Qual Problems

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September 18, 2020

1 Problems : Electricity and Magnetism

1.1 Problem 1

Consider a hollow sphere of radius a, with a charge distribution on the surface

$$\sigma(\theta, \phi) = \sum a_{\ell,m} Y_{\ell,m}(\theta, \phi). \tag{1}$$

Find, in terms of the coefficients $a_{\ell,m}$

(a) The total charnge on the surface

(b) The potential outside of the sphere.

(c) Now assume that the system has only a non-vanishing dipole momentum, $\vec{p} = p\hat{x}$. Determine the $a'_{\ell,m}s$. Write the potential as a function of θ, ϕ, and demonstrate that it has the expected form.

1.2 Problem Two

A spherical wave has an $\boldsymbol{E}$-field given by

$$\boldsymbol{E} = \frac{A\hat{\phi}}{r}\sin(\theta)\cos(kr - \omega t) \tag{2}$$

Working to learding order in $1/r$, find the (time-averaged) radiated power.

1.3 Problem Three

(Concentric conducting cylinders.) A capacitor is made out of two concentric conducting cylindrical surfaces of radii a and $b > a$. The charge on the inner conductor is $Q > 0$ and on the outer $-Q$. Uniform external magnetic field is applied in the direction of the axis of the cylinders. The magnetic flux density B is increasing with time from 0 to B_0 along the axis of the cylinder. The time dependence of its magnitude is given: $B(t)$. The magnetic field created by the currents on the cylinders is negligible.

(a) Find the magnitude of the torque experienced by the capacitor at time t and describe its direction relative to the direction of the magnetic field B_0 (same or opposite).

(b) Find the magnitude of the total angular momentum that the capacitor receives as a result by the time the magnitude of the magnetic field reaches B_0 and compare with the magnitude of the total angular momentum of the electromagnetic field using the fact that the field carries the momentum density $\mathbf{\Pi} = s/c^2 = \epsilon_0 \boldsymbol{E} \times \boldsymbol{B}$. Compare the directions of these angular momenta.