

Lab07-Amortized Analysis

CS214-Algorithm and Complexity, Xiaofeng Gao & Lei Wang, Spring 2021.

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1. Suppose we perform a sequence of n operations on a data structure in which the i th operation costs i if i is an exact power of 2, and 1 otherwise. Use an accounting method to determine the amortized cost per operation.
2. Consider an ordinary **binary min-heap** data structure with n elements supporting the instructions INSERT and EXTRACT-MIN in $O(\log n)$ worst-case time. Give a potential function Φ such that the amortized cost of INSERT is $O(\log n)$ and the amortized cost of EXTRACT-MIN is $O(1)$, and show that it works.
3. Assume we have a set of arrays A_0, A_1, A_2, \dots , where the i^{th} array A_i has a length of 2^i . Whenever an element is inserted into the arrays, we always intend to insert it into A_0 . If A_0 is full then we pop the element in A_0 off and insert it with the new element into A_1 . (Thus, if A_i is already full, we recursively pop all its members off and insert them with the elements popped from A_0, \dots, A_{i-1} and the new element into A_{i+1} until we find an empty array to store the elements.) An illustrative example is shown in Figure 1. Inserting or popping an element take $O(1)$ time.

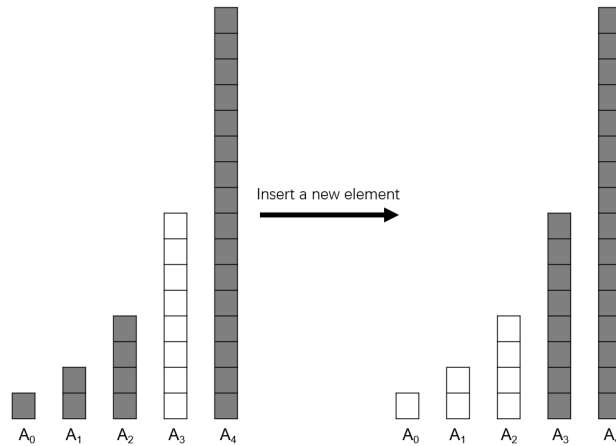


图 1: An example of making room for one new element in the set of arrays.

- (a) In the worst case, how long does it take to add a new element into the set of arrays containing n elements?
- (b) Prove that the amortized cost of adding an element is $O(\log n)$ by *Aggregation Analysis*.
- (c) If each array A_i is required to be sorted but elements in different arrays have no relationship with each other, how long does it take in the worst case to search an element in the arrays containing n elements?
- (d) What is the amortized cost of adding an element in the case of (c) if the comparison between two elements also takes $O(1)$ time?

Remark: Please include your .pdf, .tex files for uploading with standard file names.