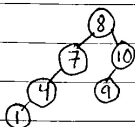


Trees

Binary search tree

- insert 8, 7, 4, 10, 9, 1 into a BST.



* if smaller go left and vice versa

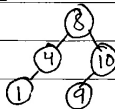
- average height of a BST is $\Theta(\sqrt{n})$

- Full tree height $\lceil \log_2(n) \rceil - 1$

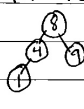
← ceiling

- degenerate is $n-1$

- delete 7



- delete 10

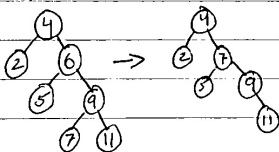


- delete 8



- When we delete the root grab the ~~value~~ ^{right} value from the ~~sub tree~~ ^{smallest}

- delete 6



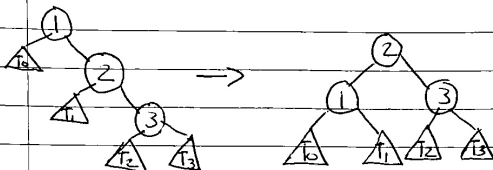
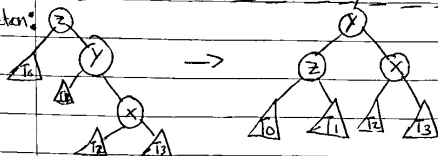
- The 7 is the smallest in the right sub tree so we lift that node up.
- For a balanced tree operations are $O(\log n)$
- for degenerate case operations are $O(n)$
- If we allow deletes average tree height is $O(\sqrt{n})$

AVL Trees Adelson-Velskii and Landis

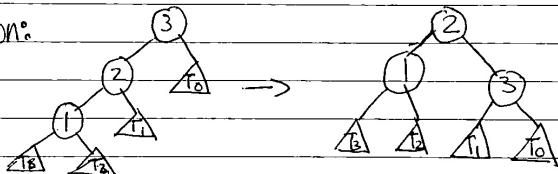
a bst with a "balance" condition

- Let y be the node where imbalance occurs.
- Let z be the parent of y
- Let x be the child of y that causes imbalance.

Left Rotation:

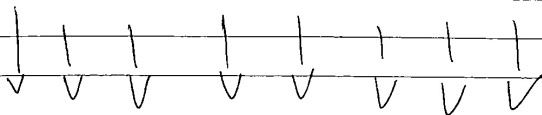
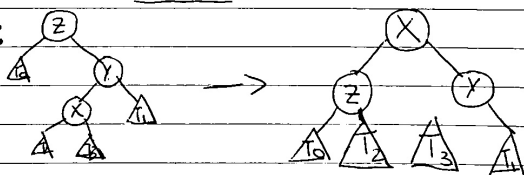


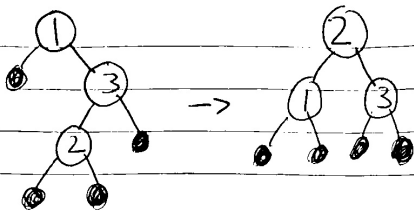
Right Rotation:



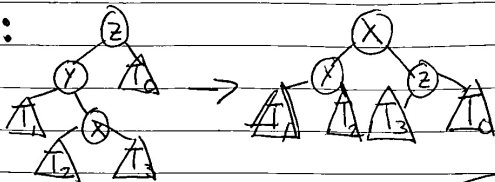
x is the middle

Right-Left:





Left-Right:



- With AVL tree with 1023 nodes should have a height around 13 or $1.44 * \log_2(1023) - 1 \approx 13.28$
- This tree works way better than without balance condition.

Splay Tree *Still is a BST* Not strictly balanced like AVL

• Data locality can be important so when we search for a node or access it we ~~move~~ ^{make} it + the root. (Done by rotations)

• Amortized cost guarantees that in consecutive operation is $O(m \log_2 n)$

• if m operations are $O(m \cdot fcn)$

• the amortized cost is $O(fcn)$

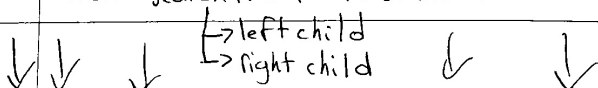
• Splaying happens when we search, insert, or delete

• During delete a node the node's parent node is splayed to the root.

Zig Rotations:

Case 1: search item is root node

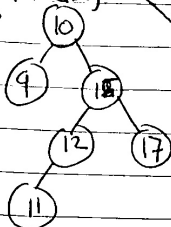
Case 2: search item is child of root node



case 3 : node has parent & grandparent (Zig-Zig)

case 4 : node has parent & grandparent not symmetrical (Zig-Zag)

find(12)



find(13)

