王6410636 黃崇嶧

$\cos(0.698) = 0.7661$, $\cos(0.733) = 0.7432$

$\cos(0.768) = 0.7193$, $\cos(0.803) = 0.6946$.

1 degree.: 取 $\cos(0.733) / \cos(0.768)$

$$P_1(x) = 0.7432 \times \frac{x - 0.768}{0.733 - 0.768} + 0.5193 \times \frac{x - 0.733}{0.768 - 0.733}$$

$$\cong 0.731591 \#$$

```
D:\桌面\新文件1.exe
請選擇插值次數 (1-4): 1
請輸入 2 個數據點 (x y):
0.733 0.7432
0.768 0.7193
請輸入要計算的x值: 0.750
f(0.750) = 0.731591

--------------------------------
Process exited after 42.61 seconds with return value 0
請按任意鍵繼續 . . .
```

Lagrange Error Bound

Let $f(x)$ be differentiable through the order $n+1$. The error between the Taylor Polynomial and $f(x)$ is bounded by:

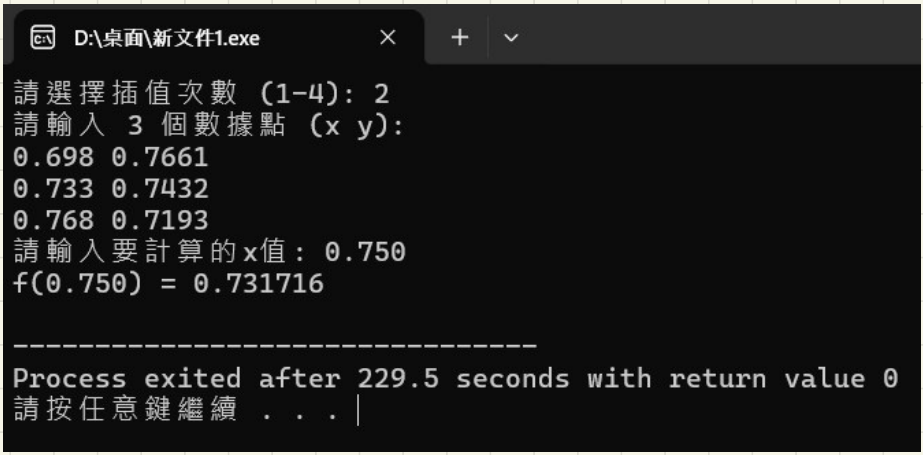
$$|R_n(x)| \leq \left| \frac{\text{MAX}\left[f^{(n+1)}(z) \right](x-c)^{n+1}}{(n+1)!} \right|$$

where z is some number between c and x.

cos
-sin
-cos
+sin
+cos

$$|R_n(x)| \le \left| \frac{MAX\{f^{(j+1)}(\cos x)\}(0.75-0)^{j}}{(j+1)!} \right| = 0.2812$$

2 degree:



$P_2(x) \cong 0.731716$

$$|R_n(x)| = \left| \frac{MAX\{|\sin(x)|\}(0.75-0)^{3}}{(j+2)!} \right| = 0.07031$$

3 degree:

```
D:\桌面\新文件1.exe
請選擇插值次數 (1-4): 3
請輸入 4 個數據點 (x y):
0.698 0.7661
0.733 0.7432
0.768 0.7193
0.803 0.6946
請輸入要計算的x值: 0.750
f(0.750) = 0.731704

範例測試：cos(0.750) 近似計算
使用3次插值結果: 0.731704 (實際值: 0.7317)

--------------------------------
Process exited after 58.56 seconds with return value 0
請按任意鍵繼續 . . .
```

$P_2(x) \cong 0.731704$

$$|R_n(x)| = \left| \frac{MAX(\cos(x)) \cdot (0.75-0)^4}{(1+3)!} \right| = 0.01318 \#$$

4 degrees:

$$P_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$$= 0.731693$$

$$|R_n(x)| = \left| \frac{MAX(-\sin(x)) \cdot (0.75)^5}{(1+4)!} \right| = 1.9775 \times 10^{-3} \#$$

2.

$f(x) = x - e^{-x} = 0$. 求 x.

$e^{-0.3} = 0.740818$ $\quad f(0.3) = 0.3 - 0.740818 = -0.440818$.

$e^{-0.4} = 0.670320$ $\quad f(0.4) = 0.4 - 0.670320 = -0.270320$

$e^{-0.5} = 0.606531$ $\quad f(0.5) = 0.5 - 0.606531 = -0.106531$

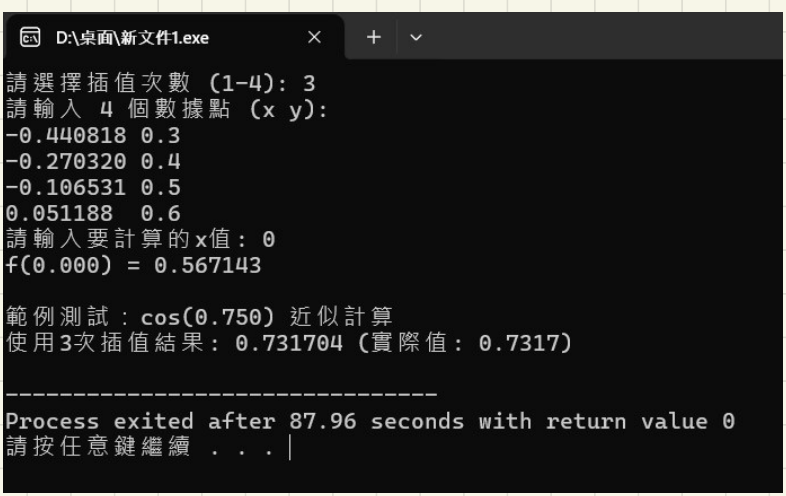
$e^{-0.6} = 0.548812$ $\quad f(0.6) = 0.6 - 0.548812 = 0.051188$

$f^{-1}(-0.440818) = 0.3$

$f^{-1}(-0.270320) = 0.4$

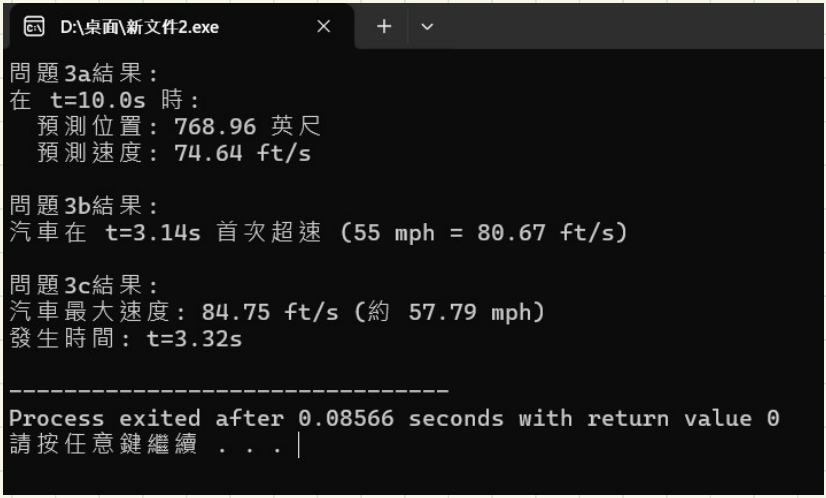
$f^{-1}(-0.106531) = 0.5$

$f^{-1}(0.051188) = 0.6$



程式方面延續HW3.1，得 $x = 0.567143$ #

3.


D:\桌面\新文件2.exe
問題3a結果：
在 t=10.0s 時：
預測位置：768.96 英尺
預測速度：74.64 ft/s
問題3b結果：
汽車在 t=3.14s 首次超速 (55 mph = 80.67 ft/s)
問題3c結果：
汽車最大速度：84.75 ft/s (約 57.79 mph)
發生時間：t=3.32s

Process exited after 0.08566 seconds with return value 0
請按任意鍵繼續 . . .