

求解 BVP: $y'' = -(x+1)y' + 2y + (1-x^2)e^{-x}$
定義域: $[0.0, 1.0]$, 邊界條件: $y(0.0)=1.0$, $y(1.0)=2.0$, $h=0.10$

--- a. 射擊法 (Shooting Method) ---

計算得到的初始斜率 $y'(a) = 0.024157$

x	y_shooting
0.00	1.000000
0.10	1.016650
0.20	1.059293
0.30	1.124476
0.40	1.209121
0.50	1.310528
0.60	1.426377
0.70	1.554712
0.80	1.693917
0.90	1.842688
1.00	2.000000

--- b. 有限差分法 (Finite Difference Method) ---

x	y_fdm
0.00	1.000000 (邊界)
0.10	1.016532
0.20	1.059102
0.30	1.124251
0.40	1.208890
0.50	1.310313
0.60	1.426194
0.70	1.554570
0.80	1.693822
0.90	1.842642
1.00	2.000000 (邊界)

--- c. 變分法 (Variational Method / FEM) ---

x	y_fem (y1+y2)
0.00	1.000000 (邊界)
0.10	0.625873
0.20	0.754565
0.30	0.890354
0.40	1.032634
0.50	1.180879
0.60	1.334658
0.70	1.493643
0.80	1.657602
0.90	1.826404
1.00	2.000000 (邊界)