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HW4

1. Determine the values $\int_1^2 e^x \sin(4x)dx$ with $h = 0.1$ by

a. Use the composite trapezoidal rule

b. Use the composite Simpsons' method

c. Use the composite midpoint rule

1.

a. $\int_a^b f(x)\,dx = \frac{\Delta x}{2}\{f(x_0) + f(x_n) + 2(f(x_1) + f(x_2) + \cdots)\}$

$\Delta x = \frac{b-a}{n} = h \qquad \therefore n = 10$

$\int_a^b f(x)\,dx = \frac{0.1}{2}\{11.58329\} \approx 0.57916$ #

4.1.a		
X	f(x)	配比
1	-2.05720247	-2.057202471
1.1	-2.8588	-2.0572
1.2	-3.3074	-6.6148
1.3	-3.2417	-6.4833
1.4	-2.5599	-5.1198
1.5	-1.2523	-2.5045
1.6	0.5773	1.1545
1.7	2.7048	5.4095
1.8	4.8014	9.6028
1.9	6.4714	12.9428
2	7.3104	7.3104
	加成	11.58329061

b. $\int_a^b f(x)\,dx = \frac{\Delta x}{3}\{f(x_0) + 4f(x_1) + 2f(x_2) + 3f(x_3) + \cdots + f(x_n)\}$

$= \frac{0.1}{3}\{11.5699\}$

≈ 0.38566 #

4.1.b		
X	f(x)	配比
1.0	-2.0572	-2.0572
1.1	-2.8588	-11.4351
1.2	-3.3074	-6.61477
1.3	-3.2417	-12.9666
1.4	-2.5599	-5.11982
1.5	-1.2523	-5.00901
1.6	0.5773	1.154544
1.7	2.7048	10.819
1.8	4.8014	9.602822
1.9	6.4714	25.88564
2.0	7.3104	7.3104
	加成	11.5699

U.

$$\int_a^b f(x)\,dx = 2h\{f(x_1)+f(x_2)+\cdots+f(x_{2n-1})\}$$

$$= 2\cdot 0.1\cdot 6.588086 \approx 1.317617 \#$$

2. Approximate $\int_1^{1.5} x^2 \ln x dx$ using Gaussian Quadrature with $n=3$ and $n=4$. Then compare the result to the exact value of the integral.

$n=3$, $x_1=-0.775$, $x_3=0.775$, $x_2=0$

$C_1=C_3=0.556$, $C_2=0.889$

$$\int_a^b f(x)dx = \int_1^{1.5} x^2 \ln x\, dx = \frac{1.5-1}{2}\sum_{i=1}^{n} c_i f\{\frac{1.5+1}{2}+\frac{1.5-1}{2}x_i\}$$

$$\approx 0.1923.$$

$n=4$. $x_1=-x_4=-0.861$. $x_2=-x_3=0.34$ $C_1=0.348$

$C_4=0.348$, $C_2=0.652$ $C_3=0.652$.

$$\int_a^b f(x) = \frac{1.5-1}{2}\sum_{i=1}^{4} c_i f(0.25x_i+1.25) \approx 0.1924 \#$$

3. Approximate $\int_0^{\pi/4}\int_{\sin x}^{\cos x}(2y\sin x+\cos^2 x)dydx$ using

a. Simpson's rule for $n=4$ and $m=4$

b. Gaussian Quadrature, $n=3$ and $m=3$

c. Compare these results with the exact value.

1, x: $h = \frac{\pi}{4}/4 = \frac{\pi}{16}$

y: $y_j = \sin x_i + j$

$$\int_{\sin x_i}^{\cos x_i} (2y\sin x_i + \cos^2 x_i)\,dy \text{ ——— 内,}$$

$$\int_0^{\frac{\pi}{4}} (\text{内})\,dx \approx 0.3827 \#$$

2, $[0, \frac{\pi}{4}] \longrightarrow [-1, 1]$ $x = \frac{\pi}{8}\xi + \frac{\pi}{8}$, $dx = \frac{\pi}{8}d\xi$

$[\sin x, \cos x] \longrightarrow [-1, 1]$ $y = \frac{\cos x - \sin x}{2}\eta + \frac{\cos x + \sin x}{2}$

$$\int_0^{\pi/4}\int_{\sin x}^{\cos x} f(x,y)\,dy\,dx \approx \frac{\pi}{8}\sum_{i=1}^{3} c_i \left(\frac{\cos x_i - \sin x_i}{2}\sum_{j=1}^{3} c_i f(x_i, y_{ij})\right)$$

$$\approx 0.3826 \#$$

3, $$\int_0^{\frac{\pi}{4}}\int_{\sin x}^{\cos x} (2y\sin x + \cos^2 x)\,dy\,dx$$

$$\approx \int_0^{\frac{\pi}{4}} (2\cos^3 x - \sin^3 x - \sin x\cos^2 x)\,dx$$

$$\approx \frac{\pi}{8} \cong 0.3929$$

4. Use the composite Simpson's rule and $n = 4$ to approximate the improper integral a) $\int_0^1 x^{-1/4} \sin x dx$, b) $\int_1^{\infty} x^{-4} \sin x dx$ by use the transform $t = x^{-1}$

1, $\int_0^1 x^{\frac{-1}{4}} dx$

sin $x=0$, 有奇點但積分收斂.

$1 \sim 0$, $\rightarrow$ $x_0 = 0$, $x_1 = 0.25$ $\ldots$ $x_4 = 1$.

$\int_0^1 f(x) dx \cong \frac{h}{3} \{ f(0) + 4f(0.25) + 2f(0.5) \ldots \}$

$\cong 0.8286$, #

2,

$\int_1^{\infty} x^{-4} \sin x \, dx$

令 $t = \frac{1}{x}$, $dx = -\frac{1}{t^2} dt$

$\int_1^{\infty} x^{-4} \sin x \, dx = \int_0^1 t^4 \sin(\frac{1}{t}) \cdot (\frac{1}{t^2}) dt$

$= \int_0^1 t^2 \sin(\frac{1}{t}) dt$

$\cong 0.1432$ #