WAVE ARBITRAGE: A DISPROOF OF THE EFFICIENT MARKET HYPOTHESIS

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ABSTRACT

Wave arbitrage is a trading strategy that has a higher expected return rate than the buy-and-hold strategy when a "fair game" dictates price fluctuations. This is a contradiction to the efficient market hypothesis. We compare the wave arbitrage and buy-and-hold strategies for SP500 stocks from late 2016 to early 2021 and demonstrate that wave arbitrage produces excess returns, even accounting for trading fees.

Keywords Efficient Market Hypothesis · Wave Arbitrage

1 Introduction

The efficient market hypothesis has been a useful way to think about markets for over half a century. According to Fama [1970],

> A market in which prices always "fully reflect" all available information is called "efficient."

There are several tests that can show that a market is inefficient. Fama [1976] summarized one of those tests with the observation, "... in an efficient market, trading rules with abnormal returns do not exist." In other words, if the efficient market hypothesis is true, it is impossible to beat the buy-and-hold strategy except through luck.

Wave arbitrage is similar to tidal power generators. As ocean waves come in or go out, the water turns a water wheel and generates electricity. With wave arbitrage, when the stock price moves up or down, the algorithm harvests some of the energy of that wave by selling when the price rises and buying when the price falls. The algorithm guarantees that a trader sells when prices are at a peak and buys when prices are at their lowest point.

The wave arbitrage strategy will only work if the market fluctuates. However, fluctuation is a defining attribute of the stock market. New information causes a market to wobble as traders react to that news, and there will always be more news. According to Brooks [2014], an acquaintance of J. P. Morgan once asked him what the market would do, to which Morgan replied, "It will fluctuate."

2 Wave Arbitrage

Consider a scenario where a trader is fully invested in two companies and at time t must decide the proportion of their assets invested in each share.

In some situations it is easy to determine whether a trader's portfolio is on the correct side of a trade. If the trader is holding only shares of stock i and the price p_i of that stock increases while p_j does not increase, then the trader is on the correct side of the trade. However, what about situations where the trader is not fully invested in one stock? We use a geometric mean to decide whether a portfolio comes out ahead or behind on any individual event.

Let s_i represent the number of shares a portfolio holds of security i. The value of the portfolio, using s_i as the basis of measurement, is $S_i = s_i + \frac{s_j p_j}{p_i}$. However, since both S_i and S_j are both reasonable representations for the value of a portfolio, we take the geometric mean of the two representations and obtain

$$G = \sqrt{\left(s_i + \frac{s_j p_j}{p_i}\right)\left(s_j + \frac{s_i p_i}{p_j}\right)} = \frac{s_i p_i + s_j p_j}{\sqrt{p_i p_j}}. \tag{1}$$

Let α_i represent the proportion of the portfolio invested in security i. At time t, the prices p_i are constant, so G has the property that for any α_i, G is a constant value. More plainly, if a trader can ignore exchange fees, then the value of G is the same both before and after each transaction.

Outside of rebalance events, G is a function of the prices p_i and p_j. To find the minimum, we take the derivative with respect to p_i:

$$\begin{aligned} G &= \frac{s_i p_i + s_j p_j}{\sqrt{p_i p_j}} \\ \frac{dG}{dp_i} &= \frac{p_j(s_i p_i - s_j p_j)}{2(p_i p_j)^{\frac{3}{2}}} \end{aligned} \tag{2}$$

Assuming prices are always greater than zero, the only root is at

$$s_i p_i = s_j p_j. \tag{3}$$

This represents the global minimum. This leads to

Lemma 1 *The sequence $\{G\}$ will monotonically increase if the only actions performed is to rebalance the portfolio so that $\alpha_i = \frac{1}{2}$.*

This observation is the key to the wave arbitrage algorithm. Fluctuations in the market will cause the value of G to change, but the value will never be less than G_t, the value of G after rebalancing $\alpha_i = \frac{1}{2}$ at time t. The trigger for rebalance events can be selected so that $G_{t+1} > G_t$ accounting for any transaction fees.

Since all share quantities and price increments have a minimum size, there is some smallest amount by which G can change. Thus, not only does G increase monotonically, it diverges.

If the portfolio is holding cash as one of its two assets, then 3 simplifies to $d = sp$. This leads to

Corollary 1 *If price p is observed at times t and $t+1$, and if the only transactions made to a cash and single share portfolio are to rebalance cash and shares to equal ratios, then the value of the portfolio will be strictly greater at time $t+1$ if at least one rebalance event has occurred.*

3 Disproof of the Efficient Market Hypothesis

Fama [1970] describes a test that can show that a market is inefficient. Fama described the total excess market value at $t+1$ as

$$V_{t+1} = \sum_{j=1}^{n} \alpha_j(\Phi_t)[r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t)]. \tag{4}$$

The value $\alpha_j(\Phi_t)$ represents the proportion of assets invested in security j, $r_{j,t+1}$ is the actual return at $t+1$, and $E(\tilde{r}_{j,t+1}|\phi_t)$ is the expected equilibrium returns given the available information Φ_t. Using the "fair game" property, meaning that price movements follow a martingale, he concludes

$$E(\tilde{V}_{t+1}|\Phi_t) = \sum_{j=1}^{n} \alpha_j(\Phi_t)E(\tilde{z}_{j,t+1}|\Phi_t) = 0. \tag{5}$$

In other words, no strategy will outperform any other strategy. Since the buy-and-hold strategy avoids all trading costs, it can be used as a benchmark. If any strategy does have a higher expected value than the buy-and-hold strategy, the market is inefficient.

While Fama [1970] put this forward as a test that can determine if a market is inefficient, he also provided this warning:

> But we should note right off that, simple as it is, the assumption that the conditions of market equilibrium can be stated in terms of expected returns elevates the purely mathematical concept of expected value to a status not necessarily implied by the general notion of market efficiency. The expected value is just one of many possible summary measures of a distribution of returns, and market efficiency per se (i.e., the general notion that prices "fully reflect" available information) does not imbue it with any special importance. Thus, the results of tests based on this assumption depend to some extent on its validity as well as on the efficiency of the market. But some such assumption is the unavoidable price one must pay to give the theory of efficient markets empirical content.

In other words, without tying the efficient market hypothesis to an assumption about expected value, the hypothesis is little more than a tautology.

Since Lemma 1 implies that $\{G\}$ monotonically increases under wave arbitrage, the goal is to determine whether the expected value of the portfolio also increases.

Inspecting the expected value of wave arbitrage, we let $S_i = s_i + \frac{s_j p_j}{p_i}$ represent the potential number of shares of security i that a portfolio can produce if all shares of security j are sold and the proceeds are used to buy shares of security i. Since 1 is the geometric mean of two quantities, we can use the inequality of arithmetic and geometric means to write

$$G = \sqrt{S_i S_j} \leq \frac{S_i + S_j}{2}. \tag{6}$$

When using wave arbitrage, since the sequence $\{G\}$ grows monotonically and diverges due to Lemma 1, the arithmetic mean of S_i and S_j will also diverge. From this we conclude that the expected number of potential shares increases:

$$E(S_{i,t+1}) > E(S_{i,t}). \tag{7}$$

Under the buy-and-hold strategy, the number of shares held does not change, but S_i is a function of the prices and can change. However, the expected returns from security i is equal to the expected returns from security j; if that were not the case, an investor could allocate all of their assets to one or the other of the securities and have an abnormally high expected return. This implies that the expected price change δ is equal for the two assets. However, if $E(p_{i,t+1}) = \delta p_{i,t}$ and $E(p_{j,t+1}) = \delta p_{j,t}$, then

$$E(S_{i,t+1}) = s_i + \frac{s_j(\delta p_{j,t})}{\delta p_{i,t}} = E(S_i). \tag{8}$$

Therefore, the expected return rate for wave arbitrage is higher than the expected return rate for buy-and-hold and Equation 5 is false. "Fair game" markets are inefficient.

4 Empirical Observation

The effect size of wave arbitrage is small but observable. The IEX [2021] dataset provides a record of all transactions on the Investors Exchange since late 2016. Using this dataset, we created a simulation that modeled both the buy-and-hold and wave arbitrage algorithms. The exchange charges a fee of $0.0009 per share traded. The simulation selects two assets and generates a buy-and-hold portfolio and a wave arbitrage portfolio and then attempts to rebalance the wave arbitrage portfolio after every transaction in the back test. The portfolio values at each time t are recorded, and after a period of one year the return is stored in a histogram.

This simulation uses 438 stocks listed on the SP500. It avoids any SP500 stock that had a split or reverse split, as well as any stock that stopped trading during the four year span. Dividends are reinvested.

This simulation does not make any attempts to avoid slippage effects, so we encourage researchers with more sophisticated back test frameworks to verify these results.

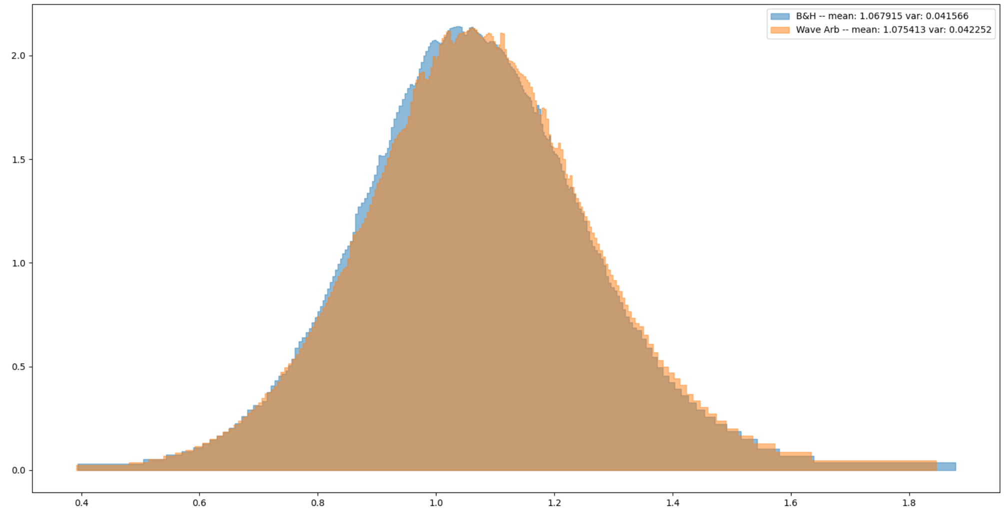

Figure 1: One Year Return Rates

Each pair of assets was modeled and over 26 billion data points for one year returns were collected into the histograms in Figure 1. The final result is that wave arbitrage outperformed the buy-and-hold strategy by an average of 0.75%.

5 Conclusions

In order to reach the conclusions in Section 3, we made the following assumptions:

- Price movements are a "fair game" and follow a martingale.
- Prices fluctuate with respect to each other.
- Prices do not go to zero.
- The efficient market hypothesis can be stated in terms of expected returns.

When prices follow a sub martingale, the disproof of Section 3 no longer works. Simulations for the sub martingale generated by using a fair coin and taking

$$p_{t+1} = \begin{cases} kp_t & \text{if heads,} \\ \frac{p_t}{k} & \text{if tails,} \end{cases} \tag{9}$$

suggest that the expected values for wave arbitrage and the buy-and-hold are equal. However, the variance of results for wave arbitrage is substantially lower than those of buy-and-hold.

The assumption that prices do not go to zero is debatable since companies go bankrupt every year. When the prices for an asset do go to zero, the value of a portfolio managed by wave arbitrage also go to zero. Additional rules should be developed to deal with this scenario. It is not clear whether the assumption that prices do not go to zero poses any serious challenge to the disproof in Section 3.

It is unclear what might happen if a substantial portion of a market is governed by wave arbitrage. Perhaps markets would become less volatile. However, since the efficient market hypothesis is false, it is also possible that other algorithms would be able to take advantage of the wave arbitrage algorithm.

References

Eugene F Fama. Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, 25(2): 383–417, 1970.

Eugene F Fama. Efficient capital markets: reply. *The Journal of Finance*, 31(1):143–145, 1976.

John Brooks. *Business adventures: twelve classic tales from the world of wall street*. Open Road Media, 2014.

IEX Market Data. `https://iextrading.com/trading/market-data/`, 2021. [Online; accessed 15-February-2021].