HW 12

1. Given the problem

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=xy,\ \ 0<x<\pi,\ \ 0<y<\pi/2$$

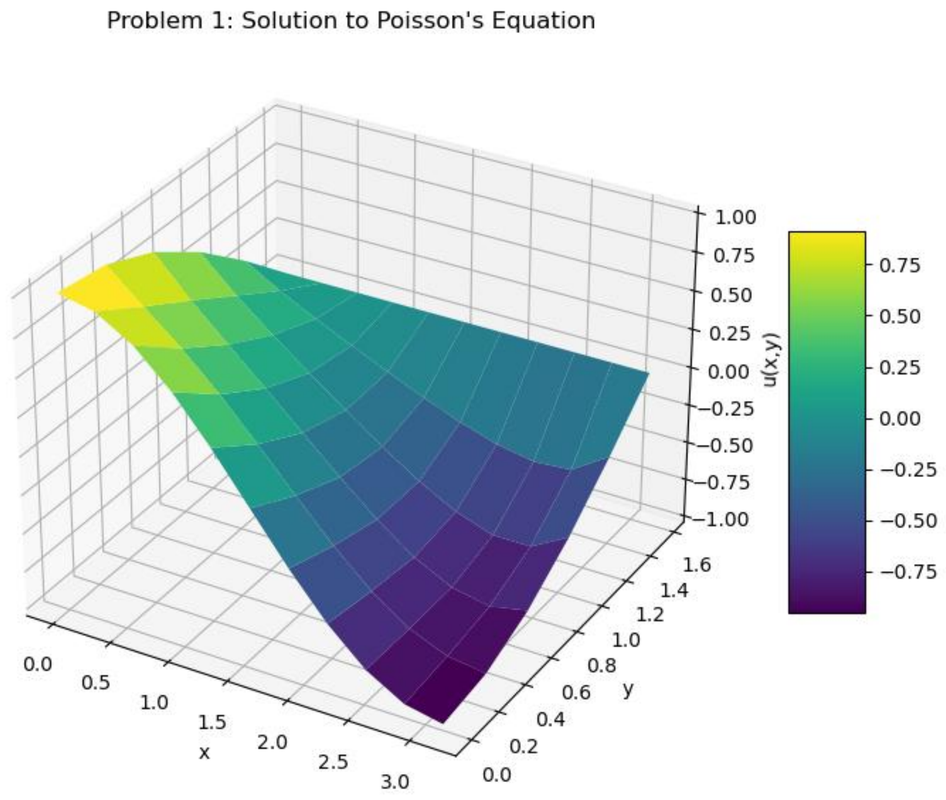
$$u(0,y)=\cos y,\ \ u(\pi,y)=-\cos y,\ \ 0\le y\le \pi/2,$$

$$u(x,0)=\cos x,\ \ u(x,\pi/2)=0,\ \ 1\le y\le 2$$

To calculate $u(x,y)$ by using $h=k=0.1\pi$.

這以用到此線上編譯為主

https://www.tutorialspoint.com/compilers/online-matplotlib-compiler.htm



2. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{1}{4K}\frac{\partial T}{\partial t}\,,\quad \frac{1}{2}\le r\le 1\,,\quad 0\le t\,,$$

$$T(1,t)=100+40t\,,\quad 0\le t\le 10\,;\qquad \frac{\partial T}{\partial r}+3T=0\quad \text{at}\quad r=\frac{1}{2}$$

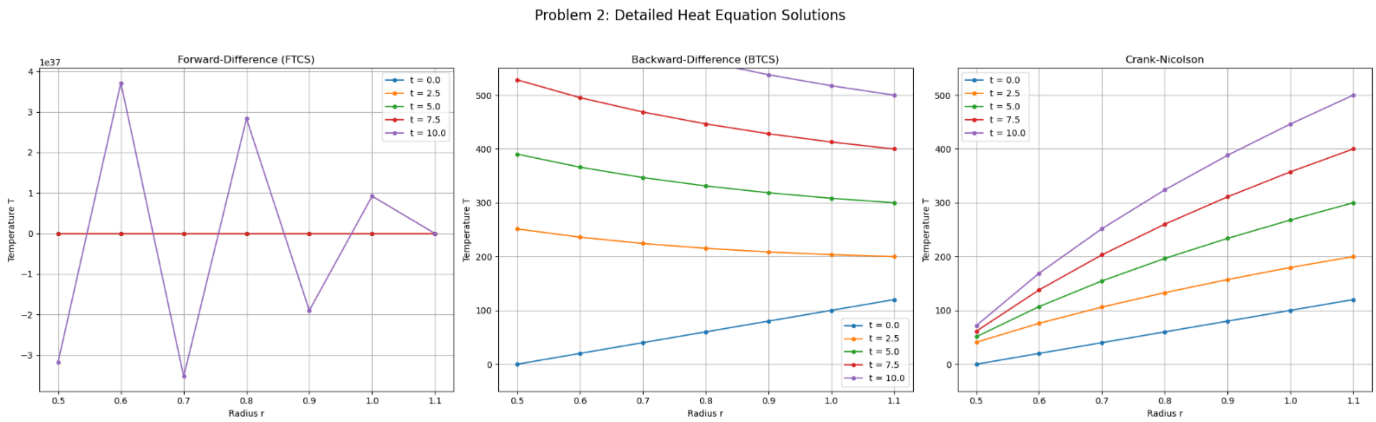
$$T(r,0)=200(r-0.5)\,,\quad 0.5\le r\le 1\,,$$

and use $\Delta t=0.5$, $\Delta r=0.1$, and $K=0.1$ to calculate $T(r,t)$

By (a) the forward-difference method

(b) the backward-difference method

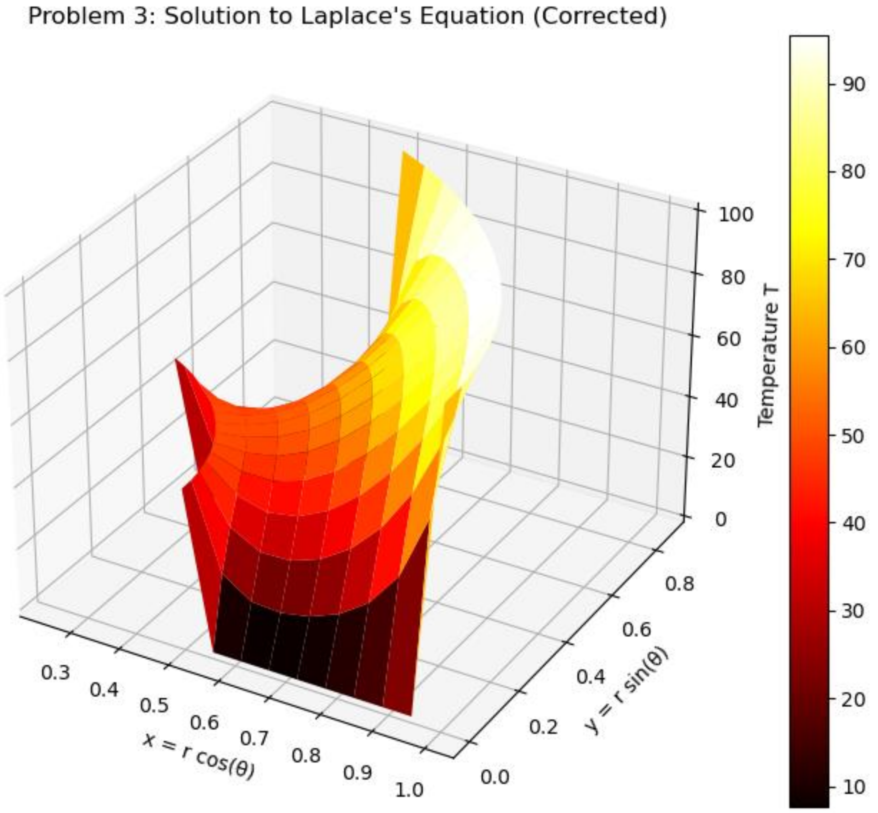
© the Crank-Nicolson algorithm.



3. Given the problem

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0,\quad \frac{1}{2}\le r\le 1,\quad 0\le t\le \pi/3,$$

$$T(r,0)=0,\quad T(r,\pi/3)=0,\quad T(1/2,\theta)=50,\quad T(1,\theta)=100.$$



4. Given the problem

$$\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}, \quad 0 \le x \le 1, \quad 0 \le t$$

$$p(0,t) = 1, \quad p(1,t) = 2, \quad p(x,0) = \cos(2\pi x), \quad \frac{\partial p}{\partial t}(x,0) = 2\pi \sin(2\pi x), \quad 0 \le x \le 1$$

To calculate p by using $\Delta x = \Delta t = 0.1$.