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Dark energy is based on a math error from 1930

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ABSTRACT

We show that the apparent magnitude of distant objects has been calculated incorrectly since 1930. We describe how this redshift correction flaw in the formula for K-corrections has propagated for over nine decades. We also re-derive the K-corrections equation. This derivation reveals that the correction for zero-points in different filters also has a flaw. We explore the consequences of properly correcting apparant magnitudes for redshift and conclude that this resolves two of the preeminent mysteries in cosmology: dark energy is not supported by observations of Type Ia supernovae, and the Hubble tension is due to a calculation error.

Keywords: Type Ia supernovae(1728) — Apparent magnitude(59)

1. INTRODUCTION

The measurement of Type Ia supernovae is one of the primary sources of data for cosmological models. The measurements involve estimating the peak magnitude in the B filter Riess et al. (1998) for the rest frame, meaning we want to know how bright the SnIa would be if its light did not undergo redshift. Redshift causes the light that would be measured by the B filter in a rest frame to be shifted to another filter. In order to observe a SnIa in an arbitrary filter and report the magnitude in the B filter we use a technique called K-corrections.

The first formal derivation of K-corrections was performed by Tolman (1930) and it failed to account for bandwidth stretching, one of the dimming effects of redshift. This error was noted by de Sitter (1934), but in Hubble & Tolman (1935) the discrepancy was noted and ignored. Oke & Sandage (1968) rederived K-corrections, but they failed to account for time dilation, one of the other dimming effects of redshift. In Kim et al. (1996), K-corrections were extended to handle additional cross-filter comparisons and address zero-point corrections, but they started their derivation by referencing the incorrect equation from Oke & Sandage (1968). The derivation in Kim et al. (1996) forms the basis for modern implementations of K-corrections. This history is explored in more depth in Section 2.

The effect of using an incorrect magnitude for SnIa is that we think objects are farther away than they really are, and the effect compounds for greater distances. Up until Riess et al. (1998), we didn't have distant enough observations for this error to matter much for cosmology models, but in the late 1990s, it was clear that our measurements for distance and redshift were not linear. This observation led to a model that utilized a cosmological constant and dark energy in order to explain the non-linear distance-redshift graph. We will show in Section 5 that correcting the flaw with K-corrections leads to a linear graph that does not need to rely on a cosmological constant.

Planck-Collaboration et al. (2020) used an alternative technique to compute the Hubble constant that is based on measurements of the cosmic microwave background. CMB based computations of $H0$ has made it evident that something was missing with our understanding of cosmology because this measurement is incompatible with the Hubble constant when measured from SnIa. As we will discuss in Section 5, fixing the error with K-corrections produces a measurement of the Hubble constant that is compatible with models based on the cosmic microwave background.

2. A BRIEF HISTORY OF K-CORRECTIONS

Observations of Type Ia supernova are essential to calibrate models that measure redshift and estimate distance. This relationship, often called the Hubble-Lemaître law, describes how quickly the universe is expanding. However, Riess

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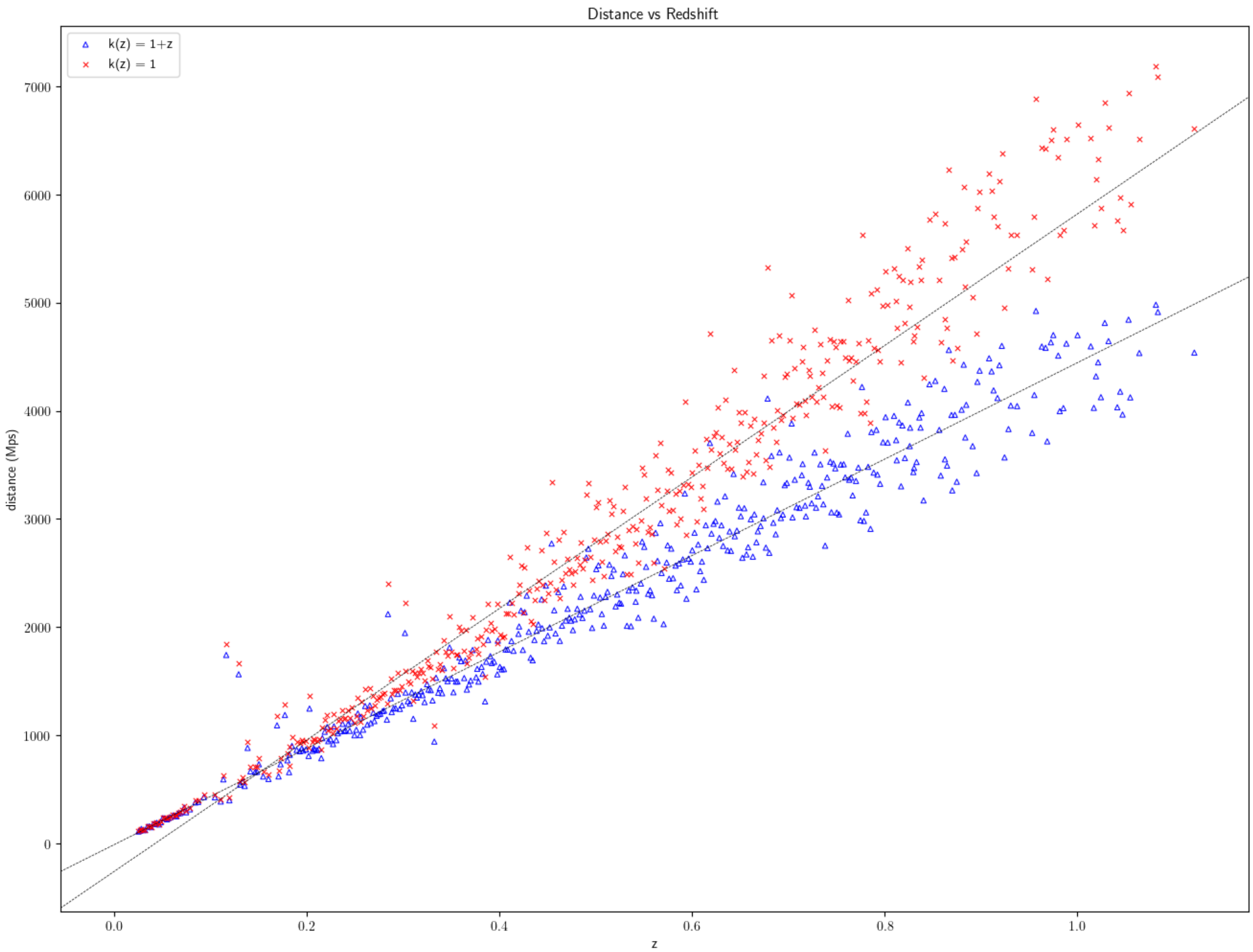

Figure 1. The relationship between distance and redshift for two treatments of magnitude data. The displayed points are roughly a third of the values in the full SnIa dataset published in DES-Collaboration et al. (2024), selected evenly to aid visibility. The $k(z) = 1$ treatment, which represents the uncorrected values, is clearly non-linear while the $k(z) = 1 + z$ treatment which fixes the K-correction redshift error, appears to be linear.

et al. (1998) and Perlmutter et al. (1999) presented evidence that there isn't a linear relationship between redshift and distance, but instead, distant objects are farther away than their redshift would predict (see Figure 1). This phenomenon implies that the acceleration of the universe is faster today than it was for old observations. Previously the cause of this phenomenon was unknown and was referred to as dark energy.

Type Ia supernovae are used to explore the relationship between distances and redshifts because these events always happen in similar ways, so the absolute brightness is roughly always the same. An analogy is to imagine someone walking in the dark and lighting matches. As long as we know how brightly a match burns at a known distance, we can estimate the distance to any match by measuring the apparent brightness before applying some geometry.

However, redshifting light dims an observation in three ways:

- The energy of a wave is inversely proportional to its wavelength, so a redshift of z means the amount of energy per photon is reduced by a factor of $\frac{1}{1+z}$.

- The bandwidth is stretched out. If a rest frame would observe photons between teh wavelengths of 400nm and 401nm, then with a redshift of $z = 1$, redshifting these photons will spread them across the wavelengths from 800nm to 802nm.

- Cosmological time dilation reduces the rate at which photons arive.

To complicate matters, we use the B-band magnitude to calculate luminosity distance. We want to know how bright a supernova appears for wavelengths around 445nm as if no redshift had occured. However, we typically need to observe the supernova with a filter that is sensitive to longer wavelengths, such as the i-band filter which is sensitive to wavelengths from 700nm-850nm (Flaugher et al. 2015). The K-corrections formula allows us to take a magnitude measured in an observation filter y and compute the magnitude in the target filter y.

The first mathematical treatments of K-corrections was performed by Tolman (1930). However, when Tolman made his derivation, he did not consider the effects of a spectra that is stretched out due to redshift. See the top two panels of Figure 2 for an example of this effect.

A few years later, de Sitter (1934), discussed all three issues that reduce the observed magnitude of a distant observation. The correction for each of these issues is identical: take a measurement for luminosity and multiply it by the factor $1 + z$.

A year later, Hubble & Tolman (1935) published a similar set of calculations for K-corrections, but these equations used $(1 + z)^2$ instead of the $(1 + z)^3$ correction term used by de Sitter. They started their derivation by copying the incorrect equation from 1930. After their derivation, they noted,

> It should be specially noted that this expression differs from the correction to m proposed by de Sitter, which contains the term $(1 + z)^3$ instead of $(1 + z)^2$. Expression (28), however, would seem to give the proper correction to use in connection with our equation (21), since it has been derived in such a way as to make appropriate allowance, first, for the double effect of nebular recession in reducing both the individual energy and the rate of arrival of photons, and then for the further circumstance that a change in spectral distribution of the energy that does arrive will lead to changes in its photographic effectiveness.

The Hubble K-corrections with the incorrect correction term have been used ever since.

By Oke & Sandage (1968), the two factors of $(1 + z)$ were attributed to the change in energy and to the spectral bandwidth elongation, which leaves time dilation as the factor that was omitted. A graph that demonstrates why it's essential to correct for both the spectra bandwidth warping and time dilation is presented in Figure 2.

The modern treatment of K-corrections is based on the work of Kim et al. (1996). This work extended the calculations of K-corrections to extend to filters beyond B and V. It also introduced a term that deals with the zero-point for the actual filters. In the modern day, filters measure the photon flux as opposed to the energy flux. Historically, bolometric devices would measure the energy flux, but modern CCD cameras effectively measure the photon flux.

Quoting Kim et al. (1996):

> Therefore, the correct K correction calculation to be used with measured photometric magnitudes is the integral photon counts:

$$K_{xy} = -2.5\log\left(\frac{\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) + 2.5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \lambda F(\lambda/(1+z)) S_y(\lambda)) d\lambda}\right). \quad (1)$$

This equation has three errors, which we will explore in Section 4.

3. DERIVATION OF K-CORRECTIONS

With modern CCD cameras, a telescope observation consists of a single value $\mathcal{F}_x$ erg/s, which represents the energy collected in filter x per second. A summary of how this works is provided by Lesser (2015). The measured energy is produced by electrons not photons, so the measured number is proportional to the number of photons. However, we need to calculate the expected number of photons collected by using a spectral energy density function F. The value $F(\lambda)$ gives the amount of energy collected by a bolometric device for the wavelength λ.

To convert the spectral energy density F to the photon density F', we need to use the Plank relation $E = \frac{hc}{\lambda}$ where E is the energy, h is Planck's constant, and c is the speed of light. This gives us

$$\begin{aligned} F(\lambda) &= F'(\lambda) \times \frac{hc}{\lambda} \\ F'(\lambda) &= \frac{\lambda F(\lambda)}{hc}. \end{aligned} \quad (2)$$

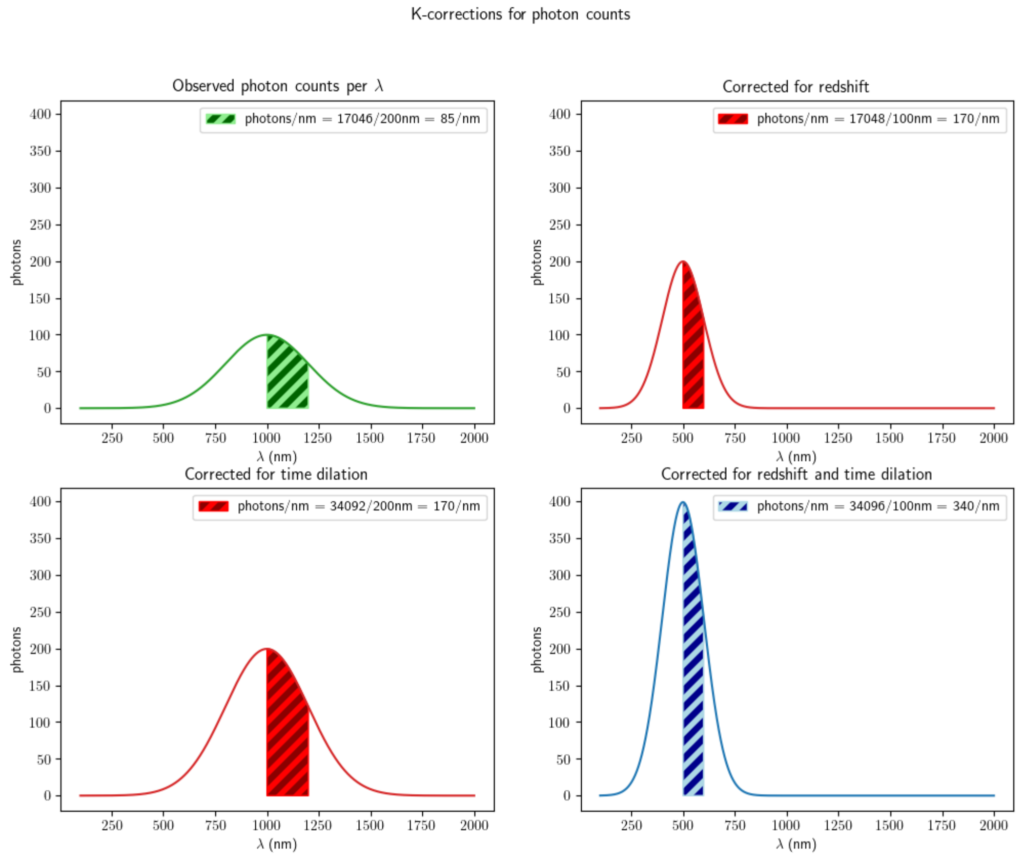

Figure 2. An example of how K-corrections take an observed magnitude and produce a rest frame magnitude. In this example, $z = 1$. The observation filter measures magnitude, which is equivalent to measuring the number of photons per nanometer. In the bottom left panel, a correction of $1 + z$ is applied which doubles the photons per nanometer. The top right panel shows the effect of correcting for the bandwidth spectrum warping effect – correcting all wavelengths for the rest frame means that the measured wavelengths have blueshifted, ideally into the cross band B filter. Applying both effects, shown in the bottom right panel, requires applying two correction factors of $1 + z$. This example omits the correction for a similar effect related to lower energy due to the Planck relation because CCD cameras obviate the need to correct for this effect.

It's important to note that for the blueshifted wavelength $\lambda/(1 + z)$, this equation produces

$$F'\left(\frac{\lambda}{1+z}\right) = \frac{\lambda}{(1+z)hc} F\left(\frac{\lambda}{1+z}\right). \tag{3}$$

However, this equation is misleading and error prone. We will want to use it to help calculate the amount of flux in an observation filter at the redshifted wavelength $\lambda \times (1 + z)$. In other words, we want to produce the photon density function R' that is the redshifted version of F'. Redshifting the photon density does two things:

- The stretching of space increases the distance between photons while they are traveling. This phenomenon appears to an observer like time dilation, although cosmological time dilation is due to a different mechanism than relativistic time dilation. This effect reduces the photon arrival rate by a factor of $1/(1 + z)$.

  In order to account for cosmological time dilation, we will need to multiply F' by $1/(1 + z)$.

- All wavelengths are changed by a factor of $1 + z$. When we integrate R' from wavelength λ_a to wavelength λ_b, the values correspond to the wavelengths $\lambda_a/(1+z)$ to $\lambda_b/(1+z)$ in F'. We will integrate over a width of $\lambda_b - \lambda_a$, but $F'(\lambda/(1 + z))$ will refer to values in a width of $(\lambda_b - \lambda_a)/(1 + z)$.

  In order to account for this bandwidth spectral warping effect, we will need to multiply F' by a second factor of $1/(1 + z)$.

Combining these two phenomena together, we can calculate the redshifted spectral energy density R using

$$\begin{aligned} R'(\lambda) &= F'\left(\frac{\lambda}{1+z}\right) \times \frac{1}{(1+z)^2} \\ R(\lambda) &= \frac{\lambda}{(1+z)hc} \times F\left(\frac{\lambda}{1+z}\right) \times \frac{1}{(1+z)^2}. \end{aligned} \tag{4}$$

We deliberately do not combine all of the factors of $1+z$ together because this form is more natural to implement in code.

In order to calculate the amount of flux $\mathcal{F}_x$ measured in filter x, we need to compute the photon density $F'(\lambda)$ and multiply it by sensitivity $S_x(\lambda)$, which represents the proportion of photons filter x will measure at wavelength λ. We then need to sum over all wavelengths, which is expressed with the equation

$$\begin{aligned} \mathcal{F}_x &= \int F'(\lambda) S_x(\lambda) d\lambda \\ &= \int \lambda F(\lambda) S_x(\lambda) d\lambda. \end{aligned} \tag{5}$$

The limits of integration are technically from 0 to ∞, but these are usually not written because the sensitivity $S(\lambda)$ is 0 for wavelengths outside of a filter's bandpass.

We will also use the energy flux $\mathcal{F}$ to magnitude m formula:

$$\begin{aligned} m_x &= -2.5\log(\mathcal{F}_x) + P_x \\ -2.5\log(\mathcal{F}_x) &= m_x - P_x. \end{aligned} \tag{6}$$

P_x represents the zero-point for the filter x on some particular telescope. In order to use consistent magnitude values across telescopes that have different light gathering abilities, we take the measured magnitude and multiply it by the ratio of the standard flux rate to the flux rate for this particular telescope and filter. For convenience, we use $P_x = -2.5\log(P'_x)$ so that we can work with flux instead of with magnitude.

Now that we have the identities in Equations 5 and 6 we will change directions and look at the definition of K-corrections K_{xy}. This value allows us to make an observation in filter y and report what the magnitude would have been in filter x if no redshift occured:

$$\begin{aligned} m_y &= M_x + \mu + K_{xy} \\ &= M_x + m_x - M_x + K_{xy} \\ &= m_x + K_{xy} \\ m_x &= m_y - K_{xy}. \end{aligned} \tag{7}$$

The second line of Equation 7 expands the distance modulus μ using $\mu = m - M$ where m is the observed magnitude and M is the absolute magnitude.

Since the K-correction is a magnitude value and we wish to work on flux values, it is convenient to define the following substitution:

$$K_{xy} = 2.5\log(K'_{xy}). \tag{8}$$

Note that this substitution omits the minus $(-)$ sign that we used on a similar substitution for P_x.

Starting with Equation 6 and then recombining the flux term $\mathcal{F}_y$ with m_y from Equation 7, we have

$$
\begin{aligned}
-2.5\log(\mathcal{F}_x) &= m_x - P_x \\
&= m_y - K_{xy} - P_x \\
&= -2.5\log(\mathcal{F}_y) + P_y - K_{xy} - P_x \\
&= -2.5\log(\mathcal{F}_y) - 2.5\log(P'_y) - 2.5\log(K'_{xy}) + 2.5\log(P_x) \\
&= -2.5\left(\log(\mathcal{F}_y) + \log(K'_{xy}) + \log(P'_y) - \log(P'_x)\right) \\
&= -2.5\log\left(\mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}\right) \\
\mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}.
\end{aligned} \tag{9}
$$

It's useful to inspect this equation to consider whether the term $\frac{P'_y}{P'_x}$ is correct, or if it might be flipped to the inverse of what it should be. However, if we rewrite Equation 9 as

$$
\mathcal{F}_x \times P'_x = \mathcal{F}_y \times P'_y \times K'_{xy} \tag{10}
$$

then the meaning is a bit more clear. The left hand side $\mathcal{F}_x \times P'_x$ is the rate of photon collection for an idealized telescope in filter x while $\mathcal{F}_y \times P'_y$ is the rate of photon collection for an idealized telescope in filter y. We are able to observe the value $\mathcal{F}_y \times P'_y$ and want to use the fudge factor K'_{xy} to produce the value $\mathcal{F}_x \times P'_x$.

We can isolate K'_{xy} and then use Equation 5 to expand

$$
\begin{aligned}
\mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x} \\
K'_{xy} &= \frac{P'_x}{P'_y} \times \frac{\mathcal{F}_x}{\mathcal{F}_y}.
\end{aligned} \tag{11}
$$

Now we use Equations 4 and 5 to calculate the fluxes $\mathcal{F}_x$ and $\mathcal{F}_y$ in terms of the spectral energy density function F. Note that $\mathcal{F}_x$ uses the rest frame spectral energy density F while $\mathcal{F}_y$ uses the redshifted spectral energy density $R(\lambda)$:

$$
\begin{aligned}
K'_{xy} &= \frac{P'_x}{P'_y} \times \frac{\mathcal{F}_x}{\mathcal{F}_y} \\
&= \frac{P'_x}{P'_y} \times \frac{\int F'(\lambda) S_x(\lambda) d\lambda}{\int R'(\lambda) S_y(\lambda) d\lambda} \\
&= \frac{P'_x}{P'_y} \times \frac{\int F'(\lambda) S_x(\lambda) d\lambda}{\int F'(\frac{\lambda}{1+z}) \times \frac{1}{(1+z)^2} S_y(\lambda) d\lambda} \\
&= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \frac{\lambda}{hc} F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{(1+z)hc} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda} \\
&= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}.
\end{aligned} \tag{12}
$$

Finally, we can use Equation 8 to convert the flux K'_{xy} back into the magnitude K_{xy}:

$$
\begin{aligned}
K_{xy} &= 2.5\log(K'_{xy}) \\
&= 2.5\log\left(\frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&= 2.5\left(\log\left(\frac{P'_x}{P'_y}\right) + \log((1+z)^2) + \log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right)\right) \\
&= 2.5\log\left(\frac{P'_x}{P'_y}\right) + 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) \\
&= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) - P_x + P_y.
\end{aligned}
\tag{13}
$$

4. CONSEQUENCES

In order to fully compare our derivation against Equation 1, we need to use the identity

$$
P = -2.5\log\left(\int \frac{\lambda}{hc} \mathcal{Z}(\lambda) S(\lambda) d\lambda\right) \tag{14}
$$

where, according to Kim et al. (1996), "$\mathcal{Z}(\lambda)$ is an idealized spectral energy distribution at $z = 0$ for which $U = B = V = R = I = 0$ in the photometric system being used." Combining this with Equation 13, we have

$$
\begin{aligned}
K_{xy} &= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) - P_x + P_y \\
&= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right) + 2.5\log\left(\int \frac{\lambda}{hc} \mathcal{Z}(\lambda) S_x(\lambda) d\lambda\right) - 2.5\log\left(\int \frac{\lambda}{hc} \mathcal{Z}(\lambda) S_y(\lambda) d\lambda\right) \\
&= 2.5\log\left(\frac{\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) + 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \frac{\lambda}{1+z} F\left(\frac{\lambda}{1+z}\right) S_y(\lambda) d\lambda}\right).
\end{aligned}
\tag{15}
$$

Three differences with Equation 1 stand out:

- The sign is different for the first term. This will have a subtle but frustrating effect for real measurements. Typically the zero-point values are similar, so this term is close to 0. However, the term isn't always zero. As we observe more distant supernovae, it is useful to switch our observational filter to a filter that is sensitive to longer wavelengths. Each observation filter will have its own small bias.

  This effect by itself can alter estimates of H_0 by several km/s / Mpc. While we present an estimate of the Hubble constant in Figure 4, we aren't terribly confident in the numbers because we were unable to correct for the zero-point correction errors or verify that these errors don't exist.

- The second term is multiplied by a 2.5 in Kim et al. (1996) but is multiplied by a 5 here. This is the manifestation of the error in Tolman (1930).

- In our derivation, the third term is has an extra $1 + z$ expression. Based on an inspection of the SN(oo)py software package presented by Burns (2024), this error is ignored and software package authors implement it as the expression is intended, not precisely as the expression is written.

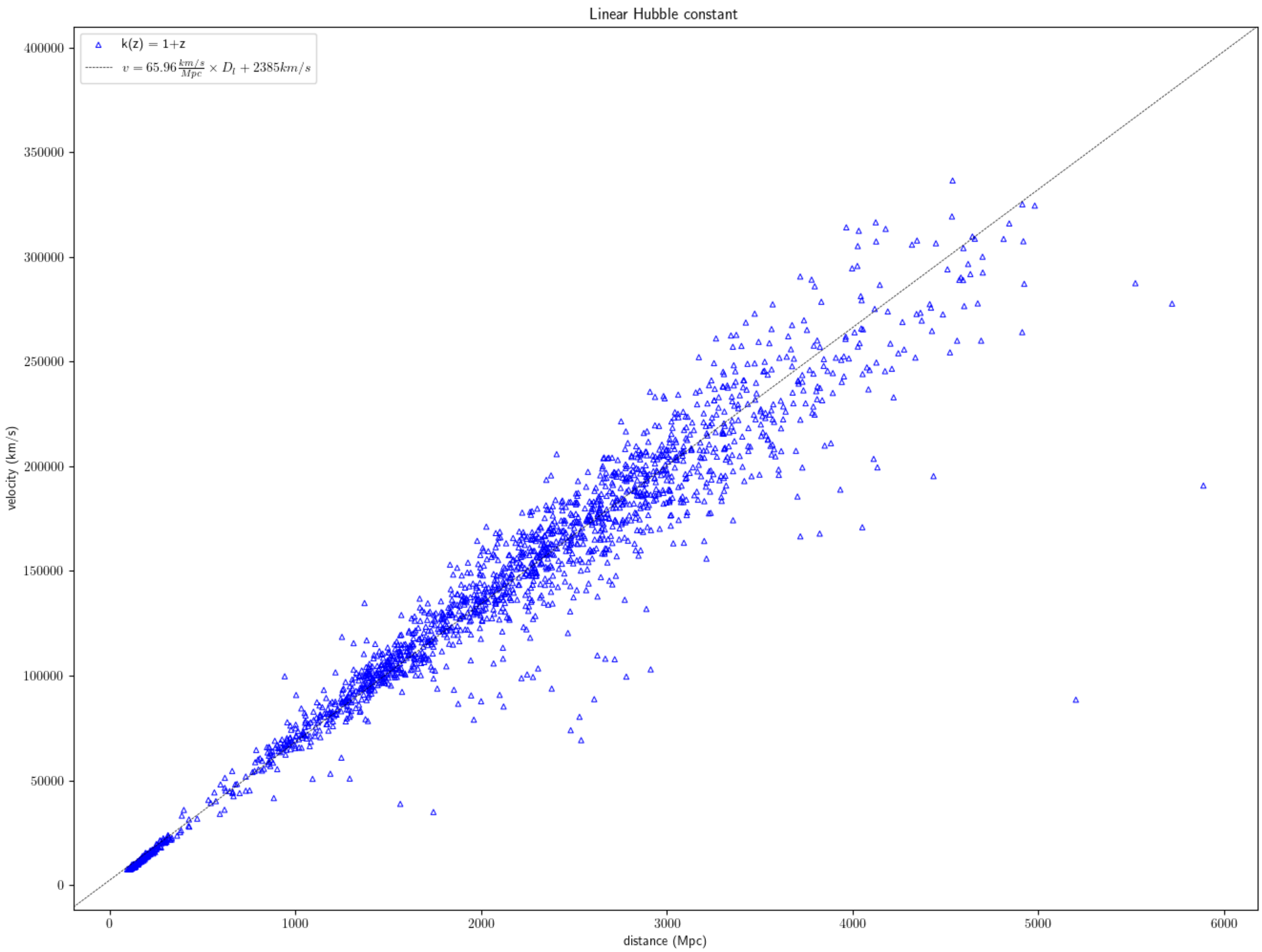

Figure 3. The relationship between expansion velocity and distance. The slope of this graph demonstrates the Hubble constant.

5. DARK ENERGY AND THE HUBBLE TENSION

As shown in Figure 1, when reported magnitudes are corrected by adding the magnitude $-2.5\log(1+z)$, there is a linear relationship between redshift and luminosity distance. This is in accordance with the Hubble-Lemaître law. As opposed to the uncorrected values, there is no visual acceleration.

Since the corrected values are approximately linear, we can use them to estimate the Hubble constant $H0$. This is demonstrated in Figure 3. Furthermore, we performed a bootstrap calculation to estimate the distribution of $H0$ estimates in Figure 4.

The value estimated here, $H0 \sim 65.94 \pm 1.29$ is highly suspicious because of the flipped zero-point corrections, but it is consistent with estimates of $H0$ that are based on the CMB. Planck-Collaboration et al. (2020) published the CMB value of $H0 \sim 67.27 \pm 0.6$, and the one sigma error bars overlap. It is interesting to note that restricting the DES observations to only those with $z < 0.1$, which presumably is a small enough redshift that all observations will have been made with the same filter, gives the estimate $H0 \sim 67.8 \pm 1.86$.

6. CONCLUSIONS

A lot of data is going to have to be reanalyzed.

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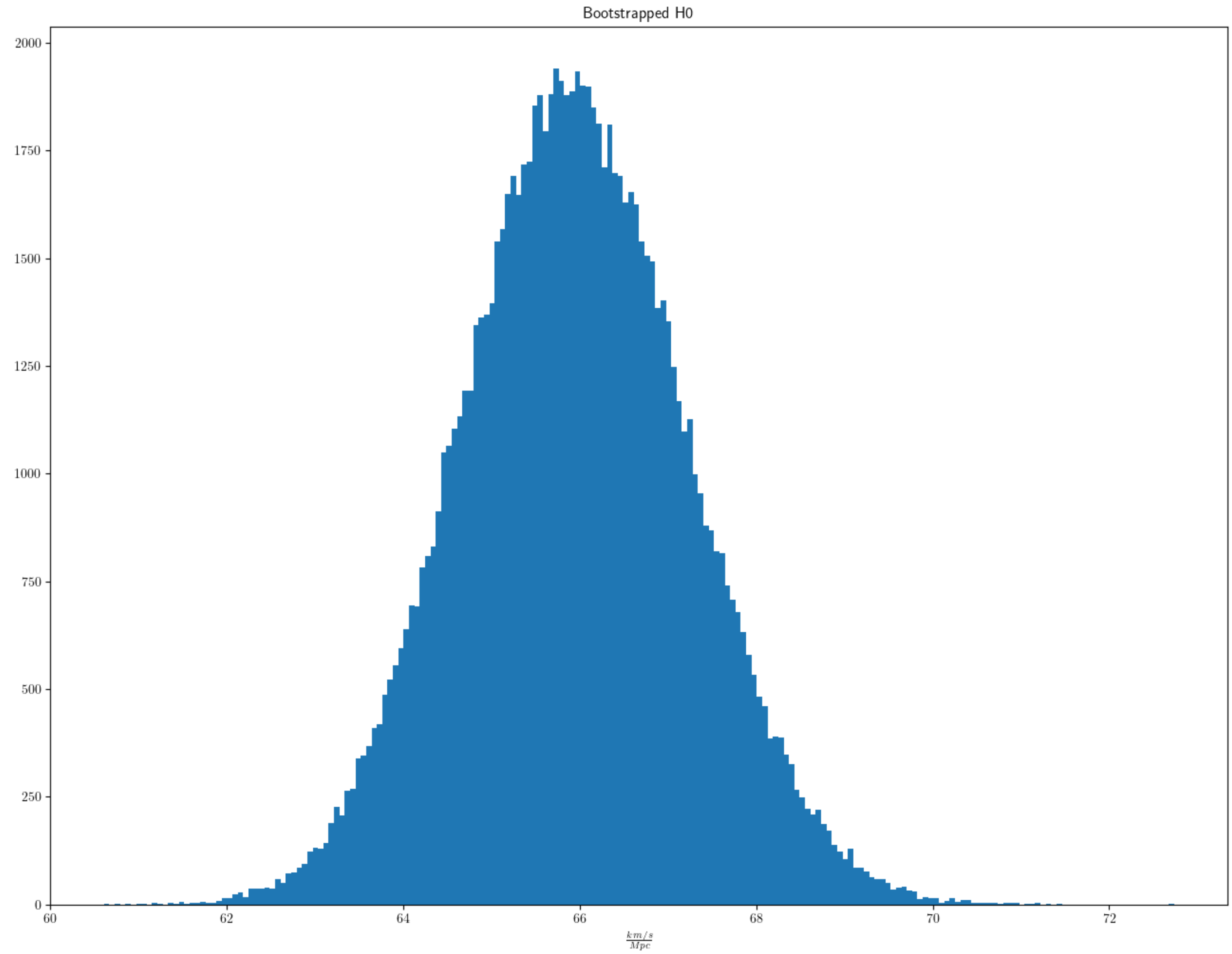

Figure 4. A histogram of 100000 bootstrap trials measuring the Hubble constant $H0$. Each trial samples an absolute Type Ia magnitude $M \sim Norm(-19.2334, 0.0404)$ based on data published by Camarena & Marra (2020). It then samples, with replacement, a population of supernovae from the dataset published by DES-Collaboration et al. (2024). Finally, it uses the non-parametric linear regression technique described by Siegel (1982). The result of the bootstrap is $H0 \sim Norm(65.94, 1.29^2)$.

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