

## HW 12

1. Given the problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = xy, \quad 0 < x < \pi, \quad 0 < y < \pi/2$$

$$u(0, y) = \cos y, \quad u(\pi, y) = -\cos y, \quad 0 \leq y \leq \pi/2,$$

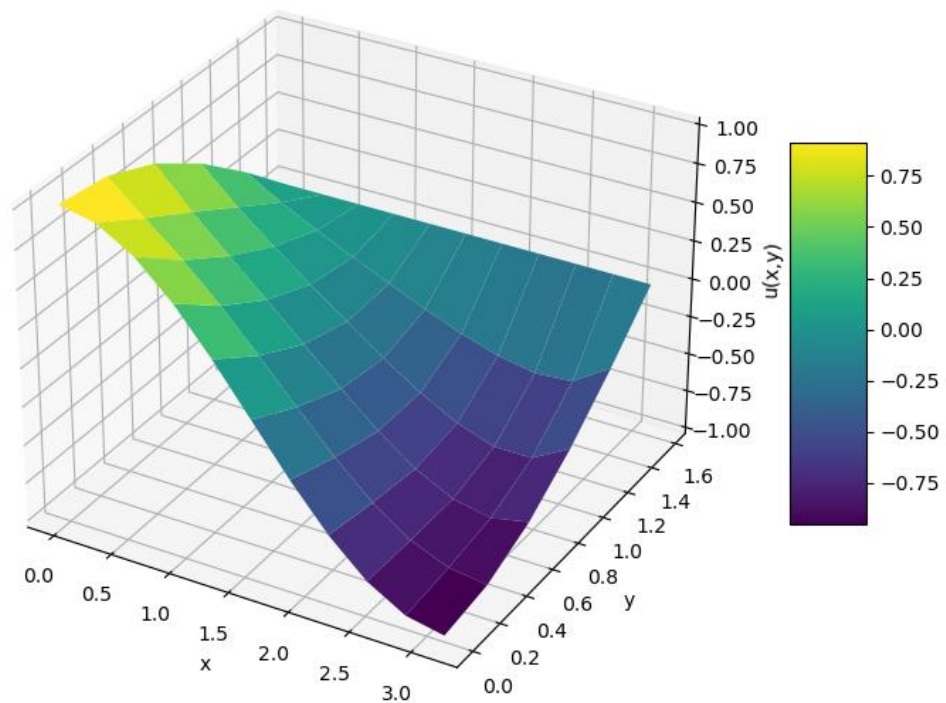
$$u(x, 0) = \cos x, \quad u(x, \pi/2) = 0, \quad 0 \leq x \leq \pi$$

To calculate  $u(x, y)$  by using  $h = k = 0.1\pi$ .

這以用到此線上編譯為主

<https://www.tutorialspoint.com/compiler/online-matplotlib-compiler.htm>

Problem 1: Solution to Poisson's Equation



2. Given the problem

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{4K} \frac{\partial T}{\partial t}, \quad \frac{1}{2} \leq r \leq 1, \quad 0 \leq t,$$

$$T(1, t) = 100 + 40t, \quad 0 \leq t \leq 10; \quad \frac{\partial T}{\partial r} + 3T = 0 \quad \text{at} \quad r = \frac{1}{2}$$

$$T(r, 0) = 200(r - 0.5), \quad 0.5 \leq r \leq 1,$$

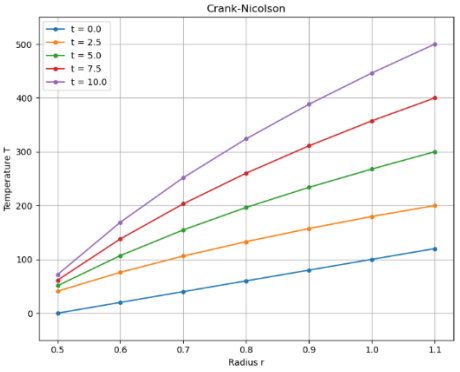
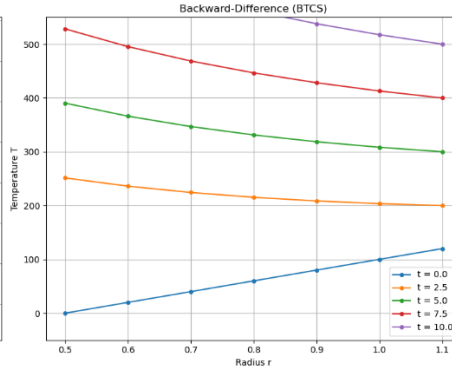
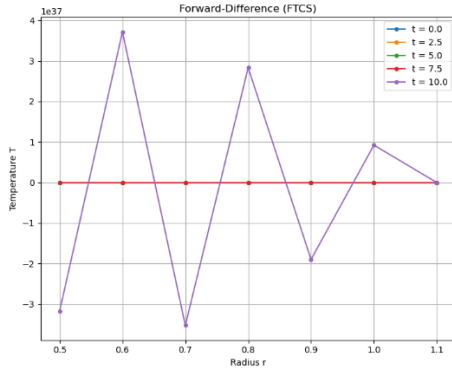
and use  $\Delta t = 0.5$ ,  $\Delta r = 0.1$ , and  $K = 0.1$  to calculate  $T(r, t)$

By (a) the forward-difference method

(b) the backward-difference method

© the Crank-Nicolson algorithm.

Problem 2: Detailed Heat Equation Solutions

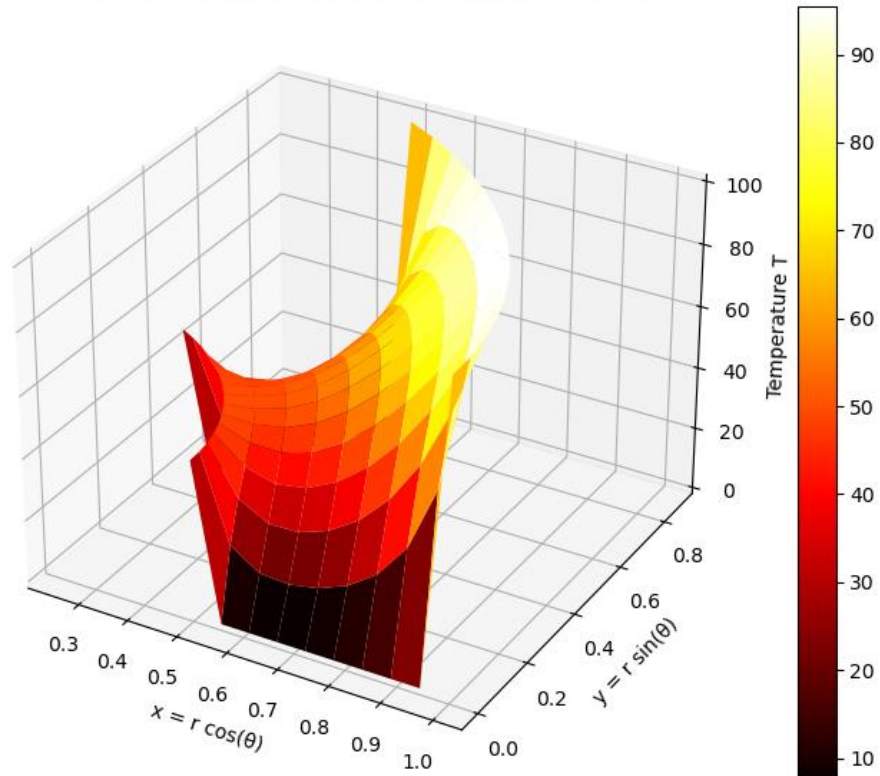


3. Given the problem

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0, \quad \frac{1}{2} \leq r \leq 1, \quad 0 \leq \theta \leq \pi/3,$$

$$T(r, 0) = 0, \quad T(r, \pi/3) = 0, \quad T(1/2, \theta) = 50, \quad T(1, \theta) = 100.$$

Problem 3: Solution to Laplace's Equation (Corrected)



4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \leq x \leq 1, \quad 0 \leq t$$

$$p(0,t)=1, \quad p(1,t)=2, \quad p(x,0)=\cos(2\pi x), \quad \frac{\partial p}{\partial t}(x,0)=2\pi \sin(2\pi x), \quad 0 \leq x \leq 1$$

To calculate  $p$  by using  $\Delta x = \Delta t = 0.1$ .

3D view of the Wave Propagation

