INDEX

λ-fat and λ-thin cubes, 187
(m, p')-Polar sets, 70
C^m-regularity condition, 84
H = W, 67
N-function, 262
complementary, 263
N-function dominance
global or near infinity, 265

Almost everywhere, 15
Anisotropic Sobolev inequality, 104
Approximation
in L^p spaces, 31
in Orlicz space $E_A(\Omega)$, 274
Approximation in $W^{m,p}(\Omega)$ by smooth functions on Ω , 66
by smooth functions on \mathbb{R}^n , 68
Approximation property, 160
Arzela-Ascoli theorem, 11
Averaging lemma of Gagliardo, 95

Banach algebra, 106 Banach lattice, 248 Banach space, 5 Besov space, 229, 254 and traces, 234, 240 homogeneous, 255 imbedding theorem for, 230 Bessel potentials, 252 Bochner integrable function, 207 Bochner integral, 206 Boundary trace, 163 Bounded continuous function space, 10 Calderón extension theorem, 156 Calderón-Zygmund inequality, 155 Capacity of a subset of a cube, 176 Cartesian product of Banach spaces, 8 Cauchy sequence, 5 Characteristic function, 15 Clarkson inequalities, 43 Closure, 2 Compact imbedding, 9, 167 for unbounded domains, 175 Compact operator, 9, 167 Compact set, 7

302 Index

Compact support, 2	Distribution function, 52, 221
Complementary N-function, 263	Domain, 1
Completely continuous operator, 9, 167	of finite width, 183
Completeness, 5	quasibounded, 173
of $L^p(\Omega)$, 29	quasicylindrical, 184
of $W^{m,p}(\Omega)$, 61	Dominance of N-functions, 265
Complete orthonormal system, 200	Dominated convergence property
Completion	of a Banach lattice, 248
of a normed space, 5	Dominated convergence theorem, 17
Complex interpolation, 247	Dual of Orlicz space $E_A(\Omega)$, 273
Complex interpolation space, 247	Dual space, 4
Cone, 81	normed, 6
Cone condition, 79, 82	of $L^p(\Omega)$, 45
uniform, 82	of $W^{m,p}(\Omega)$, 62
weak, 82	of $W_0^{m,p}(\Omega)$, 64
Continuous linear functional, 4	v
Continuous functions	Embedding, see Imbedding
between topological spaces, 3	Equimeasurable rearrangement, 221
Continuous function space, 10	Equivalence
bounded functions, 10	of J- and K-methods, 215
Hölder continuous functions, 10	of definitions of Sobolev spaces, 6
uniformly continuous functions, 10	Equivalent norm for $W_0^{m,p}(\Omega)$, 184
Convergence in mean, 270	Essentially bounded function, 26
Convex function, 261	Exact interpolation theorem, 220, 247
Convolution, 32	Extension operator, 146
Fourier transform of, 251	total, 255
Coordinate transformations	Fatou's lemma, 17
m-smooth, 77	Finite cone, 81
Cube	Finite width, 183
λ -fat or λ -thin, 187	First countable space, 9
Cusp, 115	Flow on a domain, 195
	Fourier inversion theorem, 250
Decomposition of domains, 93	Fourier transform, 250
Delta-2 (Δ_2) condition	inverse of, 250
global or near infinity, 265	Fractional order Sobolev space, 249
Delta-regular (Δ-regular), 266	Fubini's theorem, 19
Dense set, 5	Function
Derivative	essentially bounded, 26
partial, 2	measurable, 15
weak, 22	N-, 262
Dirac distribution, 20	Functional, 4
Distance between sets, 3	i diletionar, +
Distribution	Gagliardo
derivative of a, 21	averaging lemma, 95
Schwartz, 20	decomposition lemma, 93
tempered, 251	Generalized Hölder inequality, 268

Generalized Hölder space, 231	Interpolation inequality (continued)
Hölder continuity, 10	on order of smoothness, 135
generalized, 231	Interpolation pair, 208
Hölder's inequality, 24, 25	Interpolation space
converse of, 25	complex, 247
for complementary N-functions, 268	exact, 220
for mixed-norm spaces, 50	of type θ , 220
generalized, 268	Interpolation theorem
reverse, 27	exact, 220, 247
Hahn-Banach theorem, 6	Marcinkiewicz, 54
Hausdorff space, 3	Inverse Fourier transform, 250
Hilbert-Schmidt	Irregular domain
imbedding, 202	nonimbedding theorem, 111
norm, 200	Isometric isomorphism, 5
operator, 200	J-method, 211
Hilbert space, 5	discrete version, 213
-	J-norm, 208
Imbedding, 9, 80	,
best possible, 108	K-method, 209
boundary trace, 164	discrete version, 210
compact, 9, 167	K-norm, 208
noncompact, 173	L^p space, 23
of an Orlicz-Sobolev space, 284	ℓ^p space, 35
restricted, 167	Lebesgue integral, 16
Imbedding theorem	of complex-valued functions, 18
for L^p spaces, 28	Lebesgue measure, 14
for Orling appears, 260	Lebesgue space $L^p(\Omega)$, 23
for Orlicz spaces, 269	Linear functional, 4
for Sobolev spaces, 85	on $L^p(\Omega)$, 45
Inner product, 5 for $L^2(\Omega)$, 31	Lipschitz condition, 83, 93
for $W^{m,2}(\Omega)$, 61	Lipschitz spaces
Integrable function, 16	imbeddings into, 99
Integral	Locally convex, 3
Lebesgue, 16	Locally finite open cover, 82
of Banach-space-valued functions, 206	Locally integrable function, 20
Intermediate space, 208	Lorentz space, 223
classes \mathcal{H} , \mathcal{J} , and \mathcal{K} , 216	Lusin's theorem, 15
Interpolation	Marcinkiewicz
complex method, 247	
real method, 208–221	interpolation theorem, 54, 91, 226
Interpolation inequality	Maurin's theorem, 202
for L^p spaces, 27	Measurable function, 15
hybrid, 141	Measurable set, 14
involving compact subdomains, 143	Measure, 14
on degree of summability, 139	Lebesgue, 14
on degree of summatting, 139	Minkowski's inequality, 25

Minkowski's inaquality (continued)	Decomment and 7, 167
Minkowski's inequality (continued)	Precompact set, 7, 167
for integrals, 26	$\int_{\Gamma}^{P} L^{p}(\Omega)$, 38
reverse, 28	in an Orlicz space, 276
Mixed-norm space, 50	Quasi-norm, 54
Modulus of continuity	Quasibounded domain, 173
L^p , 241	Quasicylindrical domain, 184
higher order, 241	Quasicy infarical domain, 10
Mollifier, 36	Radon-Nikodym theorem, 18
for $W^{m,p}(\Omega)$, 66	Rapid decay, 192
Monotone convergence theorem, 17	Rapidly decreasing functions, 250
Multi-index, 2	Rearrangement of a function
Noncompact imbedding, 173, 186	equimeasurable decreasing, 221
Nonimbedding theorem	Reduced Sobolev inequality, 105
for irregular domains, 111	Reflexive space, 7
Norm, 4	Reflexivity
equivalent, 5, 183	of $L^p(\Omega)$, 49
in $L^p(\Omega)$, 24	of Orlicz spaces, 274
of a linear functional, 6	of Sobolev spaces, 61
of a linear operator, 9	Regularity condition, 84
of a Sobolev space, 59	Regularization, 36
Normed dual, 6	Reiteration theorem, 217
Normed space, 4	for complex interpolation, 248
Norm topology, 4	Rellich-Kondrachov theorem, 168
Troim topology,	Restricted imbedding, 167
Operator, 9	Reverse Hölder inequality, 27
strong type, 54	Reverse Minkowski inequality, 28
weak type, 54	Riesz representation theorem, 6
Open cover	for $L^p(\Omega)$, 47
locally finite, 82	for $L^1(\Omega)$, 47
Orlicz class $K_A(\Omega)$, 266	
Orlicz-Sobolev space, 281	Schwartz distribution, 20
Orlicz space, 261	Schwarz inequality, 31
Orlicz space $E_A(\Omega)$, 270	Segment condition, 68, 82
Orlicz space $L_A(\Omega)$, 268	Seminorm, 101, 135
Orthonomal system	Separability
complete, 200	of L^p spaces, 32
-	of Orlicz spaces, 274
Parallelepiped, 81	of Sobolev spaces, 61
Parallelogram law, 6	Separable space, 5
Partition of unity, 65	Sigma-algebra, 13
Permutation inequality	Simple (m, p) -extension operator, 146
for mixed norms, 51	existence of, 156
Permuted mixed norm, 50	Simple function, 15, 206
Plancherel's theorem, 251	Sobolev conjugate N-function, 282
Poincaré's inequality, 183	Sobolev imbedding theorem, 79, 84
Polar set, 70	a limiting case, 277

Scholar imhadding theorem (gentinged)	Total extension operator, 146, 255
Sobolev imbedding theorem (continued) alternate proof, 141	existence of, 147, 154
optimality of, 108	Trace, 81
sharper version, 227	boundary, 163
Sobolev's inequality, 102	of Orlicz-Sobolev functions, 287
anisotropic, 104	characterization theorem, 233
best constant, 104	Transformation of coordinates, 77
•	Triangle inequality, 207
reduced, 105	Triebel-Lizorkin space, 253
Sobolev space, 59	homogeneous, 254
of fractional order, 249	Trudinger's theorem, 277
weighted, 119	Trudinger's medicin, 277
Spiny urchin, 176	Unbounded domain
Standard cusp, 115	compact imbedding for, 175
Stein extension theorem, 154	Uniform C^m -regularity condition, 84
Stone-Weierstrass theorem, 11	Uniform cone condition, 82
Streamline, 195	Uniform convexity, 8
Strong <i>m</i> -extension operator, 146	of L^p spaces, 45
existence of, 151	of Sobolev spaces, 61
Strong local Lipschitz condition, 83, 93	Uniformly continuous function spaces, 10
Strongly measurable function, 206	- 140
Strong type operator, 54	Vandermonde determinant, 149
Sublinear operator, 54	Vector space, 3
Subspace	topological, 3
of a normed vector space, 6	Wavelet, 256
Support, 2	Weak cone condition, 82
	Weak convergence, 7
	Weak derivative, 22
Tempered distribution, 251	Weak L^p space, 53
Tesselation, 187	Weak sequential compactness, 7
Test function, 19	Weak-star topology, 4
Topological product, 3	Weak topology, 7
Topological space, 3	Weak type operator, 54, 91
Topological vector space, 3	Weighted Sobolev space, 119
locally convex, 3	•
Topology, 3	Young's theorem, 32
weak, 7	Young's inequality, 34, 35, 208, 264