Aero 421 HW 4.

1. a.

$m_1 = .684 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad k_1 = 105\times10^6 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$m_2 = .21375 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad k_2 = 58.\bar{3}\times10^6 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$\frac{1}{6}\begin{bmatrix} 1.368 & .684 & 0 \\ .684 & 1.7955 & .21375 \\ 0 & .21375 & .4275 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 105\times10^6 & -105\times10^9 & 0 \\ -105\times10^9 & 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ 0 & -58.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$u_1 = 0$

$$\frac{1}{6}\begin{bmatrix} 1.7955 & .21375 \\ .21375 & .4275 \end{bmatrix}\begin{bmatrix} \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix} + \begin{bmatrix} 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ -58.\bar{3}\times10^6 & 58.\bar{3}\times10^6 \end{bmatrix}\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{6}\begin{bmatrix} 1.7955(-\omega^2\bar{u}_2) & .21375(-\omega^2\bar{u}_3) \\ .21375(-\omega^2\bar{u}_2) & .4275(-\omega^2\bar{u}_3) \end{bmatrix}(\cos(\omega t)) + \begin{bmatrix} 163.\bar{3}\times10^6(\bar{u}_2) & -58.\bar{3}\times10^6(\bar{u}_3) \\ -58.\bar{3}\times10^6(\bar{u}_2) & 58.\bar{3}\times10^6(\bar{u}_3) \end{bmatrix}(\cos(\omega t)) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\frac{1}{6}\begin{bmatrix} 1.7955(-\omega^2) & .21375(-\omega^2) \\ .21375(-\omega^2) & .4275(-\omega^2) \end{bmatrix} + \begin{bmatrix} 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ -58.\bar{3}\times10^6 & 58.\bar{3}\times10^6 \end{bmatrix}\right)\begin{bmatrix} \bar{u}_2\cos(\omega t) \\ \bar{u}_3\cos(\omega t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det\left(\frac{1}{6}\begin{bmatrix} 1.7955(-\omega^2) & .21375(-\omega^2) \\ .21375(-\omega^2) & .4275(-\omega^2) \end{bmatrix} + \begin{bmatrix} 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ -58.\bar{3}\times10^6 & 58.\bar{3}\times10^6 \end{bmatrix}\right)$$

solved on computer

$\omega = 14530,\ 38030\ \frac{rad}{s}$

$\boxed{\omega_n = 2312.5,\ 6052.6\ \ Hz}$

b.

$\omega = 2312.5$

$$\begin{bmatrix} 1.6\times10^6 & 190,513\times10^3 \\ 190,513\times10^3 & 381,027\times10^3 \end{bmatrix} + \begin{bmatrix} 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ -58.\bar{3}\times10^6 & 58.\bar{3}\times10^6 \end{bmatrix} = \begin{bmatrix} 1.001\times10^8 & -6.582\times10^7 \\ -6.582\times10^7 & 4.326\times10^7 \end{bmatrix}$$

$$\begin{bmatrix} 1.001\times10^8 & -6.582\times10^7 \\ -6.582\times10^7 & 4.326\times10^7 \end{bmatrix}\begin{bmatrix} \bar{u}_2 \\ \bar{u}_3 \end{bmatrix}$$ system of eqns on calculator

$\boxed{\bar{u}_2 = .657\,\bar{u}_3}$

$\omega = 6052.5$

$$\begin{bmatrix} 10.962\times10^6 & 1,305\times10^6 \\ 1,305\times10^6 & 261\times10^6 \end{bmatrix} + \begin{bmatrix} 163.\bar{3}\times10^6 & -58.\bar{3}\times10^6 \\ -58.\bar{3}\times10^6 & 58.\bar{3}\times10^6 \end{bmatrix} = \begin{bmatrix} -2.696\times10^8 & -1.098\times10^8 \\ -4.477\times10^7 & -4.477\times10^7 \end{bmatrix}$$

$$\begin{bmatrix} -2.696\times10^8 & -1.098\times10^8 \\ -4.477\times10^7 & -4.477\times10^7 \end{bmatrix}\begin{bmatrix} \bar{u}_2 \\ \bar{u}_3 \end{bmatrix}$$ system of eqns on calculator

$\boxed{\bar{u}_2 = .407\,\bar{u}_3}$

c.

$f = 500000\sin(\omega t)$

$I = m_1 r_1^2 + m_2 r_2^2 \quad r_1 = .2 \quad r_2 = .55 \quad m_1 = 6\times10^{-4}\cdot .4\cdot 2850 = .684 \quad m_2 = 250\times10^{-6}\cdot .3\cdot 2850 = .21375$

$I = .092019\ kg\,m^2 = mR^2$

$R = \sqrt{\frac{I}{m_1+m_2}} = .32016\ kg\,m^2$

$J = mR^2 = .092\ kg\,m^2$

$m\ddot{x} + k_1(x - l_1\theta) + k_2(x + l_2\theta) = 0$

$J\ddot{\theta} - k_1 l_1(x - l_1\theta) + k_2 l_2(x + l_2\theta) = 0$

$.898\ddot{x} + 1.633\times10^8 x - 2.45\times10^7\theta = 0$

$.092\ddot{\theta} - 2.45\times10^7 x + 2.2\times10^7\theta = 0$

$x = \bar{x}\cos(\omega t)$, $\theta = \bar{\theta}\cos(\omega t)$

$$\begin{bmatrix} .8978 & 0 \\ 0 & .092 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 1.63\times10^8 & -2.45\times10^7 \\ -2.45\times10^7 & 2.2\times10^7 \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} .8978\cdot1.63\times10^8\omega^2 & -2.45\times10^7 \\ -2.45\times10^7 & .092\cdot2.2\times10^7\omega^2 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\det\begin{bmatrix} .8978\cdot1.63\times10^8\omega^2 & -2.45\times10^7 \\ -2.45\times10^7 & .092\cdot2.2\times10^7\omega^2 \end{bmatrix} = 0$$

$.083\omega^4 \cdot 34799\times10^4\omega^2 + 2.96\times10^5 = 0$

$\omega_1 = 10990\ rad/s \quad \omega_2 = 17350\ rad/s$

$\omega = \omega_1$

$$\begin{bmatrix} .8978\cdot1.63\times10^8\omega^2 & -2.45\times10^7 \\ -2.45\times10^7 & .092\cdot2.2\times10^7\omega^2 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$5.487\times10^7\bar{x} - 2.45\times10^7\theta = 0$

$-2.45\times10^7\bar{x} + 1.095\times10^7\theta = 0$

$\bar{x} = .447\theta$

$\omega = \omega_2$

$$\begin{bmatrix} .8978\cdot1.63\times10^8\omega^2 & -2.45\times10^7 \\ -2.45\times10^7 & .092\cdot2.2\times10^7\omega^2 \end{bmatrix}\begin{bmatrix} \bar{x} \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-1.07\times10^7\bar{x} - 2.45\times10^7\theta = 0$

$-2.45\times10^7\bar{x} \cdot 5.62\times10^6\bar{\theta} = 0$

$\bar{x} = -.229\bar{\theta}$

$$\phi = \begin{bmatrix} .447 & -.229 \\ 1 & 1 \end{bmatrix}$$

$$[m_d] = \begin{bmatrix} .447 & 1 \\ -.229 & 1 \end{bmatrix}\begin{bmatrix} .8978 & 0 \\ 0 & .092 \end{bmatrix}\begin{bmatrix} .447 & -.229 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} .2713 & 0 \\ 0 & .139 \end{bmatrix}$$

$$[k_d] = \begin{bmatrix} .477 & 1 \\ -.229 & 1 \end{bmatrix}\begin{bmatrix} 1.63\times10^8 & -2.45\times10^7 \\ -2.45\times10^7 & 2.205\times10^7 \end{bmatrix}\begin{bmatrix} .447 & -.229 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3.276\times10^7 & 0 \\ 0 & 4.184\times10^7 \end{bmatrix}$$

$\omega_1 = \sqrt{\frac{3.276\times10^7}{.271}} = 10990\ rad/s \quad \omega_2 = \sqrt{\frac{4.184\times10^7}{.139}} = 17350\ rad/s$

$$f = \begin{bmatrix} .447 & 1 \\ -.229 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 5\times10^5 \end{bmatrix} = \begin{bmatrix} 5\times10^5\sin(\omega t) \\ 5\times10^5\sin(\omega t) \end{bmatrix}$$

$$\begin{bmatrix} q_1(0) \\ q_2(0) \end{bmatrix} = \begin{bmatrix} .447 & -.0229 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} q_1'(0) \\ q_2'(0) \end{bmatrix} = \begin{bmatrix} .447 & -.0229 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$q_1(t) = \left(\frac{1.843\times10^6}{10990^2}\right)\left(\frac{\sin(\omega t)}{1-\left(\frac{\omega}{10990}\right)}\right) = .0153\left(\frac{\sin(\omega t)}{1-\left(\frac{\omega}{10990}\right)}\right)$$

$$q_2(t) = \left(\frac{3.597\times10^6}{17350}\right)\left(\frac{\sin(\omega t)}{1-\left(\frac{\omega}{17350}\right)}\right) = .0119\left(\frac{\sin(\omega t)}{1-\left(\frac{\omega}{\ }\right)}\right)$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} .447 & -.229 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1.48 & .34 \\ -1.48 & .66 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} \left(\frac{.0226}{1-\left(\frac{\omega}{10990}\right)^2} + \frac{.00403}{1-\left(\frac{\omega}{17350}\right)^2}\right)\sin(\omega t) \\ \left(\frac{.0226}{1-\left(\frac{\omega}{10990}\right)^2} + \frac{.00787}{1-\left(\frac{\omega}{17350}\right)^2}\right)\sin(\omega t) \end{bmatrix}}$$