HW6 E64106369-黄崇嶧.

1. Use Gaussian elimination and pivoting technique to solve

$$1.19x_1 + 2.11x_2 - 100x_3 + x_4 = 1.12$$

$$14.2x_1 - 0.112x_2 + 12.2x_3 - x_4 = 3.44$$

$$100x_2 - 99.9x_3 + x_4 = 2.15$$

$$15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16$$

$$\begin{bmatrix} 1.19 & 2.11 & -100 & 1 \\ 14.2 & -0.112 & 12.2 & -1 \\ 0 & 100 & -99.9 & +1 \\ 15.3 & 0.110 & -13.1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 3.44 \\ 2.15 \\ 4.16 \end{bmatrix}$$

$$\begin{bmatrix} 15.3 & 0.110 & -13.1 & -1 \\ 14.2 & -0.112 & 12.2 & -1 \\ 1.19 & 2.11 & -100 & 1 \\ 0 & 100 & -99.9 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ 3.44 \\ 1.12 \\ 2.15 \end{bmatrix}$$

$$\begin{bmatrix} 15.3 & 0.110 & -13.1 & -1 \\ 0 & -0.2139 & 24.35 & -0.07189 \\ 0 & 2.1014 & -98.9811 & 1.0777 \\ 0 & 100 & -99.9 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ -0.4209 \\ 0.7964 \\ 2.15 \end{bmatrix}$$

$$\begin{bmatrix} 15.3 & 0.110 & -13.1 & -1 \\ 0 & -0.2139 & 24.35 & -0.07189 \\ 0 & 0 & 140.3386 & 0.21594 \\ 0 & 0 & 11283.92 & -32.6091 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ -0.4209 \\ -3.3386 \\ -194.62 \end{bmatrix}$$

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\begin{bmatrix} 15.3 & 0.110 & -13.1 & -1 \\ 0 & -0.2139 & 24.35 & -0.07189 \\ 0 & 0 & 140.3386 & 0.21594 \\ 0 & 0 & 1283.92 & -32.6091 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ -0.4209 \\ -3.3386 \\ -194.62 \end{bmatrix}
$$

$$
\begin{bmatrix} 15.3 & 0.110 & -13.1 & -1 \\ 0 & -0.2139 & 24.35 & -0.07189 \\ 0 & 0 & 140.3386 & 0.21594 \\ 0 & 0 & 0 & -49.98412 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ -0.4209 \\ -3.3386 \\ 54.011 \end{bmatrix}
$$

解上述四方程式可得 $x_1 = 0.17678$

$x_2 = 0.01269$

$x_3 = -0.02066$

$x_4 = -1.08526$

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2. Find the inverse of the matrix $A$ where

$$A = \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$$

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$$\left[\begin{array}{cccc|cccc} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

(-1)
(+1)

$$\left[\begin{array}{cccc|cccc} 1 & \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{11}{4} & -\frac{3}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & -\frac{3}{4} & \frac{23}{4} & 2 & \frac{1}{4} & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$-\frac{1}{4}$
$\frac{3}{4}$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & -\frac{3}{11} & 0 & -\frac{1}{11} & \frac{4}{11} & 0 & 0 \\ 0 & -\frac{3}{4} & \frac{23}{4} & 2 & \frac{1}{4} & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{2}{11} & 0 & \frac{3}{11} & -\frac{1}{11} & 0 & 0 \\ 0 & 1 & -\frac{3}{11} & 0 & -\frac{1}{11} & \frac{4}{11} & 0 & 0 \\ 0 & 0 & \frac{61}{11} & 2 & \frac{2}{11} & \frac{3}{11} & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$+\frac{2}{11}$
$+\frac{3}{11}$
$-2$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{2}{11} & 0 & \frac{3}{11} & -\frac{1}{11} & 0 & 0 \\ 0 & 1 & -\frac{3}{11} & 0 & -\frac{1}{11} & \frac{4}{11} & 0 & 0 \\ 0 & 0 & 1 & \frac{22}{61} & \frac{2}{61} & \frac{3}{61} & \frac{11}{61} & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{4}{61} & \frac{17}{61} & -\frac{5}{61} & \frac{2}{61} & 0 \\ 0 & 1 & 0 & \frac{6}{61} & -\frac{5}{61} & \frac{23}{61} & \frac{3}{61} & 0 \\ 0 & 0 & 1 & \frac{22}{61} & \frac{2}{61} & \frac{3}{61} & \frac{11}{61} & 0 \\ 0 & 0 & 0 & \frac{183}{61} & -\frac{4}{61} & -\frac{6}{61} & -\frac{22}{61} & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{4}{61} & \frac{17}{61} & -\frac{5}{61} & \frac{2}{61} & 0 \\ 0 & 1 & 0 & \frac{6}{61} & -\frac{5}{61} & \frac{23}{61} & \frac{3}{61} & 0 \\ 0 & 0 & 1 & \frac{22}{61} & \frac{2}{61} & \frac{3}{61} & \frac{11}{61} & 0 \\ 0 & 0 & 0 & 1 & -\frac{4}{261} & -\frac{6}{261} & -\frac{22}{261} & \frac{61}{261} \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{4}{61} & \frac{73}{261} & -\frac{21}{261} & \frac{10}{261} & -\frac{4}{261} \\ 0 & 1 & 0 & \frac{6}{61} & -\frac{21}{261} & \frac{99}{261} & \frac{15}{261} & -\frac{6}{261} \\ 0 & 0 & 1 & \frac{22}{61} & \frac{10}{261} & \frac{15}{261} & \frac{55}{261} & -\frac{22}{261} \\ 0 & 0 & 0 & 1 & -\frac{4}{261} & -\frac{6}{261} & -\frac{22}{261} & \frac{61}{261} \end{array}\right]$$

$$\frac{22}{61} \times \frac{-4}{261} - \frac{2}{61} \qquad \frac{\cancel{522+88}\ 10}{\cancel{61} \times 261}$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{73}{261} & \frac{-21}{261} & \frac{10}{261} & \frac{-4}{261} \\ \frac{-21}{261} & \frac{33}{261} & \frac{15}{261} & \frac{-6}{261} \\ \frac{10}{261} & \frac{15}{261} & \frac{55}{261} & \frac{-22}{261} \\ \frac{-4}{261} & \frac{-6}{261} & \frac{-22}{261} & \frac{61}{261} \end{bmatrix} = \frac{1}{261} \begin{bmatrix} 73 & -21 & 10 & -4 \\ -21 & 99 & 15 & -6 \\ 10 & 15 & 55 & -22 \\ -4 & -6 & -22 & 61 \end{bmatrix}$$

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3. Use Crout factorization for a tri-diagonal system to solve the problem

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{Bmatrix}.$$

$$L = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ 0 & l_{32} & l_{33} & 0 \\ 0 & 0 & l_{43} & l_{44} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & 0 & 0 \\ 0 & 1 & u_{23} & 0 \\ 0 & 0 & 1 & u_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

① $l_{11} = a_{11} = 3$

② $u_{12} = a_{12}/l_{11} = -1/3$

③ $i=2$
$$\begin{cases} l_{21} = a_{21} = -1 \\ l_{22} = a_{22} - l_{21} \times u_{12} = 3 - (-1) \times (-1/3) = 8/3 \\ u_{23} = a_{23}/l_{22} = -1/\tfrac{8}{3} = -3/8 \end{cases}$$

④ $i=3$
$$\begin{cases} l_{32} = a_{32} = -1 \\ l_{33} = a_{33} - l_{32} \times u_{23} = 3 + \frac{-3}{8} = \frac{21}{8} \\ u_{34} = a_{34}/l_{33} = (-1)/\frac{21}{8} = -8/21 \end{cases}$$

⑤ $i=4$
$$\begin{cases} l_{43} = a_{43} = -1 \\ l_{44} = a_{44} - l_{43} \times u_{34} = 3 - 1 \times \frac{8}{21} = \frac{55}{21} \end{cases}$$

$$\therefore L = \begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & 8/3 & 0 & 0 \\ 0 & -1 & 21/8 & 0 \\ 0 & 0 & -1 & 55/21 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 1 & -3/8 & 0 \\ 0 & 0 & 1 & -8/21 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$Ly=b$ 求解如下：

① $l_{11}y_1 = b_1 \Rightarrow 3y_1 = 2$， $y_1 = 2/3$

$l_{21}y_1 + l_{22}y_2 \Rightarrow (-1)\times\frac{2}{3} + (\frac{8}{3})\times y_2 = 3$， $y_2 = \frac{11}{8}$

$l_{32}y_2 + l_{33}y_3 \Rightarrow \frac{-11}{8} + \frac{21}{8}y_3 = 32/8 \Rightarrow y_3 = \frac{43}{21}$

$l_{43}y_3 + l_{44}y_4 = b_4 \Rightarrow (-1)\frac{43}{21} + \frac{55}{21}y_4 = 1 \Rightarrow y_4 = \frac{64}{55}$

$\therefore y = [2/3, 11/8, 43/21, 64/55]$

$Ux=y$ 求解如下：

① $x_4 = y_4 = 64/55$

$x_3 + u_{34}x_4 = y_3 \Rightarrow x_3 + \frac{-8}{21}\times\frac{64}{55} = \frac{43}{21}$， $x_3 = \frac{959}{385}$

$x_2 + u_{23}x_3 = y_2 \Rightarrow x_2 + \frac{-3}{8}\times\frac{959}{385} = \frac{11}{8}$， $x_2 = \frac{889}{385}$

$x_1 + u_{12}x_2 = y_1 \Rightarrow x_1 + \frac{-1}{3}\times\frac{889}{385} = \frac{2}{3}$， $x_1 = \frac{553}{385}$

$x_1 + \frac{-1}{3}\times\frac{889}{385} = \frac{2}{3}$， $x_1 = \frac{553}{385}$

$\therefore x_1 = \frac{553}{385}$

$x_2 = \frac{889}{385}$

$x_3 = \frac{959}{385}$

$x_4 = \frac{64}{55}$ #