Correcting K-correction: dark energy is based on a math error from 1930

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We rederive the K-correction equation and discover that the version commonly used to calculate magnitudes fails to account for time dilation. This error traces back to 1930. We conclude that correcting apparent magnitudes for time dilation resolves two of the preeminent mysteries in cosmology: first, dark energy is not supported by observations of Type Ia supernovae and second, the Hubble tension is due to the error in the K-correction equation.

INTRODUCTION

Cosmic expansion redshifts light, so when we compute the light travel distance it is necessary to account for multiple redshift effects. We will show that modern cosmology does not correct observations for time dilation.

Most cosmological models require an estimate of distance, so this error is far reaching. However, we will focus on showing that correcting observations of Type Ia supernovae (SNe Ia) for time dilation resolves the Hubble tension. The Hubble tension, as described by Verde *et al.* [1], is an observation that independent techniques of estimating the expansion rate produce incompatible values.

In order to use SNe Ia to estimate the expansion rate of the universe, we need to identify both the redshift z and the light travel distance d_{LT}. In order to compute the light travel distance d_{LT} in parsecs (pc), we use the definition of the distance modulus [2]:

$$d_{LT} = 10^{1+(m-M)/5}\text{pc}. \tag{1}$$

The quantity m represents the apparent magnitude while M represents the absolute magnitude of a SN Ia. It is common to use the substitution $\mu = m - M$ and call μ the distance modulus.

Cosmology uses multiple distance measurements, but for measuring the expansion rate of the universe, the only one we care about is the light travel distance d_{LT}. A similar distance measurement, the luminosity distance d_L, is often discussed in conjunction of flux measurements, but d_L represents an illusory distance. In fact, $d_L = d_{LT} \times (1 + z)$, which can be thought of as the distance an object would need to be to produce the same amount of measured energy flux if redshift did not exist. The light travel distance d_{LT} represents a physical quantity while d_L does not.

Labeling m as the "apparent magnitude" is a bit misleading for distant observations because redshift will change several aspects of the observed light. However, we can correct the value m for redshift effects by using the K-correction equation. This allows us to report an apparent magnitude that represents the brightness of an object as if none of the light-altering effects of redshift were present. According to Riess *et al.* [3]:

> An appropriate K-correction quantifies the *difference* between the supernova light that falls into a standard passband (e.g., B) at $z = 0$ and that which falls into the filters we employ to observe a redshifted SN Ia.

In other words, the goal of the K-correction equation is to remove all redshift effects from the apparent magnitude m so that the distance modulus defined in Equation 1 allows us to compute the light travel distance d_{LT}.

Tolman [4] published the first formal derivation of K-correction, but he did not account for spectral bandwidth stretching, one of the dimming effects of redshift. An alternative K-correction was correctly derived by de Sitter [5]. However, in Hubble and Tolman [6], the discrepancy between the two equations was briefly discussed yet ultimately dismissed. Oke and Sandage [7] derived a K-correction equation that they noted was equivalent to earlier work, and while they accounted for spectral bandwidth stretching, they did not account for time dilation. The paper by Kim *et al.* [8], which is the basis for modern implementations of K-correction, extended the work of Oke and Sandage [7] to handle additional cross-filter comparisons and added a term that addresses zero-point corrections.

The effect of using an incorrect redshift correction for SNe Ia is that we think objects are farther away than they really are; this effect compounds for greater distances. Up until Riess *et al.* [3], we did not have distant enough observations for this error to matter much for cosmological models, but by the late 1990s, it became clear that our measurements for distance and redshift were not linear. This realization led to a model that utilized a cosmological constant Λ and dark energy in order to explain the non-linear distance-redshift graph. We will show that correcting the flaws of K-correction leads to a linear graph that does not need to rely on a cosmological constant.

Planck-Collaboration *et al.* [9] used an alternative technique to compute the Hubble constant H_0 that is based on measurements of the cosmic microwave background (CMB). CMB-based computations of H_0 have made it evident that something is missing with our understanding of cosmology because this technique produces a significantly different value than when the constant is derived using SNe Ia. Fixing the errors with K-correction

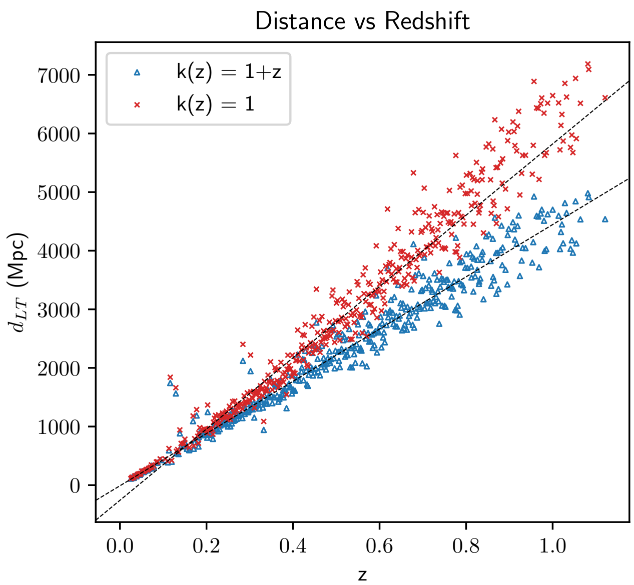

FIG. 1. The relationship between light travel distance d_{LT} and redshift z for two treatments of the magnitude data. The red $k(z) = 1$ treatment, which represents the uncorrected values, is clearly non-linear while the blue $k(z) = 1 + z$ treatment, which fixes the K-correction redshift error, appears to be linear. The displayed points are roughly a third of the values in the full SNe Ia dataset published in DES-Collaboration *et al.* [11], selected with uniform redshift spacing to aid visibility.

produces a measurement of the Hubble constant that is compatible with models based on the CMB.

A BRIEF HISTORY OF K-CORRECTION

Observational data from SNe Ia are essential for the calibration of models that use redshift to estimate distance. The approximately linear relationship between redshift and light travel distance, often called the Hubble-Lemaître law, describes how quickly the universe is expanding. However, Riess *et al.* [3] and Perlmutter *et al.* [10] presented evidence that there is not a linear relationship between redshift and distance, but instead, distant objects are farther away than their redshift would predict (see Figure 1). This phenomenon implies that the acceleration of the universe is faster today than it was for old observations. The cause of this phenomenon was unknown and was referred to as dark energy.

SNe Ia are used to explore the relationship between distance and redshift because these events have a roughly constant absolute magnitude which allows us to use the apparent magnitude to estimate distance.

However, redshifting dims the brightness of an observation in three ways:

- The energy of a wave is inversely proportional to its wavelength; thus a redshift of z means the amount of energy per photon is reduced by a factor of $1/(1 + z)$.
- The bandwidth stretches. If the rest frame observes photons in the bandwidth from 400nm and 401nm, then a redshift of $z = 1$ stretches the wavelengths to the bandwidth from 800nm to 802nm. The number of photons per bandwidth is reduced by a factor of $1/(1 + z)$.
- Cosmological time dilation reduces the rate at which photons arrive. The number of photons that arive per second is reduced by a factor of $1/(1 + z)$.

To complicate matters, we use the B band magnitude to calculate light travel distance. We want to know how bright a supernova appears for wavelengths around 445nm as if no redshift had occurred. However, we typically need to observe the supernova with a filter that is sensitive to longer wavelengths, such as the i band filter which is sensitive to wavelengths from 700nm-850nm [12]. The K-correction formula allows us to take a magnitude measured in an observation filter y and compute the magnitude in the target filter x. Even if the observation filter is the same as the target filter, the K-correction equation is necessary to remove redshift effects.

The first mathematical treatment of K-correction was performed by Tolman [4]. However, when Tolman made his derivation, he did not consider the effects of a spectrum that is stretched due to redshift.

A few years later, de Sitter [5] discussed all three issues that reduce the observed brightness of a distant observation. The correction for each of these issues is identical: take a flux measurement and multiply it by the factor $1 + z$.

A year later, Hubble and Tolman [6] published a similar set of calculations where they stated:

> It should be specially noted that this expression differs from the correction to m proposed by de Sitter, which contains the term $(1+z)^3$ instead of $(1 + z)^2$. Expression (28), however, would seem to give the proper correction to use in connection with our equation (21), since it has been derived in such a way as to make appropriate allowance, first, for the double effect of nebular recession in reducing both the individual energy and the rate of arrival of photons, and then for the further circumstance that a change in spectral distribution of the energy that does arrive will lead to changes in its photographic effectiveness.

However, even though they considered all three dimming effects of redshift, they started their derivation by copying the incorrect equation from 1930. The incorrect correction term has been used ever since.

In a paper published by Oke and Sandage [7], the two factors of $1+z$ were attributed to the change in energy and to the spectral bandwidth elongation, which leaves time dilation as the factor that was omitted.

The modern treatment of K-correction is based on the work of Kim *et al.* [8]. This work extended the work of Oke and Sandage [7] to apply to filters beyond the blue B and visible V bands. It also introduced a term that deals with the zero-point correction for the actual filters. Historically, bolometric devices would measure the energy flux, but modern charge-coupled device (CCD) cameras effectively measure the photon flux, so Kim *et al.* [8] derived a form of the K-correction equation that uses photon flux. This is the form of the K-correction equation that is used by modern astrophysics, but we will show that this equation still contains the time dilation error that produces the dark energy phenomenon.

DERIVATION OF K-CORRECTION

The K-correction K_{xy} uses a spectral energy density template F that specifies the expected spectrum for a SN Ia observation. It also uses the observed redshift z and the observed photon flux magnitude m_y measured in filter y.

With modern CCD cameras, a telescope observation consists of a single value $\mathcal{F}_y$ erg/s, which represents the energy collected per second in filter y. Photogenerated electrons are collected in potential wells, which means that the energy measured in this filter is proportional to the number of photons [13]. However, we need to calculate the expected number of photons collected by using a spectral energy density function F.

To convert the spectral energy density F to the photon density F', we need to use the Plank relation $E = hc/\lambda$ where E is the energy, h is Planck's constant, and c is the speed of light. This gives us

$$\begin{aligned} F(\lambda) &= F'(\lambda) \times \frac{hc}{\lambda} \\ F'(\lambda) &= \frac{\lambda F(\lambda)}{hc}. \end{aligned} \tag{2}$$

It is important to note that for the blueshifted wavelength $\lambda/(1+z)$, this equation produces

$$F'\left(\frac{\lambda}{1+z}\right) = \frac{\lambda}{(1+z)hc} F\left(\frac{\lambda}{1+z}\right). \tag{3}$$

However, this equation is misleading and error prone. We will want to use it to help calculate the flux in an observation filter at the redshifted wavelength $\lambda \times (1+z)$. In other words, we want to produce the photon density function R' that is the redshifted version of F'. Redshifting the photon density does two things:

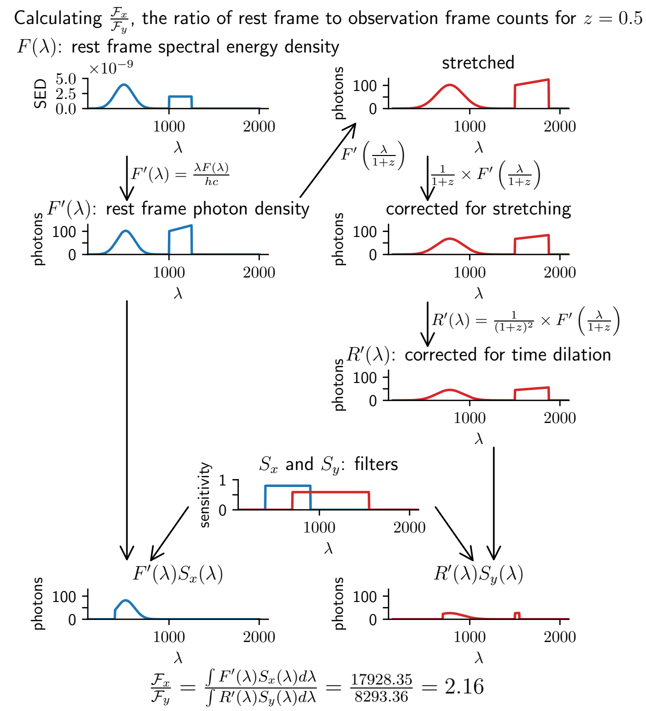

FIG. 2. The value $\mathcal{F}_x/\mathcal{F}_y$ is equal to the number of photons we want to report in the target filter divided by the number of photons we measure in the observation filter. Deriving this value starts with $F(\lambda)$, the spectral energy density, which we then convert to $F'(\lambda)$, the photon density for the rest frame. Starting at the top right corner of the figure we show three steps used to compute $R'(\lambda)$, the photon density in the observation frame. We first stretch the photon density, but this step inflates counts. In the second step, we correct for the inflated counts by multiplying by $1/(1+z)$; at this point, the total count equals the total count of $F'(\lambda)$. Finally, in the third step we finish calculating the observation frame photon density $R'(\lambda)$ by accounting for time dilation, which also reduces photon counts by a factor of $1/(1+z)$. We can compute the expected ratio of photons by totalling the area under the curves in the bottom two panels. Dark energy occurs when we omit the step that corrects for time dilation.

- All wavelengths are increased by a factor of $1+z$. When we integrate R' from wavelength λ_a to wavelength λ_b, the values correspond to the wavelengths $\lambda_a/(1+z)$ to $\lambda_b/(1+z)$ in F'. We will integrate over a width of $\lambda_b - \lambda_a$, but $F'(\lambda/(1+z))$ will refer to values in a width of $(\lambda_b - \lambda_a)/(1+z)$.

  In order to account for the spectral bandwidth stretching effect, we multiply F' by $1/(1+z)$.

- The stretching of space increases the distance between photons while they are traveling. To an observer, this phenomenon appears like time dilation, although cosmological time dilation is due to a different mechanism than relativistic time dilation. This effect reduces the photon arrival rate by a factor of $1/(1+z)$.

In order to account for cosmological time dilation, we multiply F' by a second factor of $1/(1+z)$.

Combining these two phenomena, we can calculate the redshifted spectral energy density R using

$$\begin{aligned} R'(\lambda) &= F'\left(\frac{\lambda}{1+z}\right) \times \frac{1}{(1+z)^2} \\ R(\lambda) &= \frac{\lambda}{(1+z)hc} \times F\left(\frac{\lambda}{1+z}\right) \times \frac{1}{(1+z)^2}. \end{aligned} \tag{4}$$

We deliberately do not combine all of the factors of $1+z$ together because this form is more natural to implement in code.

In order to calculate the flux $\mathcal{F}_x$ measured in filter x, we need to compute the photon density $F'(\lambda)$ and multiply it by the sensitivity $S_x(\lambda)$, which represents the proportion of photons with wavelength λ that filter x will measure. We then sum over all wavelengths, which is expressed with the equation

$$\begin{aligned} \mathcal{F}_x &= \int F'(\lambda) S_x(\lambda) d\lambda \\ &= \int \lambda F(\lambda) S_x(\lambda) d\lambda. \end{aligned} \tag{5}$$

We will also use the flux $\mathcal{F}$ to magnitude m formula:

$$\begin{aligned} m_x &= -2.5\log(\mathcal{F}_x) + P_x \\ -2.5\log(\mathcal{F}_x) &= m_x - P_x. \end{aligned} \tag{6}$$

P_x represents the zero-point for the filter x on some particular telescope. In order to use consistent magnitude values across telescopes that have different light gathering abilities, we take the measured magnitude and multiply it by the ratio of the standard flux rate to the flux rate for this particular telescope and filter. For convenience, we use $P_x = -2.5\log(P'_x)$ so that we can work with flux instead of with magnitude.

Now that we have the identities in Equations 5 and 6, we will change directions and look at the definition of K-correction K_{xy}. This value allows us to make an observation in filter y and report what the magnitude would have been in filter x if no redshift occurred:

$$\begin{aligned} m_y &= M_x + \mu + K_{xy} \\ &= M_x + m_x - M_x + K_{xy} \\ &= m_x + K_{xy} \\ m_x &= m_y - K_{xy}. \end{aligned} \tag{7}$$

The second line of Equation 7 expands the distance modulus μ using $\mu = m - M$ where m is the observed magnitude and M is the absolute magnitude.

Since the K-correction is a magnitude value and we wish to work on flux values, it is convenient to define the following substitution:

$$K_{xy} = 2.5\log(K'_{xy}). \tag{8}$$

Note that this substitution omits the minus $(-)$ sign that we used on a similar substitution for P_x.

Starting with Equation 6 and then recombining the flux term $\mathcal{F}_y$ with m_y from Equation 7, we have

$$\begin{aligned} -2.5\log(\mathcal{F}_x) &= m_x - P_x \\ &= m_y - K_{xy} - P_x \\ &= -2.5\log(\mathcal{F}_y) + P_y - K_{xy} - P_x \\ &= -2.5\log(\mathcal{F}_y) - 2.5\log(P'_y) \\ &\quad - 2.5\log(K'_{xy}) + 2.5\log(P'_x) \\ &= -2.5\left(\log(\mathcal{F}_y) + \log(K'_{xy}) \right. \\ &\quad \left. + \log(P'_y) - \log(P'_x)\right) \\ &= -2.5\log\left(\mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}\right) \\ \mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}. \end{aligned} \tag{9}$$

We can isolate K'_{xy} and then use Equation 5 to expand

$$\begin{aligned} \mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x} \\ K'_{xy} &= \frac{P'_x}{P'_y} \times \frac{\mathcal{F}_x}{\mathcal{F}_y}. \end{aligned} \tag{10}$$

We now use Equations 4 and 5 to calculate the fluxes $\mathcal{F}_x$ and $\mathcal{F}_y$ in terms of the spectral energy density function F. Note that $\mathcal{F}_x$ uses the rest frame spectral energy density F, while $\mathcal{F}_y$ uses the redshifted spectral energy density $R(\lambda)$:

$$\begin{aligned} K'_{xy} &= \frac{P'_x}{P'_y} \times \frac{\mathcal{F}_x}{\mathcal{F}_y} \\ &= \frac{P'_x}{P'_y} \times \frac{\int F'(\lambda) S_x(\lambda) d\lambda}{\int R'(\lambda) S_y(\lambda) d\lambda} \\ &= \frac{P'_x}{P'_y} \times \frac{\int F'(\lambda) S_x(\lambda) d\lambda}{\int F'\left(\frac{\lambda}{1+z}\right) \times \left(\frac{1}{1+z}\right)^2 S_y(\lambda) d\lambda} \\ &= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \left(\frac{\lambda}{hc}\right) F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{(1+z)hc} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda} \\ &= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \left(\frac{\lambda}{1+z}\right) F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}. \end{aligned} \tag{11}$$

Finally, we use Equation 8 to convert K'_{xy} back into the magnitude K_{xy}:

$$
\begin{aligned}
K_{xy} &= 2.5\log(K'_{xy}) \\
&= 2.5\log\left(\frac{P'_x}{P'_y} \times (1+z)^2\right. \\
&\qquad \left.\times \frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \left(\frac{\lambda}{1+z}\right) F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&= 2.5\left(\log\left(\frac{P'_x}{P'_y}\right) + \log\left((1+z)^2\right)\right. \\
&\qquad \left. + \log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \left(\frac{\lambda}{1+z}\right) F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right)\right) \\
&= 2.5\log\left(\frac{P'_x}{P'_y}\right) + 5\log(1+z) \\
&\quad + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&\quad - P_x + P_y.
\end{aligned} \tag{12}
$$

CONSEQUENCES

The modern version of the K-correction equation was presented by Kim *et al.* [8]:

$$
\begin{aligned}
K_{xy} = &-2.5\log\left(\frac{\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) \\
&+ 2.5\log(1+z) \\
&+ 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \lambda F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \quad \text{(incorrect).}
\end{aligned} \tag{13}
$$

In order to fully compare Equation 12 against Equation 13, we need to convert the zero-point value P_x into an expression that describes the accumulation of flux $\mathcal{F}_x$. We start with Equation 6, set $m_x = 0$, and solve for the zero-point value P_x:

$$
\begin{aligned}
m_x &= -2.5\log(\mathcal{F}_x) + P_x \\
0 &= -2.5\log(\mathcal{F}_x) + P_x \\
P_x &= 2.5\log(\mathcal{F}_x).
\end{aligned} \tag{14}
$$

Next, we convert the measured flux $\mathcal{F}_x$ to energy flux F_x using Equation 5 before renaming $F(\lambda) = \mathcal{Z}(\lambda)$ to indicate that we are measuring the idealized zero-point flux.

$$
\begin{aligned}
P_x &= 2.5\log(\mathcal{F}_x(\lambda)) \\
&= 2.5\log\left(\int \lambda F(\lambda) S_x(\lambda) d\lambda\right) \\
&= 2.5\log\left(\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda\right).
\end{aligned} \tag{15}
$$

Combining this with Equation 12, we have

$$
\begin{aligned}
K_{xy} &= 5\log(1+z) \\
&\quad + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&\quad - P_x + P_y \\
&= 5\log(1+z) \\
&\quad + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&\quad - 2.5\log\left(\int \frac{\lambda}{hc} \mathcal{Z}(\lambda) S_x(\lambda) d\lambda\right) \\
&\quad + 2.5\log\left(\int \frac{\lambda}{hc} \mathcal{Z}(\lambda) S_y(\lambda) d\lambda\right) \\
&= -2.5\log\left(\frac{\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) \\
&\quad + 5\log(1+z) \\
&\quad + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right).
\end{aligned} \tag{16}
$$

Two differences with Equation 13 stand out:

- The second term is multiplied by 2.5 in Kim *et al.* [8], but is multiplied by 5 here. This is the manifestation of the error in Tolman [4] and leads to the conclusion that the expansion of the universe is accelerating.
- In our derivation, the third term has an extra $1+z$ expression. Based on an inspection of the SN(oo)py software package presented by Burns *et al.* [14] and the SNANA software package presented by Kessler *et al.* [15], this error is ignored and software package authors implement it as intended, not precisely as written.

CONCLUSIONS

As shown in Figure 1, when reported magnitudes are corrected by adding the magnitude $-2.5\log(1+z)$, there

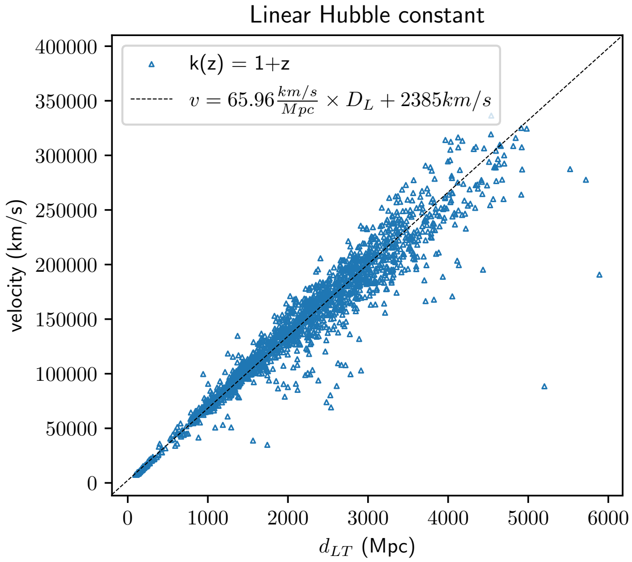

FIG. 3. The relationship between expansion velocity and light travel distance. The slope of this graph demonstrates the Hubble constant. The data for these SN Ia observations comes from the Dark Energy Survey [16], but the magnitudes are corrected to account for time dilation by adding $-2.5\log(1+z)$.

is a linear relationship between redshift and light travel distance. This is in accordance with the Hubble-Lemaître law. As opposed to the uncorrected values, there is no visual acceleration. Since the corrected values are approximately linear, we can use them to estimate the Hubble constant H_0 which is demonstrated in Figure 3.

The value estimated here, $H_0 = 65.94 \pm 1.3$, is consistent with estimates of H_0 that are based on the CMB. Planck-Collaboration *et al.* [17] published the CMB value of $H_0 = 67.27 \pm 0.6$, and the one sigma error bars overlap.

Since the K-correction error has corrupted all astronomical measurements that depend on both magnitude and redshift, a lot of data needs to be reanalysed. Cosmological parameters, such as those used by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric or the Λ-CDM model, can only be recalculated once the data is cleaned.

The reasons that the K-correction errors went undiscovered for so long should also be explored. Many hints existed that something was wrong, yet the problems persisted.

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