WAVE ARBITRAGE: A DISPROOF OF THE EFFICIENT MARKET HYPOTHESIS

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ABSTRACT

Wave arbitrage is a trading strategy that has a higher expected return rate than the buy-and-hold strategy when the market model is a martingale. This is a contradiction to the efficient market hypothesis. We compare the wave arbitrage and buy-and-hold strategies for SP500 stocks from late 2016 to early 2021 and demonstrate that wave arbitrage produces excess returns, even accounting for trading fees.

Keywords Efficient Market Hypothesis · Wave Arbitrage

1 Introduction

The efficient market hypothesis has been a useful way to think about markets for over fifty years. According to Fama [1970],

> A market in which prices always "fully reflect" all available information is called "efficient."

There are several tests that can show that a market is inefficient. Fama [1976] summarized one of those tests with the observation, "... in an efficient market, trading rules with abnormal returns do not exist." In other words, if the efficient market hypothesis is true, it is impossible to beat the buy-and-hold strategy except through luck.

Wave arbitrage is similar to tidal power generators. As ocean waves come in or go out, the water turns a water wheel and generates electricity. With wave arbitrage, when the stock price moves up or down, the algorithm harvests some of the energy of that wave by selling when the price rises and buying when the price falls.

The wave arbitrage strategy will only work if the market fluctuates. However, fluctuation is a defining attribute of the stock market. New information causes a market to wobble as traders react to that news, and there will always be more news. According to Brooks [2014], an acquaintance of J. P. Morgan once asked him what the market would do, to which Morgan replied, "It will fluctuate."

2 Wave Arbitrage

Consider a scenario where a trader is fully invested in two companies and at time t must decide the proportion of their assets invested in each share.

In some situations it's easy to determine whether a trader's portfolio is on the correct side of a trade. If the trader is holding only shares of stock i and the price of that stock p_i increases while p_j does not increase, then the trader is on the correct side of the trade. However, what about situations where the trader is not fully invested in one stock? We use a geometric mean to decide whether a portfolio comes out ahead or behind on any individual event.

Let s_i represent the number of shares in security 1. The value of the portfolio, using s_i as the basis of measurement, is $s_i + \frac{s_j p_j}{p_i}$. However, since both s_i and s_j are both reasonable bases for measurement, we take the geometric mean of the two representations and obtain

$$G = \sqrt{\left(s_i + \frac{s_j p_j}{p_i}\right)\left(s_j + \frac{s_i p_i}{p_j}\right)} = \frac{s_i p_i + s_j p_j}{\sqrt{p_i p_j}} \tag{1}$$

Let α_i represent the proportion of the portfolio invested in security i. At time t, the prices p_i are constant, so G has the property that for any α_i, G is a constant value. More plainly, if a trader can ignore exchange fees, then the value of G is the same both before and after each transaction.

Outside of rebalance events, G is a function of the prices p_i. To find the minimum, we take the derivative with respect to p_i:

$$G = \frac{s_i p_i + s_j p_j}{\sqrt{p_i p_j}}$$
$$\frac{dG}{dp_i} = \frac{p_j (s_i p_i - s_j p_j)}{2(p_i p_j)^{\frac{3}{2}}} \tag{2}$$

Assuming prices are always greater than zero, the only root is at

$$s_i p_i = s_j p_j. \tag{3}$$

This represents the global minimum. This gives us the following:

Lemma *The value G will monotonically increase if the only actions performed is to rebalance the portfolio it so that $\alpha_i = \frac{1}{2}$.*

This observation leads to the wave arbitrage algorithm. Fluctuations in the market will cause the value of G to change, but the value will never be less than G_t, the value of G after rebalancing $\alpha_i = \frac{1}{2}$ at time t. The trigger for rebalance events can be selected so that $G_{t+1} > G_t$ accounting for any transaction fees.

If the portfolio is holding cash as one of its two assets, then 3 simplifies to $d = sp$. This leads to

Corollary *If price p is observed at times t and $t+1$, then if the only transactions made to the portfolio are to rebalance cash and shares to equal ratios, then the value of the portfolio will be strictly greater at time $t+1$ if at least one rebalance event has occurred.*

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