E64106267黃崇暉

(a)

```
=== 雅可比法求解 Ax = b ===
A 矩陣:
  4.0  -1.0   0.0  -1.0   0.0   0.0
 -1.0   4.0  -1.0   0.0  -1.0   0.0
  0.0  -1.0   4.0   0.0   1.0  -1.0
 -1.0   0.0   0.0   4.0  -1.0  -1.0
  0.0  -1.0   0.0  -1.0   4.0  -1.0
  0.0   0.0  -1.0   0.0  -1.0   4.0
b 向量: 迭代 -1: b = [  0.000000,  -1.000000,   9.000000,   4.000000,   8.000000,   6.000000]
初始 x^(0): x: [  0.000000,   0.000000,   0.000000,   0.000000,   0.000000,   0.000000]
最大迭代次數: 10

迭代 1: x = [  0.000000,  -0.250000,   2.250000,   1.000000,   2.000000,   1.500000]
迭代 2: x = [  0.187500,   0.812500,   2.062500,   1.875000,   2.562500,   2.562500]
迭代 3: x = [  0.671875,   0.953125,   2.453125,   2.328125,   3.312500,   2.656250]
迭代 4: x = [  0.820313,   1.359375,   2.324219,   2.660156,   3.484375,   2.941406]
迭代 5: x = [  1.004883,   1.407227,   2.454102,   2.811523,   3.740234,   2.952148]
迭代 6: x = [  1.054688,   1.549805,   2.404785,   2.924316,   3.792725,   3.048584]
迭代 7: x = [  1.118530,   1.563049,   2.451416,   2.973999,   3.880676,   3.049377]
迭代 8: x = [  1.134262,   1.612656,   2.432938,   3.012146,   3.896606,   3.083023]
迭代 9: x = [  1.156200,   1.615952,   2.449768,   3.028473,   3.926956,   3.082386]
迭代 10: x = [  1.161106,   1.633231,   2.442845,   3.041386,   3.931703,   3.094181]

達到最大迭代次數 10，可能未完全收斂 (最大差異: 1.73e-002)。

--- 雅可比法最終解 (近似值) ---
迭代 -1: x_final = [  1.161106,   1.633231,   2.442845,   3.041386,   3.931703,   3.094181]

--------------------------------
Process exited after 0.07874 seconds with return value 0
請按任意鍵繼續 . . .
```

(b)

```
=== 高斯-賽德爾法求解 Ax = b ===
A 矩陣:
  4.0  -1.0   0.0  -1.0   0.0   0.0
 -1.0   4.0  -1.0   0.0  -1.0   0.0
  0.0  -1.0   4.0   0.0   1.0  -1.0
 -1.0   0.0   0.0   4.0  -1.0  -1.0
  0.0  -1.0   0.0  -1.0   4.0  -1.0
  0.0   0.0  -1.0   0.0  -1.0   4.0
b 向量: b: [  0.000000,  -1.000000,   9.000000,   4.000000,   8.000000,   6.000000]
初始 x^(0): x: [  0.000000,   0.000000,   0.000000,   0.000000,   0.000000,   0.000000]
最大迭代次數: 10

迭代 1: x = [  0.000000,  -0.250000,   2.187500,   1.000000,   2.187500,   2.593750]
迭代 2: x = [  0.187500,   0.890625,   2.574219,   2.242188,   3.431641,   3.001465]
迭代 3: x = [  0.783203,   1.447266,   2.504272,   2.804077,   3.813202,   3.079369]
迭代 4: x = [  1.062836,   1.595078,   2.465311,   2.988852,   3.915824,   3.095284]
迭代 5: x = [  1.145982,   1.631779,   2.452810,   3.039273,   3.941584,   3.098598]
迭代 6: x = [  1.167763,   1.640539,   2.449388,   3.051986,   3.947781,   3.099292]
迭代 7: x = [  1.173131,   1.642575,   2.448522,   3.055051,   3.949230,   3.099438]
迭代 8: x = [  1.174407,   1.643039,   2.448312,   3.055769,   3.949561,   3.099468]
迭代 9: x = [  1.174702,   1.643144,   2.448263,   3.055933,   3.949636,   3.099475]
迭代 10: x = [  1.174769,   1.643167,   2.448251,   3.055970,   3.949653,   3.099476]

達到最大迭代次數 10，可能未完全收斂 (最大差異: 6.72e-005)。

--- 高斯-賽德爾法最終解 (近似值) ---
x_final: [  1.174769,   1.643167,   2.448251,   3.055970,   3.949653,   3.099476]

--------------------------------
Process exited after 0.1131 seconds with return value 0
請按任意鍵繼續 . . .
```

(c)

```
=== SOR 法求解 Ax = b (w = 1.20) ===
A 矩陣：
  4.0  -1.0   0.0  -1.0   0.0   0.0
 -1.0   4.0  -1.0   0.0  -1.0   0.0
  0.0  -1.0   4.0   0.0   1.0  -1.0
 -1.0   0.0   0.0   4.0  -1.0  -1.0
  0.0  -1.0   0.0  -1.0   4.0  -1.0
  0.0   0.0  -1.0   0.0  -1.0   4.0
b 向量：b: [  0.000000,  -1.000000,   9.000000,   4.000000,   8.000000,   6.000000]
初始 x^(0): x: [  0.000000,   0.000000,   0.000000,   0.000000,   0.000000,   0.000000]
最大迭代次數：10

迭代 1: x = [  0.000000,  -0.300000,   2.610000,   1.200000,   2.670000,   3.384000]
迭代 2: x = [  0.270000,   1.425000,   2.819700,   2.857200,   4.165860,   3.218868]
迭代 3: x = [  1.230660,   1.879866,   2.415922,   3.213176,   4.060401,   3.099123]
迭代 4: x = [  1.281781,   1.651458,   2.423870,   3.089756,   3.940021,   3.089343]
迭代 5: x = [  1.166008,   1.628678,   2.448626,   3.040660,   3.939600,   3.098599]
迭代 6: x = [  1.167600,   1.641012,   2.450278,   3.053608,   3.950046,   3.100377]
迭代 7: x = [  1.174866,   1.644355,   2.448350,   3.056865,   3.950470,   3.099571]
迭代 8: x = [  1.175393,   1.643393,   2.448078,   3.056257,   3.949672,   3.099411]
迭代 9: x = [  1.174816,   1.643091,   2.448233,   3.055918,   3.949592,   3.099465]
迭代 10: x = [  1.174740,   1.643151,   2.448261,   3.055955,   3.949653,   3.099481]

達到最大迭代次數 10，可能未完全收斂 (最大差異：7.68e-005)。

--- SOR 法 (w = 1.20) 最終解 (近似值) ---
x_final: [  1.174740,   1.643151,   2.448261,   3.055955,   3.949653,   3.099481]

--------------------------------
Process exited after 0.07341 seconds with return value 0
請按任意鍵繼續 . . .
```

(d)

```
=== 最速下降法 (共軛梯度法筆記版本) 求解 Ax = b ===
A 矩陣:
  4.0  -1.0   0.0  -1.0   0.0   0.0
 -1.0   4.0  -1.0   0.0  -1.0   0.0
  0.0  -1.0   4.0   0.0   1.0  -1.0
 -1.0   0.0   0.0   4.0  -1.0  -1.0
  0.0  -1.0   0.0  -1.0   4.0  -1.0
  0.0   0.0  -1.0   0.0  -1.0   4.0
b 向量: b: [  0.000000,  -1.000000,   9.000000,   4.000000,   8.000000,   6.000000]
初始 x^(0): x: [  0.000000,   0.000000,   0.000000,   0.000000,   0.000000,   0.000000]
最大迭代次數: 20

迭代  1: t_k =   0.326733, ||v_k|| =  14.071247
迭代  1: x = [  0.000000,  -0.326733,   2.940594,   1.306931,   2.613861,   1.960396]
迭代  2: t_k =   0.229358, ||v_k|| =   8.633862
迭代  2: x = [  0.224816,   1.017622,   2.082205,   2.074485,   2.725134,   2.811972]
迭代  3: t_k =   0.265424, ||v_k|| =   4.395626
迭代  3: x = [  0.806850,   1.007448,   2.553503,   2.463061,   3.522347,   2.695040]
迭代  4: t_k =   0.229073, ||v_k|| =   2.760334
迭代  4: x = [  0.862540,   1.431899,   2.316671,   2.731533,   3.539793,   2.991847]
迭代  5: t_k =   0.266906, ||v_k|| =   1.453772
迭代  5: x = [  1.052916,   1.429606,   2.481424,   2.856458,   3.805660,   2.962241]
迭代  6: t_k =   0.229126, ||v_k|| =   0.925083
迭代  6: x = [  1.069963,   1.572025,   2.403634,   2.946959,   3.811540,   3.062626]
迭代  7: t_k =   0.267057, ||v_k|| =   0.488772
迭代  7: x = [  1.133825,   1.571234,   2.459335,   2.988698,   3.901124,   3.053191]
迭代  8: t_k =   0.229111, ||v_k|| =   0.311624
迭代  8: x = [  1.139468,   1.619198,   2.433207,   3.019250,   3.903100,   3.087030]
迭代  9: t_k =   0.267065, ||v_k|| =   0.164704
迭代  9: x = [  1.160987,   1.618927,   2.451981,   3.033297,   3.933299,   3.083875]
迭代 10: t_k =   0.229104, ||v_k|| =   0.105036
```

```
迭代 11: t_k =   0.267064, ||v_k|| =   0.055516
迭代 11: x = [  1.170137,   1.635001,   2.449506,   3.048334,   3.944144,   3.094218]
迭代 12: t_k =   0.229101, ||v_k|| =   0.035405
迭代 12: x = [  1.170775,   1.640450,   2.446539,   3.051808,   3.944368,   3.098062]
迭代 13: t_k =   0.267063, ||v_k|| =   0.018713
迭代 13: x = [  1.173221,   1.640419,   2.448672,   3.053403,   3.947799,   3.097704]
迭代 14: t_k =   0.229101, ||v_k|| =   0.011934
迭代 14: x = [  1.173436,   1.642255,   2.447672,   3.054574,   3.947875,   3.099000]
迭代 15: t_k =   0.267063, ||v_k|| =   0.006308
迭代 15: x = [  1.174260,   1.642245,   2.448391,   3.055112,   3.949031,   3.098879]
迭代 16: t_k =   0.229101, ||v_k|| =   0.004023
迭代 16: x = [  1.174333,   1.642864,   2.448054,   3.055507,   3.949057,   3.099316]
迭代 17: t_k =   0.267063, ||v_k|| =   0.002126
迭代 17: x = [  1.174610,   1.642861,   2.448296,   3.055688,   3.949446,   3.099275]
迭代 18: t_k =   0.229101, ||v_k|| =   0.001356
迭代 18: x = [  1.174635,   1.643069,   2.448183,   3.055821,   3.949455,   3.099422]
迭代 19: t_k =   0.267063, ||v_k|| =   0.000717
迭代 19: x = [  1.174729,   1.643068,   2.448264,   3.055882,   3.949586,   3.099409]
迭代 20: t_k =   0.229101, ||v_k|| =   0.000457
迭代 20: x = [  1.174737,   1.643138,   2.448226,   3.055927,   3.949589,   3.099458]

達到最大迭代次數 20，可能未完全收斂 (最大 x 差異: 7.03e-005, 殘差範數: 4.57e-004)。

--- 共軛梯度法之最終解 (近似值) ---
x_final: [  1.174737,   1.643138,   2.448226,   3.055927,   3.949589,   3.099458]

--------------------------------
Process exited after 0.09559 seconds with return value 0
請按任意鍵繼續 . . .
```

經網路計算機計算後得以下對應各未知數之結果

$x_0$ = 1.1747885622231171
$x_1$ = 1.6431735803463556
$x_2$ = 2.4482480869915424
$x_3$ = 3.055980668546114
$x_4$ = 3.949657672170761
$x_5$ = 3.099476439790576

誤差範圍經比對與以上之計算答案均小於 1%