上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

作业 3

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2024 年 4 月 19 日

目　录

1　第 1 题

Prove that two successive 2D rotations are additive:

$$R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2) \tag{1}$$

证明　根据二维旋转的定义有

$$\begin{aligned} R(\theta_1)R(\theta_2) &= \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{pmatrix} \\ &= \begin{pmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix} \\ &= R(\theta_1 + \theta_2) \end{aligned}$$

即二维旋转是加性的。 ■

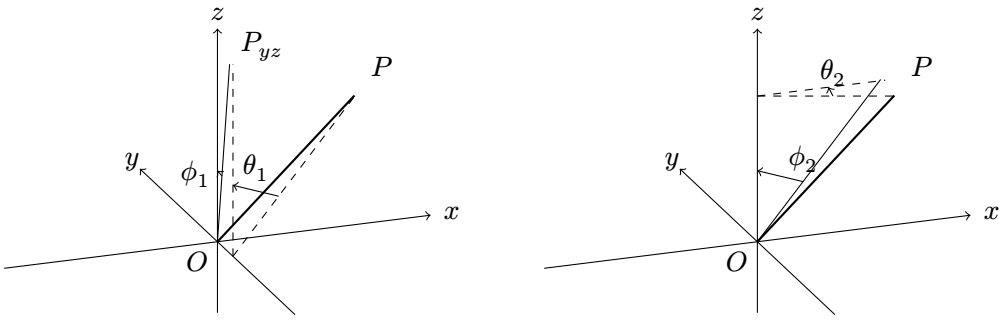
2　第 2 题

Consider a line from the origin of a right-handed coordinate system to the point $P(x, y, z)$. Find the transformation matrices needed to rotate the line into the positive z axis in two different ways, and show by

algebraic manipulation that, in each case, the point P does go to the z axis. For each method, calculate the sines and cosines of the angles of rotation.

a. Rotate about the y axis into the (y, z)plane, then rotate about the x axis into the z axis.

b. Rotate about the z axis into the (x, z)plane, then rotate about the y axis into the z axis.



(a) 先绕 y 轴旋转，再绕 x 轴旋转　　(b) 先绕 z 轴旋转，再绕 y 轴旋转

图 1　两种旋转情况

解　如图 1a 所示，先绕 y 轴旋转 θ_1，再绕 x 轴旋转 ϕ_1，角度正方向从面向坐标轴的方向看，其中

$$\sin\theta_1 = -\frac{x}{\sqrt{x^2+z^2}} \qquad \cos\theta_1 = \frac{z}{\sqrt{x^2+z^2}} \tag{2}$$

$$\sin\phi_1 = \frac{y}{\sqrt{x^2+y^2+z^2}} \qquad \cos\phi_1 = \frac{\sqrt{x^2+z^2}}{\sqrt{x^2+y^2+z^2}} \tag{3}$$

则根据三维旋转的定义有

$$\begin{aligned}
R_x(\phi_1)R_y(\theta_1)P &= \begin{pmatrix} 1 & & \\ & \cos\phi_1 & -\sin\phi_1 \\ & \sin\phi_1 & \cos\phi_1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & & \sin\theta_1 \\ & 1 & \\ -\sin\theta_1 & & \cos\theta_1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\theta_1 & & \sin\theta_1 & \\ \sin\phi_1\sin\theta_1 & \cos\phi_1 & -\sin\phi_1\cos\theta_1 & \\ -\cos\phi_1\sin\theta_1 & \sin\phi_1 & \cos\phi_1\cos\theta_1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} x\cos\theta_1 + z\sin\theta_1 \\ \sin\phi_1(x\sin\theta_1 - z\cos\theta_1) + y\cos\phi_1 \\ \cos\phi_1(-x\sin\theta_1 + z\cos\theta_1) + y\sin\phi_1 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \\ \sqrt{x^2+y^2+z^2} \\ 1 \end{pmatrix}
\end{aligned}$$

即最终落在了 z 轴上。

如图 1b 所示，先绕 z 轴旋转 θ_2，再绕 y 轴旋转 ϕ_2，有

$$\sin\theta_2 = -\frac{y}{\sqrt{x^2+y^2}} \qquad \cos\theta_2 = \frac{x}{\sqrt{x^2+y^2}} \tag{4}$$

$$\sin\phi_2 = -\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} \qquad \cos\phi_2 = \frac{z}{\sqrt{x^2+y^2+z^2}} \tag{5}$$

则根据三维旋转的定义有

$$\begin{aligned}
R_y(\phi_2)R_z(\theta_2)P &= \begin{pmatrix} \cos\phi_2 & & \sin\phi_2 & \\ & 1 & & \\ -\sin\phi_2 & & \cos\phi_2 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & & \\ \sin\theta_2 & \cos\theta_2 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\phi_2\cos\theta_2 & -\cos\phi_2\sin\theta_2 & \sin\phi_2 & \\ \sin\theta_2 & \cos\theta_2 & & \\ -\sin\phi_2\cos\theta_2 & \sin\phi_2\sin\theta_2 & \cos\phi_2 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos\phi_2(x\cos\theta_2 - y\sin\theta_2) + z\sin\phi_2 \\ x\sin\theta_2 + y\cos\theta_2 \\ \sin\phi_2(-x\cos\theta_2 + y\sin\theta_2) + z\cos\phi_2 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \\ \sqrt{x^2+y^2+z^2} \\ 1 \end{pmatrix}
\end{aligned}$$

即最终也落在了 z 轴上。

3 第 3 题

Try to build a BSP tree for the graph below. For easier grading, please choose vertex at each step according to the alphabetical order.

解 其中 d 被 b 切割为 d_1 和 d_2，如图 3 所示，建立后的 BSP 树如图 4 所示。

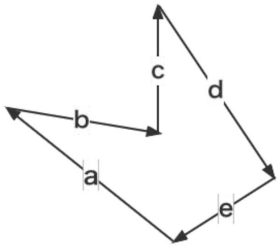


图 2　problem

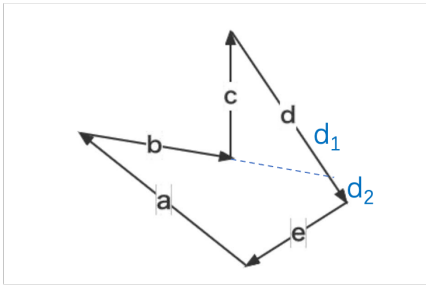


图 3　切割后的边

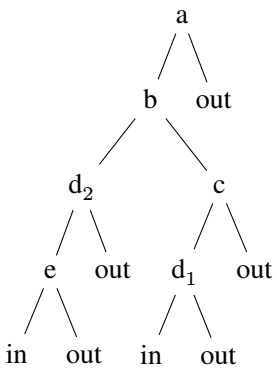


图 4　BSP 树