作业 9

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1(1). **解**

$$\mathbf{A}_1 = \begin{pmatrix} 7 & 3 & -2 \\ 3 & 4 & -1 \\ -2 & -1 & 3 \end{pmatrix}$$

计算如表所示:

k	$\mathbf{u}_k^\top$	$\max(\mathbf{v}_k)$
0	$(1, 1, 1)$	
1	$(1, 0.75, 0)$	8.00000
2	$(1, 0.64864865, -0.2972973)$	9.25000
3	$(1, 0.61756374, -0.37110482)$	9.54054
4	$(1, 0.60879835, -0.38883968)$	9.59490
5	$(1, 0.60641274, -0.39309539)$	9.60407
6	$(1, 0.60577683, -0.39412075)$	9.60543
7	$(1, 0.60560975, -0.39436892)$	9.60557

故主特征值与其对应的特征向量为

$$\lambda_1 \approx 9.605, \quad \mathbf{x}_1 = (1, 0.60560975, -0.39436892)^\top$$

3. **解** 令

$$\mathbf{B} = \mathbf{A} - 6\mathbf{E} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & -5 \end{pmatrix}$$

取

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

以避免对角线上的 0 元素，进行 LU 分解，$\mathbf{PB} = \mathbf{LU}$，有

$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & \frac{4}{5} & 1 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} 2 & -3 & 1 \\ 0 & \frac{5}{2} & -\frac{11}{2} \\ 0 & 0 & \frac{27}{5} \end{pmatrix}$$

根据反幂法迭代公式，

$$\begin{cases} \mathbf{L}\mathbf{y}_k = \mathbf{P}\mathbf{u}_{k-1} \\ \mathbf{U}\mathbf{v}_k = \mathbf{y}_k \\ \mathbf{u}_k = \frac{\mathbf{v}_k}{\max(\mathbf{v}_k)} \end{cases}$$

根据 $\mathbf{U}_1\mathbf{v}_1 = (1,1,1)^\top$，有

$$\mathbf{v}_1 = (1.61851852, 0.80740741, 0.18518519)^\top, \quad \mathbf{u}_1 = (1, 0.49885584, 0.11441648)^\top$$

则进行如下的迭代过程：

k	$\mathbf{u}_k^\top$	$\max(\mathbf{v}_k)$
1	$(1, 0.49885584, 0.11441648)$	1.6185
2	$(1, 0.5349076, 0.2761807)$	0.74294
3	$(1, 0.51810545, 0.23348783)$	0.78759
4	$(1, 0.52470794, 0.24451802)$	0.77284
5	$(1, 0.52225069, 0.24155724)$	0.77757
6	$(1, 0.52312807, 0.24237021)$	0.77602
7	$(1, 0.52282154, 0.24214025)$	0.77653
8	$(1, 0.522927, 0.24220711)$	0.77636
9	$(1, 0.52289107, 0.24218715)$	0.77642
10	$(1, 0.52290323, 0.24219325)$	0.77640
11	$(1, 0.52289914, 0.24219135)$	0.77640

最终可得 $\lambda \approx \frac{1}{0.77640} + 6 = 7.28799$，特征向量为 $(1, 0.52289914, 0.24219135)$。

9(1). **解**

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

根据 QR 方法迭代公式，

$$\mathbf{A}_k = \mathbf{Q}_k\mathbf{Q}_k$$
$$\mathbf{A}_{k+1} = \mathbf{R}_k\mathbf{Q}_k$$

有如下的迭代过程：

k	$\mathbf{A}_k$	k	$\mathbf{A}_k$
1	$\begin{pmatrix} 1.00000 & -2.19089 & 0.00000 \\ -2.19089 & -0.66667 & 1.19257 \\ 0.00000 & 1.19257 & 2.66667 \end{pmatrix}$	11	$\begin{pmatrix} 3.36420 & -0.21413 & 0.00000 \\ -0.21413 & -2.34899 & 0.25820 \\ 0.00000 & 0.25820 & 1.98479 \end{pmatrix}$
2	$\begin{pmatrix} 1.27586 & 2.33263 & 0.00000 \\ 2.33263 & -0.56957 & 1.20507 \\ 0.00000 & 1.20507 & 2.29371 \end{pmatrix}$	12	$\begin{pmatrix} 3.36830 & 0.15067 & 0.00000 \\ 0.15067 & -2.35745 & 0.21783 \\ 0.00000 & 0.21783 & 1.98915 \end{pmatrix}$
3	$\begin{pmatrix} 1.81951 & -2.29349 & 0.00000 \\ -2.29349 & -0.86236 & 1.06119 \\ 0.00000 & 1.06119 & 2.04285 \end{pmatrix}$	13	$\begin{pmatrix} 3.37031 & -0.10599 & 0.00000 \\ -0.10599 & -2.36259 & 0.18375 \\ 0.00000 & 0.18375 & 1.99228 \end{pmatrix}$
4	$\begin{pmatrix} 2.40694 & 2.00767 & -0.00000 \\ 2.00767 & -1.34810 & 0.88307 \\ 0.00000 & 0.88307 & 1.94116 \end{pmatrix}$	14	$\begin{pmatrix} 3.37131 & 0.07456 & 0.00000 \\ 0.07456 & -2.36581 & 0.15498 \\ 0.00000 & 0.15498 & 1.99450 \end{pmatrix}$
5	$\begin{pmatrix} 2.84138 & -1.59080 & 0.00000 \\ -1.59080 & -1.76248 & 0.73085 \\ 0.00000 & 0.73085 & 1.92110 \end{pmatrix}$	15	$\begin{pmatrix} 3.37180 & -0.05244 & 0.00000 \\ -0.05244 & -2.36789 & 0.13070 \\ 0.00000 & 0.13070 & 1.99609 \end{pmatrix}$
6	$\begin{pmatrix} 3.09885 & 1.18594 & 0.00000 \\ 1.18594 & -2.02892 & 0.60933 \\ 0.00000 & 0.60933 & 1.93006 \end{pmatrix}$	16	$\begin{pmatrix} 3.37204 & 0.03689 & 0.00000 \\ 0.03689 & -2.36926 & 0.11021 \\ 0.00000 & 0.11021 & 1.99722 \end{pmatrix}$
7	$\begin{pmatrix} 3.23554 & -0.85692 & 0.00000 \\ -0.85692 & -2.18082 & 0.51118 \\ 0.00000 & 0.51118 & 1.94528 \end{pmatrix}$	17	$\begin{pmatrix} 3.37216 & -0.02595 & 0.00000 \\ -0.02595 & -2.37019 & 0.09293 \\ 0.00000 & 0.09293 & 1.99802 \end{pmatrix}$
8	$\begin{pmatrix} 3.30467 & 0.61009 & 0.00000 \\ 0.61009 & -2.26397 & 0.43031 \\ 0.00000 & 0.43031 & 1.95930 \end{pmatrix}$	18	$\begin{pmatrix} 3.37222 & 0.01825 & 0.00000 \\ 0.01825 & -2.37082 & 0.07836 \\ 0.00000 & 0.07836 & 1.99860 \end{pmatrix}$
9	$\begin{pmatrix} 3.33897 & -0.43143 & 0.00000 \\ -0.43143 & -2.30938 & 0.36279 \\ 0.00000 & 0.36279 & 1.97041 \end{pmatrix}$	19	$\begin{pmatrix} 3.37225 & -0.01284 & -0.00000 \\ -0.01284 & -2.37125 & 0.06607 \\ 0.00000 & 0.06607 & 1.99900 \end{pmatrix}$
10	$\begin{pmatrix} 3.35588 & 0.30415 & 0.00000 \\ 0.30415 & -2.33460 & 0.30604 \\ 0.00000 & 0.30604 & 1.97873 \end{pmatrix}$	20	$\begin{pmatrix} 3.37227 & 0.00903 & 0.00000 \\ 0.00903 & -2.37156 & 0.05570 \\ 0.00000 & 0.05570 & 1.99929 \end{pmatrix}$

此时,

$$\tilde{\mathbf{Q}}_{20} = \begin{pmatrix} 0.28304 & -0.51361 & -0.80999 \\ 0.33341 & 0.84455 & -0.41901 \\ 0.89929 & -0.15146 & 0.41029 \end{pmatrix}$$

列向量即为对应的特征向量。

10. **解**

$$\mathbf{A}=\begin{pmatrix}1&1&1\\2&-1&-1\\2&-4&5\end{pmatrix}$$

$\alpha_1=(1,2,2)^\top$，范数 $\|\alpha_1\|_2=3$，取 $\mathbf{y}_1=(-3,0,0)^\top$，有 $\mathbf{u}_1=\frac{\alpha_1-\mathbf{y}_1}{\|\alpha_1-\mathbf{y}_1\|}=\frac{1}{\sqrt{6}}(2,1,1)^\top$，则

$$\mathbf{H}_1=\mathbf{E}-2\mathbf{u}\mathbf{u}^\top=\begin{pmatrix}-\frac{1}{3}&-\frac{2}{3}&-\frac{2}{3}\\-\frac{2}{3}&\frac{2}{3}&-\frac{1}{3}\\-\frac{2}{3}&-\frac{1}{3}&\frac{2}{3}\end{pmatrix}$$

则

$$\mathbf{A}_2=\mathbf{H}_1\mathbf{A}_1=\begin{pmatrix}-3&3&-3\\0&0&-3\\0&-3&3\end{pmatrix}$$

$\tilde{\alpha}_2=(0,-3)^\top$，范数 $\|\tilde{\alpha}_2\|_2=3$，取 $\tilde{\mathbf{y}}_2=(-3,0)^\top$，有 $\tilde{\mathbf{u}}_2=\frac{\tilde{\alpha}_2-\tilde{\mathbf{y}}_2}{\|\tilde{\alpha}_2-\tilde{\mathbf{y}}_2\|}=\frac{1}{\sqrt{2}}(1,-1)^\top$，则

$$\tilde{\mathbf{H}}_2=\mathbf{E}-2\tilde{\mathbf{u}}_2\tilde{\mathbf{u}}_2^\top=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \mathbf{H}_2=\begin{pmatrix}1&\\&\tilde{\mathbf{H}}_2\end{pmatrix}=\begin{pmatrix}1&&\\&&1\\&1&\end{pmatrix}$$

故

$$\mathbf{A}_3=\mathbf{H}_2\mathbf{A}_2=\begin{pmatrix}-3&3&-3\\&-3&3\\&&-3\end{pmatrix}=\mathbf{R}$$

而

$$\mathbf{Q}=\mathbf{H}_1\mathbf{H}_2=\begin{pmatrix}-\frac{1}{3}&-\frac{2}{3}&-\frac{2}{3}\\-\frac{2}{3}&-\frac{1}{3}&\frac{2}{3}\\-\frac{2}{3}&\frac{2}{3}&-\frac{1}{3}\end{pmatrix}$$