

The Mathematics of Music Theory

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A conjecture of Polya and Szego states that among the polygons with the same number of sides and area, the regular polygon minimizes the first eigenvalue of the Dirichlet Laplacian. This conjecture has only been proven in the cases of three and four sides. In this paper introduce a computational approach to checking the Polya-Szego conjecture for five or more sides.

[16%] Assignment Outline

Introduction

Consider the eigenvalue solutions for the Laplace operator with Dirichlet boundary conditions for any open, bounded set $\Omega \subset \mathbb{R}^2$

$$\begin{cases} -\Delta u = \lambda u \\ u = 0 \end{cases} \quad (1)$$

Due to Rellich's compactness lemma TODO TODO, the spectrum of the Dirichlet Laplacian consists only of discrete eigenvalues

$$0 < \lambda_1(\Omega) \leq \lambda_1(\Omega) \leq \lambda_1(\Omega) \leq \dots \rightarrow +\infty$$

which can be ordered by their multiplicity [Hen06]. The first eigenvalue λ_1 , which is also called the fundamental tone, is of particular importance.

In 1877, Rayleigh conjectured that for all domains with a fixed area, the fundamental tone is minimized by the disc [Dan11]. This was eventually proven in any euclidean space, using a technique called Schwarz rearrangement [Hen06].

Bibliography

References

- [1] Daniel Daners. “Krahn’s proof of the Rayleigh conjecture revisited”. In: *Archiv der Mathematik* (2011), pp. 187–199.
- [2] Antoine Henrot. *Extremum Problems for Eigenvalues of Elliptic Operators*. Birkhauser, 2006.