第 9 次作业

Log Creative

May 9, 2020

8. 设$B = P(P(P(\varnothing)))$。

$B = P(P(P(\varnothing))) = P(P(\{\varnothing\})) = P(\{\varnothing, \{\varnothing\}\}) = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$

(1) 是否$\varnothing \in B$? 是否$\varnothing \subseteq B$?

解. $\varnothing \in B$ 是一个元素

$\varnothing \subseteq B$ 空集包含于任何集合

(2) 是否$\{\varnothing\} \in B$? 是否$\{\varnothing\} \subseteq B$?

解. $\{\varnothing\} \in B$ 是一个元素

$\{\varnothing\} \subseteq B$，其元素空集是$B$的一个元素

(3) 是否$\{\{\varnothing\}\} \in B$? 是否$\{\{\varnothing\}\} \subseteq B$?

解. $\{\{\varnothing\}\} \in B$ 是一个元素

$\{\{\varnothing\}\} \subseteq B$ 其元素$\{\varnothing\}$是B的一个元素

12. 设全集$E = \{1, 2, 3, 4, 5\}$，集合$A = \{1, 4\}, B = \{1, 2, 5\}, C = \{2, 4\}$。求下列集合。

(1) $A \cap -B$.

解. $A \cap -B = \{1, 4\} \cap \{3, 4\} = \{4\}$

(2) $(A \cap B) \cup -C$.

解. $(A \cap B) \cup -C = \{1\} \cup \{1, 3, 5\} = \{1, 3, 5\}$

(3) $-(A \cap B)$.

解. $-(A \cap B) = -\{1\} = \{2, 3, 4, 5\}$

(4) $P(A) \cap P(B)$.

解. $P(A) \cap P(B) = P(A \cap B) = \{\varnothing, \{1\}\}$

(5) $P(A) - P(B)$.

解. $P(A) - P(B) = \{\varnothing, \{1\}, \{4\}, \{1, 4\}\} - \{\varnothing, \{1\}, \{2\}, \{5\}, \{1, 2\}, \{1, 5\}, \{2, 5\}, \{1, 2, 5\}\} = \{\{4\}, \{1, 4\}\}$

14. 写出下列集合

(1) $\cup\{\{3,4\},\{\{3\},\{4\}\},\{3,\{4\}\},\{\{3\},4\}\}$

解. $\cup\{\{3,4\},\{\{3\},\{4\}\},\{3,\{4\}\},\{\{3\},4\}\} = \{3,4,\{3\},\{4\}\}$

(2) $\cap\{\{1,2,3\},\{2,3,4\},\{3,4,5\}\}$

解. $\cap\{\{1,2,3\},\{2,3,4\},\{3,4,5\}\} = \{3\}$

17. 设A、B和C是任意的集合，证明:

(1) $(A-B)-C = A-(B\cup C)$

证明.

$$\begin{aligned}(A-B)-C &= (A\cap -B)-C\\ &= (A\cap -B)\cap -C\\ &= A\cap(-B\cap -C)\\ &= A\cap -(B\cup C)\\ &= A-(B\cup C)\end{aligned}$$

□

(4) $A\subseteq C\wedge B\subseteq C \Leftrightarrow A\cup B\subseteq C$

证明.

$$\begin{aligned}A\subseteq C\wedge B\subseteq C &= (\forall x)(x\in A\to x\in C)\wedge(\forall x)(x\in B\to x\in C)\\ &= (\forall x)((x\in A\to x\in C)\wedge(x\in B\to x\in C))\\ &= (\forall x)((x\in A\vee x\in B)\to x\in C)\\ &= (\forall x)(x\in A\cup B\to x\in C)\\ &= A\cup B\subseteq C\end{aligned}$$

□

(6) $A\cap B=\varnothing \Leftrightarrow A\subseteq -B \Leftrightarrow B\subseteq -A$

证明. **a.** $A\cap B=\varnothing$. 设$x\in A \Rightarrow x\in A-\varnothing = A-(A\cap B) = A\cap -B \Rightarrow x\in -B$

$A\subseteq -B$.

b. $A\subseteq -B$. $\forall x$，$x\in B\Rightarrow x\notin -B\Rightarrow x\notin A(A\subseteq -B)\Rightarrow x\in -A$

$B\subseteq -A$.

c. $B\subseteq -A$. $\forall x$，$x\in A\Rightarrow x\notin -A\Rightarrow x\notin B(B\subseteq -A)$

$\forall x$，$x\in B\subseteq -A\Rightarrow x\notin A$

$A\cap B=\varnothing$

$A\cap B=\varnothing\Rightarrow A\subseteq -B\Rightarrow B\subseteq -A\Rightarrow A\cap B=\varnothing$

$A\cap B=\varnothing\Leftrightarrow A\subseteq -B\Leftrightarrow B\subseteq -A$

□

27. 足球队有38人，篮球队有15人，排球队有20人，三个队队员共58人，其中3人同时参加三个队，问同时参加两个队的人有几个。

解. 设足球队队员的集合为A，篮球队队员的集合为B，排球队队员的集合为C

$|A| = 38, |B| = 15, |C| = 20, |A \cup B \cup C| = 58, |A \cap B \cap C| = 3$

包括同时参加三个队的人：

$|A \cap B| + |B \cap C| + |A \cap C| - 2|A \cap B \cap C| = |A| + |B| + |C| - |A \cup B \cup C| + |A \cap B \cap C| - 2|A \cap B \cap C| = 38 + 15 + 20 - 58 - 3 = 12$