MSDA Math Bridge Week 2
Calculus-Based Probability Theory
Logan Thomson

1. pdf $f(x) - \begin{cases} cx\ if\ 0 \le x \le 4 \\ 0 \text{ otherwise} \end{cases}$

a) $1 = \int_0^4 cx\,dx = \left[c\frac{x^2}{2}\right]_0^4 dx = \left[c\frac{16}{2}\right] - \left[c\frac{0}{2}\right] = c8$

$1 = c8;\quad c = \frac{1}{8}$

b) $P(-1 \le x \le 1) = \int_{-1}^1 cx\,dx = \left[c\frac{x^2}{2}\right]_{-1}^1 dx = c\left[\frac{1}{2}\right] - \left[\frac{1}{2}\right] = \frac{1}{8}$

c) $P(x > 2) = \int_2^4 cx\,dx = \left[c\frac{x^2}{2}\right]_2^4 dx = c\left[\frac{16}{2}\right] - \left[\frac{4}{2}\right] = \frac{1}{8}(6) = \frac{3}{4}$

d) $P(x < 3|x > 1)$

e) $E|X| = \int_0^4 x(cx)\,dx = \left[c\frac{x^3}{3}\right]_0^4 dx = \left[\frac{c64}{3}\right] - \left[\frac{c0}{3}\right] = \frac{8}{3} = 2.33$

f)

2. pdf $f(x) - \begin{cases} \frac{3}{2}x^2\ if -1 < x < 1 \\ 0 \text{ otherwise} \end{cases}$

a. $P(-2 < x < 0) = \int_{-2}^0 \frac{3}{2}x^2\,dx = \left[\frac{3}{2}\frac{x^3}{3}\right]_{-2}^0 dx = \left[\frac{x^3}{2}\right]_{-2}^0 dx = [0] + \left[\frac{1}{2}\right] = \frac{1}{2}$

b. $P(x > -.5) = \int_{-.5}^1 \frac{3}{2}x^2\,dx = \left[\frac{3}{2}\frac{x^3}{3}\right]_{-.5}^1 dx = \left[\frac{x^3}{2}\right]_{-.5}^1 dx = \left[\frac{1}{2}\right] - \left[\frac{-\frac{1}{8}}{2}\right] = \left[\frac{1}{2}\right] + \left[\frac{\frac{1}{8}}{2}\right] = \frac{\frac{9}{8}}{2} = \frac{9}{16}$

c.

d. $E|X| = \int_{-1}^1 x(\frac{3}{2}x^2)\,dx = \int_{-1}^1 (\frac{3}{2}x^3)\,dx = \left[\frac{3x^4}{8}\right]_{-1}^1 dx = \left[\frac{3x^4}{8}\right]_{-1}^1 = \left[\frac{3(1)}{8}\right] - \left[\frac{3(1)}{8}\right] = 0$

e.

f.

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3.

4.