

Probing New Physics with MeV Telescopes

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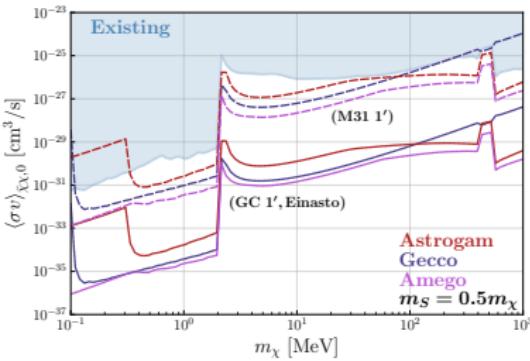


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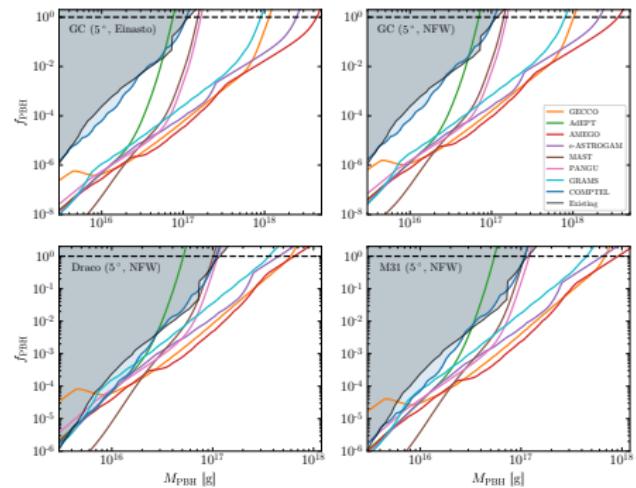
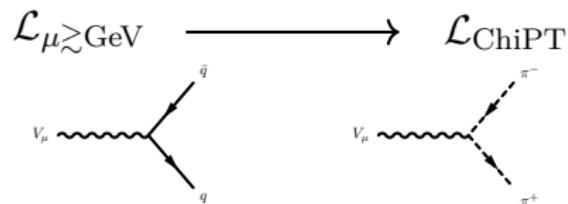
Preview

- Dark matter with $m_\chi <$ GeV is an exciting prospect
- Exciting upcoming MeV γ -ray telescopes could probe the dark sector to unprecedented sensitivity
- New tools developed explicitly for studying MeV physics

Preview



Dictionary



Overview

① Motivation

② MeV Dark Matter

③ Primordial Black Holes

④ Future Work

⑤ Conclusions

Why MeV Dark Matter?

Motivation

- ① For decades, WIMPs have been the de facto DM candidates

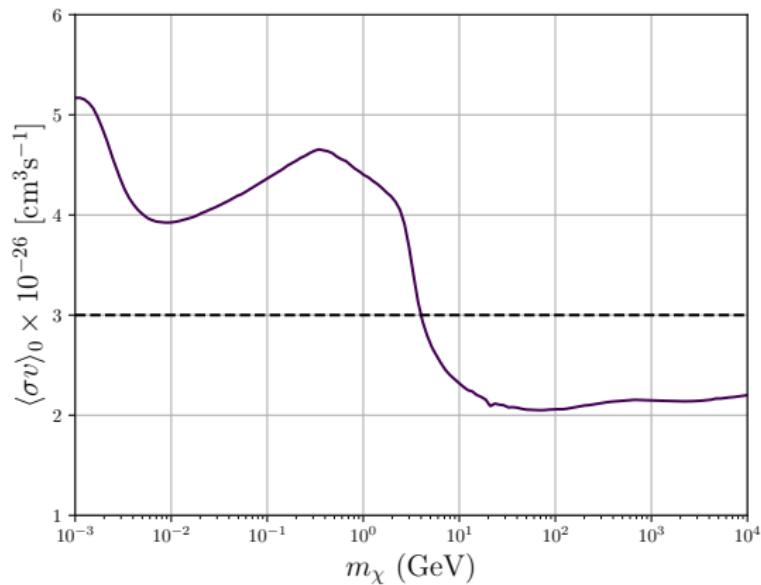
Motivation

- ① For decades, WIMPs have been the de facto DM candidates
 - ① **Natuarally produce DM with correct relic density via freeze-out**

$$\frac{\Omega_\chi h^2}{0.12} \sim \left(\frac{2 \times 10^{-9} \text{GeV}^{-2}}{\langle \sigma v \rangle} \right) \left(\frac{80}{g_*} \right)^{1/2} \left(\frac{x_f}{23} \right)$$

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 - ① Naturally produce DM with correct relic density via freeze-out
 - ② Expect new physics at EW scale (e.g. Naturalness + Hierarchy Problem)
 - ③ **Models with NP at EW scale often accommodate EW scale DM candidate (e.g. MSSM)**

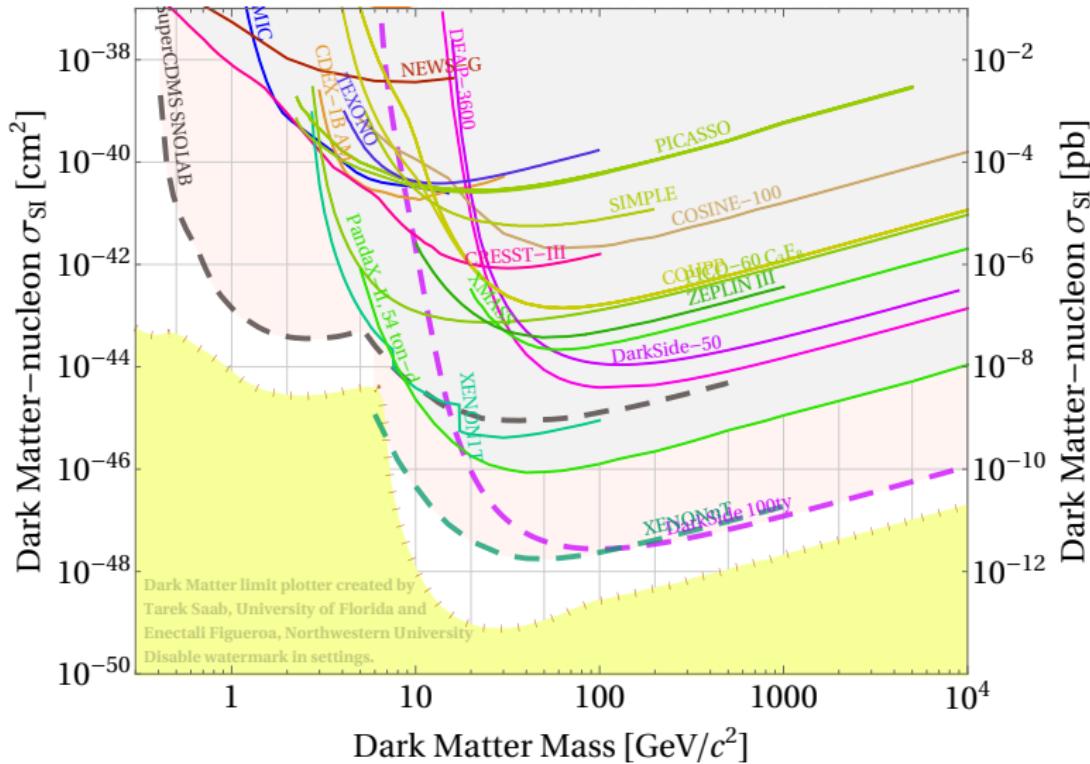
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- ② **No evidence for WIMPs**

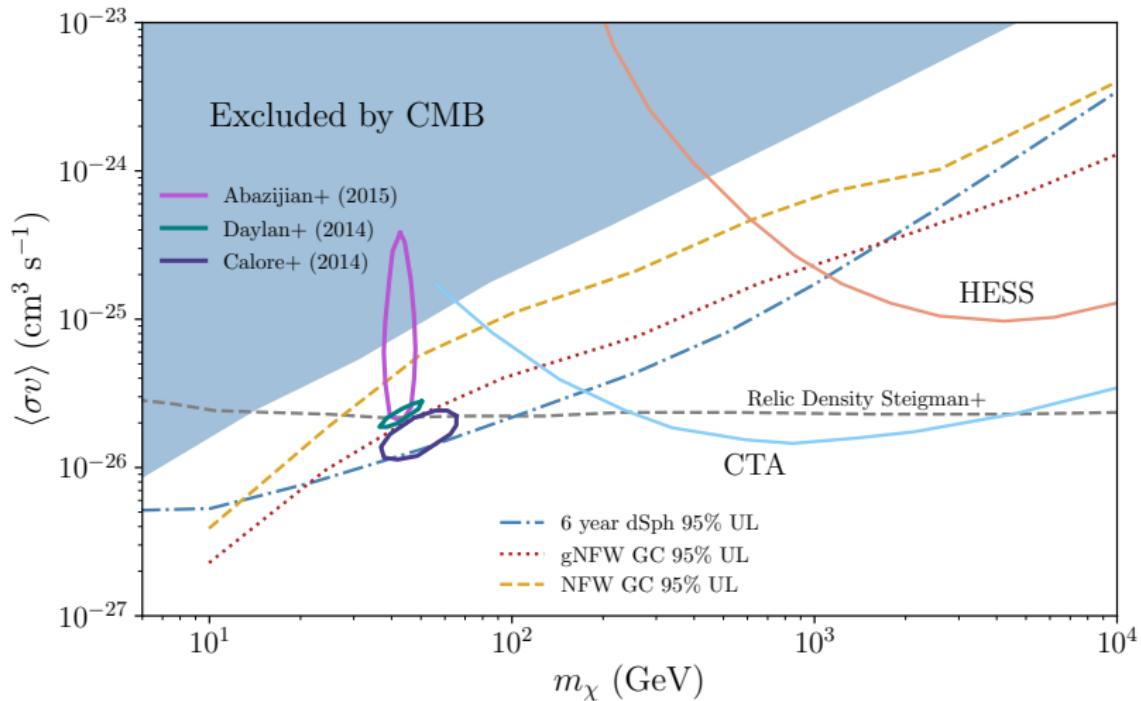
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 - ③ Models with NP at EW scale often accommodate EW scale DM candidate (e.g. MSSM)
- ② No evidence for WIMPs
- ③ **Experiments are putting tight constraints on WIMP models**

Motivation



Motivation



Alternative to WIMPs: MeV DM

① **No WIMPs** \implies Explore different mass ranges/mediators

Alternative to WIMPs: MeV DM

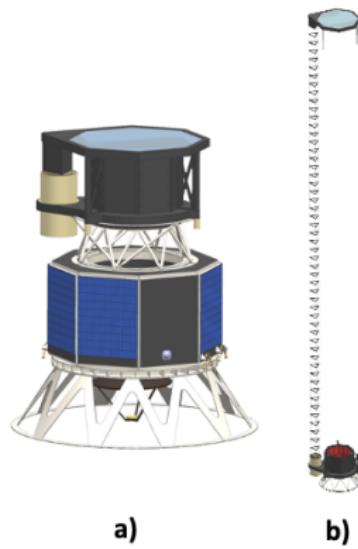
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Alternative to WIMPs: MeV DM

- ① No WIMPs \implies Explore different mass ranges/mediators
- ② MeV masses much less constrained by current direct and indirect detection experiments
- ③ Exciting upcoming opportunities to probe MeV DM via indirect detection: AS-Astrogam, AMEGO, **GECCO**



GECCO



Coded
Aperture Mask

Incident photon flux is modulated while passing through the Mask and creates an image on the CdZnTe detector plane.

Mask deployment cylinder

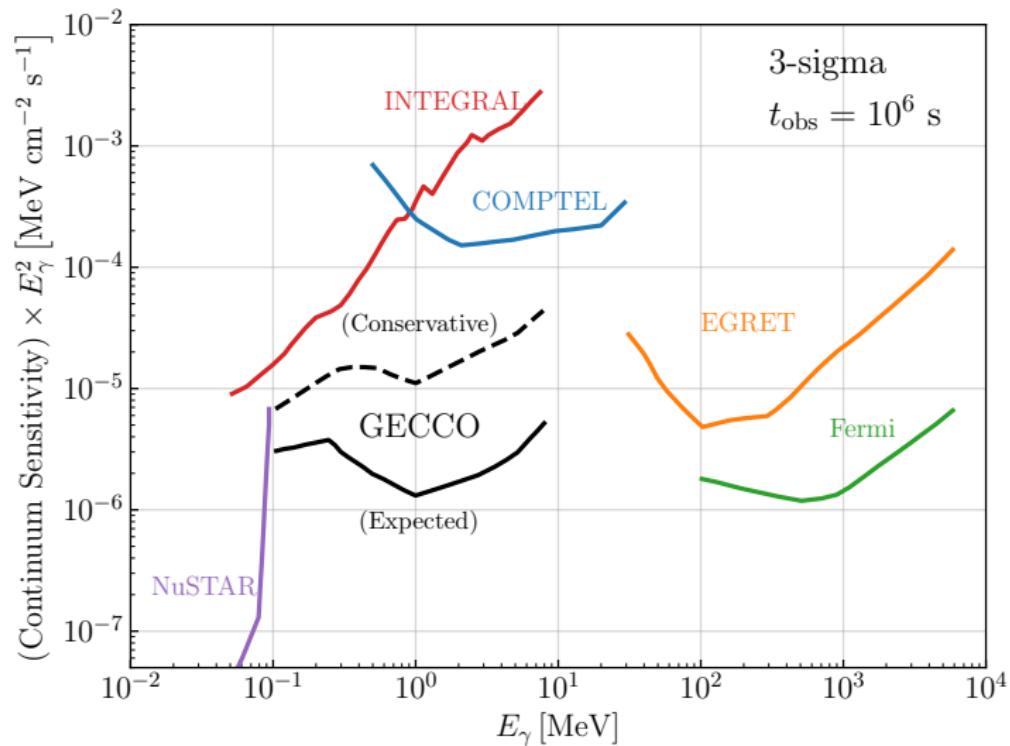
BGO shield provides absorption of natural background photons and vetoes production of background photons by charged cosmic rays

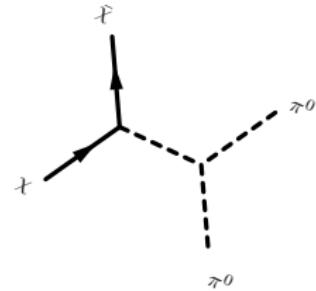
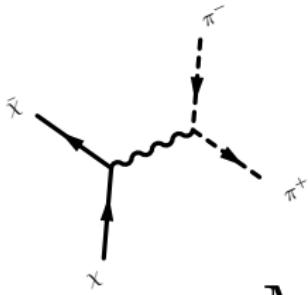
Plastic scintillator anticoincidence detector above the CdZnTe Imager provides protection against charged cosmic rays

The CdZnTe Imager provides detection of incident photons with a position resolution of <1mm and with energy resolution of ~1%.

CsI 5-cm thick log calorimeter measures energy escaping from CdZnTe Imager

GECCO Sensitivity





MeV Dark Matter: From GeV to MeV



A. Coogan, S. Profumo, **LM**: arXiv:1907.11846

A. Coogan, S. Profumo, **LM**: arXiv:2104.06168

A. Coogan, A. Moiseev, S. Profumo, **LM**: arXiv:2101.10370

Framework

- Dark Matter models with:

Annihilating-DM : $0.1 \text{ MeV} \lesssim m_\chi \lesssim 250 \text{ MeV}$

Decaying-DM : $0.1 \text{ MeV} \lesssim m_\chi \lesssim 500 \text{ MeV}$

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(more on this later)

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- Compute realistic spectra and branching fractions by matching quark interactions onto the chiral Lagrangian
- **Developed public, open-source python package for comprehensive analysis of MeV DM models**



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- Constrainers for popular MeV telescopes: COMPTEL, EGRET, Fermi, INTEGRAL, ADEPT, AMEGO, MAST, PANGU, AS-Astrogam, GECCO (easy to implement new ones)



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- Facilities to compute constraints from CMB and various PHENO constraints

Simplified Models

Models: $\mu \gtrsim \text{GeV}$

- Dark Matter: $[\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y]$ -Neutral Dirac Fermion

$$\mathcal{L}_\chi = i\bar{\chi}(\gamma^\mu \partial_\mu - m_\chi)\chi$$

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- Give portal to SM via a new mediator

$$\mathcal{L}_{\chi(\text{int})} \supset \begin{cases} g_{S\chi} S \bar{\chi} \chi & \text{Scalar Mediator} \\ ig_{P\chi} P \bar{\chi} \gamma^5 \chi & \text{Pseudo-Scalar Mediator} \\ g_{V\chi} V_\mu \bar{\chi} \gamma^\mu \chi & \text{Vector Mediator} \\ g_{A\chi} A_\mu \bar{\chi} \gamma^\mu \gamma^5 \chi & \text{Axial-Vector Mediator} \end{cases}$$

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- Focus on scalar and vector mediator cases
- Also consider RH-neutrino with mixing with a single SM neutrino

$$\begin{pmatrix} \hat{\nu}_k \\ \hat{\bar{\nu}} \end{pmatrix} = \begin{pmatrix} -i \cos \theta & \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_k \\ \bar{\nu} \end{pmatrix}$$

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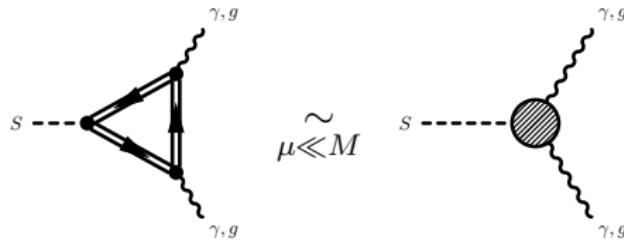
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$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{\text{SM}} + \mathcal{L}_\chi - \frac{1}{2} (\partial_\mu S)^2 - V(S) - g_{S\chi} S \bar{\chi} \chi \\ & - S \sum_f g_{Sf} \bar{f} f + \frac{S}{\Lambda} \left(g_{SF} \frac{\alpha_{\text{EM}}}{4\pi} F_{\mu\nu} F^{\mu\nu} + g_{SG} \frac{\alpha_s}{4\pi} G_{\mu\nu}^a G^{\mu\nu,a} \right)\end{aligned}$$

Higgs Portal: $\mu \gtrsim \text{GeV}$

- Assume the scalar mediator mixes with SM Higgs

$$\begin{pmatrix} \hat{h} \\ \hat{S} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}$$

Higgs Portal: $\mu \gtrsim \text{GeV}$

- Assume the scalar mediator mixes with SM Higgs

$$\hat{h} = \cos \theta h - \sin \theta S$$

- Induces interactions between SM and S

$$-\frac{h}{v_h} \sum_{\psi} m_{\psi} \bar{\psi} \psi + \dots \rightarrow -\sin \theta \frac{S}{v_h} \sum_{\psi} m_{\psi} \bar{\psi} \psi + \dots$$

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- Resulting scalar Lagrangian with dimension-5 operators for $\mu \gtrsim \text{GeV}$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\chi} - \frac{1}{2} (\partial_{\mu} S)^2 - V(S) - g_{S\chi} S \bar{\chi} \chi$$

$$- S \sum_f \cancel{g_{Sf}} \bar{f} f + \frac{S}{\Lambda} \left(\cancel{g_{SF}} \frac{\alpha_{\text{EM}}}{4\pi} F_{\mu\nu} F^{\mu\nu} + \cancel{g_{SG}} \frac{\alpha_s}{4\pi} G_{\mu\nu}^a G^{\mu\nu,a} \right)$$

$$\cancel{g_{Sf}} = \frac{m_f}{v_h} \sin \theta, \quad \cancel{g_{SF}} = \frac{5}{6} \sin \theta, \quad \cancel{g_{SG}} = -3 \sin \theta, \quad \Lambda = v_h$$

Vector Mediator: $\mu \gtrsim \text{GeV}$

- Add massive U(1) vector V_μ via Stueckelberg (or SSB):

$$\begin{aligned} & -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}(\partial_\mu\sigma + m_V V_\mu)^2 - \frac{1}{2\xi}(\partial_\mu V^\mu - \xi m\sigma)^2 \\ & \longrightarrow \frac{1}{2}V_\mu \left[\left(\square + m_V^2 \right) g^{\mu\nu} - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] V_\nu + \dots \end{aligned}$$

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- Charge SM and DM:

$$\mathcal{L} \supset V_\mu g_{V\chi} V_\mu \bar{\chi} \gamma^\mu \chi + \sum_f g_{Vf} V_\mu \bar{f} \gamma^\mu f$$

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- Vector Lagrangian for $\mu \gtrsim \text{GeV}$

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\chi + \frac{1}{2}V_\mu & \left[(\square + m_V^2)g^{\mu\nu} - \partial^\mu\partial^\nu \right] V_\nu \\ & + g_{V\chi} V_\mu \bar{\chi} \gamma^\mu \chi + \sum_f g_{Vf} V_\mu \bar{f} \gamma^\mu f \end{aligned}$$

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$$eQ A_\mu \bar{\psi} \gamma^\mu \psi \rightarrow -e\epsilon Q V_\mu \bar{\psi} \gamma^\mu \psi + \dots$$

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- Result

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \mathcal{L}_\chi + \frac{1}{2} V_\mu \left[(\square + m_V^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] V_\nu \\ & + g_V \chi V_\mu \bar{\chi} \gamma^\mu \chi + \sum_f \textcolor{red}{g_V f} V_\mu \bar{f} \gamma^\mu f \end{aligned}$$

$$\textcolor{red}{g_V f} = -\epsilon e Q_f$$

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Charged Currents:

$$\mathcal{J}_{\mu}^{+} = \sum_i \nu_i^{\dagger} \bar{\sigma}_{\mu} \ell_i + \sum_{i,j} V_{ij}^{\text{CKM}} u_i^{\dagger} \bar{\sigma}_{\mu} d_j$$

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Neutral Currents:

$$J_\mu^Z = \frac{1}{c_W} \sum_f g_{f,L}^Z f^\dagger \bar{\sigma}_\mu f + \frac{1}{c_W} \sum_{\bar{f}} g_{f,R}^Z \bar{f}^\dagger \bar{\sigma}_\mu \bar{f}$$

$$g_{f,L}^Z = T_f^3 - Q_f s_W^2$$

$$g_{f,R}^Z = -Q_f s_W^2$$

Decent to MeV Scale

Moving below 1 GeV

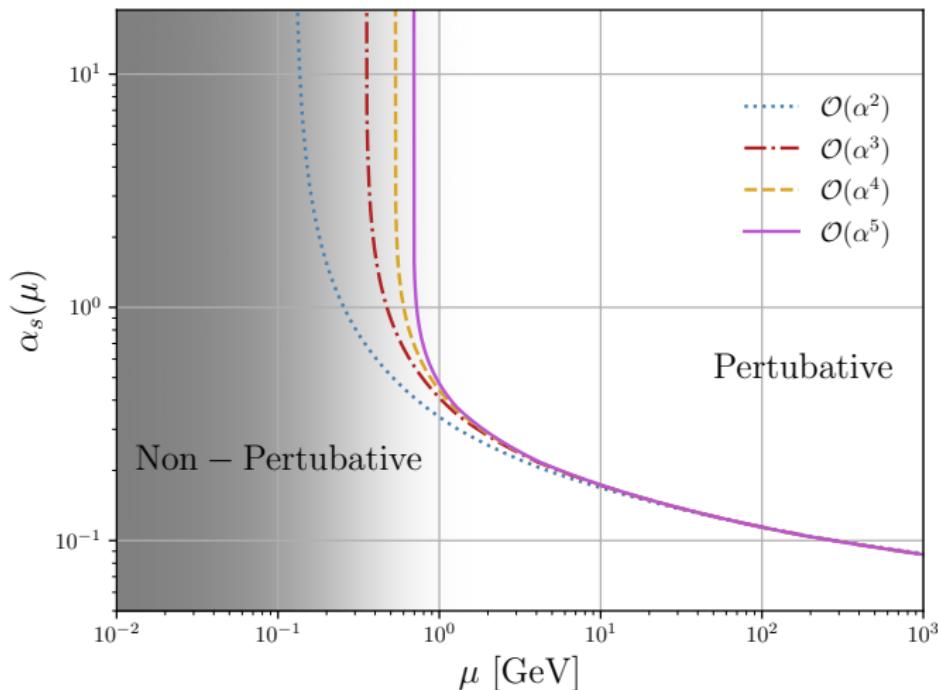
Now that we have the Lagrangians above 1 GeV, we need to determine the Lagrangians below 1 GeV

$$\mathcal{L}_{\mu > 1 \text{ GeV}}$$



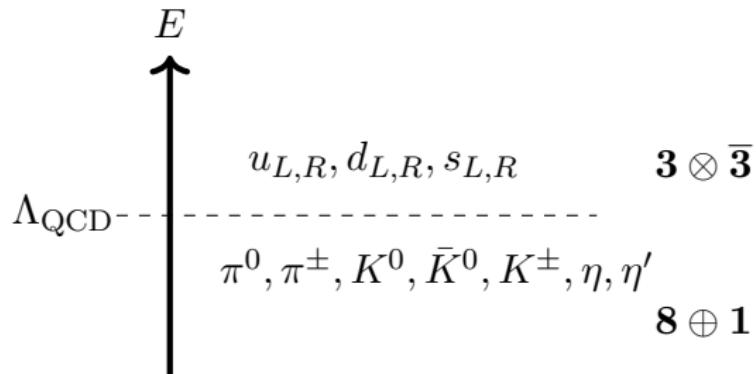
$$\mathcal{L}_{\mu < 1 \text{ GeV}} \sim \mathcal{L}_{\text{ChiPT}}$$

Moving below 1 GeV



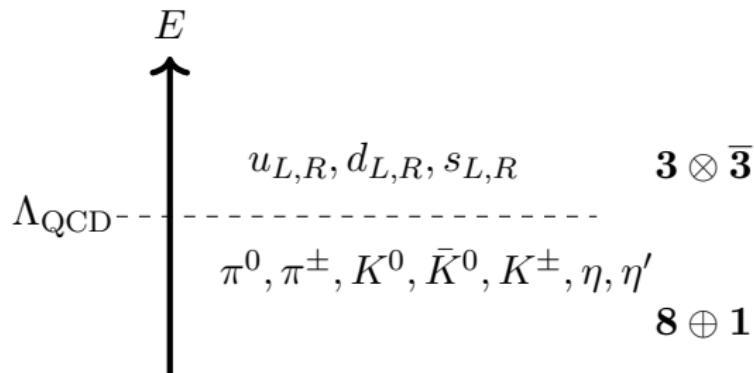
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- Use an effective Lagrangian below 1 GeV : Chiral Lagrangian

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- Symmetric under global $SU(3)_L \otimes SU(3)_R$ symmetry in chiral limit

$$\mathbf{q} \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow e^{i\theta_L^a \lambda^a / 2} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \bar{\mathbf{q}} \equiv \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix} \rightarrow e^{i\theta_R^a \lambda^a / 2} \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$$

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- CCWZ tells us how to construct bottom-up Lagrangian for pseudo-Goldstones generated from symmetry breaking

Chiral Lagrangian

The Chiral Lagrangian is

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}\left(D_\mu \Sigma^\dagger D^\mu \Sigma\right) + \frac{f_\pi^2}{4} \text{Tr}\left(\chi \Sigma^\dagger + \Sigma \chi^\dagger\right)$$

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where

$$\Sigma = \exp\left(\frac{i\sqrt{2}}{f_\pi} \Pi^a \lambda_a\right), \quad \Sigma \rightarrow U_R \Sigma U_L^\dagger$$

Π^a are the NBG

λ_a Gell-Mann matrices

$$\sqrt{2} \Pi^a \lambda_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

Chiral Lagrangian

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$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}\left(\textcolor{red}{D}_\mu \Sigma^\dagger \textcolor{red}{D}^\mu \Sigma\right) + \frac{f_\pi^2}{4} \text{Tr}\left(\chi \Sigma^\dagger + \Sigma \chi^\dagger\right)$$

where

$$\textcolor{red}{D}_\mu \Sigma = \partial_\mu \Sigma - i \textcolor{blue}{r}_\mu \Sigma + i \Sigma \textcolor{violet}{l}_\mu$$

and $\textcolor{violet}{l}_\mu$ and $\textcolor{blue}{r}_\mu$ are left- and right-handed currents associated with a local $\textcolor{magenta}{SU}(3)_L \otimes \textcolor{blue}{SU}(3)_R$ symmetry

$$\textcolor{violet}{l}_\mu \rightarrow U_L \textcolor{violet}{l}_\mu U_L^\dagger$$

$$\textcolor{blue}{r}_\mu \rightarrow U_R \textcolor{blue}{r}_\mu U_R^\dagger$$

$$D_\mu \Sigma \rightarrow U_R (\textcolor{red}{D}_\mu \Sigma) U_L^\dagger$$

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where

$$\chi = 2B_0(s + ip), \quad \chi \rightarrow U_R \chi U_L^\dagger$$

and s, p are the scalar and pseudo-scalar current densities and

$$B_0 = \frac{m_\pi^2}{m_u + m_d} \approx 2600 \text{ MeV}$$

Without any external fields,

$$s = \text{diag}(m_u, m_d, m_s)$$

Matching

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- Below 1GeV $\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\chi\text{PT}}$
- Enforce correlation functions of external fields match above and below 1 GeV
- Match $\ell_\mu, \mathbf{r}_\mu, \mathbf{s}$ and \mathbf{p} in both theories
- Operators that need to be matched:

$$S\bar{\mathbf{q}}\mathbf{G}_{Sq}\mathbf{q}, \quad \bar{\mathbf{q}}\gamma^\mu(\ell_\mu P_L + \mathbf{r}_\mu P_R)\mathbf{q}, \quad SG_{\mu\nu}^a G^{a,\mu\nu}$$

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- Matched onto ChiPT mass term:

$$-\bar{\mathbf{q}}(\mathbf{M}_q + S\mathbf{G}_{Sq})\mathbf{q} \rightarrow \frac{f_\pi^2}{4} \text{Tr}[\chi \Sigma^\dagger + \text{c.c.}], \quad \chi = 2B(\mathbf{M}_q + S\mathbf{G}_{Sq})$$

Matching: $\bar{\mathbf{q}}\gamma^\mu(\ell_\mu P_L + \mathbf{r}_\mu P_R)\mathbf{q}$

- Current transform as

$$\mathbf{r}_\mu \rightarrow \mathbf{U}_R \mathbf{r}_\mu \mathbf{U}_R^\dagger, \quad \ell_\mu \rightarrow \mathbf{U}_L \ell_\mu \mathbf{U}_L^\dagger$$

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- Matching currents is done using via a connection

$$D_\mu \Sigma = \partial_\mu \Sigma - i \mathbf{r}_\mu \Sigma + i \Sigma \ell_\mu$$

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- Vector Mediator

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- Additional term from chiral anomaly (Wess-Zumino-Witten):

$$-\frac{e^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \frac{\pi^0}{f_\pi} \rightarrow -\frac{e^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \frac{\pi^0}{f_\pi} - \frac{\epsilon e^2}{16\pi^2} F_{\mu\nu} \tilde{V}^{\mu\nu} \frac{\pi^0}{f_\pi}$$

Matching: $S G_{\mu\nu}^a G^{\mu\nu,a}$

- Use trace anomaly and RG invariance of scale divergence:

$$\partial_\mu d^\mu = \theta_\mu^\mu$$

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- Scale divergence for $\mu > \text{GeV}$

$$\partial_\mu d^\mu \sim \frac{\beta}{2g_s} G^2 + \sum_q (1 - \gamma_m) m_q \bar{q} q + \dots$$

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- Scalar interaction is:

$$g_{SG} \frac{\alpha}{4\pi} \frac{S}{\Lambda} G^2 \rightarrow -\frac{2g_{SG}}{\beta_0} \frac{S}{\Lambda} \partial_\mu d^\mu + \frac{2g_{SG}}{\beta_0} \frac{S}{\Lambda} \sum_q m_q \bar{q} q + \dots$$

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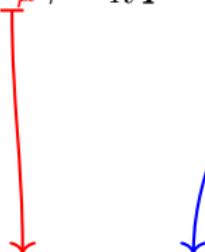
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- Matched onto ChiPT Lagrangian by computing $\partial_\mu d^\mu$

$$\partial_\mu d^\mu = -\frac{f_\pi^2}{2} \text{Tr}\left[(D_\mu \Sigma)^\dagger (D_\mu \Sigma)\right] - f_\pi^2 \text{Tr}\left[\chi \Sigma^\dagger + \text{c.c.}\right]$$

Matching: Dictionary

$$\mathcal{L}_{\mu > \text{GeV}} \supset \bar{q} \color{red}{r_\mu} \gamma^\mu P_R q + \bar{q} \color{blue}{\ell_\mu} \gamma^\mu P_L q + \bar{q} s q + \phi \frac{\alpha}{4\pi} G_{\mu\nu}^a G^{\mu\nu,a}$$



$$D_\mu \Sigma = \partial_\mu \Sigma - i \color{red}{r_\mu} \Sigma + i \Sigma \color{blue}{\ell_\mu}$$

$$\chi = 2B_0 \left(s + \left(1 - \frac{2}{\beta_0} \phi \right) M_q \right)$$

$$\mathcal{L}_{\mu < \text{GeV}} \supset \frac{f_\pi^2}{4} \text{Tr} \left[(D_\mu \Sigma)^\dagger (D_\mu \Sigma) \right] + \frac{f_\pi^2}{4} \text{Tr} \left[\chi \Sigma^\dagger + \text{c.c.} \right]$$

$$- \frac{f_\pi^2}{\beta_0} \phi \text{Tr} \left[(D_\mu \Sigma)^\dagger (D_\mu \Sigma) \right] + \frac{2f_\pi^2}{\beta_0} \phi \text{Tr} \left[\chi \Sigma^\dagger + \text{c.c.} \right]$$

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$$D_\mu \Sigma = \partial_\mu \Sigma - i r_\mu \Sigma + i \Sigma \ell_\mu$$

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$$\mathcal{L}_{\mu > \text{GeV}} \supset \bar{q} r_\mu \gamma^\mu P_R q + \bar{q} \ell_\mu \gamma^\mu P_L q + \bar{q} \textcolor{orange}{s} q + \phi \frac{\alpha}{4\pi} G_{\mu\nu}^a G^{\mu\nu,a}$$

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$$\begin{aligned} \mathcal{L}_{\mu < \text{GeV}} &\supset \frac{f_\pi^2}{4} \text{Tr} \left[(\mathcal{D}_\mu \Sigma)^\dagger (\mathcal{D}_\mu \Sigma) \right] + \frac{f_\pi^2}{4} \text{Tr} \left[\chi \Sigma^\dagger + \text{c.c.} \right] \\ &\quad - \frac{f_\pi^2}{\beta_0} \phi \text{Tr} \left[(\mathcal{D}_\mu \Sigma)^\dagger (\mathcal{D}_\mu \Sigma) \right] + \frac{2f_\pi^2}{\beta_0} \phi \text{Tr} \left[\chi \Sigma^\dagger + \text{c.c.} \right] \end{aligned}$$

From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching - vector

- Below a GeV

$$\begin{aligned}\mathcal{L}_V \supset & g_{V\chi} V_\mu \bar{\chi} \gamma^\mu \chi + \sum_\ell g_{V\ell} V_\mu \bar{\ell} \gamma^\mu \ell \\ & + \frac{f_\pi^2}{4} \text{Tr}\left((\textcolor{red}{D}_\mu \Sigma)^\dagger \textcolor{red}{D}_\mu \Sigma\right) + \frac{f_\pi^2}{4} \text{Tr}\left(\textcolor{blue}{\chi} \Sigma^\dagger + \Sigma \textcolor{blue}{\chi}^\dagger\right)\end{aligned}$$

with

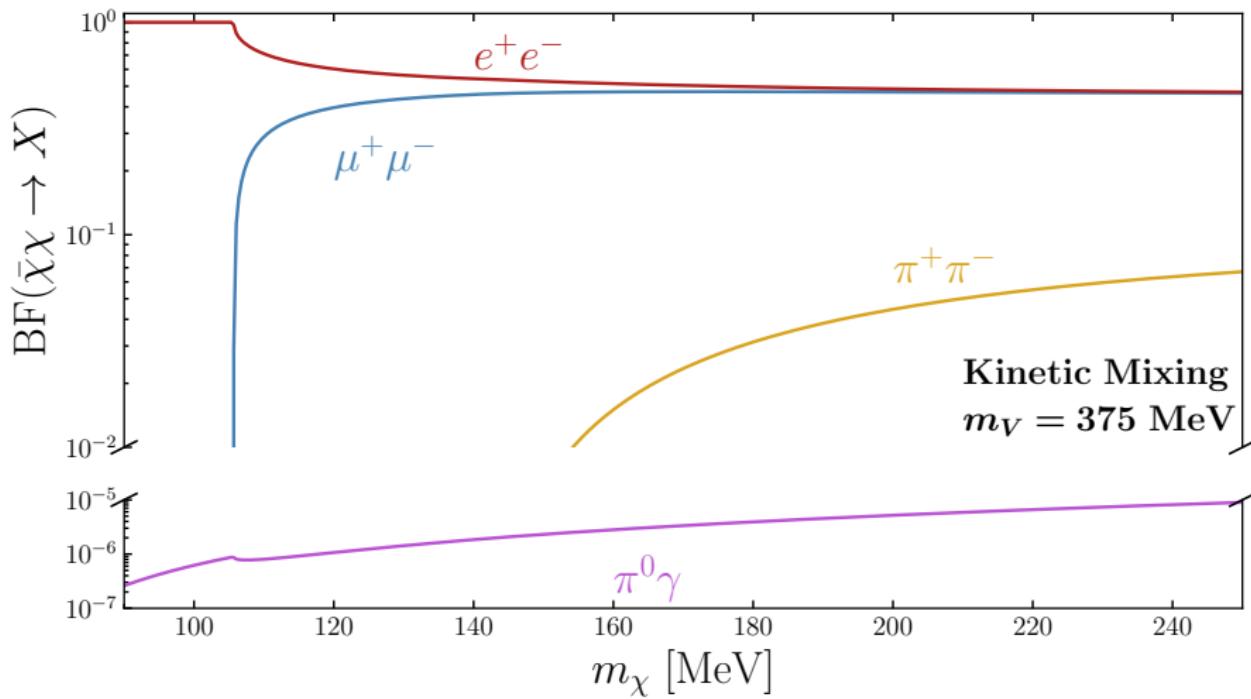
$$D_\mu \Sigma = \partial_\mu \Sigma - i \textcolor{red}{r}_\mu \Sigma + i \Sigma \textcolor{orange}{\ell}_\mu$$

$$\chi = 2B_0 \textcolor{teal}{s}$$

$$\textcolor{teal}{s} = \text{diag}(m_u, m_d, m_s)$$

$$r_\mu = \ell_\mu = -e A_\mu \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) + V_\mu \text{diag}(g_{Vu}, g_{Vd}, g_{Vs})$$

From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching - vector



From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching- scalar

- Below 1 GeV, we have

$$\begin{aligned}\mathcal{L}_S \supset g_{S\chi} S \bar{\chi} \chi + g_{fV} \frac{S}{v} \sum_{\ell} m_{\ell} \bar{\ell} \ell + \frac{\alpha_{\text{EM}}}{4\pi\Lambda} g_{SF} S F^2 \\ + \frac{f_{\pi}^2}{4} \text{Tr}((\textcolor{red}{D}_{\mu} \Sigma)^{\dagger} \textcolor{red}{D}_{\mu} \Sigma) + \frac{f_{\pi}^2}{4} \text{Tr}(\textcolor{blue}{\chi} \Sigma^{\dagger} + \Sigma \textcolor{blue}{\chi}^{\dagger}) \\ + \frac{2g_G}{9v} S \left(\frac{f_{\pi}^2}{2} \text{Tr}((D_{\mu} \Sigma)^{\dagger} (D^{\mu} \Sigma)) + f_{\pi}^2 \text{Tr}(\chi \Sigma^{\dagger} + \Sigma \chi^{\dagger}) \right)\end{aligned}$$

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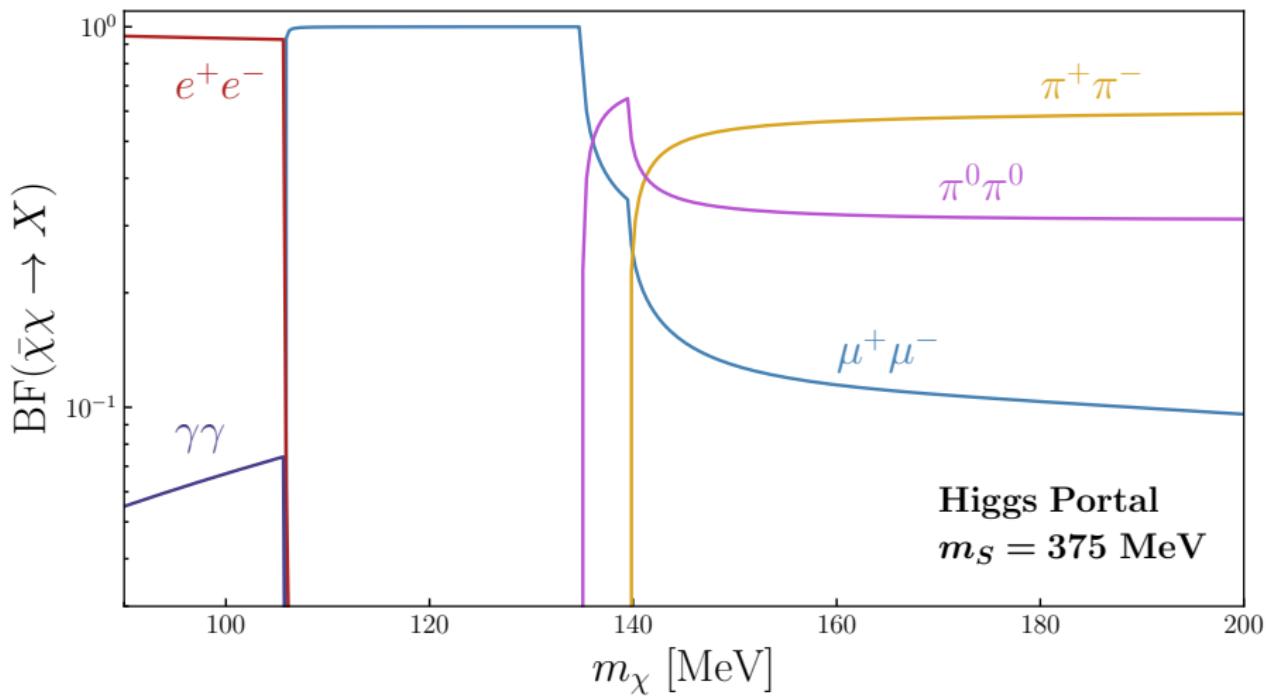
$$D_{\mu} \Sigma = \partial_{\mu} \Sigma - i \textcolor{red}{r}_{\mu} \Sigma + i \Sigma \textcolor{brown}{\ell}_{\mu}$$

$$\chi = 2B_0 \textcolor{teal}{s}$$

$$\textcolor{teal}{s} = \text{diag}(m_u, m_d, m_s) \left(1 + g_{SF} \frac{S}{v} \right)$$

$$\textcolor{red}{r}_{\mu} = \textcolor{brown}{\ell}_{\mu} = -e A_{\mu} \text{diag} \left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} \right)$$

From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching- scalar



From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching- **RHN**

$$\mathcal{L}_{\bar{\nu}(\text{int})} = \frac{f_\pi^2}{4} \text{Tr} \left[|\partial_\mu \Sigma - i r_\mu \Sigma + i \Sigma \ell_\mu|^2 \right]$$

Currents:

$$r_\mu = -\frac{8G_F}{\sqrt{2}} \mathbf{G}_R \mathbf{R}_\mu^0, \quad \ell_\mu = -\frac{4G_F}{\sqrt{2}} \left(2\mathbf{G}_L \mathbf{L}_\mu^0 + \mathbf{V}^\dagger \mathbf{L}_\mu^- \right)$$

$$\mathbf{L}_\mu^0 = \frac{\sin(2\theta)}{4c_W} \delta_{ik} \left(\nu_i^\dagger \bar{\sigma}_\mu \bar{\nu} + \text{c.c.} \right) + \frac{1}{2c_W} \left(-1 + 2s_W^2 \right) \ell_i^\dagger \bar{\sigma}_\mu \ell_i$$

$$\mathbf{L}_\mu^- = \sin \theta \delta_{ik} \ell_i^\dagger \bar{\sigma}_\mu \bar{\nu}$$

$$\mathbf{R}_\mu^0 = \frac{s_W^2}{c_W} \bar{\ell}_i^\dagger \bar{\sigma}_\mu \bar{\ell}_i$$

From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching- **RHN**

$$\mathcal{L}_{\bar{\nu}(\text{int})} = \frac{f_\pi^2}{4} \text{Tr} \left[|\partial_\mu \Sigma - i r_\mu \Sigma + i \Sigma \ell_\mu|^2 \right]$$

Currents:

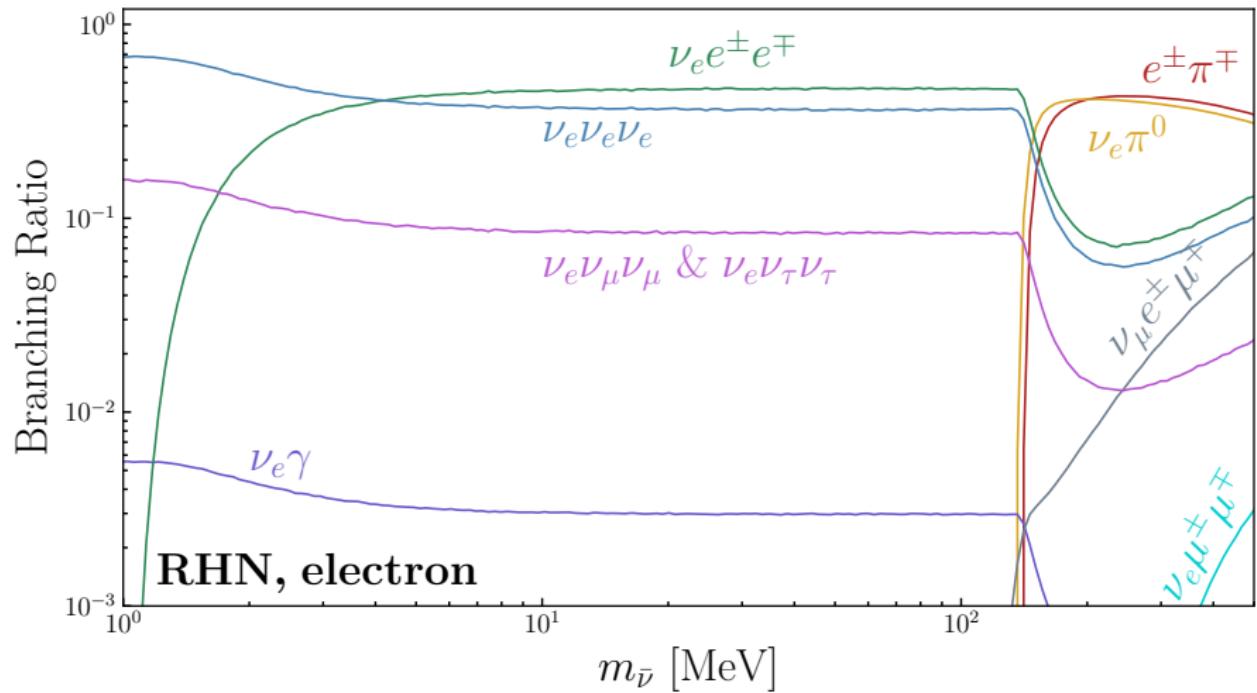
$$r_\mu = 2\mathbf{G}_R R_\mu^0, \quad \ell_\mu = 2\mathbf{G}_L L_\mu^0 + \mathbf{V}^\dagger L_\mu^-$$

$$\mathbf{G}_R = -\frac{s_W^2}{3c_W} \text{diag}(2, -1, -1)$$

$$\mathbf{G}_L = \frac{1}{2c_W} \text{diag}(1, -1, -1) + \mathbf{G}_R$$

$$\mathbf{V} = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

From $\mu > 1\text{GeV}$ to $\mu < 1\text{GeV}$: Matching- RHN



Validity of ChiPT

Validity of ChiPT

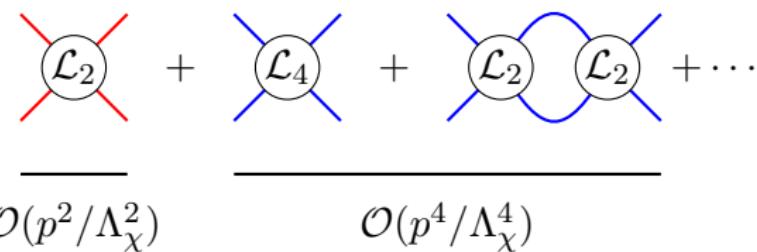
- Chiral perturbation theory has a limited range of validity

Validity of ChiPT

- Chiral perturbation theory has a limited range of validity
- The chiral expansion is

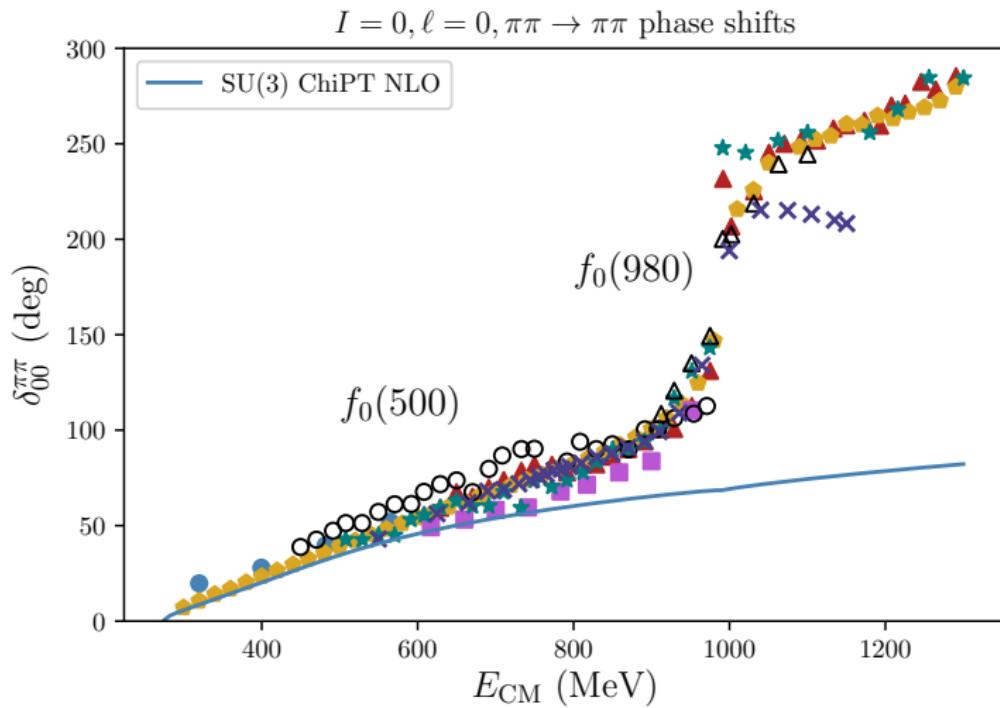
$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \dots$$

$$\mathcal{M} \sim \left(\frac{p^2}{\Lambda_\chi^2} \right) \mathcal{M}^{(2)} + \left(\frac{p^2}{\Lambda_\chi^2} \right)^2 \mathcal{M}^{(4)} + \dots$$

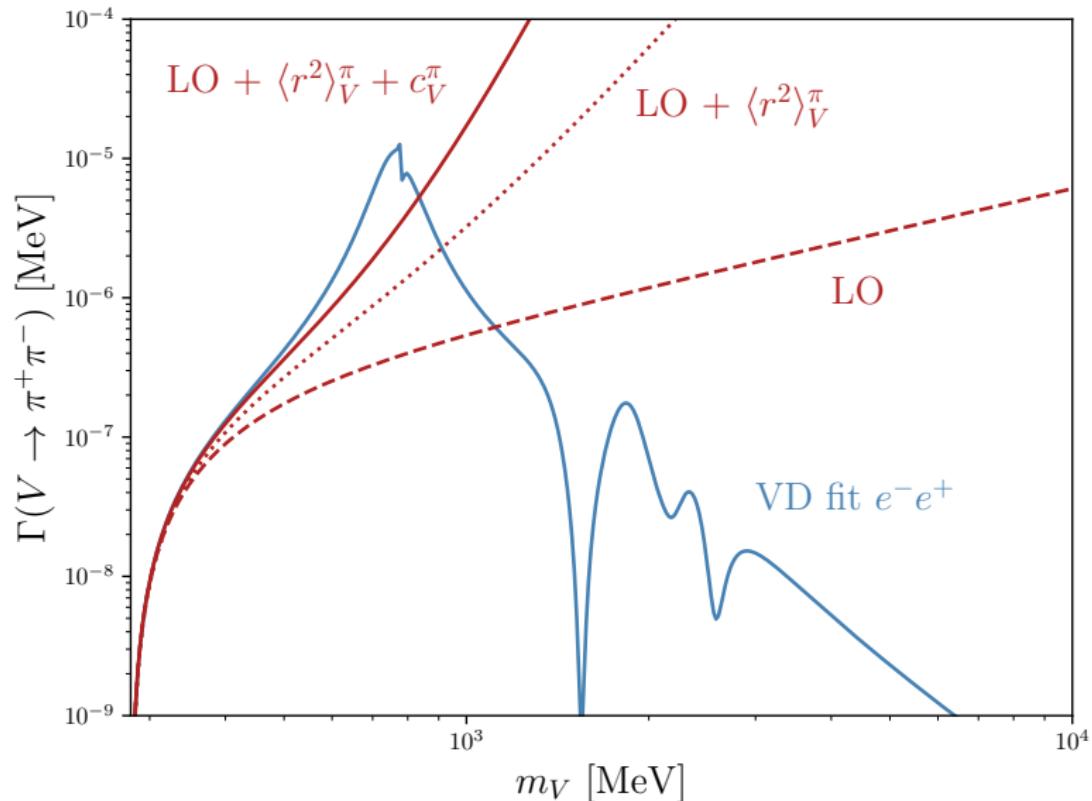


where $\Lambda_\chi \approx 4\pi f_\pi \approx 1.2 \text{ GeV}$

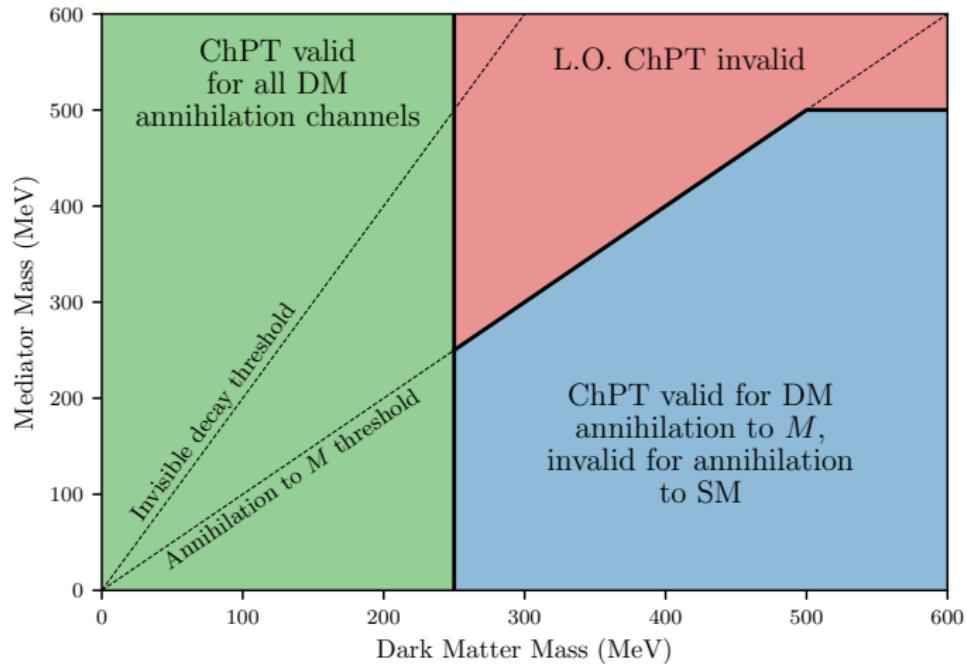
Validity of ChiPT



Validity of ChiPT



Validity of ChPT



Indirect Detection

- Gamma ray flux observed by detector

$$\frac{d\Phi}{dE_\gamma} = \frac{\Delta\Omega}{4\pi m_\chi^a} \cdot \left[\frac{1}{\Delta\Omega} \int d\Omega \int_{\text{LOS}} d\ell \rho_\chi^a \right] \cdot \Gamma \cdot \frac{dN}{dE_\gamma}$$

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Integral along detector's "Line-of-sight" of dark matter density of target with angular size $\Delta\Omega$

$a = 2$ for annihilating DM

$a = 1$ for decaying DM

Target	[MeV 2 cm $^{-5}$ sr $^{-1}$]		[MeVcm $^{-2}$ sr $^{-1}$]	
	$J(1')$	$J(5^\circ)$	$D(1')$	$D(5^\circ)$
Galactic Center (NFW)	6.972×10^{32}	1.782×10^{30}	4.84×10^{26}	1.597×10^{26}
Galactic Center (Einasto)	5.987×10^{34}	4.965×10^{31}	4.179×10^{27}	2.058×10^{26}
Draco (NFW)	3.418×10^{30}	8.058×10^{26}	5.949×10^{25}	1.986×10^{24}
M31 (NFW)	1.496×10^{31}	1.479×10^{27}	3.297×10^{26}	4.017×10^{24}

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DM interaction rate:

$$\text{Annihilating DM : } \Gamma = \frac{\langle \sigma v \rangle}{2f_\chi}$$

$$\text{Decaying DM : } \Gamma = \frac{1}{\tau}$$

Indirect Detection

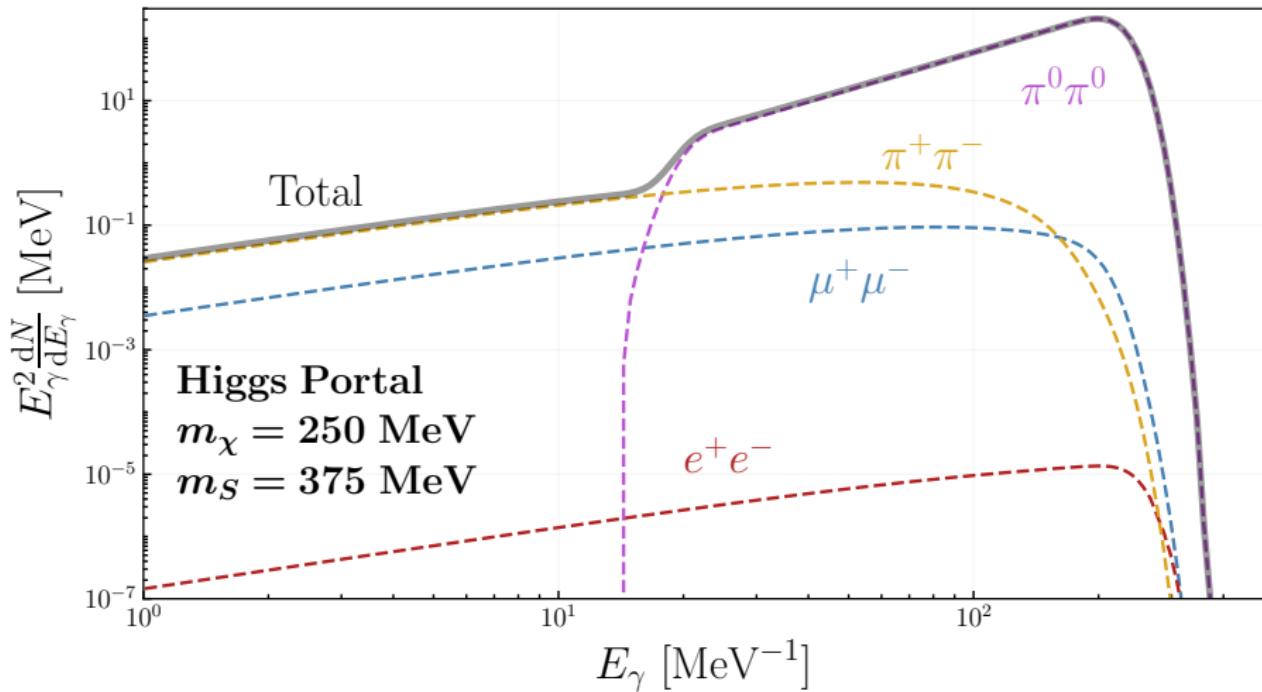
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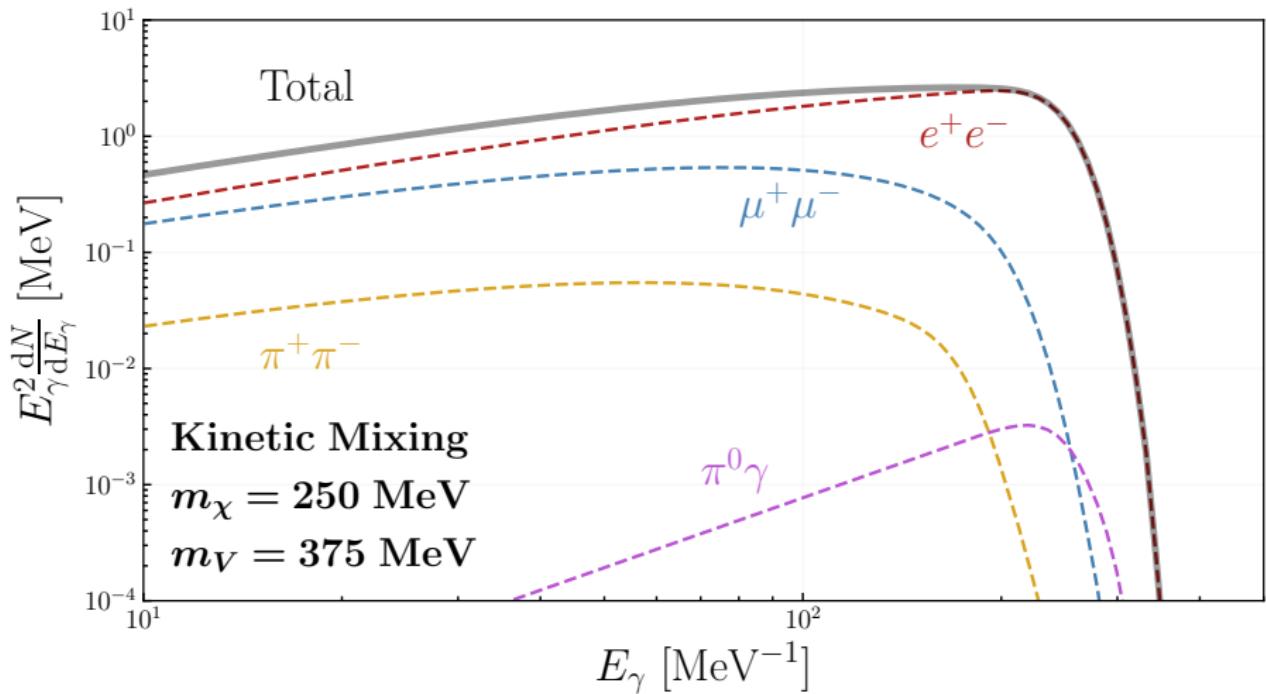
Photon spectrum per annihilation/decay:

$$\frac{dN}{dE_\gamma} = \sum_X \text{BR}(\bar{\chi}\chi \rightarrow \gamma + X) \frac{dN_{\bar{\chi}\chi \rightarrow \gamma+X}}{dE_\gamma}$$

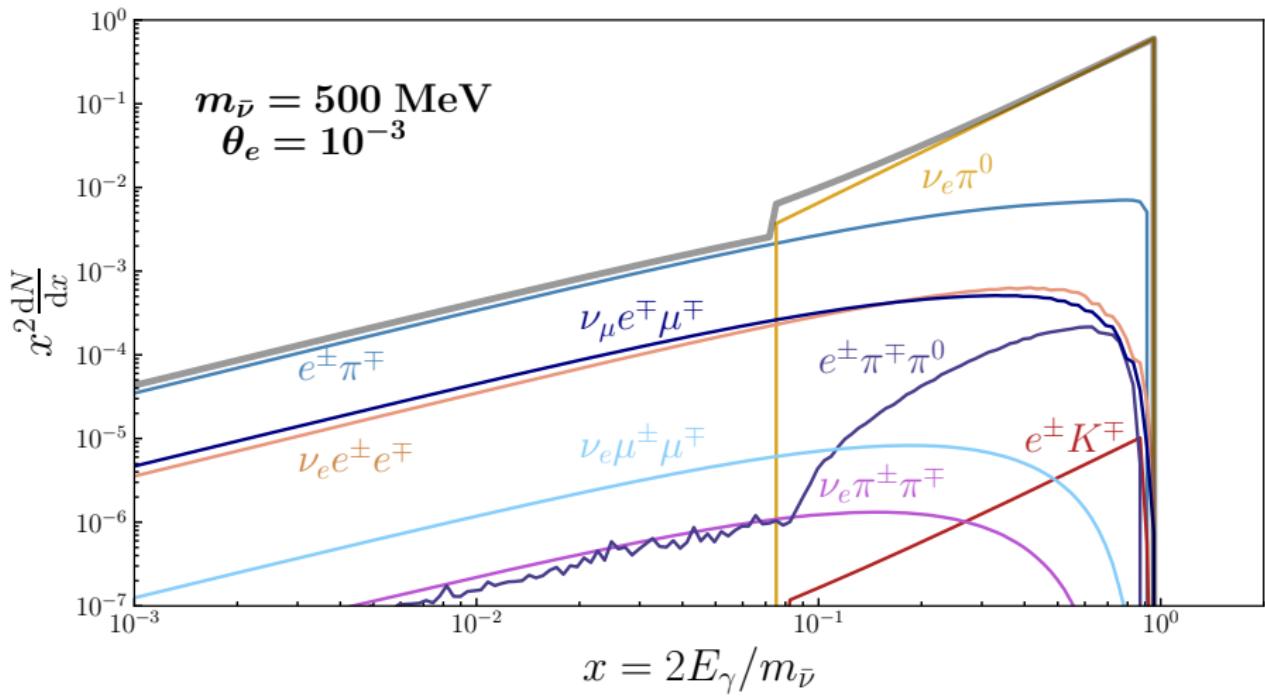
Indirect Detection



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- Including detector energy resolution

$$\frac{d\bar{\Phi}}{dE_\gamma} = \int d\tilde{E}_\gamma R_\epsilon(E_\gamma|\tilde{E}_\gamma) \frac{d\Phi}{dE_\gamma}$$

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Detector energy resolution \sim Gaussian:

$$R_\epsilon(E_\gamma|\tilde{E}_\gamma) \sim \frac{1}{\sqrt{2\pi}} \frac{1}{\epsilon\tilde{E}} \exp\left(-\frac{1}{2}\left(\frac{\tilde{E}-E}{\epsilon\tilde{E}}\right)^2\right)$$

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- Observed photon count in energy bin $(E_{\min}^{(i)}, E_{\max}^{(i)})$

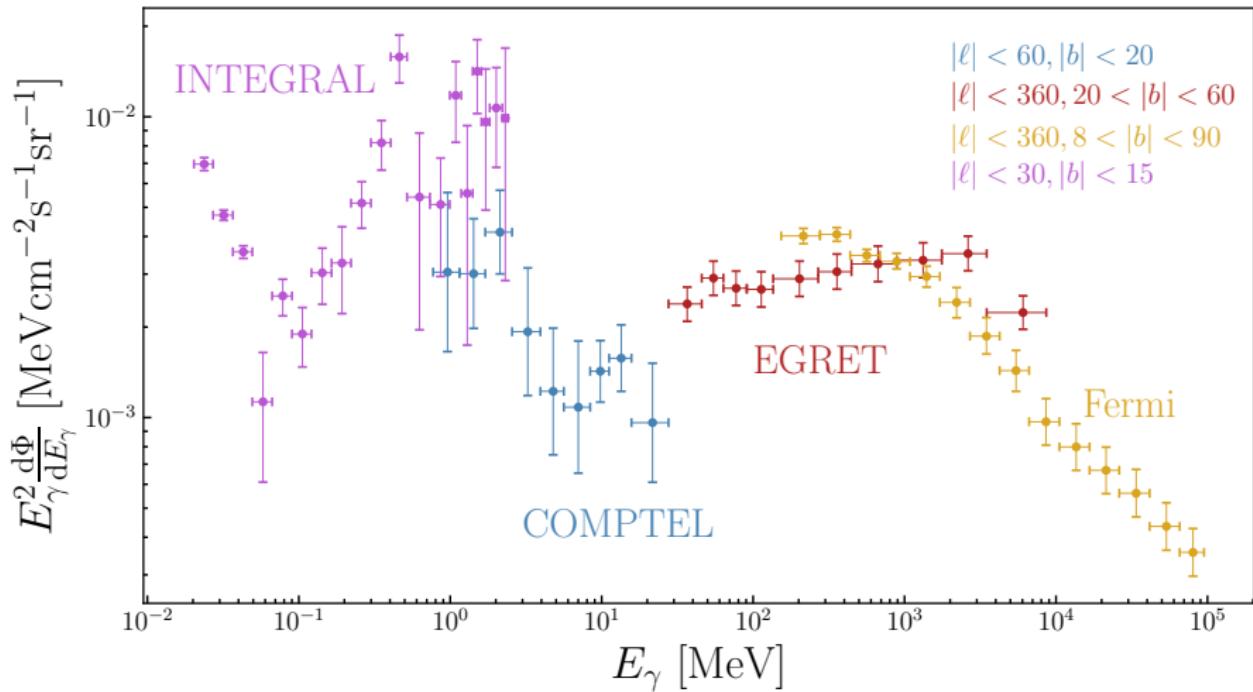
$$N_\gamma = \int_{E_{\min}^{(i)}}^{E_{\max}^{(i)}} dE_\gamma T_{\text{obs}} A_{\text{eff}}(E_\gamma) \frac{d\bar{\Phi}}{dE_\gamma}$$

Constraining

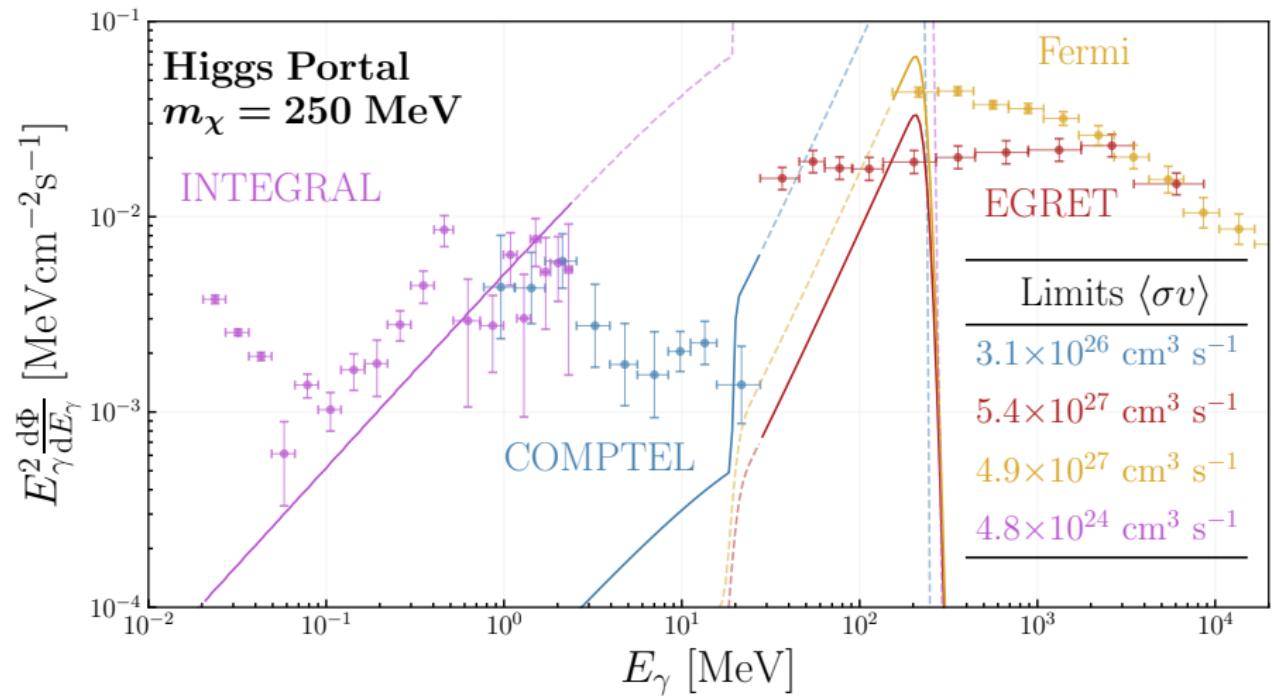
- For telescopes with reported data, constrain by asserting DM signal no greater than twice the upper error

$$\left[\int_{E_{\text{low}}^{(i)}}^{E_{\text{high}}^{(i)}} dE_\gamma \frac{d\Phi_\gamma}{dE_\gamma} \right] \leq \Phi_\gamma^{(i)} + 2\delta\Phi_\gamma^{(i)}, \quad i \in 1, \dots, N_{\text{bins}}$$

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$$N_{S,\text{bkg}}(a,b) = \int_a^b dE_\gamma \frac{d\Phi_{\gamma}^{S,\text{bkg}}}{dE_\gamma}$$

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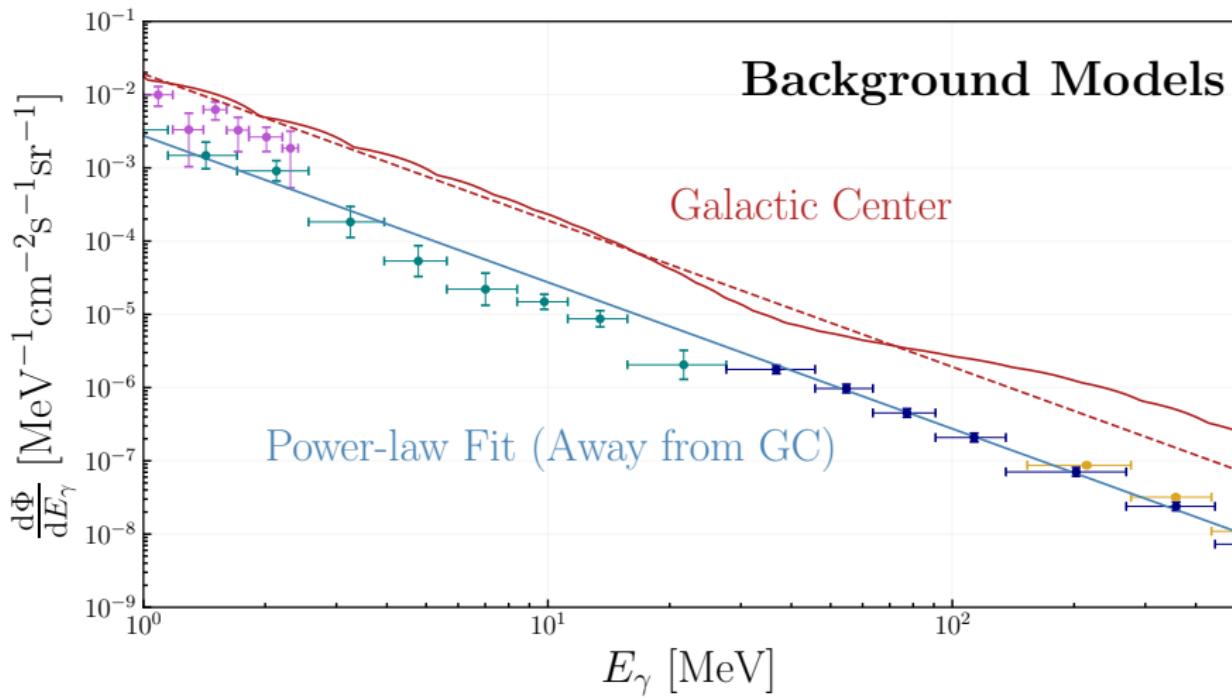
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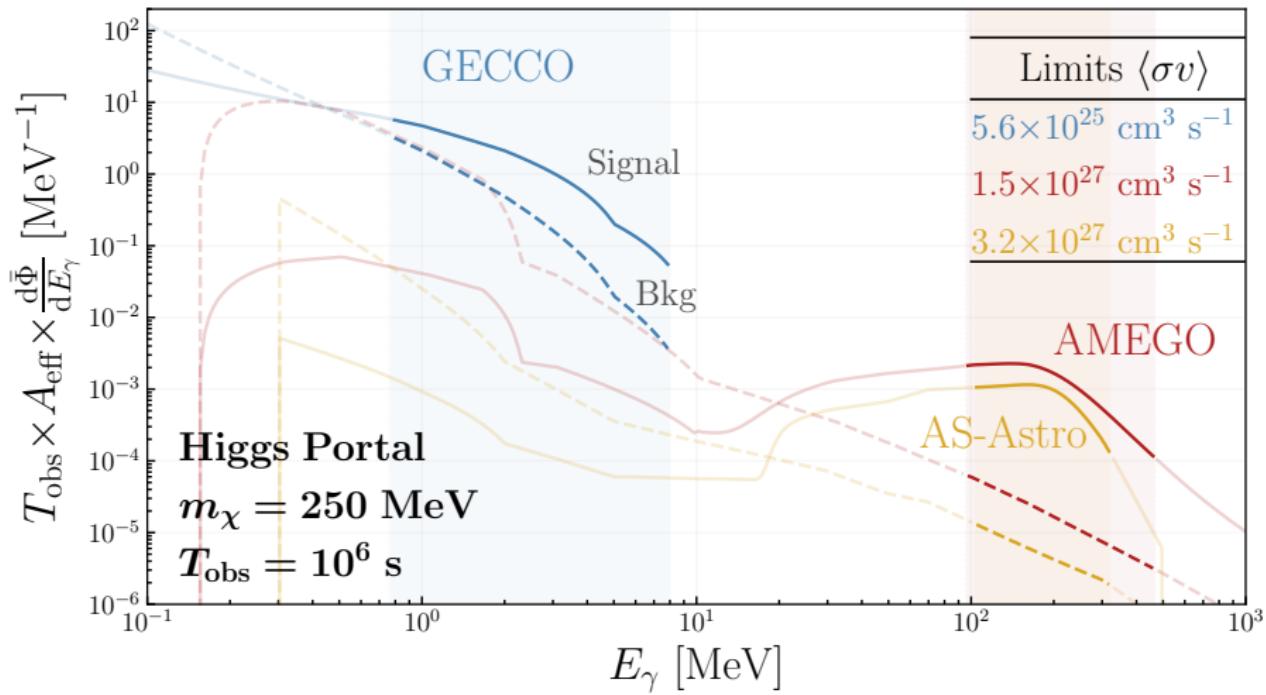
- Use more sophisticated model for Galactic center with bremsstrahlung, π^0 and inverse-Compton computed with GALPROP

Constraining

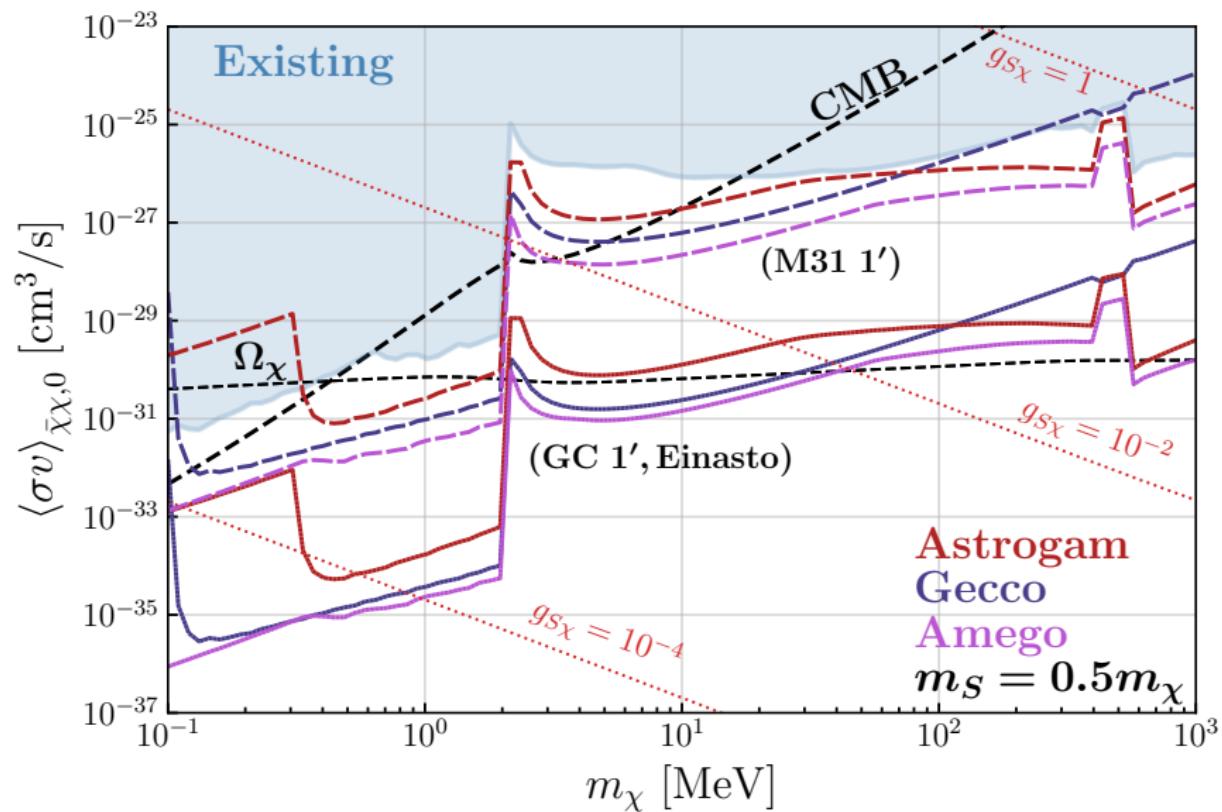
Background Models



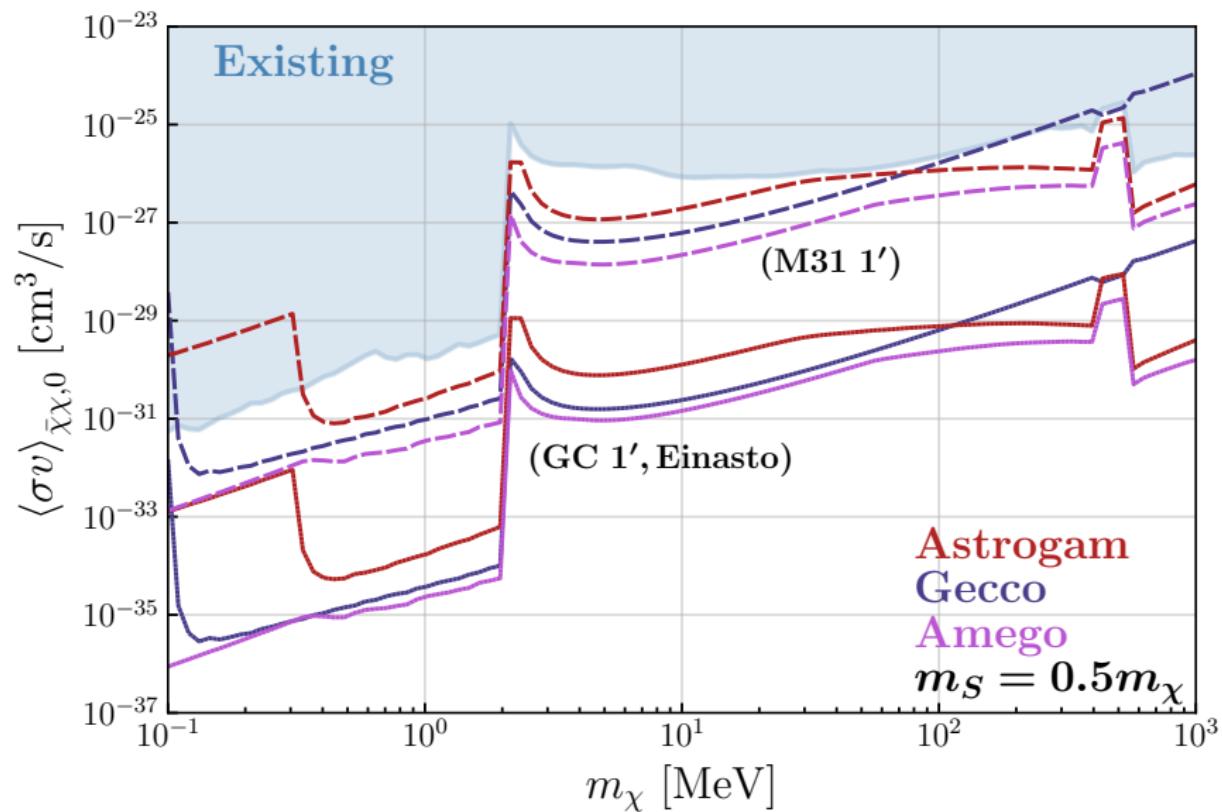
Constraining



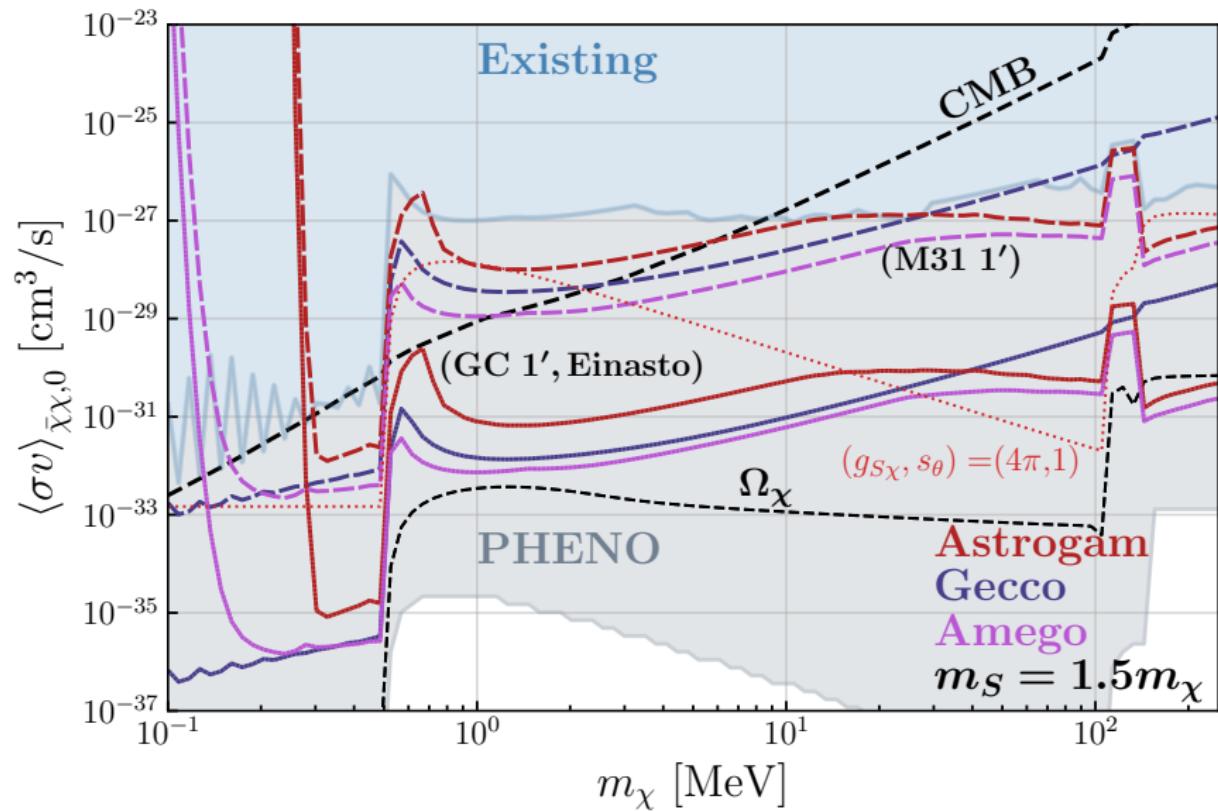
Higgs Portal Constraints



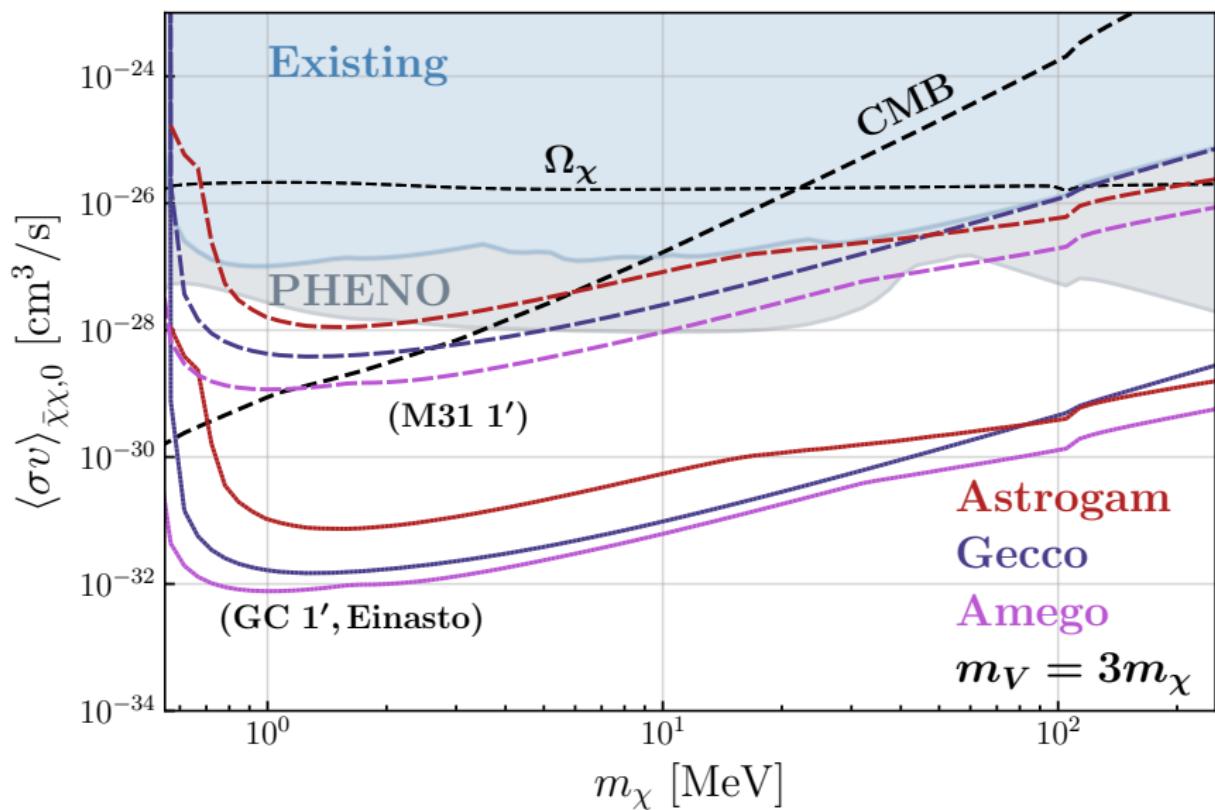
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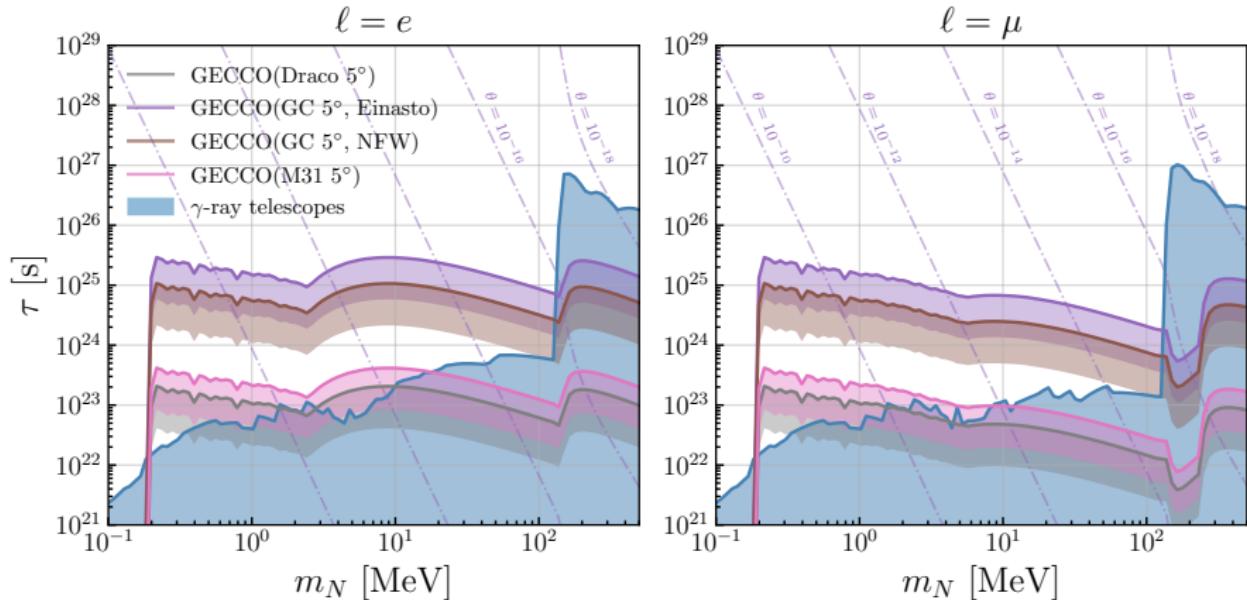
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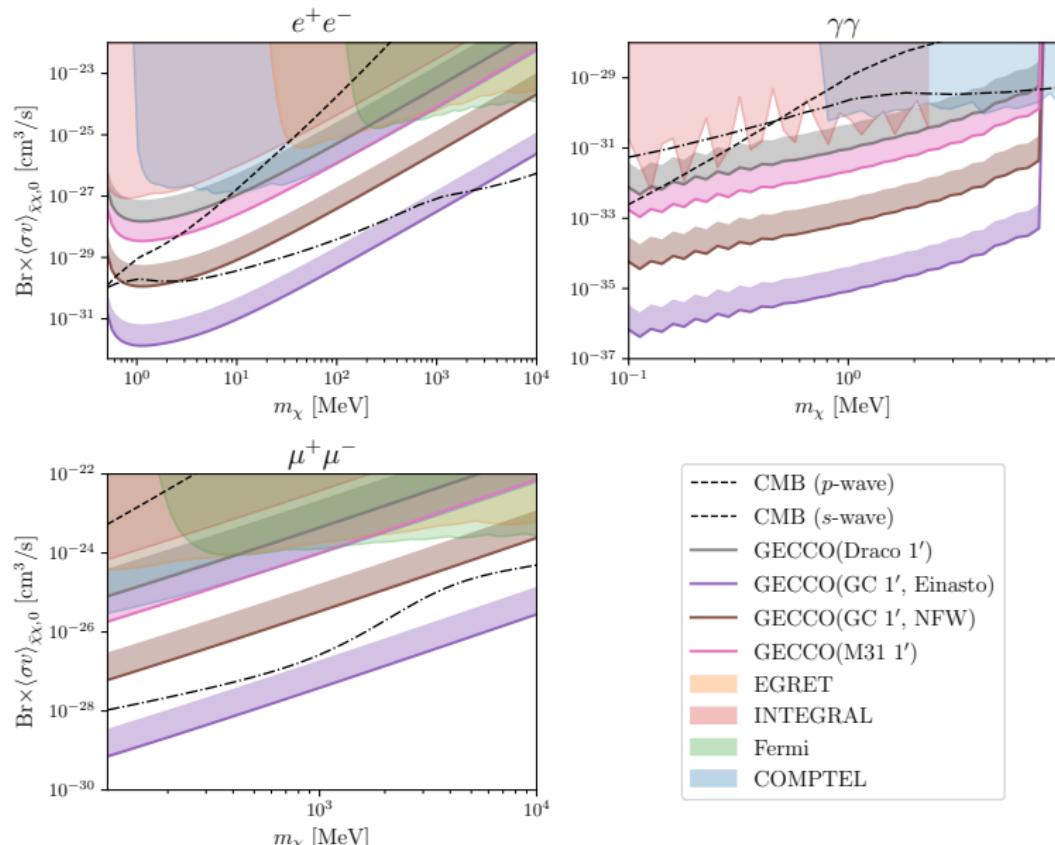
Kinetic Mixing Constraints



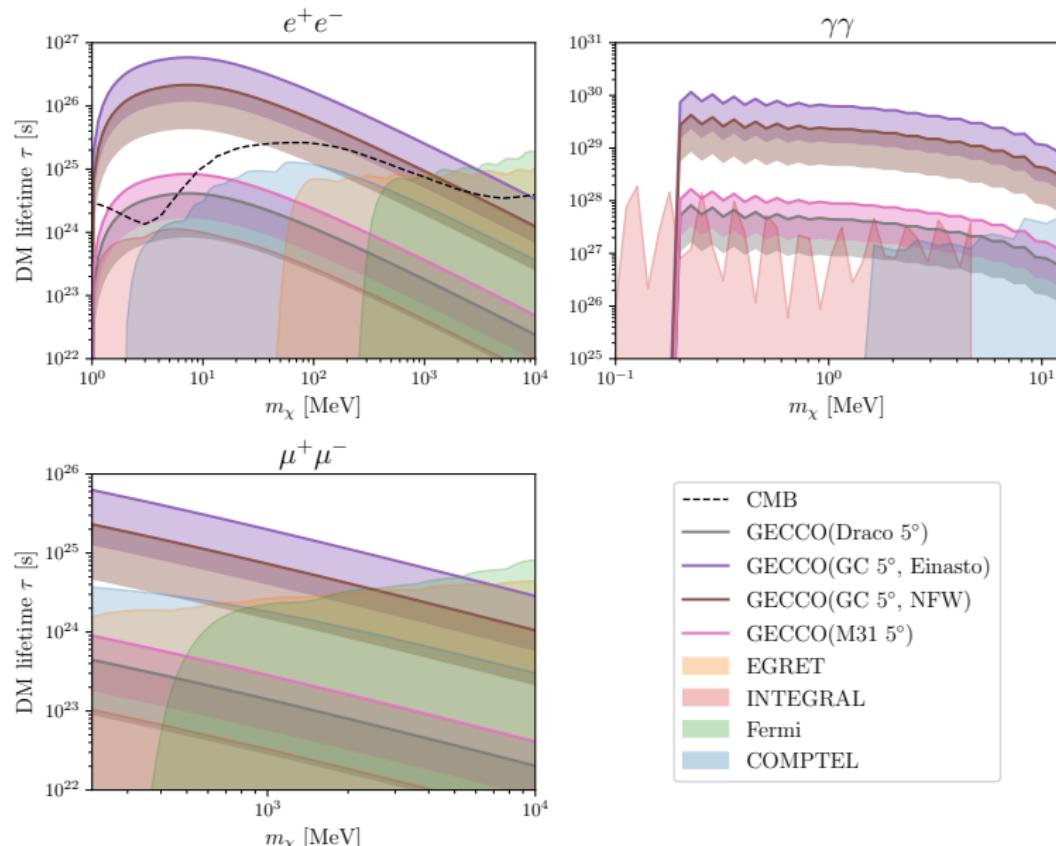
RH Neutrino Constraints



Model Independent Constraints



Model Independent Constraints



Primodial Black Holes

- A. Coogan, S. Profumo, LM: arXiv:2010.04797
- A. Coogan, S. Profumo, LM: arXiv:2101.10370

Hawking Radiation

- ① Can MeV telescopes be used to probe Hawking radiation for PBH?

Hawking Radiation

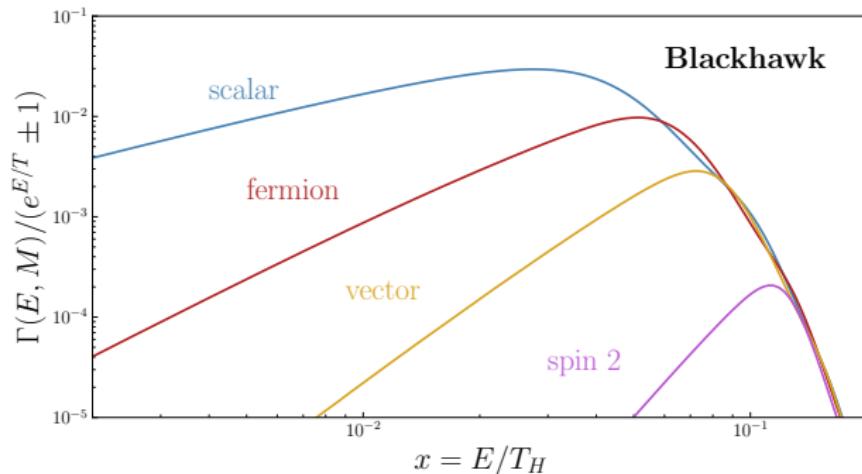
- ① Can MeV telescopes be used to probe Hawking radiation for PBH?
- ② Primary emission rates

$$\frac{\partial^2 N}{\partial E_i \partial t} = \frac{1}{2\pi} \frac{\Gamma(E_i, M)}{e^{E_i/T_H} - (-1)^{2s}}, \quad T_H = \frac{M_{\text{pl}}^2}{8\pi M_H}$$

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- ③ Total photon spectrum

$$\begin{aligned} \frac{\partial^2 N}{\partial E_\gamma \partial t} &= \frac{\partial^2 N_{\gamma, \text{primary}}}{\partial E_\gamma \partial t} + \sum_{i=e^\pm, \mu^\pm, \pi^\pm} \int dE_i \frac{\partial^2 N_{i, \text{primary}}}{\partial E_i \partial t} \frac{dN_\gamma^{\text{FSR}}}{dE_\gamma} \\ &\quad + \sum_{i=\mu^\pm, \pi^0, \pi^\pm} \int dE_i \frac{\partial^2 N_{i, \text{primary}}}{\partial E_i \partial t} \frac{dN_\gamma^{\text{decay}}}{dE_\gamma} \end{aligned}$$

Hawking Radiation: Photon Spectrum

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Primary spectra:

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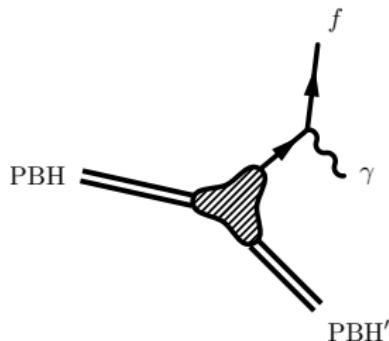
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FSR: ($x = E_\gamma/E_i$)

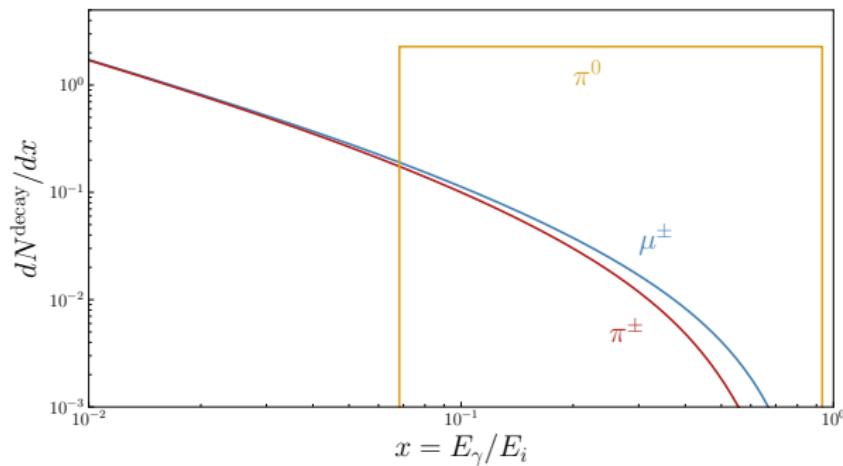
$$\frac{\partial N_\gamma^{\text{FSR}}}{\partial E_\gamma} = \frac{\alpha_{\text{EM}}}{2\pi E_i} P_{\gamma \leftarrow i}(x) \left[\log\left(\frac{1-x}{\mu_i^2}\right) - 1 \right]$$

$$P_{\gamma \leftarrow i} = \begin{cases} \frac{2(1-x)}{x}, & i = \pi^\pm, \\ \frac{1+(1-x)^2}{x}, & i = e^\pm, \mu^\pm, \end{cases}$$

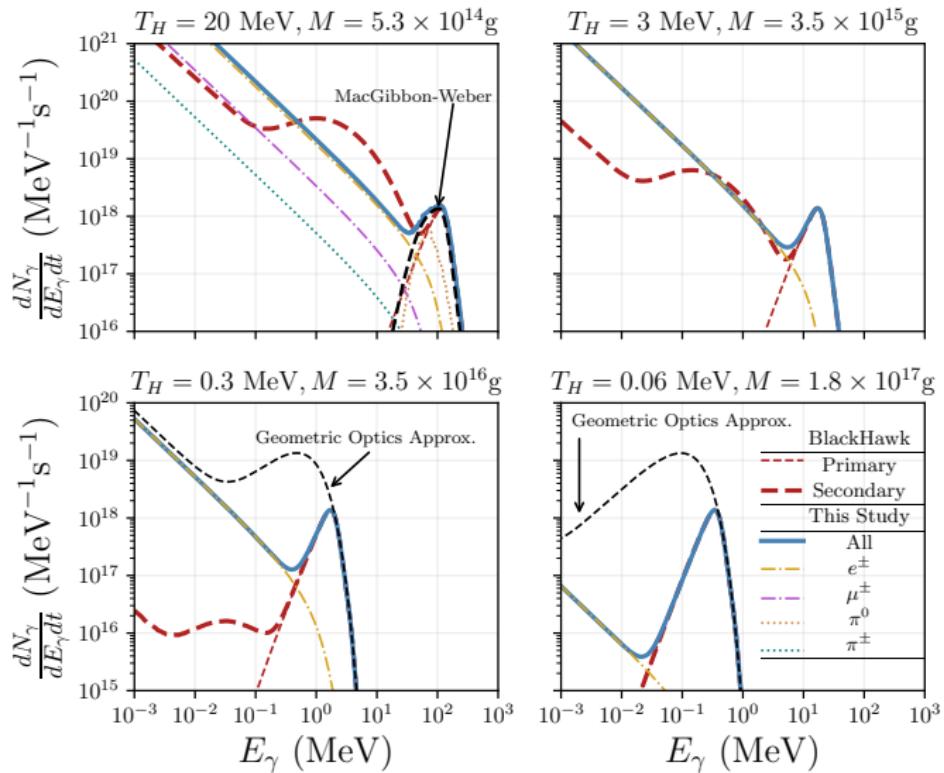


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Hawking Radiation: Photon Spectrum



Constraining f_{PBH}

- Given a fraction of DM in the form of (monochromatic) PBHs $f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{CDM}}$ observed gamma-ray spectrum is:

$$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{1}{4\pi} \int_{\text{LOS}} d\ell \frac{\partial^2 N_\gamma}{\partial E_\gamma \partial t} f_{\text{PBH}} \frac{\rho_{\text{DM}}}{M}$$

Constraining f_{PBH}

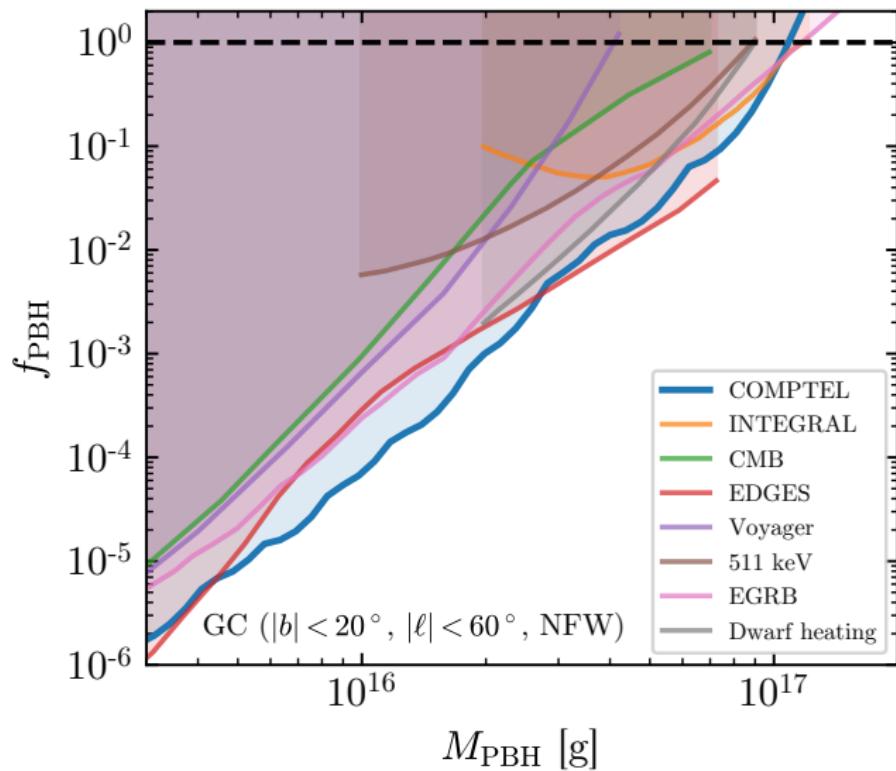
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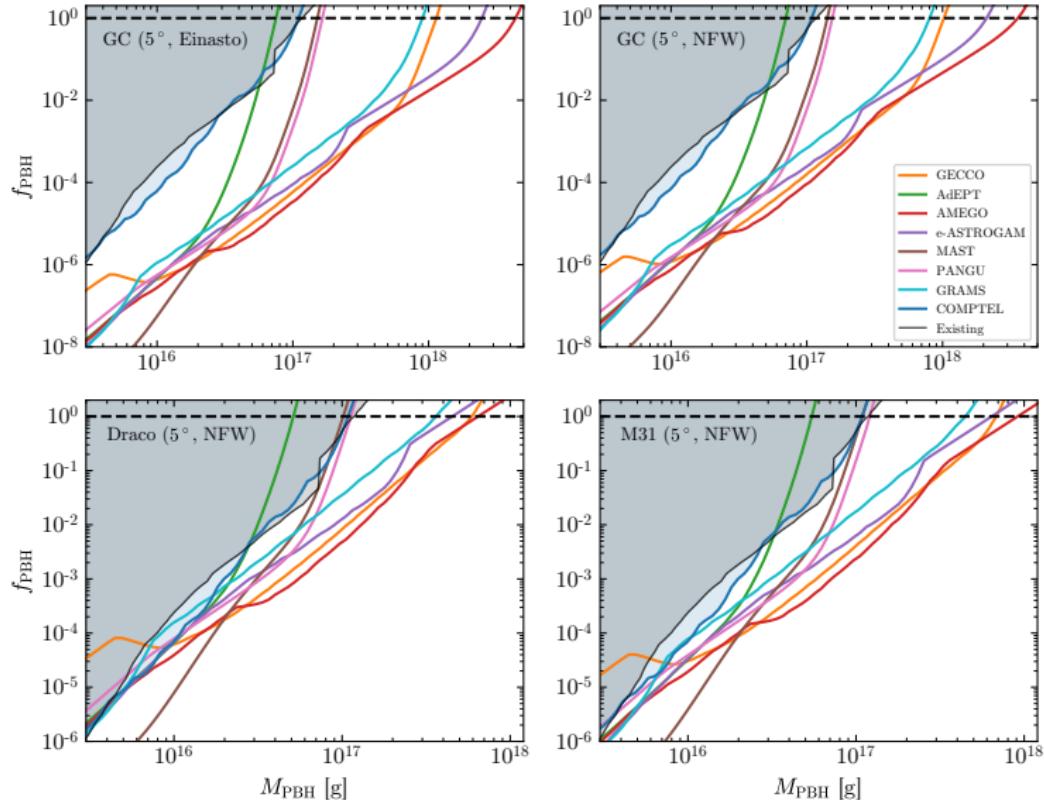
- As with decaying DM, number of observed photons:

$$N_\gamma = T_{\text{obs}} \int_{E_{\text{min}}}^{E_{\text{max}}} dE_\gamma A_{\text{eff}} \int d\tilde{E}_\gamma R_\epsilon(E_\gamma, \tilde{E}_\gamma) \frac{d\Phi}{dE_\gamma}$$

Constraining f_{PBH}



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Future Work

Extending Hazma to GeV

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- Idea: assume DM couplings to quarks via

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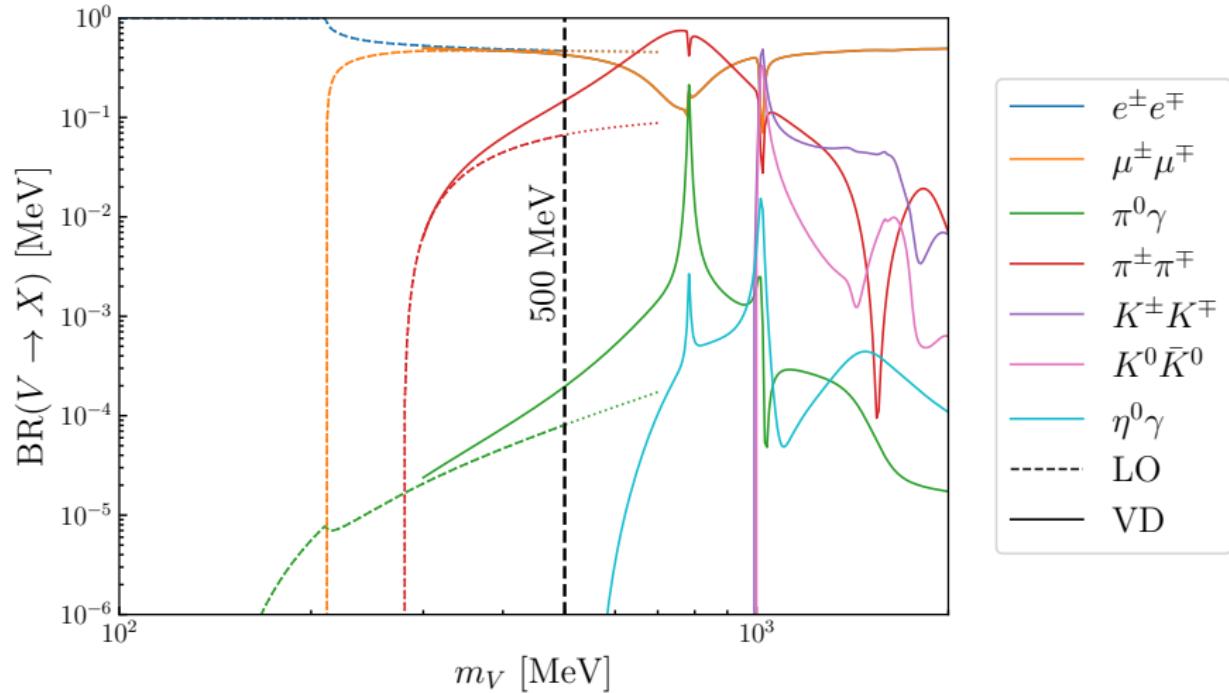
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- How much does this change?

Extending Hazma to GeV



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- Recipe for translating models from quark-level to chiral Lagrangian is straight-forward
- Hazma:** New open-source, user-friendly python package to explore/constrain MeV DM models
- Upcoming MeV telescopes could increase sensitivity to MeV DM models by factor ~ 100

Conclusions

- Without including non-perturbative effects, it is crucial to restrict use of chiral Lagrangian to $\sqrt{s} \lesssim 500$
- Recipe for translating models from quark-level to chiral Lagrangian is straight-forward
- Hazma:** New open-source, user-friendly python package to explore/constrain MeV DM models
- Upcoming MeV telescopes could increase sensitivity to MeV DM models by factor ~ 100
- MeV telescopes could also detect Hawking radiation for $M_{\text{PBH}} \sim 10^{15} - 10^{18}$ g

Thanks

Thanks to everyone who has helped and encouraged me throughout Graduate School...



Happy Holidays!



Honorable Mentions

Large-Nightmare
One-Loop Charge Breaking 2HDM
Asymptotic Analysis of Boltzmann Equation

Large-Nightmare Dark Matter

Stefano Profumo, Dean J. Robinson, LM:
arXiv:2010.03586

Theory

- Consider an $SU(N)$ gauge theory with a single dark (effectively massless, $m_{\tilde{q}} \ll \Lambda$) “quark”
- We take $N \gg 1$ and assume $g_{\text{dark}} \sim 1/\sqrt{N}$ (large-N limit)
- Two stable states: $\tilde{\eta}'$ ($\bar{q}q$) and $\tilde{\Delta}$ ($N\tilde{q}$)
- The $\tilde{\eta}'$ is very light while the $\tilde{\Delta}$ very heavy

State	Mass	Lifetime	$U(1)_V$
$\tilde{\eta}'$	$\sim \Lambda/\sqrt{N}$	stable	0
$\tilde{\Delta}$	$\sim N\Lambda$	stable	N
$\tilde{\omega}$	$\sim \Lambda$	N^2/Λ	0
\tilde{G}	$\sim \text{few } \Lambda$	N^2/Λ	0

Interactions

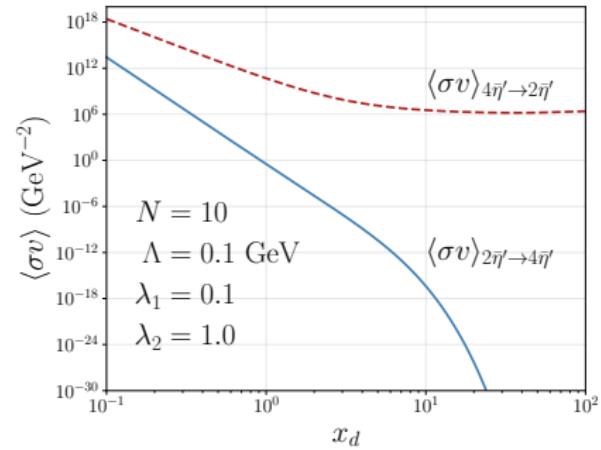
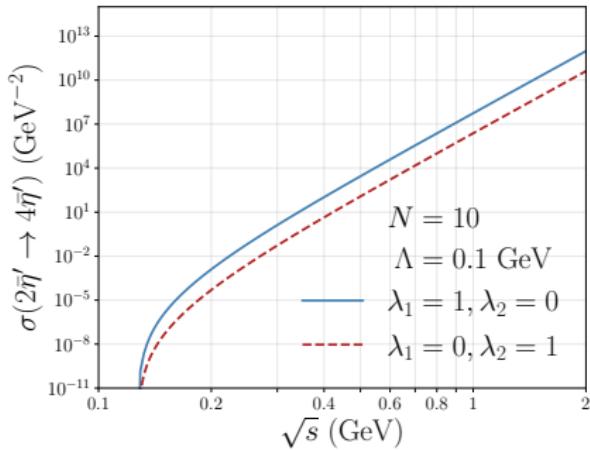
- Interactions for the $\tilde{\eta}$ are roughly:

$$\sigma_{2\tilde{\eta} \rightarrow 2\tilde{\eta}}(s) \sim \frac{\pi^3 s^3 |\lambda_1|^2}{4\Lambda^8 N^2}, \quad \sigma_{2\tilde{\eta} \rightarrow 4\tilde{\eta}}(s) \sim \frac{\pi^3 s^7}{48\Lambda^{16} N^4} \left| 10\lambda_1^2 + \lambda_2 \right|^2$$

- Interactions for the $\tilde{\Delta}$:

$$\sigma_{\tilde{\eta}\tilde{\eta} \rightarrow \bar{\tilde{\Delta}}\tilde{\Delta}}(s) \sim \frac{e^{-2cN}}{64\pi N^2 \Lambda^2}, \quad \sigma_{\tilde{\Delta}\tilde{\Delta} \rightarrow \tilde{\Delta}\tilde{\Delta}}(s) \sim \frac{4\pi^3}{\Lambda^2}$$

Interactions



Thermal Evolution of Dark Sector

- If a theory is thermally decoupled from the SM, it may have a different temperature
- Total entropy in dark and SM sector will be conserved
- Ratios of entropies densities are then constant:

$$\text{const} = \frac{s_d}{s_{\text{SM}}} = \frac{h_d(T_d)T_d^3}{h_{\text{SM}}(T_{\text{SM}})T_{\text{SM}}^3}$$

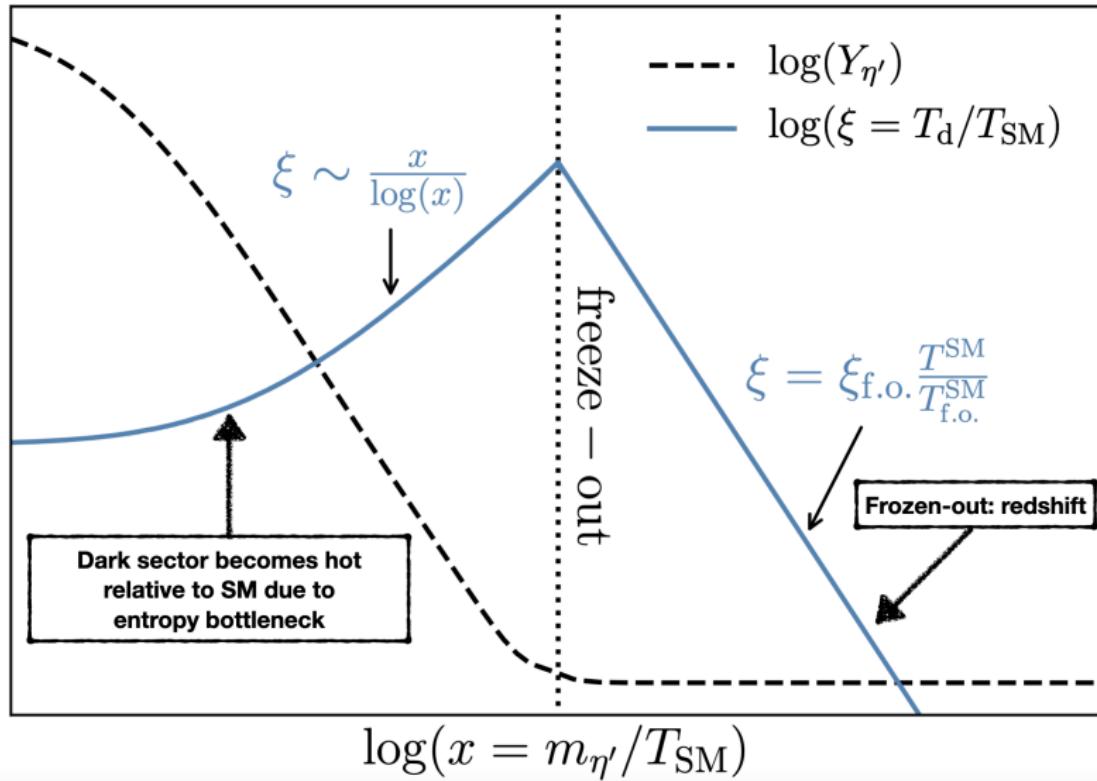
- We can determine dark temperature at late times if we know ratio at some early time

$$\xi(T_{\text{SM}}) \equiv \frac{T_d}{T_{\text{SM}}} = \left(\frac{h_{\text{SM}}}{h_{\text{SM}}^\infty} \frac{h_d^\infty}{h_d(\xi T_{\text{SM}})} \right)^{1/3} \xi^\infty$$

- As long as dark sector is in thermal equilibrium, it becomes exponentially hot relative to SM bath

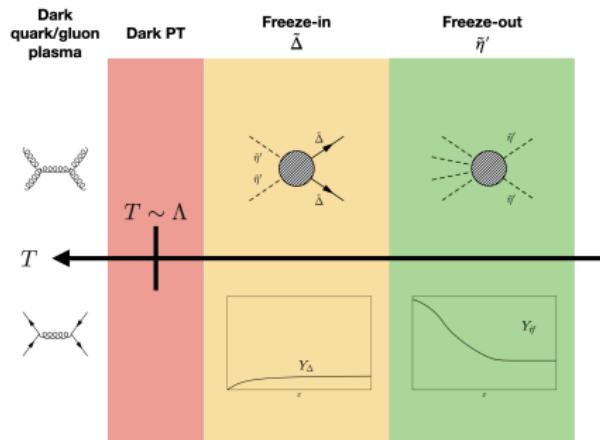
$$h(x = m/T) \sim x^3 K_3(x) \sim x^{5/2} x^{-x}, \quad (x \rightarrow \infty)$$

Thermal Evolution of Dark Sector



Cosmic Evolution

- ① High temperatures: dark quark-gluon plasma
- ② $T \sim \Lambda$: dark quark and gluons confine to $\tilde{\eta}'$ and $\tilde{\Delta}$
- ③ $n_{\tilde{\Delta}}$ initially suppress due to difficulty in forming
- ④ $\tilde{\Delta}$ s are frozen in via $2\tilde{\eta}' \rightarrow \bar{\tilde{\Delta}}\tilde{\Delta}$
- ⑤ $\tilde{\eta}'$ annihilate via $4\tilde{\eta}' \rightarrow 2\tilde{\eta}'$



Experimental Handels

- Measurements from bullet cluster and shapes of halos put tight constraints on self-interaction cross section

$$\sigma_{\text{SI}} \lesssim \frac{\text{barn}}{\text{GeV}}$$

- BBN and CMB constrain the effective number of neutrino constraints:

$$\Delta N_{\text{eff}} < 0.3$$

- If the $\tilde{\eta}'$ is in equilibrium for too long, we affect N_{eff}

$$N_{\text{eff}}^{\text{CMB}} \sim 3.046 + \frac{4}{7} \left(\frac{11}{4} \right)^{4/3} g_d^{\text{CMB}} \xi_{\text{CMB}}^4, \quad N_{\text{eff}}^{\text{BBN}} \sim 3 + \frac{4}{7} g_d^{\text{BBN}} \xi_{\text{BBN}}^4$$

Relic Densities

- $\tilde{\eta}'$ relic density can be approximated using entropy conservation and instantaneous freeze-out

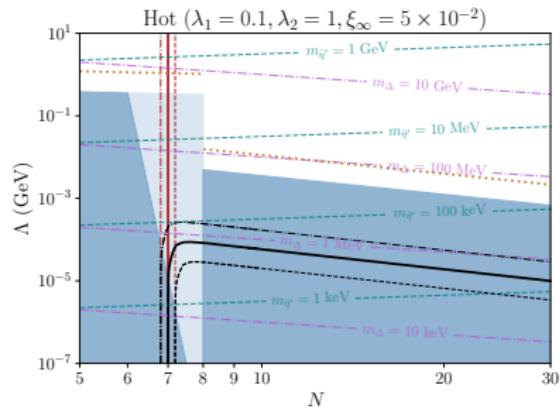
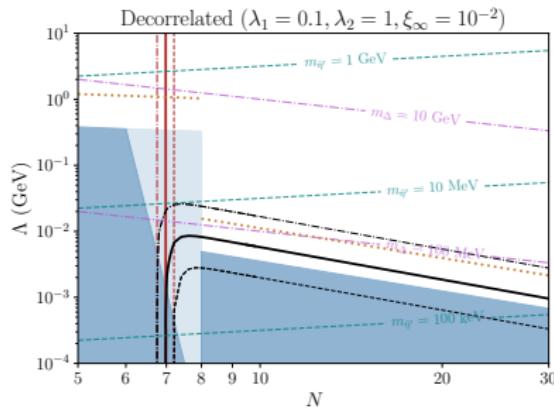
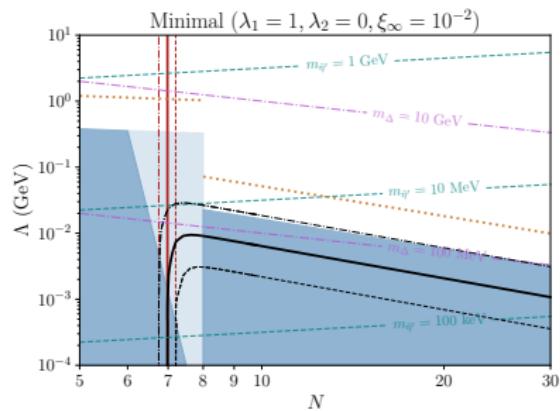
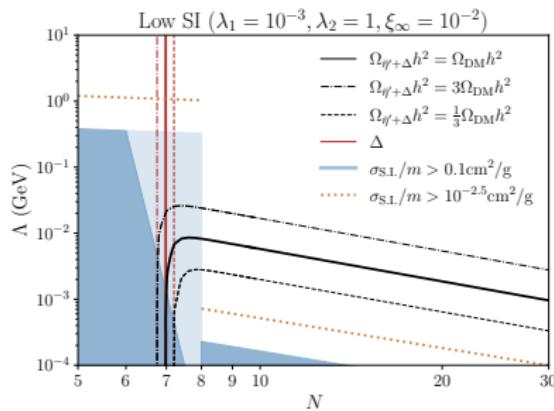
$$r_s = \frac{h_d}{h_{\text{SM}}} \xi^3 \sim \frac{N^2}{100} \xi_\infty^3, \quad Y_{\tilde{\eta}'} \sim \frac{n_{\tilde{\eta}'}}{s_{\text{SM}}} = \frac{r_s}{x^{d,f}}$$

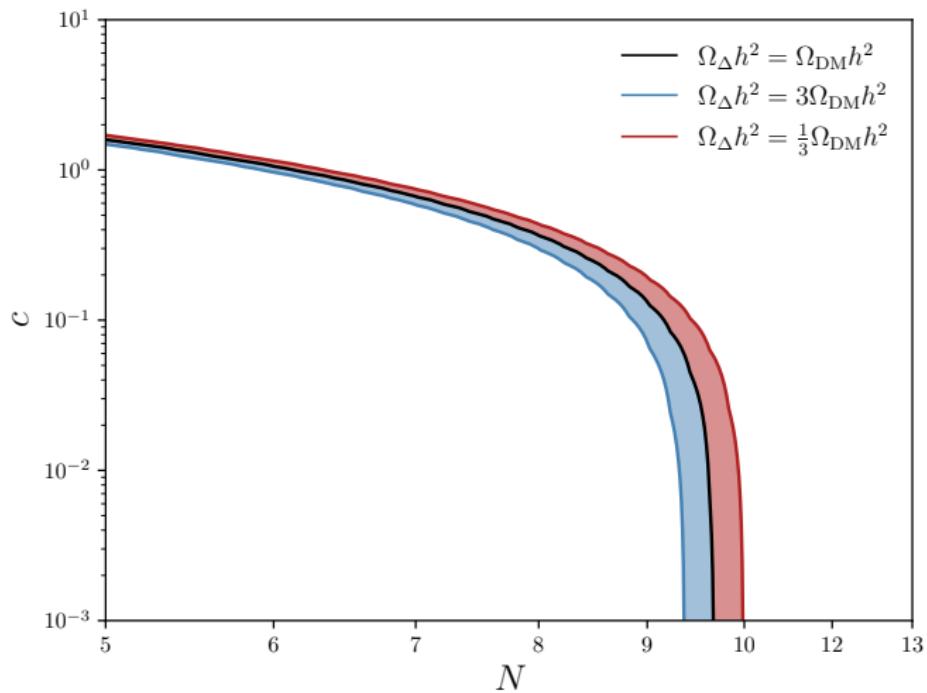
- Putting together:

$$\Omega_{\tilde{\eta}'} h^2 \sim 0.12 \left(\frac{10}{x_{d,f} + 1} \right) \left(\frac{\xi_\infty}{10^{-2}} \right)^3 \left(\frac{\Lambda}{20 \text{ MeV}} \right) \left(\frac{N}{10} \right)^{3/2}$$

- $\tilde{\Delta}$ relic density from direct integration of Boltzmann equation:

$$\Omega_{\tilde{\Delta}} h^2 \sim (\text{const.}) N^{3/2} e^{-2(c+1)N}$$





One-Loop Charge-Breaking Minima in the Two-Higgs Doublet Model

Pedro Ferreira, Stefano Profumo, LM:
arXiv:1910.08662

Tree-Level Charge-Breaking Minima in the THDM

- Possible to show that tree-level THDM potential with softly broken \mathbb{Z}_2 yeilds either an EW or CB minimum, **but not both**

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$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_1 + i c_2 \\ r_1 + i i_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3 + i c_4 \\ r_2 + i i_2 \end{pmatrix}$$

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$$\text{EW : } \langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$\text{CB : } \langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha \\ \bar{v}_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \bar{v}_2 \end{pmatrix}$$

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- Difference between vacua

$$V_{\text{CB}} - V_{\text{EW}} = \frac{M_{H^\pm}^2}{2(v_1^2 + v_2^2)} \left[(v_1 \bar{v}_1 - v_2 \bar{v}_2)^2 + \alpha^2 v_1^2 \right]$$

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- Does this hold at 1-loop?

One-Loop Corrections

- One-loop corrections are included using the effective potential:

$$V_{\text{eff}}(\bar{\phi}) = V_{\text{tree}}(\bar{\phi}) + \frac{\hbar}{64\pi^2} \sum_i (-1)^{2s_i} n_i [M_i^2(\bar{\phi})]^2 \left[\log\left(\frac{M_i^2(\bar{\phi})}{\mu^2}\right) - c_i \right]$$

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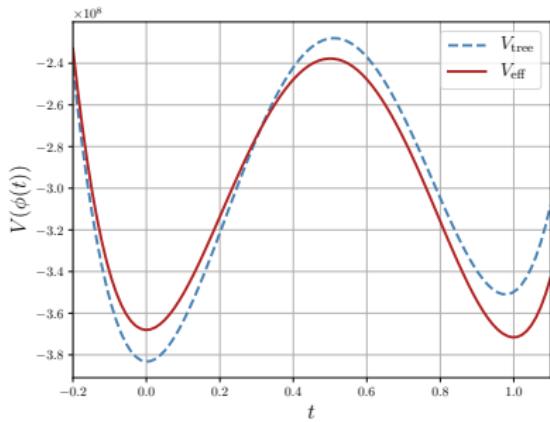
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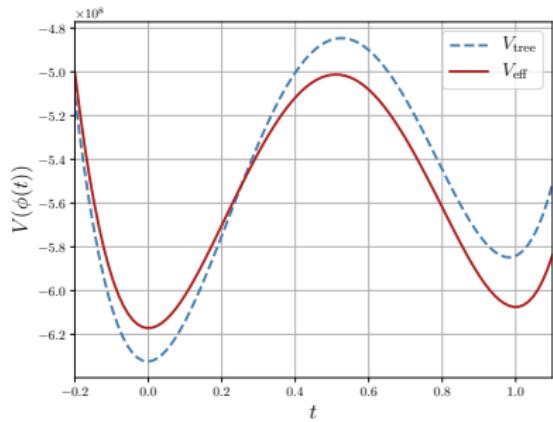
- Results: **there exists parameters with simultaneous CB and EW minima**
- Tend to occur when $V_{\text{CB}} \sim V_{\text{EW}}$

One dimensional slices of the effective scalar potential

$$\phi(t) = (1-t)\phi_{\text{EW}} + t\phi_{\text{CB}}$$
$$\phi(0) = \phi_{\text{EW}}, \quad \phi(1) = \phi_{\text{CB}}$$

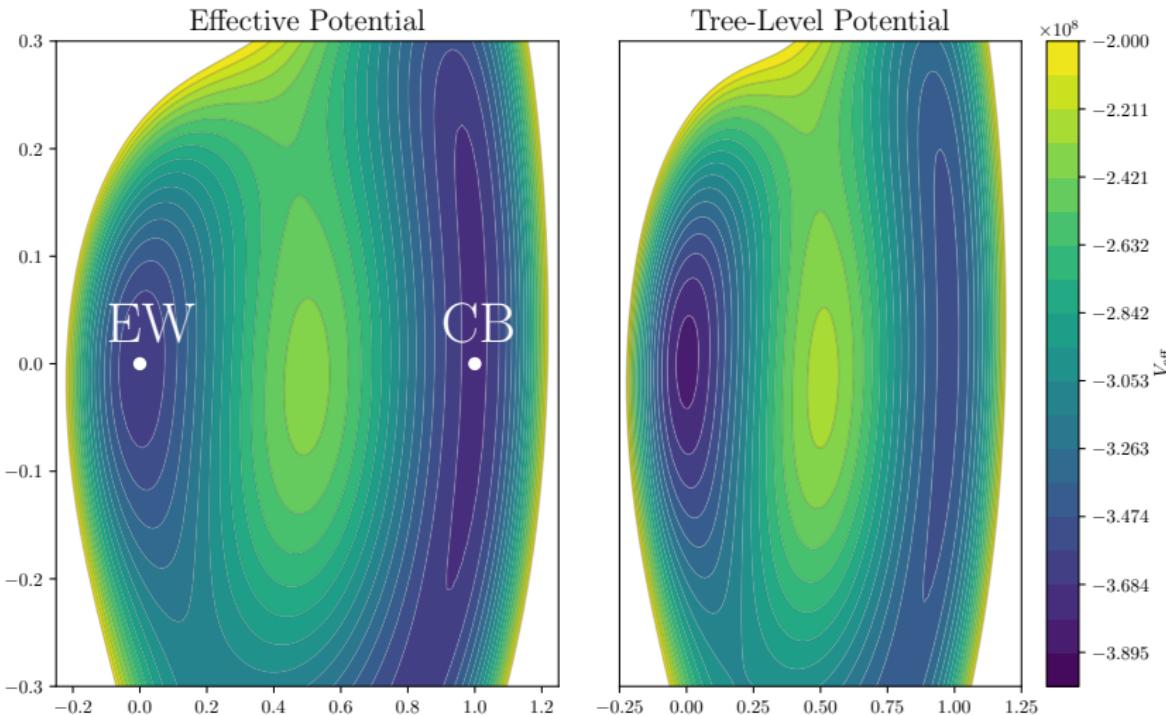


(a) $V_{\text{eff}}(\phi_{\text{EW}}) < V_{\text{eff}}(\phi_{\text{CB}})$

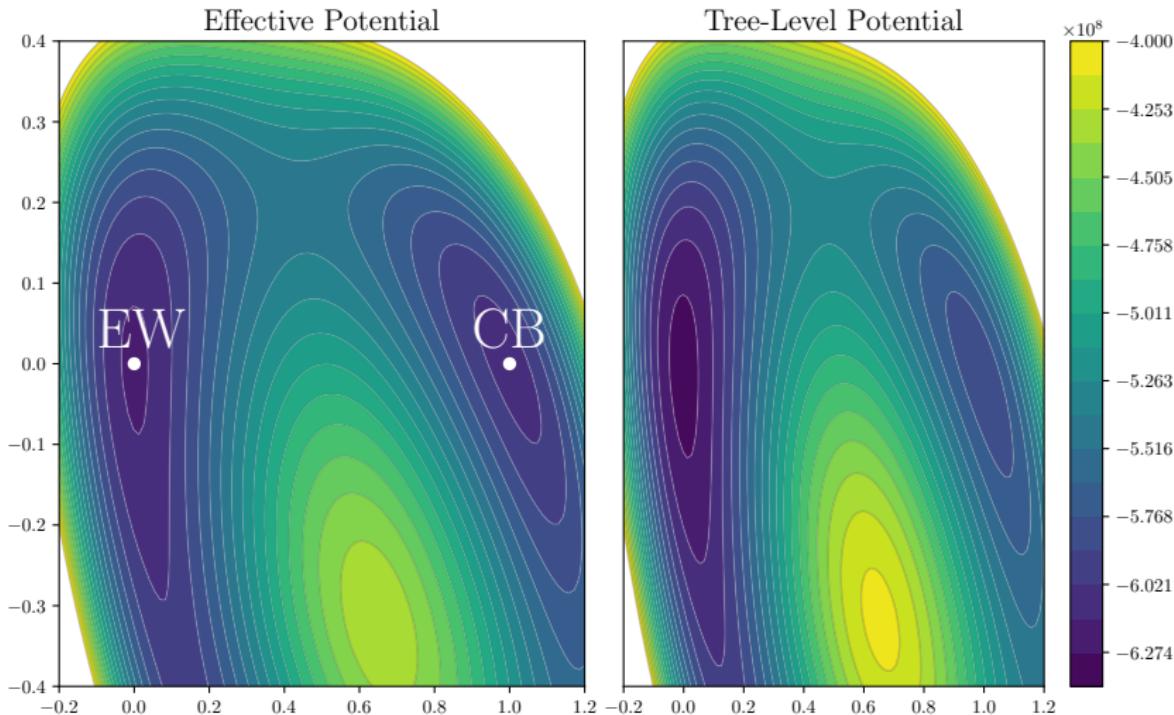


(b) $V_{\text{eff}}(\phi_{\text{CB}}) < V_{\text{eff}}(\phi_{\text{EW}})$

$$V_{\text{eff}}(\phi_{CB}) < V_{\text{eff}}(\phi_{EW})$$



$$V_{\text{eff}}(\phi_{EW}) < V_{\text{eff}}(\phi_{CB})$$



Asymptotic analysis of the Boltzmann equation for dark matter relic abundance

**Hiren H. Patel, Jaryd F. Ulbricht, LM:
arXiv:2009.04012**

Asymptotic Analysis of the Boltzmann Equation

- ① Dark Matter relic abundance determined using first moment of Boltzmann equation

$$\frac{dY}{dx} = -\lambda f(x) \left[Y^2 - Y_{\text{eq}} \right],$$

$$\lambda f(x) = \sqrt{\frac{\pi}{45}} \frac{m_\chi M_{\text{pl}}}{x^2} \frac{h}{\sqrt{g}} \left(1 + \frac{1}{3h} \frac{dh}{dx} \right) \langle \sigma v_{\text{M}\varnothing\text{l}} \rangle$$

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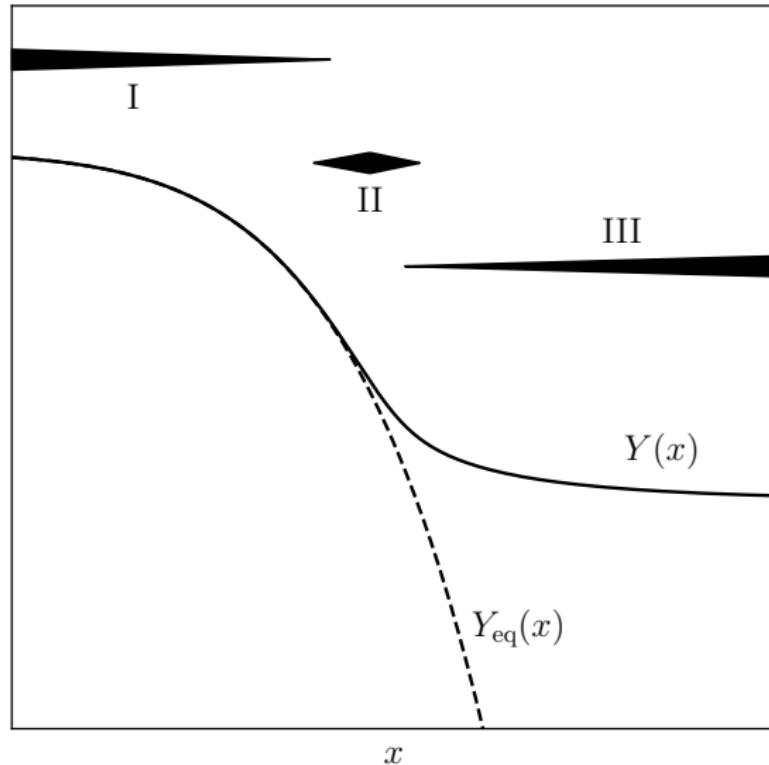
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- ② Numerical solutions are time consuming/difficult (very stiff equation)
- ③ Standard analysis of Gondolo, Gemini makes estimating errors in approximations difficult
- ④ Asymptotic analysis gives method for arbitrary accurate results with error estimates

Asymptotic Analysis of the Boltzmann Equation



Asymptotic Analysis of the Boltzmann Equation

