HW8

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1. Given the data as listed below

x	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3
y	102.6	113.2	130.1	142.1	167.5	195.1	224.9	256.8

a. Construct the least squares approximation of degree two and compute the error.

b. Construct the least squares approximation of the form be^{ax} and compute the error.

c. Construct the least squares approximation of the form bx^n and compute the error.

$n=8$, $\sum x_i = 40.2$, $\sum x_i^2 = 206.74$, $\sum x_i^3 = 1087.5$ $\sum x_i^4 = 5843.7$,

$\sum y_i = 1332.3$, $\sum x_i y_i = 7011.21$ $\sum x_i^2 y_i = 37691.57$

(a).

令 $P(x) = ax^2+bx+c$

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & \sum 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i^2 y_i \\ \sum x_i y_i \\ \sum y_i \end{bmatrix}$$

$$\begin{bmatrix} 5843.8 & 1087.5 & 206.7 \\ 1087.5 & 206.7 & 40.2 \\ 206.7 & 40.2 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 37691.57 \\ 7011.21 \\ 1332.3 \end{bmatrix}$$

$\therefore a = 4.94,\ b = 16.02,\ c = -41.59$

$\therefore P(x) = 4.94x^2 + 16.02x - 41.59$

誤差如下表所示相對誤差小於1%

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	1	2	3	4	5	6	7	8	SUM
Y	102.60	113.20	130.10	142.10	167.50	195.10	224.90	256.80	1332.30
X	4.00	4.20	4.50	4.70	5.10	5.50	5.90	6.30	40.20
XX	16.00	17.64	20.25	22.09	26.01	30.25	34.81	39.69	206.74
XXX	64.00	74.09	91.13	103.82	132.65	166.38	205.38	250.05	1087.49
XXXX	256.00	311.17	410.06	487.97	676.52	915.06	1211.74	1575.30	5843.82
XY	410.40	475.44	585.45	667.87	854.25	1073.05	1326.91	1617.84	7011.21
XXY	1641.60	1996.85	2634.53	3138.99	4356.68	5901.78	7828.77	10192.39	37691.57
(a)Y-valu	101.53	112.836	130.535	142.829	168.601	195.955	224.889	255.4046	1332.58
(a) error	1.07	0.36	0.44	0.73	1.10	0.86	0.01	1.40	5.96

(b) $y = be^{(ax)}$, $\ln(y) = \ln(b) + ax$

設 $Y = \ln(y)$, $B = \ln(b)$, $A = a$.

$\therefore Y = aX + b$ $\begin{bmatrix} \Sigma X_i^2 & \Sigma X_i \\ \Sigma X_i & n \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \Sigma X_i Y_i \\ \Sigma Y_i \end{bmatrix}$

得 $a = 0.4036$, $b = e^{(3.0662)} = 20.99$

誤差如下表所示相對誤差小於1%#

	1	2	3	4	5	6	7	8	SUM
Y	102.60	113.20	130.10	142.10	167.50	195.10	224.90	256.80	1332.30
X	4.00	4.20	4.50	4.70	5.10	5.50	5.90	6.30	40.20
XX	16.00	17.64	20.25	22.09	26.01	30.25	34.81	39.69	206.74
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XXXX	256.00	311.17	410.06	487.97	676.52	915.06	1211.74	1575.30	5843.82
XY	410.40	475.44	585.45	667.87	854.25	1073.05	1326.91	1617.84	7011.21
XXY	1641.60	1996.85	2634.53	3138.99	4356.68	5901.78	7828.77	10192.39	37691.57
(a)Y-valu	101.53	112.84	130.54	142.83	168.60	195.96	224.89	255.40	1332.58
(a) error	1.07	0.36	0.44	0.73	1.10	0.86	0.01	1.40	5.96
ln(Y)	4.63	4.73	4.87	4.96	5.12	5.27	5.42	5.55	40.54
X*ln(Y)	18.52	19.86	21.91	23.30	26.12	29.00	31.95	34.95	205.62
(b)Y-valu	105.07	113.88	128.50	139.27	163.61	192.19	225.78	265.23	1333.51
(b) error	2.47	0.68	1.60	2.83	3.89	2.91	0.88	8.43	23.68

(c) $y = bx^a$, $\ln(y) = \ln(b) + a\ln x$

原 $\ln(x)$ 原 $\ln(b)$

$\Rightarrow Y = a\underline{x} + \underline{b}$

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum x_i Y_i \\ \sum x_i \end{bmatrix}$$

得 $a = 2.01$, $b = 1.84$,

誤差如下表所示相對誤差小於1%#

Y	102.60	113.20	130.10	142.10	167.50	195.10	224.90	256.80	1332.30
X	4.00	4.20	4.50	4.70	5.10	5.50	5.90	6.30	40.20
XX	16.00	17.64	20.25	22.09	26.01	30.25	34.81	39.69	206.74
XXX	64.00	74.09	91.13	103.82	132.65	166.38	205.38	250.05	1087.49
XXXX	256.00	311.17	410.06	487.97	676.52	915.06	1211.74	1575.30	5843.82
XY	410.40	475.44	585.45	667.87	854.25	1073.05	1326.91	1617.84	7011.21
XXY	1641.60	1996.85	2634.53	3138.99	4356.68	5901.78	7828.77	10192.39	37691.57
(a)Y-value	101.53	112.84	130.54	142.83	168.60	195.96	224.89	255.40	1332.58
(a) error	1.07	0.36	0.44	0.73	1.10	0.86	0.01	1.40	5.96
ln(Y)	4.63	4.73	4.87	4.96	5.12	5.27	5.42	5.55	40.54
X*ln(Y)	18.52	19.86	21.91	23.30	26.12	29.00	31.95	34.95	205.62
(b)Y-value	105.07	113.88	128.50	139.27	163.61	192.19	225.78	265.23	1333.51
(b) error	2.47	0.68	1.60	2.83	3.89	2.91	0.88	8.43	23.68
ln(X)	1.39	1.44	1.50	1.55	1.63	1.70	1.77	1.84	12.82
ln(Y)	4.63	4.73	4.87	4.96	5.12	5.27	5.42	5.55	40.54
ln(X)*ln(Y)	6.42	6.79	7.32	7.67	8.34	8.99	9.61	10.21	65.36
ln(X)*ln(X)	1.92	2.06	2.26	2.39	2.65	2.91	3.15	3.39	20.74
(c)Y-value	103.18	113.81	130.74	142.68	168.14	195.69	225.35	257.11	1336.69
(c) error	0.58	0.61	0.64	0.58	0.64	0.59	0.45	0.31	4.39

2. Find the least squares polynomial approximation of degree two on the interval $[-1,1]$ for the function $f(x)=\frac{1}{2}\cos x+\frac{1}{4}\sin 2x$

$$\begin{bmatrix} \int x^0\,dx & \int x^1\,dx & \int x^2\,dx \\ \int x^1\,dx & \int x^2\,dx & \int x^3\,dx \\ \int x^2\,dx & \int x^3\,dx & \int x^4\,dx \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}=\begin{bmatrix} \int x^0 f(x)\,dx \\ \int x^1 f(x)\,dx \\ \int x^2 f(x)\,dx \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{3} \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}=\begin{bmatrix} 0.8414 \\ 0.2176 \\ 0.2391 \end{bmatrix}$$

$\therefore a_0=-0.2279,\ a_1=0.156,\ a_2=0.4984$

$$P(x)=-0.2279x^2+0.156x+0.4984$$

3. Determine the discrete least squares trigonometric polynomial S_4 using $m = 16$ for $f(x) = x^2 \sin x$ on the interval $[0,1]$.

b. Compute $\int_0^1 S_4(x)dx$

c. Compare the integral in part (b) to $\int_0^1 x^2 \sin x dx$

d. Compute the error $E(S_4)$

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--- Problem 3: Discrete Least Squares Trigonometric Polynomial S4 ---
   Generating 32 data points for f(x) = x^2*sin(x) on [0.0, 1.0]...

   Trigonometric polynomial S4 coefficients:
   a0 = 0.420554
   a1 = -0.098026
   a2 = 0.002905
   a3 = 0.013280
   a4 = -0.018841
   b1 = 0.239837
   b2 = -0.129085
   b3 = 0.085778

   S4(z) = 0.210277 -0.098026*cos(1z) +0.239837*sin(1z) +0.002905*cos(2z) -0.129085*sin(2z) +0.013280*cos(3z) +0.085778*sin(3z) -0.018841
*cos(4z)

b. Integral of S4(x)dx from 0 to 1 = a0/2 = 0.210277

c. True integral of x^2*sin(x)dx from 0 to 1 = 0.223244
   Difference between S4 integral and true integral = 0.012967

d. Error E(S4) = sum [y_j - S4(z_j)]^2 = 2.817284e-01
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