第 6 次作业

Log Creative

April 10, 2020

1. 证明下列等值式和蕴含式.

(1) $\neg(\exists x)(\exists y)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y)) = (\forall x)(\forall y)((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \to \neg R(x,y))$

证明.

$$
\begin{aligned}
&(\forall x)(\forall y)((P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \to \neg R(x,y)) \\
&\quad =(\forall x)(\forall y)(\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y)) \vee \neg R(x,y)) \\
&\quad =(\forall x)(\forall y)\neg(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y)) \\
&\quad =(\forall x)\neg(\exists x)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y)) \\
&\quad =\neg(\exists x)(\exists x)(P(x) \wedge P(y) \wedge Q(x) \wedge Q(y) \wedge R(x,y))
\end{aligned}
$$

□

(3) $(\forall x)(P(x) \vee q) \to (\exists x)(P(x) \wedge q) = ((\exists x)\neg P(x) \wedge \neg q) \vee ((\exists x)P(x) \wedge q)$

证明.

$$
\begin{aligned}
&(\forall x)(P(x) \vee q) \to (\exists x)(P(x) \wedge q) \\
&\quad =\neg((\forall x)(P(x) \vee q)) \vee ((\exists x)(P(x) \wedge q)) \\
&\quad =(\exists x)\neg(P(x) \vee q) \vee ((\exists x)P(x) \wedge q) \\
&\quad =(\exists x)(\neg P(x) \wedge \neg q) \vee ((\exists x)P(x) \wedge q)
\end{aligned}
$$

□

(5) $(\forall x)P(x) \to q = (\exists x)(P(x) \to q)$

证明.

$$
\begin{aligned}
&(\forall x)P(x) \to q \\
&\quad =\neg((\forall x)P(x)) \vee q \\
&\quad =(\exists x)\neg P(x) \vee q \\
&\quad =(\exists x)(\neg P(x) \vee q) \\
&\quad =(\exists x)(P(x) \to q)
\end{aligned}
$$

□

2. 判断下列各公式哪些是普遍有效的并给出证明，不是普遍有效的举出反例。

(2) $((\exists x)P(x) \leftrightarrow (\exists x)Q(x)) \to (\exists x)(P(x) \leftrightarrow Q(x))$

解. 不是普遍有效的。

在$\{1,2\}$上分析，$P(1)=Q(2)=\mathrm{T}, P(2)=Q(1)=\mathrm{F}$，左边成立，但右边发现并不存在一个$x_0 \in \{1,2\}$使得$P(x) \leftrightarrow Q(x)=\mathrm{T}$。

(4) $(\forall x)(P(x) \to Q(x)) \to ((\exists x)P(x) \to (\forall x)Q(x))$

解. 不是普遍有效的。

在$\{1,2\}$上分析，$P(1)=Q(1)=\mathrm{F}, P(2)=Q(2)=\mathrm{T}$，$P(2) \to Q(1)=\mathrm{F}$

4. 求下列(2)的前束范式，(9)的 Skolem 范式（只含$\forall$）

(2) $(\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge ((\exists u)Q(x,u) \to (\exists w)Q(y,w)))$

解.

$$
\begin{aligned}
&(\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge ((\exists u)Q(x,u) \to (\exists w)Q(y,w))) \\
&\quad =(\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge (\neg((\exists u)Q(x,u)) \vee (\exists w)Q(y,w))) \\
&\quad =(\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge ((\forall u)(\neg Q(x,u)) \vee (\exists w)Q(y,w))) \\
&\quad =(\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge ((\forall u)(\exists w)(\neg Q(x,u) \vee Q(y,w)))) \\
&\quad =(\forall x)(\forall y)(\forall z)(\forall u)(\exists w)(P(x,y,z) \wedge (\neg Q(x,u) \vee Q(y,w)))
\end{aligned}
$$

(9) $(\forall x)(P(x) \to (\exists y)Q(x,y)) \vee (\forall z)R(z)$

解.

$$
\begin{aligned}
&(\forall x)(P(x) \to (\exists y)Q(x,y)) \vee (\forall z)R(z) \\
&\quad =(\forall x)(\neg P(x) \vee (\exists y)Q(x,y)) \vee (\forall z)R(z) \\
&\quad =(\forall x)(\exists y)(\neg P(x) \vee Q(x,y)) \vee (\forall z)R(z) \\
&\quad =(\forall x)(\exists y)(\forall z)(\neg P(x) \vee Q(x,y) \vee R(z)) \\
&\quad =(\forall x)(\forall z)(\neg P(x) \vee Q(x,f(x)) \vee R(z))
\end{aligned}
$$