Wave Arbitrage: A Disproof of the Efficient Market Hypothesis

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Wave arbitrage is a trading strategy that harvests the wave motion of stocks to increase the expected portfolio return. The algorithm is closely related to the 50:50 balanced portfolio strategy. We prove that the expected portfolio return for using wave arbitrage on two assets with the same expected rate of return is greater than the expected portfolio return of the buy-and-hold strategy. This is a counterexample to the efficient market hypothesis, which leads us to conclude that markets that fluctuate are not efficient.

I. Introduction

Wave arbitrage is closely related to the 50:50 portfolio. In it's traditional form, the strategy is to annually rebalance the portfolio, 50% in stocks and 50% in either cash or bonds. Even though this strategy is conservative, it has been observed to perform well, as explored by Carlson (2015). The key to wave arbitrage is to isolate and emphasize one of the factors that makes the 50:50 portfolio perform well. In fact, the expected value of wave arbitrage is higher than the buy-and-hold strategy, which is a counterexample to the efficient market hypothesis.

The efficient market hypothesis has been a useful way to think about markets for over fifty years. According to Fama (1970),

> A market in which prices always "fully reflect" all available information is called "efficient."

There are several tests that can show that a market is inefficient. Fama (1976) summarized one of those tests with the observation, "... in an efficient market, trading rules with abnormal returns do not exist." In other words, if the efficient market hypothesis is true, it is impossible to beat the buy-and-hold strategy except through luck.

Wave arbitrage is similar to tidal power generators. As ocean waves come in or go out, the water turns a water wheel and generates electricity. With wave arbitrage, when the stock price moves up or down, the algorithm harvests some of the energy of that wave by selling when the price rises and buying when the price falls.

The wave arbitrage strategy will only work if the market fluctuates. However, fluctuation is a defining attribute of the stock market. New information causes a market to wobble as traders react to that news, and there will always be more news. According to Brooks (2014), an aquaintance of J. P. Morgan once asked him what the market would do, to which Morgan replied, “It will fluctuate.”

II. Wave Arbitrage

A common way to evaluate a trading algorithm is to find a risk-free benchmark and then use that benchmark to compute the algorithm's Sharpe ratio, as described by Sharpe (1994). However, the Sharpe ratio doesn't reflect on whether an individual trade was advantageous.

In some situations it's easy to determine whether a trader's portfolio is on the correct side of a trade. If the trader is holding only stocks and the value of those stocks surges, the trader is on the correct side of the trade. Similarly, if the trader is only holding cash and the value of a stock plummets, the trader is also on the correct side of the trade.

However, what about situations where the trader has part of their portfolio in stocks and part of their portfolio in cash? It's useful to use a geometric mean to explore this relationship. If a portfolio is holding d dollars, s stocks of some company, and the exchange rate per share is p, then the value of the portfolio in dollars is $d + sp$ while the value of the portfolio in shares is $\frac{d}{p} + s$. Then, the geometric mean g of these two valuations for the portfolio is given by

$$g = \sqrt{(d + sp) \times \left(\frac{d}{p} + s\right)}. \tag{1}$$

This equation has the property that for any price p, if the trader is able to change their asset allocation without incurring trading fees, then all possible allocation ratios will provide the same geometric mean g.

We now prove the following lemma:

LEMMA 1: *The value g will monotonically increase if the only action performed on the portfolio is to rebalance it to a 50:50 ratio, given that the value of the asset never reaches zero.*

Outside of rebalance events the geometric mean is a function of the price p, so we can identify the global minimum:

$$g = \sqrt{(d+sp)\left(\frac{d}{p}+s\right)}$$

$$\frac{dg}{dp} = \frac{\frac{-d^2}{p^2}+s^2}{2\sqrt{(d+sp)\left(\frac{d}{p}+s\right)}} \tag{2}$$

set to zero

$$0 = \frac{-2d^2}{p^2}+s^2$$

$$d = sp$$

Coincidentally, sp is the value of the portfolio currently in stocks while d is the value of the portfolio that is currently in dollars, so the geometric mean is at a global minimum when the two halves of the portfolio are balanced. This means that after the portfolio has been rebalanced, any movement to the stock price will increase g.

This observation leads to the wave arbitrage algorithm. We can select a trigger that depends on the market fluctuating. For example, if the value of the shares in the portfolio is 0.5% higher or lower than the value of the cash, then rebalance. As long as the price fluctuates enough to repeatedly hit this trigger, the geometric mean will increase, assuming that the value of the asset never reaches zero.

Even in the presence of fees, the rebalance trigger can be selected so that the value of g at subsequent rebalance events is still monotonically increasing.

Lemma 1 gives us the following:

COROLLARY 1: *For all prices $p > 0$, if the price p is observed at two different times and a rebalance event occurred between those two times, then the value of the portfolio will be greater at the later of those two times.*

III. The portfolio mirror

It is instructive to have a graph that shows how the rebalancing operation works. Figure 1 shows the portfolio mirror. The basic idea of the portfolio mirror is that instead of graphing the number of dollars, we will instead graph the dollar value of the currently held shares. Instead of graphing the number of shares, we will instead graph the number of shares we could purchase with the currently held cash.

The portfolio mirror has one key advantage: when the price moves, the location of the portfolio moves. This movement maps out a hyperbola.

We are able to graph the rebalance lines, which represent all possible portfolio configurations given that the price is fixed and we are willing to make a trade.

The rebalance lines correspond to the geometric mean g, so the price movement hyperbola is tangent to the rebalance line when the portfolio is balanced.

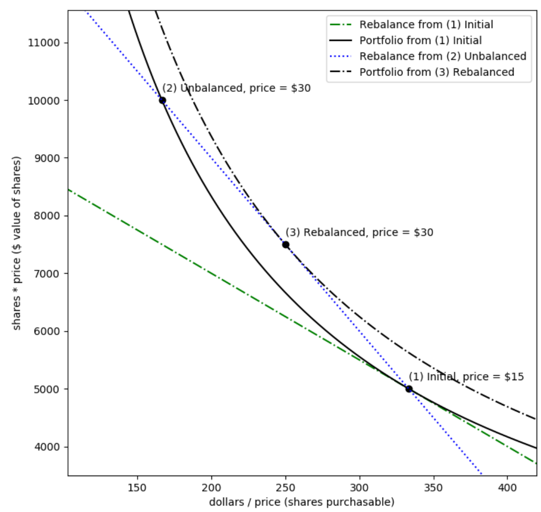

Figure 1. The Portfolio Mirror

Note: The portfolio is balanced when it starts at (1). As the price increases, the portfolio reaches (2) where it is unbalanced. The operation of rebalancing takes the portfolio to (3).

The example in Figure 1 shows a scenario where the portfolio starts in a balanced state, but then as the price increases, the portfolio moves up the price movement hyperbola. Rebalancing will move the portfolio halfway toward the initial location. This process will jump the portfolio to a price movement hyperbola that is farther away from the origin.

IV. The efficient market hypothesis

Fama (1976) described multiple tests that can show that a market is inefficient. The one we make use of is based on the observation that in an efficient market, it is impossible to beat the buy-and-hold strategy. More formally,

$$\sum_{j=1}^{n} \alpha_j(\Phi_{t-1}) E(\tilde{R}_{jt}|\Phi_{t-1}) = \sum_{j=1}^{n} \alpha_j(\Phi_{t-1}) E_m(\tilde{R}_{jt}|\Phi_{t-1}^m). \tag{3}$$

The value $E(\tilde{R}_{jt}|\Phi_{t-1})$ is the true expected return while $E_m(\tilde{R}_{jt}|\Phi_{t-1}^m)$ is the market assessed expected return. $\alpha_j(\Phi_{t-1})$ is a function of the information Φ_{t-1} and represents the proportion of assets that are allocated to asset j.

This provides a test for market efficiency. In an efficient market, no allocation of funds is expected to outperform a buy-and-hold strategy. Thus, if wave arbitrage provides an expected return higher than the buy-and-hold strategy for some market, that market is not efficient.

The expected return of an asset might a submartingale, i.e. the return is greater than or equal to zero. However, since returns are defined in terms of cash, the expected return of cash is zero. In such a scenario, the return from the wave arbitrage effect may be less than the expected return from the asset itself.

This presents something of a problem, but only if we use wave arbitrage to trade between an asset and cash. However, we can also use wave arbitrage to trade between two non-cash assets. In order to explore this, we need to reinterpret Equation (1) so that instead of using dollars d, we have two types of shares, s_1 and s_2. We can achieve this by making the substitutions $d = s_2$ and $p = \frac{p_1}{p_2}$. This gives us

$$g = \sqrt{\left(s_1 + \frac{s_2 p_2}{p_1}\right) \times \left(s_2 + \frac{s_1 p_1}{p_2}\right)} = \frac{s_1 p_1 + s_2 p_2}{\sqrt{p_1 p_2}} \tag{4}$$

This equation is minimized when $s_1 p_1 = s_2 p_2$. We can use a similar rebalance trigger; for example, if $s_1 p_1$ is 0.5% higher or lower than $s_2 p_2$, then we can rebalance.

From here, we are prepared to prove the following theorem:

THEOREM 1: *If $E(R_{s_1 t}|\Phi_{t-1}) = E(R_{s_2 t}|\Phi_{t-1})$, then the expected return rate of wave arbitrage is greater than the expected return rate of the buy-and-hold strategy, given that the market fluctuates enough to repeated trigger the wave arbitrage rebalancing.*

The expected return rate for the buy-and-hold strategy is given by the expression $\sum_{j=1}^{2} \alpha_j(\Phi_{t-1}) E(\tilde{R}_{jt}|\Phi_{t-1})$, and since the expected return rates of the two stocks is equal, this reduces to $E(\tilde{R}_{s_1 t}|\Phi_{t-1})$.

For the wave arbitrage strategy, we start by observing that since the market is fluctuating enough to repeatedly trigger the rebalancing event, then if a rebalance event occurred at time time $t-1$, we can always find a time t where a rebalance event occurs. From Lemma 1, we know the that g is increasing. That is, $g_t > g_{t-1}$.

In order to simplify manipulating Equation (4), we can make a normalizing assumption. That is, even if the share price of the two assets is different, we can create batches of shares so that the number of batches b_1 made up of s_1 is the same as the number of batches b_2 made up of s_2. At that point, the prices per batch will be equal. This gives us $g = b_1 + b_2 = 2b$.

Since $g_t > g_{t-1}$, we have $b_t > b_{t-1}$. This is important because b represents the number of shares held in the portfolio. For the buy-and-hold strategy, $b_t = b_{t-1}$ because the number of shares held never changes. Since the wave arbitrage algorithm will hold more batches of shares than the buy-and-hold algorithm, and since the expected value of a batch at time t is greater than or equal to the value at $t-1$, the expected value of wave arbitrage is higher than the expected value of the buy-and-hold strategy.

There are a few situations where this disproof doesn't apply. If all assets with a specific return rate fluctuate in the same way, wave arbitrage will fail to work.

V. Empirical observation

The effect of wave arbitrage is small, but it is observable. In this section, we use IEX (2019) to compare the buy-and-hold strategy against wave arbitrage over the past three years. This dataset provides a record of all trades on the IEX exchange for the past three years. Most of the S&P 500 stocks, listed as of 2019-12-19, are traded on that exchange.

We randomly selected two separate symbols from this list and then used the tick data simulate both the buy-and-hold and the wave arbitrage algorithms. We modeled a flat fee of $0.001 per share since fees on IEX will generally be lower than that figure.

For both algorithms, we allocated half of the assets to the first of the two randomly selected symbols and half of the assets to the other symbol. At this point, we never altered the allocation ratio of the buy-and-hold portfolio. For the wave arbitrage algorithm, we rebalanced the portfolio whenever the value of one asset exceeded the value of the other by at least 0.5%. The dataset provided a three year window, and we recorded the percentage delta of each strategy for all one year intervals within that window. The results are summarized in Figure 2.

This experiment shows an effect size of 0.48%. This result should not be taken as conclusive evidence that wave arbitrage will always have a higher return rate

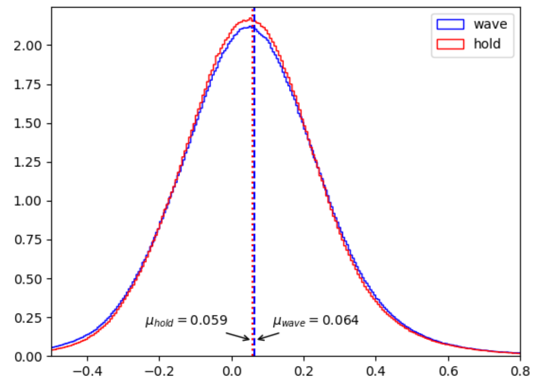

FIGURE 2. ONE YEAR RETURN RATES FOR BUY-AND-HOLD VS WAVE ARBITRAGE STRATEGIES.

Note: The displayed histograms are composed of 29 million observations for each of the buy-and-hold (hold) and wave arbitrage (wave) strategies. The 99.95% confidence interval is [0.05882, 0.05899] for μ_{hold} and is [0.06354, 0.06367] for μ_{wave}. The bootstrap estimate was generated from a rolling window with 29 million observations. On each iteration, the oldest observation was removed and a replacement observation was randomly selected. Every 100th iteration, the mean of the buffer was recorded for the bootstrap distribution. A total of 1 million observations were collected for the boostrap distribution.

than the buy-and-hold strategy. However, based on the results of Theorem 1, given enough time, some positive effect size will exist.

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