Dynamics of Decision Making

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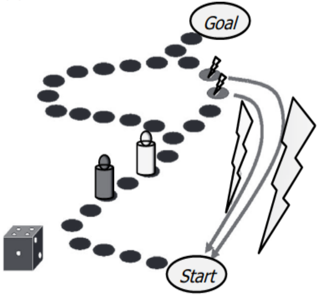
Introduction

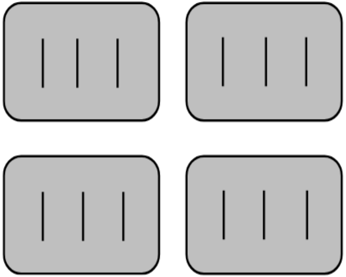
We make multiple decisions in daily life. (Examples...)

(a)



Board games [1]

(b)



Perceptual decision making

[1] Platt ML, Huettel SA. *Risky business: the neuroeconomics of decision making under uncertainty.* Nat Neurosci. 2008 Apr

Introduction

Generally, decision making requires:[2]

1. a suitable **representation** of inputs and potential outcomes as well as of the values attached to the options
2. a **selection** process that picks one of the options
3. potentially also some **feedback** that enables learning so as to achieve improved performance over several trials

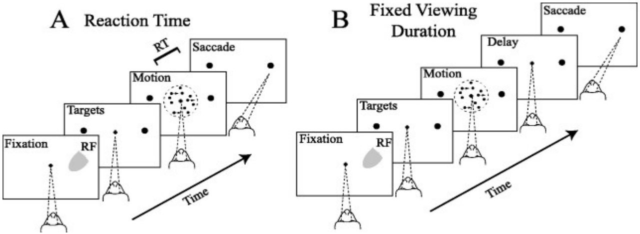
Here, we focus on the dynamic selection between different options in the context of perceptual decision making, driven by the convenience of measuring neuronal activity.

[2] Rangel et all. *A framework for studying the neurobiology of value-based decision making*. Nat Rev Neurosci. 2008

Contents

Perceptual decision making

A classical study of perceptual decision making in monkeys [3]



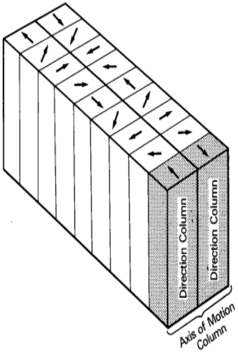
The stimulus consists of a random pattern of moving dots, where most of the dots move coherently in the same direction. The monkey has been trained to indicate the perceived motion direction by saccadic eye movements to one of two targets.

[3] Salzman et al. *Cortical microstimulation influences perceptual judgements of motion direction.* Nature. 1990

Perceptual decision making

Biologists have found that:

Different neurons in the middle temporal visual area (MT) respond to different directions of motion, and clusters of neighboring neurons share receptive fields with a similar preferred direction of motion.

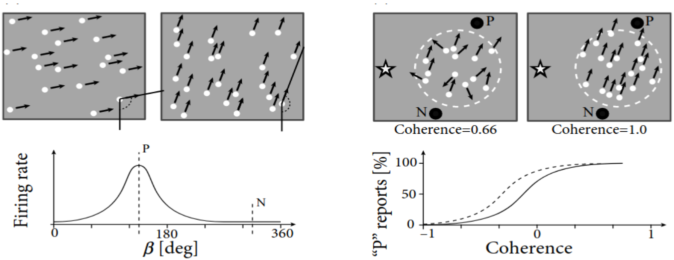


[4]

[4] Albright et al. *Columnar organization of directionally selective cells in visual area MT of the macaque* J Neurophysiol. 1984

Perceptual decision making

The location of the receptive field and the preferred direction of motion is determined by varying the movement angle and the location of the random dot stimulus. Once the receptive properties of the local MT neurons have been determined, only two different classes of stimuli are used.[5]



[5] Salzman et al. *Cortical microstimulation influences perceptual judgements of motion direction.* Nature. 1990

Perceptual decision making

Conclusion: the perceptual decision of the monkey relies on the motion information represented in the activity of MT neurons.[6]

After that: neuroscientists have continuously conducted such monkey perceptual decision making experiments, leading to many new discoveries[7] [8], including the modeling of decision making.

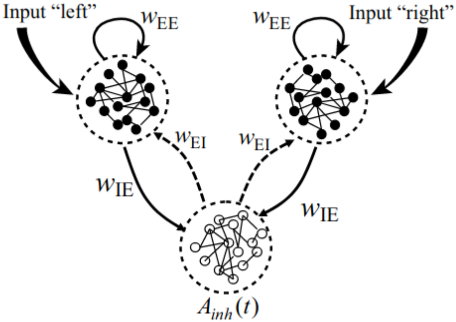
[6] Salzman et al. *Cortical microstimulation influences perceptual judgements of motion direction.* Nature. 1990

[7] Roitman et al. *Response of neurons in the lateral intraparietal area during a combined visual discrimination reaction time task.* 2002

[8] Rangel et all. *A framework for studying the neurobiology of value-based decision making.* Nat Rev Neurosci. 2008

Competition through common inhibition

The essential features of the experiments of Roitman and Shadlen (2002) can be described by a simple model of decision making where two excitatory populations compete with each other through a shared inhibitory population. [9]

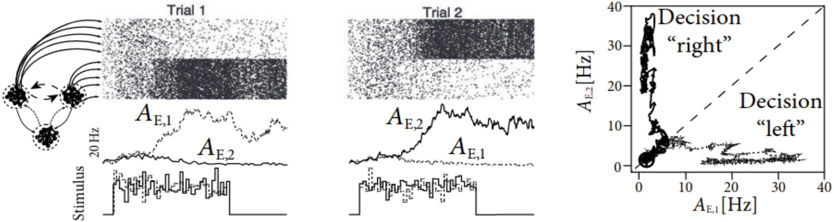


Note: Input "left" brings excitatory population 1 activity $A_{E,1}(t)$. Each excitatory population has connection weights w_{EE}. The inhibitory population receives input of strength w_{IE} from the two excitatory populations and sends back inhibition of strength w_{EI}.

[9] Wang etal. *Anatomical, physiological, molecular and circuit properties of nest basket cells in the developing somatosensory cortex.* 2002

Competition through common inhibition

The input into each population is described as a mean plus some noise.
The following picture shows presentations of an unbiased stimulus, where the input to the left and right populations has an equal mean but different realizations of noise. [10]



In the absence of stimulation, the activity of both excitatory populations exhibits a low firing rate of less than 5 Hz (circle). Upon stimulation, the dynamics converge.

[10] Wang etal. *Anatomical,physiological,molecular and circuit properties of nest basket cells in the developing somatosensory cortex*.2002

Dynamics of decision making

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Next, we will present a mathematical analysis of decision-making in models of interacting populations, divided into the following four parts.

P1. Give the rate equations for a model with three populations, two excitatory ones which interact with a common inhibitory population.

P2. Reduce the rate model with three populations to a simplified system described by two differential equations.

P3. Analysis the fixed points of the two-dimensional dynamical system in the phase plane for several situations relevant to experiments on decision making.

P4. Generalize the formalism of competition through shared inhibition to the case of K competing populations.

P1. Model with three populations

1. About rate models:

Let $A(t)$ represent the average firing rate of a population of homogeneous neurons, and let the input potential be denoted as $h(t)$. Let F be the gain function, then the population activity can be expressed as:[11]

$$A(t) = F(h(t)).$$

We use an exponential kernel to perform convolution to represent $h(t)$:

$$h(t) = \frac{R}{\tau_m} \int_0^\infty e^{-\frac{s}{\tau_m}} I(t-s)ds,$$

where τ_m is the membrane time constant and R is the membrane resistance.

[11] Wulfram Gerstner, Werner M. Kistler, Richard Naud, Liam Paninski. *Neuronal Dynamics* Cambridge University Press. 2014

P1. Model with three populations

By applying integration by parts, the above expression is equivalent to the equation

$$\tau_m \frac{dh(t)}{dt} = -h + RI(t),$$

Therefore, the general form of the rate model is:[12]

$$\begin{aligned} A(t) &= F(h(t)) \\ \tau_m \frac{dh(t)}{dt} &= -h + RI(t) \end{aligned} \tag{1}$$

[12] Wulfram Gerstner, Werner M. Kistler, Richard Naud, Liam Paninski. *Neuronal Dynamics* Cambridge University Press. 2014

P1. Model with three populations

2. Rate model with three populations:

Let g_E and g_{inh} be the gain functions of excitatory and inhibitory neurons respectively.

$$
\begin{aligned}
A_{E,1} &= g_E(h_{E,1}) \\
A_{E,2} &= g_E(h_{E,2}) \\
A_{inh} &= g_{inh}(h_{inh}) \\
\tau_E \frac{dh_{E,1}}{dt} &= -h_{E,1} + w_{EE} g_E(h_{E,1}) + w_{EI} g_{inh}(h_{inh}) + RI_1 \\
\tau_E \frac{dh_{E,2}}{dt} &= -h_{E,2} + w_{EE} g_E(h_{E,2}) + w_{EI} g_{inh}(h_{inh}) + RI_2 \\
\tau_{inh} \frac{dh_{inh}}{dt} &= -h_{inh} + w_{IE} g_E(h_{E,1}) + w_{IE} g_E(h_{E,2})
\end{aligned}
\tag{2}
$$

P2. Effective inhibition

The system of three populations is still complex. To reduce it to two dimensions and utilize phase plane analysis tools, we make the following assumptions.

Assumption 1.

The membrane time constant $\tau_{inh} \ll \tau_E$, i.e. we consider the limit of a separation of time scales $\tau_{inh} \ll \tau_E \to 0$. Therefore we can treat the dynamics of h_{inh} in as instantaneous, so that the inhibitory potential is always at its fixed point

$$h_{inh} = w_{IE}[g_E(h_{E,1}) + g_E(h_{E,2})]$$

According to the expression of the convolution kernel, such an assumption implies that the current decay in the excitatory neuron population is slower than that in the inhibitory neuron population, which is consistent with the fact that excitatory synapses typically possess **NMDA receptors**.

P2. Effective inhibition

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After assuming the separation of time scales between inhibition and excitation, the dynamical equations for $h_{E,1}$ and $h_{E,2}$ become:

$$\tau_E \frac{dh_{E,1}}{dt} = -h_{E,1} + w_{EE}g_E(h_{E,1}) + w_{EI}g_{inh}(w_{IE}[g_E(h_{E,1}) + g_E(h_{E,2})]) + RI_1$$
$$\tau_E \frac{dh_{E,2}}{dt} = -h_{E,2} + w_{EE}g_E(h_{E,2}) + w_{EI}g_{inh}(w_{IE}[g_E(h_{E,1}) + g_E(h_{E,2})]) + RI_2$$

P2. Effective inhibition

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Assumption 2.

The second assumption is not absolutely necessary, but it makes the remaining two equations more transparent. We assume a linear gain function

$$g_{inh}(h_{inh}) = \gamma h_{inh}$$

with a slope factor $\gamma > 0$. Then equations for $h_{E,1}$ and $h_{E,2}$ become:

$$\begin{aligned}\tau_E \frac{dh_{E,1}}{dt} &= -h_{E,1} + (w_{EE} - \alpha) g_E(h_{E,1}) - \alpha g_E(h_{E,2}) + RI_1 \\ \tau_E \frac{dh_{E,2}}{dt} &= -h_{E,2} + (w_{EE} - \alpha) g_E(h_{E,2}) - \alpha g_E(h_{E,1}) + RI_2\end{aligned} \tag{3}$$

where we have introduced a parameter $\alpha = -\gamma w_{EI} w_{IE} > 0$.

P2. Effective inhibition

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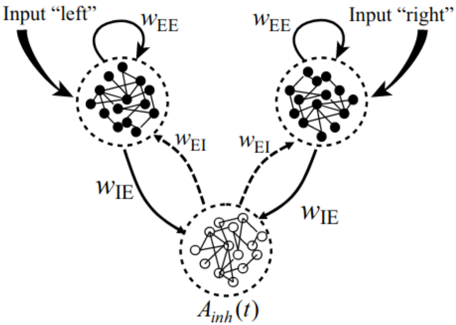
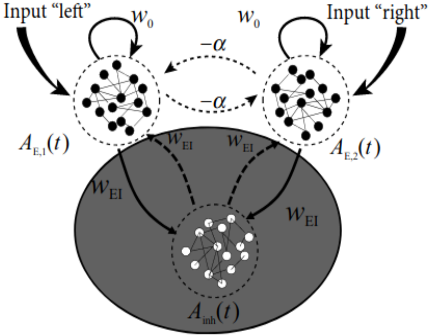
Thus, the model of three populations has been replaced by a model with two excitatory populations that interact with an effective inhibitory coupling of strength α.

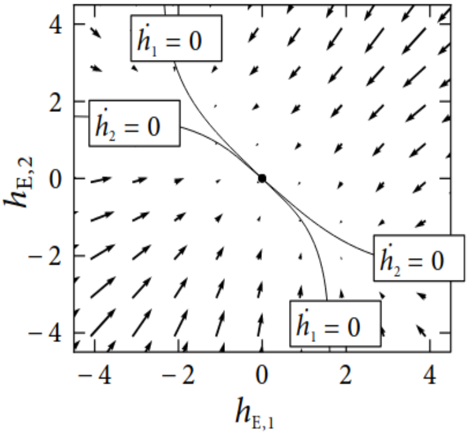




P3. Phase plane analysis

$I_1 = I_2 = 0$:

There exists only a single fixed point $h_{E,1} = h_{E,2} \approx 0$, corresponding to a small level of spontaneous activity.



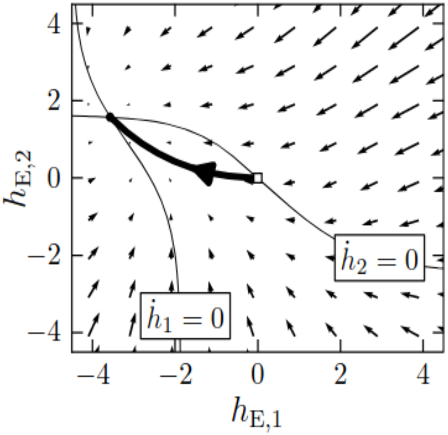
13

[13] Wulfram Gerstner, Werner M. Kistler, Richard Naud, Liam Paninski. *Neuronal Dynamics*. Cambridge University Press, 2014

P3. Phase plane analysis

$I_1 > 0 = I_2$:

The fixed point moves to an asymmetric position where population 1 exhibits much stronger activity than population 2.



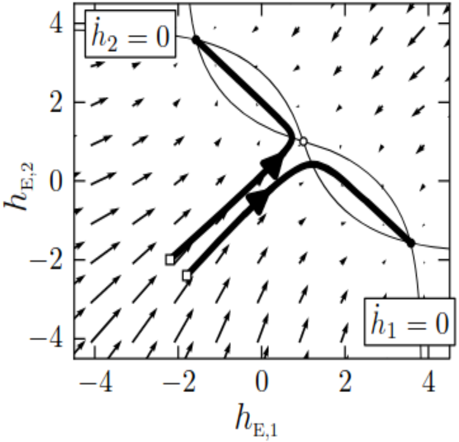
14

[14] Wulfram Gerstner, Werner M. Kistler, Richard Naud, Liam Paninski. *Neuronal Dynamics* Cambridge University Press. 2014

P3. Phase plane analysis

$I_1 = I_2 > 0$:

Three fixed points exist and the symmetric one is a saddle point. The two other fixed points occur at equivalent positions symmetrically to the left and right of the diagonal.

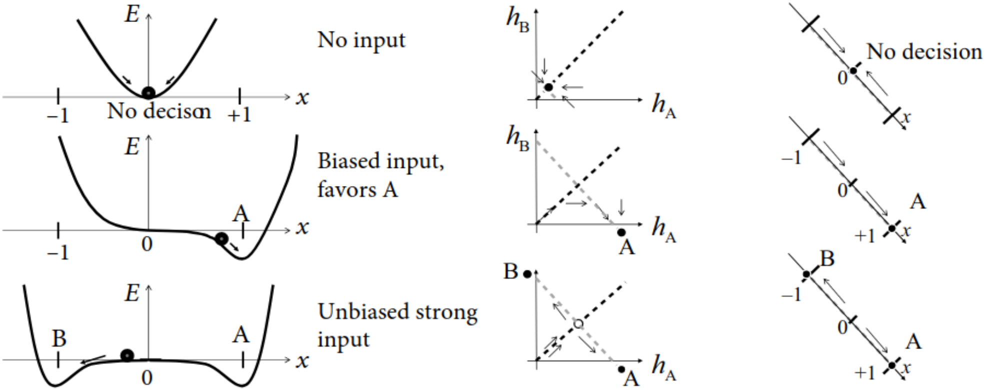


15

[15] Wulfram Gerstner, Werner M. Kistler, Richard Naud, Liam Paninski. *Neuronal Dynamics*. Cambridge University Press. 2014

P3. Phase plane analysis

Decisions correspond to a ball rolling down an energy landscape, plotted as a function of a formal decision variable x.



P4. Formal winner-take-all networks

Each outcome is represented by one population of excitatory neurons. We work with an effective inhibition of strength $\alpha > 0$ between the K pools of neurons and with a self-interaction of strength w_0 within each pool of neurons.

$$\begin{aligned} A_k(t) &= g(h_k(t)) \\ \tau\frac{dh_k(t)}{dt} &= -h_k(t) + w_0 g(h_k(t)) - \alpha\sum_{j\neq k} w_{kj}\, g(h_j(t)) + RI_k \end{aligned} \tag{4}$$

Such networks are called **winner-take-all** networks, which are a standard topic of artificial neural networks. [16] [17] [18]

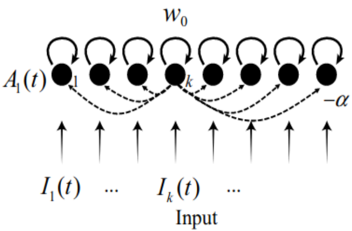
[16] Hertz, J., Krogh, A., and Palmer, R. G. *Introduction to the Theory of Neural Computation.* 1991

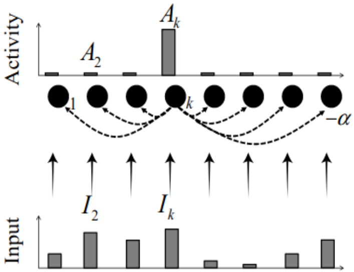
[17] Kohonen, T. *Self-Organization and Associative Memory.* Springer-Verlag. 1984

[18] Haykin, S. *Neural Networks. Prentice* 1994

P4. Formal winner-take-all networks

Each artificial neuron receives an input I_k, has a positive feedback of magnitude w_0 onto itself but inhibits with strength α all other neurons.





Under fixed $I_k > 0$, the network converges to a state where only a single "winner" neuron is active, i.e., **the one which receives the strongest input**.

P4. Formal winner-take-all networks

We consider an arbitrary network of K neuronal populations $1 \leq j \leq K$ with population rate $A_j = g(h_j) \geq 0$ where g is a gain function with derivative $g' > 0$ and h follows the dynamics

$$\tau \frac{dh_j}{dt} = -h_j + RI_j + \sum_k w_{jk} g(h_k)$$

with fixed inputs I_j. If the coupling is symmetric, i.e., $w_{ij} = w_{ji}$, then the energy[19]

$$E = -\sum_i \sum_j w_{ij} A_i A_j - \sum_i A_i R I_i + \sum_i \int_0^{A_i} g^{-1}(a) da$$

is **a Liapunov function** of the dynamics.

[19] Cohen et.al. *Absolute stability of global pattern formation and parallel memory storage by competitive neural networks.* 1983

P4. Formal winner-take-all networks

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We exploit the fact that $w_{ij} = w_{ji}$ and $\frac{dA_i}{dt} = g'(h_i)\frac{dh_i}{dt}$ so as to find

$$\frac{dE}{dt} = -\sum_i \left[\sum_j w_{ij} A_j\right] g'(h_i)\frac{dh_i}{dt} - \sum_i RI_i g'(h_i)\frac{dh_i}{dt} + \sum_i g^{-1}(A_i) g'(h_i)\frac{dh_i}{dt}$$

$$= -\tau \sum_i g'(h_i)\left[\frac{dh_i}{dt}\right]^2 \leq 0$$

Furthermore, since the neuronal gain function stays below a biologically sustainable firing rate $g(x) \leq A_{max}$, the energy is bounded from below.

Summary

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1. Decisions are prepared and **made in the brain** so that numerous physiological correlates of decision making can be found in the human and monkey cortex.
2. An influential computational **model** describes decision making **as the competition** of several populations of excitatory neurons which share a common pool of inhibitory neurons.
3. Under suitable conditions, the explicit model of inhibitory neurons can be replaced by an effective **inhibitory coupling** between excitatory populations.
4. In a rate model, the competitive interactions between two excitatory populations can be understood using **phase plane analysis**.
5. Equivalently, the decision process can be described as downward motion in an energy landscape which plays the role of a **Liapunov function**.

Thank You