DARK ENERGY IS BASED ON A MATH ERROR FROM 1930

A PREPRINT

Logan P. Evans
loganpevans@gmail.com

December 27, 2024

ABSTRACT

We show that the magnitude of distant objects has been calculated incorrectly since 1930. We describe how this math error has propagated for over nine decades. Finally, we explore the consequences of correcting this flaw and resolve two of the preeminent mysteries in cosmology: dark energy is not supported by observations of Type Ia supernovae, and the Hubble tension is due to a calculation error.

Keywords Cosmological Parameters · Dark Energy · Luminosity Distance · Hubble tension

1 Introduction

The measurement of Type Ia supernovae is one of the primary sources of data for cosmological models. The measurements involve estimating the peak magnitude in the B filter Riess et al. [1998] for the rest frame, meaning we want to know how bright the SnIa would be if its light didn't undergo redshift. Redshift causes the light that would be mesaured by the B filter in a rest frame to be shifted to another filter. In order to observe a SnIa in an arbitrary filter and report the magnitude in the B filter we use a technique called K-corrections.

The first formal derivation of K-corrections was performed by Tolman [1930], and it failed to account for bandwidth stretching, one of the dimming effects of redshift. This error was noted by de Sitter [1934], but in Hubble and Tolman [1935] the descrepancy was noted and ignored. Oke and Sandage [1968] rederived K-corrections, but they failed to account for time dilation, one of the other dimming effects of redshift. In Kim et al. [1996], K-corrections were extended to handle additional cross-filter comparisons and address zero-point corrections, but they started their derivation by referencing the incorrect equation from Oke and Sandage [1968]. Hsiao et al. [2007] expanded on the work done by Kim et al. [1996] by providing expected SnIa template spectra. Finally, Burns et al. [2010] implemented the popular software package Sn(OO)py which implemented the SnIa template spectra into K-corrections. This software package computes K-corrections differently from prior work, and in order to match prior values, the code adds an extra term. This history is explored in more depth in Section 2.

The effect of using an incorrect magnitude for SnIa is that we think objects are farther away than they really are, and the effect componds for greater distances. Up until Riess et al. [1998], we didn't have distant enough observations for this error to matter much for cosmology models, but in the late 1990s, it was clear that our measurements for distance and redshift were not linear. This observation led to a model that utilized a cosmological constant and dark energy in order to explain the non-linear distance-redshift graph. We will show in Section 6 that correcting the flaw with K-corrections leads to a linear graph that does not need to rely on a cosmological constant.

Since Aghanim et al. [2020], it has been evident that something was missing with our understanding of cosmology. This research used an alternative technique based on measurements of the cosmic microwave background to measure the Hubble constant, but as summarized in Perivolaropoulos and Skara [2022], this measurement was incompatable with the Hubble constant measured from SnIa. As we will discuss in Section 8, fixing the error with K-corrections produces a measurement of the Hubble constant that is compatible with models based on the cosmic microwave background.

2 A brief history of K-corrections

Observations of Type Ia supernova are essential to calibrate models that measure redshift and estimate distance. This relationship, often called the Hubble-Lemaître law, describes how quickly the universe is expanding. However, Riess et al. [1998] and Perlmutter et al. [1999] presented evidence that there isn't a linear relationship between redshift and distance, but instead, distant objects are farther away than their redshift would predict (see Figure 1). This phenomenon implies that the acceleration of the universe is faster today than it was for old observations. Previously the cause of this phenomenon was unknown and was referred to as dark energy.

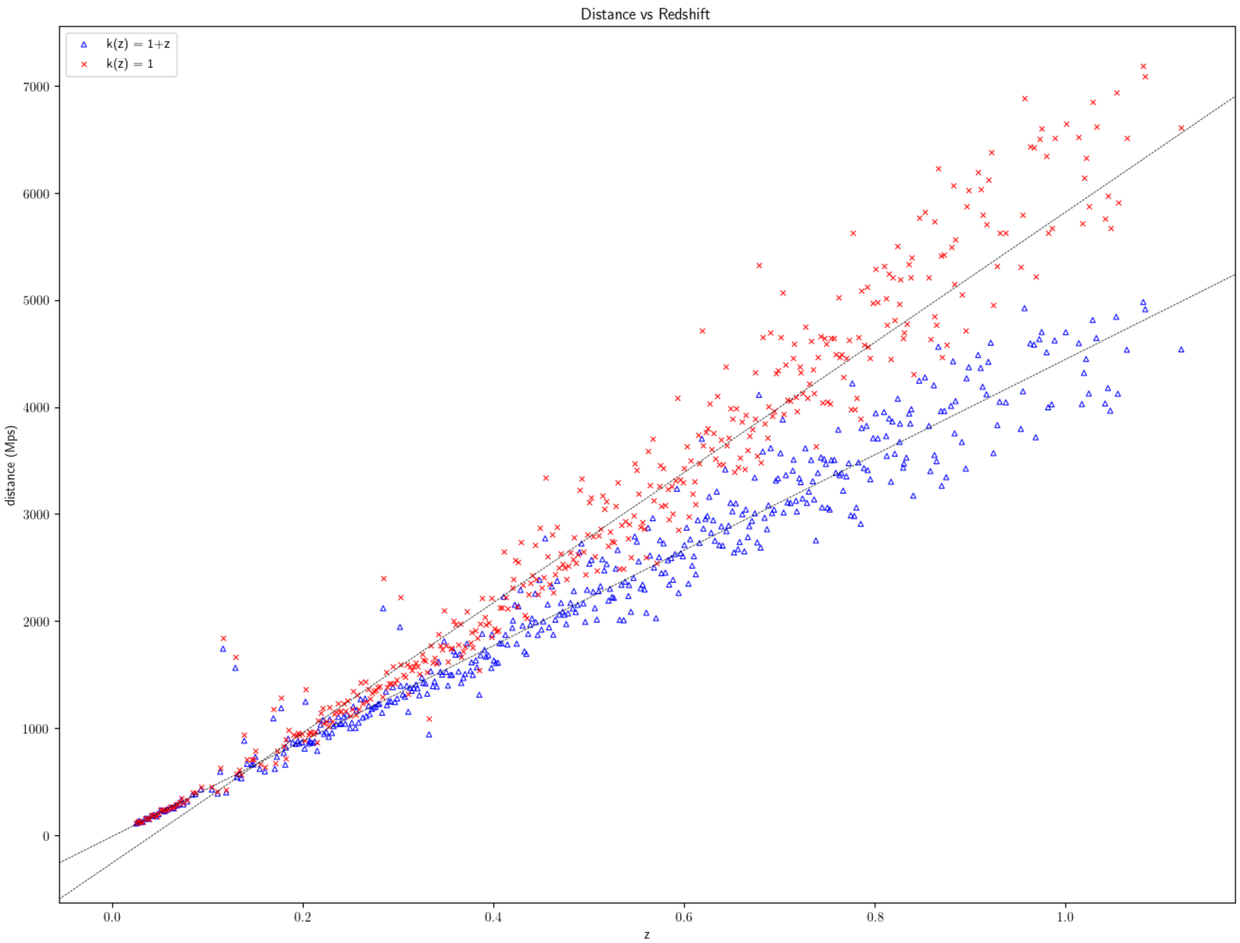

Figure 1: The relationship between distance and redshift for two treatments of magnitude data. The displayed points are roughly a third of the values in the full DES dataset, selected evenly to aid visibility. The $k(z) = 1$ treatment is clearly non-linear while the $k(z) = 1 + z$ treatment appears to be linear.

Type Ia supernova are used to explore the relationship between distances and redshifts because these events always happen in similar ways, so the absolute brightness is roughly always the same. An analogy is to imagine someone walking in the dark and lighting matches. As long as we know how brightly a match burns at a known distance, we can estimate the distance to any match by measuring the appearant brightness before applying some geometry.

Measuring the apperant brightness of a Type Ia supernova is non-trivial. The aspect of the process that we will explore here concerns how redshift affects the light we observe. This problem is often referred to as K-corrections, and one of the first mathematical treatments of the problem was performed by Tolman [1930]. However, when Tolman made his derivation, he did not consider the effects of a spectra that is stretched out due to redshift.

A few years later, de Sitter [1934], discussed all three issues that reduce the observed magnitude of a distant observation. The correction for each of these issues is identical: take a measurement for luminosity and multiply it by the factor $1 + z$.

A year later, Hubble and Tolman [1935] published a similar set of calculations for K-corrections, but these equations used $(1+z)^2$ instead of the $(1+z)^3$ correction term used by de Sitter. They started their derivation by copying the incorrect equation from 1930. After their derivation, they noted,

> It should be specially noted that this expression differs from the correction to m proposed by de Sitter, which contains the term $(1+z)^3$ instead of $(1+z)^2$. Expression (28), however, would seem to give the proper correction to use in connection with our equation (21), since it has been derived in such a way as to make appropriate allowance, first, for the double effect of nebular recession in reducing both the individual energy and the rate of arrival of photons, and then for the further circumstance that a change in spectral distribution of the energy that does arrive will lead to changes in its photographic effectiveness.

The Hubble K-corrections with the incorrect correction term have been used ever since.

By Oke and Sandage [1968], the two factors of $(1+z)$ were attributed to the change in energy and to the spectral bandwidth elongation, which leaves time dilation as the factor that was omitted. A graph that demonstrates why it's essential to correct for both the spectra bandwidth warping and time dilation is presented in Figure 2.

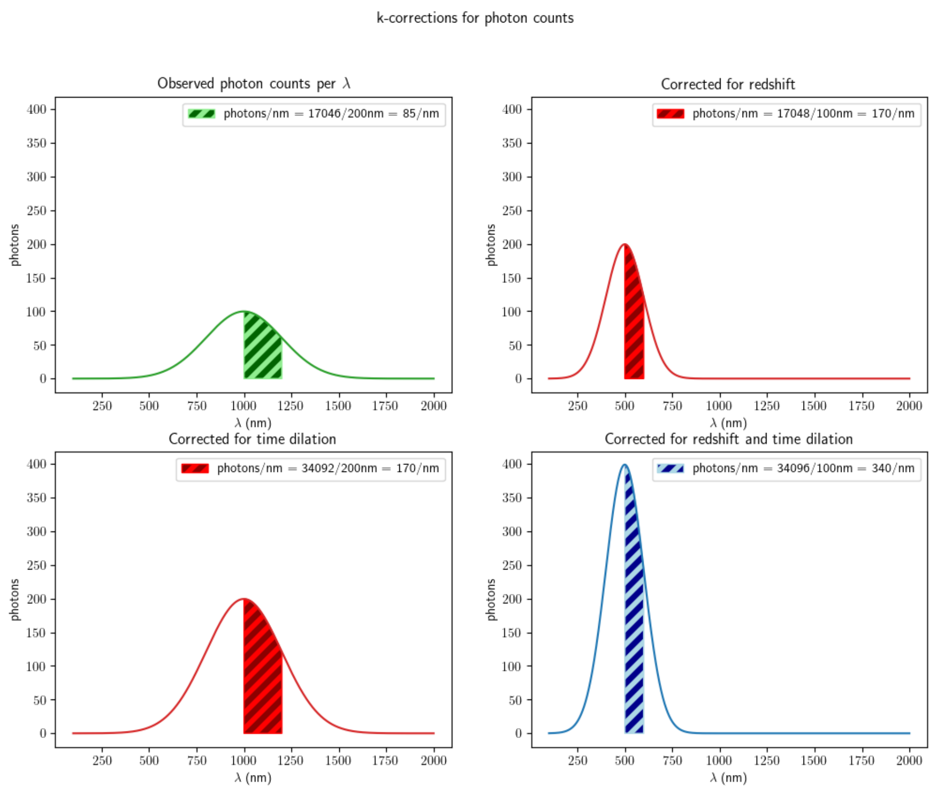

Figure 2: An example of how K-corrections take an observed magnitude and produce a rest frame magnitude. In this example, $z = 1$. The observation filter measures magnitude, which is equivalent to measuring the number of photons per nanometer. In the bottom left panel, a correction of $1+z$ is applied which doubles the photons per nanometer. The top right panel shows the effect of correcting for the bandwidth spectrum warping effect – correcting all wavelengths for the rest frame means that the measured wavelengths have blueshifted, ideally into the cross band B filter. Applying both effects, shown in the bottom right panel, requires applying two correction factors of $1+z$. This example omits the correction for a similar effect related to lower energy due to the Planck relation because CCD cameras obviate the need to correct for this effect.

The modern treatment of K-corrections is based on the work of Kim et al. [1996]. This work extended the calculations of K-corrections to extend to filters beyond B and V. It also introduced a term that deals with the zero-point for the actual filters. In the modern day, filters measure the photon flux as upposed to the energy flux. Historically, bolometric devices would measure the energy flux, but modern CCD cameras effectively measure the photon flux.

Quoting Kim et al. [1996]:

> Therefore, the correct K correction caluculation to be used with measured photometric magnitudes is the integral photon counts:
>
> $$K_{xy} = -2.5\text{log}\left(\frac{\int \lambda \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \lambda \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) + 2.5\text{log}(1+z) + 2.5\text{log}\left(\frac{\int \lambda F(\lambda) S_x(\lambda) d\lambda}{\int \lambda F(\lambda/(1+z)) S_y(\lambda)) d\lambda}\right). \tag{1}$$

This equation has three errors, which we will explore in Section 4.

3 Derivation of K-corrections

With modern CCD cameras, a telescope observaion consists of a single value $\mathcal{F}_x$ erg/s, which represents the energy collected in filter x per second. A summary of how this works is provided by Lesser [2015]. The measured energy is produced by electrons not photons, so the measured number is proportional to the number of photons. However, we need to calculate the expected number of photons collected by using a spectral energy density function F. The value $F(\lambda)$ gives the amount of energy collected by a bolometric device for the wavelength λ.

To convert the spectral energy density F to the photon density F', we need to use the Plank relation $E = \frac{hc}{\lambda}$ where E is the energy, h is Planck's constant, and c is the speed of light. This gives us

$$\begin{aligned} F(\lambda) &= F'(\lambda) \times \frac{hc}{\lambda} \\ F'(\lambda) &= \frac{\lambda F(\lambda)}{hc}. \end{aligned} \tag{2}$$

It's important to note that for the blueshifted wavelength $\lambda/(1+z)$, this equation produces

$$F'\left(\frac{\lambda}{1+z}\right) = \frac{\left(\frac{\lambda}{1+z}\right) F\left(\frac{\lambda}{1+z}\right)}{hc}. \tag{3}$$

However, this equation is misleading and error prone. We will want to use it to help calculate the amount of flux in an observation filter at the redshifted wavelength $\lambda \times (1+z)$. In other words, we want to produce the photon density function R' that is the redshifted version of F'. Redshifting does two things:

- The stretching of space increases the distance between photons while they are traveling. This phenomenon appears to an observer like time dilation, although cosmological time dilation is due to a different mechanism than relativistic time dilation. This effect reduces the photon arrival rate by a factor of $1/(1+z)$.
  In order to account for cosmological time dilation, we will need to multiply F' by $1/(1+z)$.
- All wavelengths are changed by a factor of $1+z$. When we integrate R' from wavelength λ_a to wavelength λ_b, the values correspond to the wavelengths $\lambda_a/(1+z)$ to $\lambda_b/(1+z)$ in F'. We will integrate over a width of $\lambda_b - \lambda_a$, but $F'(\lambda/(1+z))$ will refer to values in a width of $(\lambda_b - \lambda_a)/(1+z)$.
  In order to account for this bandwidth spectral warping effect, we will need to multiply F' by a second factor of $1/(1+z)$.

Combining these two phenomena together, we can calculated the redshifted spectral energy density R using

$$\begin{aligned} R'(\lambda) &= F'(\lambda/(1+z)) \times \frac{1}{(1+z)^2} \\ R(\lambda) &= \frac{F(\lambda/(1+z))}{(1+z)hc} \times \frac{1}{(1+z)^2}. \end{aligned} \tag{4}$$

In order to calculate the amount of flux $\mathcal{F}_x$ measured in filter x, we need to compute the photon density $F'(\lambda)$ and multiply it by sensity $S_x(\lambda)$, which represents the proportion of photons filter x will measure at wavelength λ. We then need to sum over all wavelengths, which is expressed with the equation

$$
\begin{aligned}
\mathcal{F}_x &= \int F'(\lambda) S_x(\lambda) d\lambda \\
&= \int \lambda F(\lambda) S_x(\lambda) d\lambda.
\end{aligned} \tag{5}
$$

The limits of integration are techinically from 0 to ∞, but these are usually not written because the sensitivity $S(\lambda)$ is 0 for wavelengths outside of a filter's bandpass.

We will also use the energy flux $\mathcal{F}$ to magnitude m formula:

$$
\begin{aligned}
m_x &= -2.5\log(\mathcal{F}_x) + P_x \\
-2.5\log(\mathcal{F}_x) &= m_x - P_x.
\end{aligned} \tag{6}
$$

P_x represents the zero-point for the filter x on some particular telescope. In order to use consistant magnitude values across telescopes that have different light gathering abilities, we take the measured magnitude and multiply it by the ratio of the standard flux rate to the flux rate for this particular telescope and filter. For convenience, we use $P_x = -2.5\log(P'_x)$ so that we can work with flux instead of with magnitude.

Now that we have the identities in Equations 5 and 6 we will change directions and look at the definition of K-corrections K_{xy}. This value allows us to make an observation in filter y and report what the magnitude would have been in filter x if no redshift occured:

$$
\begin{aligned}
m_y &= M_x + \mu + K_{xy} \\
&= M_x + m_x - M_x + K_{xy} \\
&= m_x + K_{xy} \\
m_x &= m_y - K_{xy}.
\end{aligned} \tag{7}
$$

The second line of Equation 7 expands the distance modulus μ using $\mu = m - M$ where m is the observed magnitude and M is the absolute magnitude.

Since the K-correction is a magnitude value and we wish to work on flux values, it is convenient to define the following substitution:

$$
K_{xy} = 2.5\log(K'_{xy}). \tag{8}
$$

Note that this substitution omits the minus $(-)$ sign.

Starting with Equation 6 and then recombining the flux term $\mathcal{F}_y$ with m_y from Equation 7, we have

$$
\begin{aligned}
-2.5\log(\mathcal{F}_x) &= m_x - P_x \\
&= m_y - K_{xy} - P_x \\
&= -2.5\log(\mathcal{F}_y) + P_y - K_{xy} - P_x \\
&= -2.5\log(\mathcal{F}_y) - 2.5\log(P'_y) - 2.5\log(K'_{xy}) + 2.5\log(P_x) \\
&= -2.5\left(\log(\mathcal{F}_y) + \log(K'_{xy}) + \log(P'_y) - \log(P'_x)\right) \\
&= -2.5\log\left(\mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}\right) \\
\mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x}.
\end{aligned} \tag{9}
$$

It's useful to inspect this equation to consider whether the term $\frac{P'_y}{P'_x}$ is correct, or if it might be flipped to the inverse of what it should be. However, if we rewrite Equation 9 as

$$
\mathcal{F}_x \times P'_x = \mathcal{F}_y \times P'_y \times K'_{xy} \tag{10}
$$

then the meaning is a bit more clear. The left hand side $\mathcal{F}_x \times P'_x$ is the rate of photon collection for an idealized telescope in filter x while $\mathcal{F}_y \times P'_y$ is the rate of photon collection for an idealized telescope in filter y. We are able to observe the value $\mathcal{F}_y \times P'_y$ and want to use the fudge factor K'_{xy} to produce the value $\mathcal{F}_x \times P'_x$.

We can isolate K'_{xy} and then use Equation 5 to expand

$$\begin{aligned}\mathcal{F}_x &= \mathcal{F}_y \times K'_{xy} \times \frac{P'_y}{P'_x} \\ K'_{xy} &= \frac{\mathcal{F}_x P'_x}{\mathcal{F}_y P'_y}.\end{aligned} \tag{11}$$

Now we use Equations 4 and 5 to calculate the fluxes $\mathcal{F}_x$ and $\mathcal{F}_y$ in terms of the spectral energy density function F. Note that $\mathcal{F}_x$ uses the rest frame spectral energy density F while $\mathcal{F}_y$ uses the redshifted spectral energy density $R(\lambda)$:

$$\begin{aligned}K'_{xy} &= \frac{\mathcal{F}_x P'_x}{\mathcal{F}_y P'_y} \\ &= \frac{P'_x \int F'(\lambda) S_x d\lambda}{P'_y \int R'(\lambda) S_y d\lambda} \\ &= \frac{P'_x}{P'_y} \times \frac{\int F'(\lambda) S_x d\lambda}{\int F'(\lambda/(1+z)) \times \frac{1}{(1+z)^2} d\lambda} \\ &= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \frac{\lambda F(\lambda)}{hc} S_x d\lambda}{\int \frac{F(\lambda/(1+z))}{(1+z)hc} S_y d\lambda} \\ &= \frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}.\end{aligned} \tag{12}$$

Finally, we can use Equation 8 to convert the flux K'_{xy} back into the magnitude K_{xy}:

$$\begin{aligned}K_{xy} &= 2.5\log(K'_{xy}) \\ &= 2.5\log\left(\frac{P'_x}{P'_y} \times (1+z)^2 \times \frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right) \\ &= 2.5\left(\log\left(\frac{P'_x}{P'_y}\right) + \log((1+z)^2) + \log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right)\right) \\ &= 2.5\log\left(\frac{P'_x}{P'_y}\right) + 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right) \\ &= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right) - P_x + P_y.\end{aligned} \tag{13}$$

4 Consequences

In order to fully compare our derivation against Equation 1, we need to use the identity

$$P = -2.5\log\left(\int \mathcal{Z}(\lambda) S_x(\lambda) d\lambda\right) \tag{14}$$

where, according to Kim et al. [1996], "$\mathcal{Z}(\lambda)$ is an idealized spectral energy distribution at $z = 0$ for which $U = B = V = R = I = 0$ in the photometric system being used." Combining this with Equation 13, we have

$$
\begin{aligned}
K_{xy} &= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right) - P_x + P_y \\
&= 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right) + 2.5\log\left(\int \mathcal{Z}(\lambda) S_x(\lambda) d\lambda\right) - 2.5\log\left(\int \mathcal{Z}(\lambda) S_y(\lambda) d\lambda\right) \\
&= 2.5\log\left(\frac{\int \mathcal{Z}(\lambda) S_x(\lambda) d\lambda}{\int \mathcal{Z}(\lambda) S_y(\lambda) d\lambda}\right) + 5\log(1+z) + 2.5\log\left(\frac{\int \lambda F(\lambda) S_x d\lambda}{\int \frac{\lambda}{1+z} F(\lambda/(1+z)) S_y d\lambda}\right).
\end{aligned}
\tag{15}
$$

```python
import snpy
import numpy

# Use an arbitrary epoch for the spectral energy density template.
wave, flux = snpy.kcorr.get_SED(day=9)

# Use an arbitrary redshift.
z = 0.5

# Use the B and R filters from the Lick telescope as examples.
rest_filter = snpy.fset["Bkait"]
obs_filter = snpy.fset["Rkait"]

# Compute the desired rest magnitude.
m_b = rest_filter.synth_mag(wave, flux)

# Compute the observed magnitude after redshift effects.
m_r = obs_filter.synth_mag(wave, flux, z=z)

# TODO(lpe): Wait, why is m_r brighter than m_b? Shouldn't the fix term have
# the opposite sign?
# Huh... m_r is brighter than m_b even if SED is forced to be flat.

got_k = snpy.kcorr.K(wave, flux, rest_filter, obs_filter, z=z)[0]

# From the definition of k-corrections:
want_k = m_r - m_b

def print_results():
  """
  >>> print_results()
  got_k (-0.679138363247608) != want_k (-1.1193665108868114)
  2.5*log10(1+z) == 0.4402281476392031
  got_k - 2.5*log10(1+z) (-1.1193665108868112) == want_k (-1.1193665108868114)
  """
  print(f"got_k ({got_k}) != want_k ({want_k})")
  print(f"2.5*log10(1+z) == {2.5 * numpy.log10(1+z)}")
  fixed = got_k - 2.5*numpy.log10(1+z)
  print(f"got_k - 2.5*log10(1+z) ({fixed}) == want_k ({want_k})")
```

Listing 1: A demonstration that K-corrections calculated by snpy are off by `2.5*log10(1+z)`. The roundoff error is roughly `2e-16`.

5 The dimming effects of redshift

The magnitude measurements for Type Ia supernova goes through a long chain of data processing. The data used here was collected by the Dark Energy Survey Collaboration, as summarized in DES-Collaboration et al. [2024] and described in more detail by Vincenzi et al. [2024]. However, the data processing does not explicitly correct for either redshift or time dilation.

In the big bang model, there are multiple phenomena associated with redshift that we might expect to reduce the apparent magnitude of a Type Ia supernova.

5.1 Recessional velocity redshift

The energy carried by a photon is inversely proportional to wavelength, given by the Planck relation

$$E = \frac{hc}{\lambda} \tag{16}$$

where E is energy, h is the Planck constant, c is the speed of light, and λ is the wavelength.

As redshift increases the wavelength of a photon, the energy decreases.

The supernova data collected by the DES Collaboration used a CCD camera, a photon counting device, as described by Flaugher et al. [2015]. Kim et al. [1996] noted that photometric measurements that depend on bolometers will need to be corrected for the reduced energy level of redshifted light, but with a photon counting device, this correction is not necessary.

While the redshift phenomenon should impact the light detected from distant supernova, it should not impact the magnitude measurements so it does not need to be explicitly corrected.

5.2 Time dilation

The second phenomenon is that time dilation for objects moving quickly relative to our observational rest frame will reduce the rate at which photons are being emitted. Instead of changing the properties of individual photons, time dilation reduces the count of photons by a factor of $\frac{1}{1+z}$ where z is the redshift.

This phenomenon will not be addressed by the nuances of any measuring device, so it must be explicitly corrected.

5.3 Stretching of space

If space itself is stretching, it will both increase the wavelength of photons and also reduce the density of those photons. If space is stretching at a constant rate, the effect would be indistinguishable from the redshift and time dilation created by high relative recessional velocities. However, an accelerating expansion of the universe may indicate a non-constant rate of stretching. This would manifest by distant objects having a greater distance per redshift than nearby objects.

This non-linear streching-of-space model would be indistinguishable from a scenario where a constant force is pushing all objects away from each other.

5.4 Tired light

An alternative to the big bang theory is the tired light hypothesis, as described by Zwicky [1929] and Shao [2013]. The idea is that distant objects are mostly stationary relative to us, but the energy of light is lost as it travels through space. A feature of the tired light hypothesis is that since distant objects do not have a high relative velocity to us, they should not show time dilation.

However, as shown by Blondin et al. [2008] and White et al. [2024], distant supernova do experience time dilation. Based on this, we can reject the tired light hypothesis and assume the big bang model.

6 Correcting magnitude for time dilation

Luminosity distance D_L, is the apparent distance of an object based on the observed luminosity, also known as the flux F. This does not take into account any movement of the observed object between the time when the light was emitted and the light is observed.

To derive the luminosity distance from these measurements, we start by computing the flux F. Magnitude m is defined on a logarithmic scale where magnitude 1 has 100 times the brightness of magnitude 6, leading to

$$F = \frac{1}{\sqrt[5]{100}^{m-1}}. \tag{17}$$

This is proportional to the number of photons detected by a telescope. We can find the corrected flux F^* by multiplying by $k(z)$, the redshift correction factor. Time dilation of quickly moving objects reduces the number of photons by a factor of $\frac{1}{1+z}$, so we have

$$\begin{aligned} F^* &= F \times k(z) \\ &= F(1+z). \end{aligned} \tag{18}$$

To compute the corrected magnitude m^*, we can solve

$$\begin{aligned} F^* &= \frac{1}{\sqrt[5]{100}^{m^*-1}} \\ m^* &= m - \frac{\ln{(z+1)}}{\ln{(\sqrt[5]{100})}}. \end{aligned} \tag{19}$$

From here, we can use the standard distance modules μ defined as

$$\mu = m^* - M \tag{20}$$

where M is the absolute magnitude. The luminosity distance D_L in parsecs can then be calculated as

$$D_L = 10^{1+\frac{\mu}{5}}. \tag{21}$$

7 Linear distance vs redshift relationship

The theory of an accelerating expansion is based on there being a non-linear relationship between redshift and distance. More specifically, old objects (the ones that are more distant) should have more distance per redshift than newer objects.

As shown in Figure 1, this non-linear relationship is only observed when $k(z) = 1$, meaning that magnitude is not corrected for time dilation.

In contrast, when $k(z) = 1 + z$, time dilation is accounted for and there is a linear relationship between redshift and distance. A linear model rules out an accelerated expansion.

8 Hubble tension

Previous studies use F instead of $F*$. To the best of our knowledge, no studies that depend on the distance of Type Ia supernova, reaching back at least to Kim et al. [1996], Riess et al. [1998], and Perlmutter et al. [1999], have accounted for time dilation.

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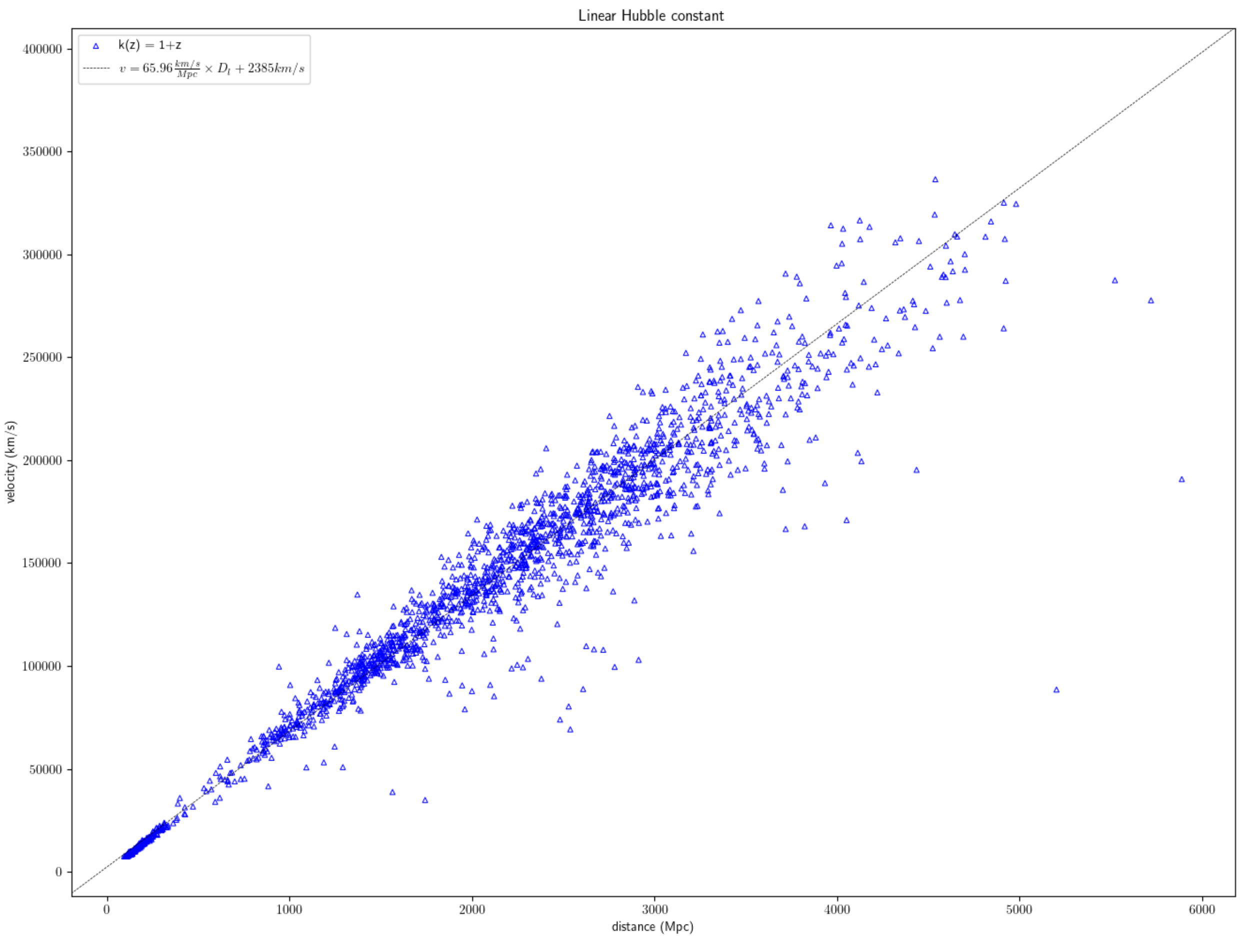

Figure 3: The relationship between expansion velocity and distance. The slope of this graph demonstrates the Hubble constant.

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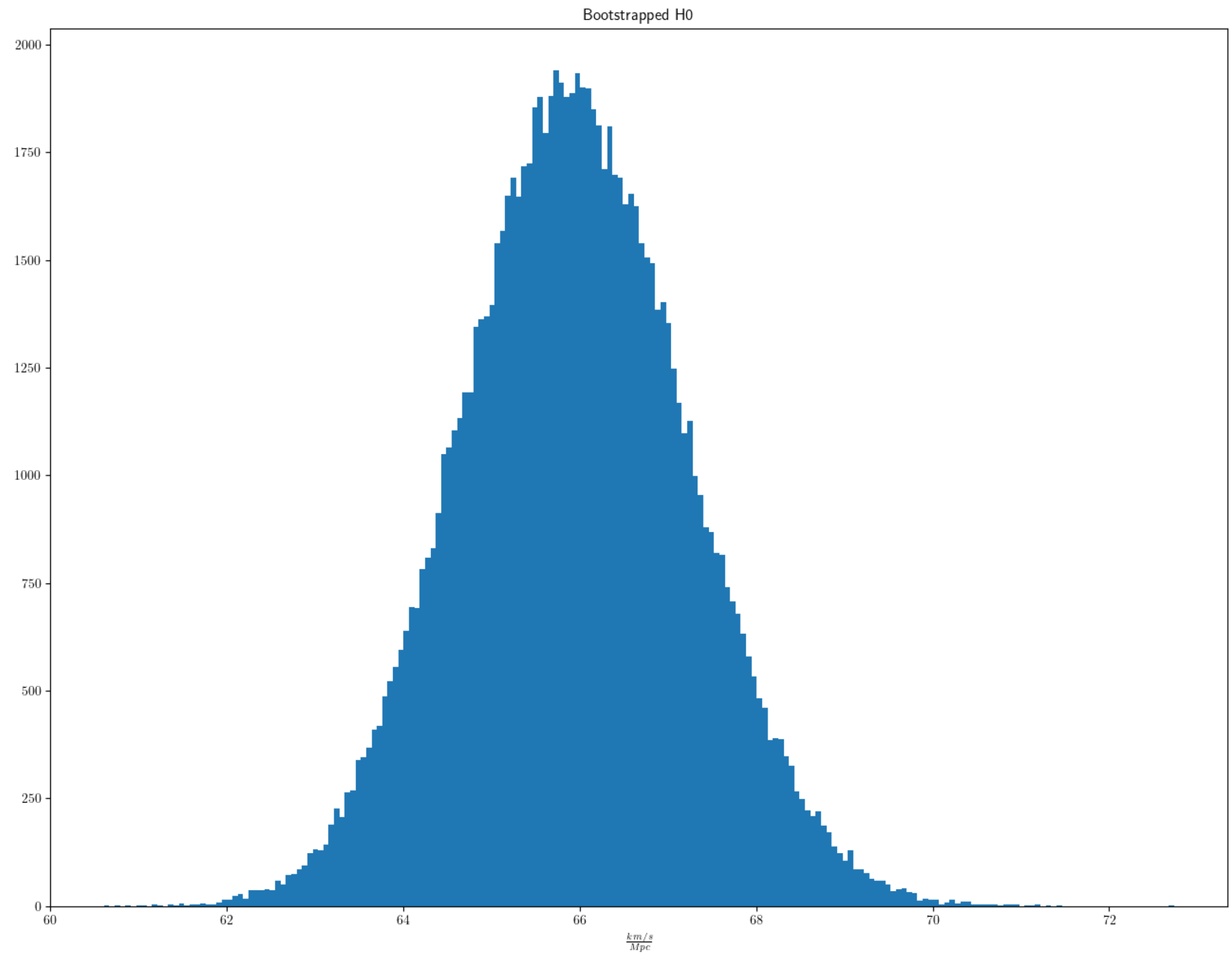

Figure 4: A histogram of 100000 bootstrap trials measuring the Hubble constant $H0$. Each trial samples an absolute Type Ia magnitude $M \sim Norm(-19.2334, 0.0404)$ based on data published by Camarena and Marra [2020]. It then samples, with replacement, a population of supernovae from the dataset published by DES-Collaboration et al. [2024]. Finally, it uses the non-parametric linear regression technique described by Siegel [1982]. The result of the bootstrap is $H0 \sim Norm(65.94, 1.29)$.

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