
	TCS	**0.3829**	**0.3660**	**0.3498**	**0.3261**	**0.2550**	**0.2184**	**0.2165**	0.2077	**0.3469**	**0.3312**	**0.3044**	**0.2940**	**0.2377**	**0.2069**	**0.1754**	**0.1594**
12	NP	0.5752	0.5752	0.5752	0.5752	0.6374	0.6374	0.6374	0.6374	0.5274	0.5274	0.5274	0.5274	0.6317	0.6317	0.6317	0.6317
	CGA	0.5316	0.4806	0.4545	0.3935	0.4710	0.4163	0.3334	0.2961	0.5021	0.4903	0.4753	0.4669	0.5757	0.5767	0.5721	0.5756
	DRIMUX	0.4878	0.4348	0.4192	**0.3409**	0.4068	0.3591	**0.2055**	**0.1508**	0.4402	0.4290	0.4196	0.4159	0.5861	0.5808	0.5817	0.5711
	LBS	0.5173	0.5043	0.4999	0.4819	0.4054	0.3884	0.3797	0.3694	0.4689	0.4590	0.4496	0.4259	0.5965	0.5825	0.5544	0.5309
	PTC	0.4702	0.4507	0.4400	0.4253	0.4223	0.4012	0.4052	0.3735	0.4297	0.4268	0.4021	0.3939	0.4950	0.4679	0.4460	0.4268
	LCAC	/	/	/	/	/	/	/	/	/	/	/	/	0.4045	0.3646	0.3320	0.3088
	TCS	**0.4197**	**0.4007**	**0.3796**	0.3792	**0.3355**	**0.3206**	0.3076	0.2850	**0.3590**	**0.3579**	**0.3462**	**0.3470**	**0.3445**	**0.3054**	**0.2700**	**0.2656**
15	NP	0.5752	0.5752	0.5752	0.5752	0.6374	0.6374	0.6374	0.6374	0.5274	0.5274	0.5274	0.5274	0.6317	0.6317	0.6317	0.6317
	CGA	0.5455	0.5032	0.4697	0.4464	0.4711	0.4213	0.3746	0.3355	0.5077	0.4994	0.4877	0.4837	0.5687	0.5629	0.5697	0.5610
	DRIMUX	0.4945	0.4607	**0.4299**	**0.4268**	0.4237	0.3956	**0.3520**	**0.2864**	0.4486	0.4385	0.4314	0.4288	0.6046	0.5963	0.5933	0.5888
	LBS	0.5260	0.4963	0.4874	0.4903	0.4258	0.4192	0.4105	0.4092	0.4898	0.4756	0.4552	0.4489	0.6029	0.5644	0.5610	0.5323
	PTC	0.4957	0.4500	0.4741	0.4745	0.4573	0.4500	0.4386	0.4536	0.4642	0.4498	0.4284	0.4474	0.5637	0.5221	0.5018	0.4936
	LCAC	/	/	/	/	/	/	/	/	/	/	/	/	0.4835	0.4758	0.4498	0.4289
	TCS	**0.4520**	**0.4457**	0.4355	0.4280	**0.3865**	**0.3716**	0.3577	0.3682	**0.3731**	**0.3695**	**0.3620**	**0.3585**	**0.4289**	**0.3766**	**0.3472**	**0.3255**

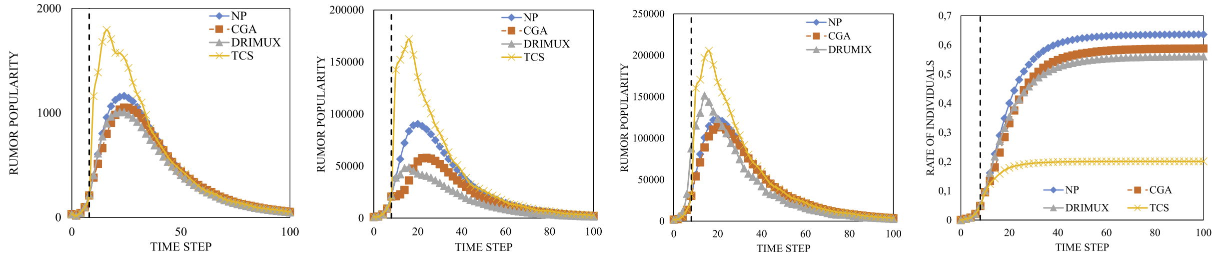


(a) **Evolution of the rumor popularity.**

(b)**Evolution the ratio of infected individuals.**

(c)**Evolution the ratio of rumor's believers.**

Fig. 8. The impact of different types of strategies on the propagation process when $k = 15\%$ **for different detection times of the rumor** $t_{det} = \{2, 4, 8, 12, 15\}$, **represented in dash line, on Twitter.** It illustrates the impact of different types of strategies on the popularity of the rumor, the number of infected nodes, and the number of individuals who believe in the rumor.

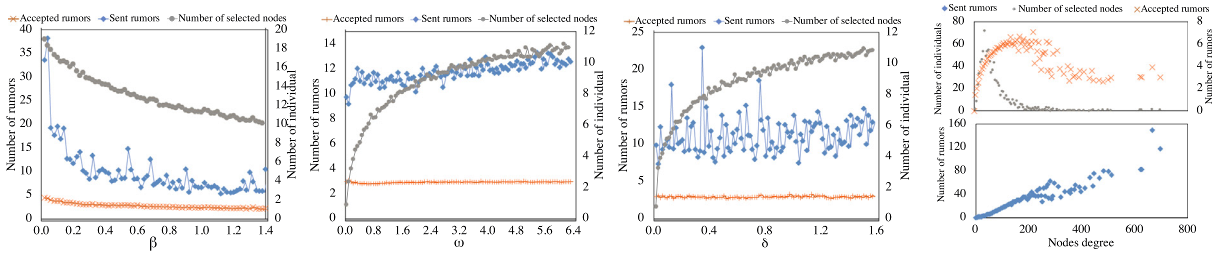


(a) **Evolution of the rumor popularity on Facebook.** (b) **Evolution of the rumor popularity on Twitter.** (c) **Evolution of the rumor popularity onYouTube.** (d) Evolution of the ratio of believers of the rumor

Fig. 9. The impact of the different types of strategies on the rumor propagation process when $k = 10\%$ **and** $t_{det} = 8$ **in multiplex of OSNs.** It illustrates the impact of different type of strategies on the popularity of the rumor and the number of individuals who believes the rumor in the multiplex OSNs.

algorithm and DRIMUX present slower propagation compared to the TCS. This can be explained with Fig. 8(a) and (b); the TCS aims to raise the awareness of individuals and shows an increasing ratio of the people who have heard about the rumor as well as the popularity of the rumor. This strategy does not slow the propagation during the early-stage but prevents the adoption of the rumor to reduce its impact in the long term. From top to bottom, the experiments exhibit the effect of the detection time on the RIM strategies. We observe that the TCS significantly reduce the impact of the rumor at an early-stage detection and is followed by DRIMUX. However, as detection time increases, it is seen that the performance of the three strategies in minimizing the influence of the rumor is reduced. Furthermore, Fig. 9 displays the efficiency of the three strategies in reducing the influence of the rumor in multiplex OSNs. It is observed in Fig. 9(a)–(c) that TCS increases the rumor popularity in all the layers of the multiplex and thus complements the findings of previous experiments. However, it is seen that DRIMUX and CGA reduce the popularity of the rumor only in the Twitter layer since the strategies are applied in this layer. Finally, it is illustrated that the proposed strategy presents the best results in reducing the influence of the rumor as compared to other strategies in multiplex OSNs.

Overall, Table 2 shows that all algorithms significantly reduce the impact of the rumor in the early-stage detection of the rumor compared to late-stage detection. Similarly, a substantial decrease in the impact of the rumor is observed for the high values of RNSasT. We start the analysis on single network experiments when RNSasT $k = 5\%$ and 10%. It can be seen that the truth campaign algorithms (TCS and PTC) performed the best among the strategies in the three networks and significantly reduced the impact of the rumor when discovered in the early-stage $t_{det} = 2, 4$ compared to middle and late-stage detections $t_{det} = 8, 12, 15$. This observation can be explained by the fact that the TCS initiates the most influential nodes to start a truth campaign. Hence, when the rumor is detected in the early-stages, the selected nodes will contribute significantly to diminish the influence of the rumor even when RNSasT is low. Resultantly, FRB is reduced in the final stage. However, in the late-stage detection, even though the TCS exhibits better results, the PTC exhibits low performance than DRIMUX in the case of Facebook or Slashdot. Moreover, CGA, DRIMUX, and LBS have a higher efficiency when RNSasT is small in this case (Twitter and Facebook network). This is due to the fact that in the early stage detection with low values of RNSasT, removing links blocking nodes will be more efficient since the rumor did not infect a large number of nodes. Nevertheless, it is seen in the early-stage detection that LBS has better performance then DRIMUX. When RNSasT $k = 15\%$ and 20%, we perceive that the TCS has better results among the strategies in the three OSNs for $t_{det} = 2, 4, 8,$ followed by DRIMUX and then PTC. However, in the late-stage detection $t_{det} = 12, 15,$ we can see that DRIMUX has the best performance in some cases (see Table 2). Presumably, this observation is explained by the fact that, when the rumor is detected at a relatively late-stage, it would have already infected a significant portion of the nodes in the entire network, and individuals will have already lost interest in the rumor. Additionally, blocking link strategy seems to have less efficiency then the blocking nodes strategy since a large number of nodes are infected in this case, and blocking a link will not prevent the spread of the rumor. The strategy of raising the awareness of individuals during the late-stage is less efficient since the rumor would have a considerable influence. Furthermore, since the Slashdot is the densest network followed by Twitter and Facebook, and this is illustrated by its average degree of connection per node (see Table 1). The rumor will spread faster in Slashdot compared to the other networks. Hence, these networks' features will improve the result of TCS and PTC by amplifying the propagation of the truth campaign. As a consequence, the TCS accomplishes the best results in all the scenarios in Slashdot. However, we observe a relatively lower performance compared to DRIMUX on Twitter and Facebook for late-stage detection. Moreover, for the multiplex OSNs propagation, for the results in Table 2 indicates that truth campaign strategies TCS, PTC, and LCAC present significantly better performance than other strategies in all scenarios. Since DRIMUX, LBS, and CGA work under the closed-world assumption, they are not designed for reducing the impact of the rumor in multiplex OSNs structure. In other words, blocking a node in one network will not reduce its influence in the other networks; however, its activities will remain the same in the other networks. Thus, blocking nodes or links strategies are effective only under closed-world assumptions. Moreover, TCS presents a higher performance than the LCAC and PTC. Since contrary to these two methods, the TCS selects the most influential nodes according to their characteristics and authority in the network based on the



(a) **The influence of β on the distribution of the number of nodes selected by TCS and the number of sent/accepted rumors.** (b) **The influence of ω on the distribution of the number of nodes selected by the TCS and the number of sent/accepted rumors.** (c) **The influence of δ on the distribution of the number of nodes selected by the TCS and the number of sent/accepted rumors.** (d) **The influence of the degree node on the distribution of the number of nodes selected by the TCS and the number of sent/accepted rumors.**

Fig. 10. The influence of different factors on the distribution of the number of nodes selected by the TCS and the number of sent/accepted rumors.

HISBmodel. Therefore, we can conclude that the proposed strategy presents the best results in minimizing the influence of the rumors in multiplex OSNs.

5.2.3. Statistical analysis

Statistical analysis is presented in this section to consolidate our findings to provide insights into the role of the human factors in the rumor influence minimization. The results of the analysis are presented in Fig. 10 and illustrate the impact of different factors on the number of sent/accepted rumors and the distribution of the number of nodes selected by TCS in function of the human factors. To obtain these results, We ran simulations for 5000 times; however, this time, the human factors of each node were randomly assigned different values for each simulation to avoid any biased results. The individuals' characteristics values are assigned with a uniform random distribution to provide a homogeneous population, where $\omega \in [\pi/128, 2\pi]$, $\beta \in [0.02, 1.4]$, and $\delta \in [\pi/128, \pi/2]$. Following this, we subdivided the range of values of each factor studied into 100 equal intervals and counted the number of rumors sent/accepted per node according to each interval. As a result, we obtained the impact human factors (β, ω and δ) and the node's degree on the number of sent/accepted rumor illustrated respectively in Fig. 10. A similar logic has been employed to illustrate the impact of human factors (β, ω, and δ) and the node's degree on the distribution of the number of nodes selected by TCS in function of the human factors.

First, It is seen from Fig. 10(b)–(d) that human factors do not influence the number of accepted rumors. This is a consequence of the initial hypothesis in the rumor acceptance rules. However, nodes' degree has a significant influence on the number of accepted rumors and reflects our rumor acceptance rules and the hypothesis regarding the individuals' authority in the network (see Section 3.4). The number of accepted rumors increased when the degree nodes increases; when it attains a particular peak value, it gradually decreases. This phenomenon is explained by the fact that the more links a node has, the more chances there are of receiving rumors and consequently of accepting more rumors. However, the more a node has links, the authority of a node in the network increases, and this decreases the chances of accepting a rumor. This result is in accordance with our initial hypothesis about the role of the user's authority in human interactions.

Second, in our designing assumption, the sending probability of a rumor has a direct relationship with the attraction of individuals toward rumors and is highly influenced by human factors. The number of sent rumors increases depending on the extent to which an individual is attracted to the rumor. We witness an increasing dependence between the number of sent rumors and ω and δ, while there is a decreasing dependence between β and the number of sent rumors. Moreover, this result shows a more significant impact of β on the number sent rumors then ω and δ. These results are in accordance with our initial hypothesis, the theoretical proof in Proposition 1, and the experimental results obtained in Section 5.1.2; the more individuals are attracted to the rumors, the higher are their the chances of sending the rumors. Moreover, increasing dependence on the number of sent rumors on the degree of nodes is also observed since the chances of sending a rumor are higher if the node has higher number of links.

Finally, Fig. 10(b), (c) and (d), illustrate the influence of the human factors on the selected nodes by the TCS. Nodes with low values of β and high values of ω and δ are observed to be the most likely to be selected by the TCS. The TCS aims to select the most influential node to spread the truth campaign; as shown before, nodes with low values of β and high values of ω and δ are the most likely to send the rumors and are then most likely to be selected by TCS. It is seen in Fig. 10(a), the TCS does not select nodes with a higher degree. However, the number of selected nodes by TCS for each interval of the degree of nodes increased when the degree of nodes increases, upon reaching a peak value, then it gradually decreases. This peak value is obtained at around [35, 40], which are the nodes that are most likely to accept rumors and influence other users. This peak value depends on the number of nodes and the distribution of the degree of nodes in the network. It is generally assumed that nodes with a higher degree are the most influential nodes in a network and hence are the most susceptible to be selected by the RIM strategy. However, even though high degree nodes have great authority in the network, they are less susceptible to be influenced by other users due to the celebrity effect [9,11]. Thus, the TCS aims to select nodes that are most susceptible to be infected by the rumor and are the most likely to infect other individuals to spread the truth campaign.

5.2.4. Discussion

As elaborated above, the proposed method has been tested in four different real networks in diverse scenarios and by considering various situations in real-world circumstances. Moreover, the TCS has been compared with the latest proposed methods that have similar and different characteristics regarding their approach. The results of the experiments confirm that the proposed strategy presents the best results among all strategies in single and multiplex OSNs. The meta-analysis of the obtained results in Table 2 reveals that the TCS achieves an average of 52% in reducing the number of individuals who could be reached by the rumor in all the scenarios on Twitter, Facebook, and Slashdot networks compared to 41% reached by DRIMUX. While in multiplex OSNs, TCS achieved 71% reduction of the impact of the rumor, followed by 63% and 52% for LCAC and PTC respectively, where DRIMUX has realized only 12% performance in this case. Moreover, In these experiments, we introduced several scenarios by varying the detection time of the rumor and the budgets, which illustrate different situations. First, the detection time of the rumor represents the efficiency of the rumor detection method, which reflects the performance of these methods to detect the rumor in a short period. Also, We set various budgets for the RIM algorithms, which set a threshold for the number of individuals that could be selected for the RIM strategy. The evidence from this scenarios provides proof that the TCS performs the best in the worst case when $t_{det} = 15$ and $k = 5\%$ with an average of 31% in reducing the number of individuals could be reached by the rumor, in which DRIMUX achieved the second-best results in this with 17%. In the best case when $t_{det} = 2$ and $k = 20\%$, TCS in this an average performance of 69% reduction of the impact of the rumor compared to the 41% obtained by DRIMUX. These results confirm the effectiveness of the proposed strategy in selecting accurately target nodes based on the HISBmodel to minimize the influence of the rumor in a single network or considering multiplex OSNs. The criteria for selecting the candidate nodes by the TCS depends on the ability of the individual to influence other users based on the human characteristics and their social interactions, which has been testified in the statistical analysis of Section 5.2.3. For instance, the TCS does not favor high degree nodes in their criteria for selecting target nodes; however, the selected nodes are based on the most influential nodes that could easily be infected and infect other nodes.

6. Conclusions and future work

Considering the threat presented by the spread of rumors on online social networks (OSNs) to society, this work highlighted the need for and proposed a strategy for addressing the rumor propagation problem in OSNs. It concentrated on both designing a realistic rumor diffusion model as well as limiting the spreading of rumors. Therefore, we proposed a novel rumor propagation model on multiplex OSNs based on individual and social behaviors known as the HISBmodel. This model takes into account various human factors, such as individual opinions, social influences, and behaviors. Based on this model, we proposed a truth campaign strategy (TCS) to minimize the influence of the rumor in multiplex OSNs from the perspective of a network inference using the survival theory. This strategy selects the most influential nodes and then start a truth campaign to raise individual awareness and prevent the spreading of rumors. Accordingly, we proposed a greedy algorithm based on the likelihood principle that guarantees an approximation within 63% of the optimal solution. We systematically conducted experiments on real datasets to evaluate the performance of our model and to gauge the effectiveness of the proposed influence minimization strategy. First, the results illustrated that our model depicts the evolution of rumor propagation more realistic than other models since it reproduces all the trends of rumor propagation mentioned in the literature and real-world rumor propagation. Moreover, the experiments showed that the proposed model highlights the impact of human factors accurately in accordance with the works of the literature. Second, the comparison analysis showed the efficiency of our strategy in minimizing the influence of the rumor compared to other methods in the literature. In the best-case scenario, these results showed that the proposed method achieves an average of 69% reduction in the number of people most likely to be infected by the rumor. In the worst-case scenario, our method can reduce the impact of the rumor by 31% on an average compared to 17% of the second-best results of the methods of the literature. These results confirm the efficiency of the proposed strategy in accurately selecting target nodes based on the HISBmodel in minimizing the influence of the rumor in a single network or by considering multiplex OSNs. The criteria for selecting the candidate nodes by the TCS lies in the ability of the individual to influence other users based on human characteristics and social interactions.

For future works, we are considering estimating the human factors from real user profile information for accurate results based on psychological and sociological studies on OSNs. We propose an improvement in the propagation process of this model for multiple rumors propagation to introduce a model for the breaking news rumors. Moreover, we consider introducing the topological structure of network features such as community structures in rumor influence minimization strategies for better results.

Declaration of Competing Interest

None

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